

## Lorentz-Lorenz shift in a Bose-Einstein condensate

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We study the quantum field theory of light-matter interactions for quantum degenerate atomic gases at low light intensity. We argue that the contact interactions between atoms emerging in the dipole gauge may be ignored. Specifically, they are canceled by concurrent infinite level shifts of the atoms. Our development yields the classic Lorentz-Lorenz local-field shift of the atomic resonance. [S1050-2947(97)03209-5]

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The understanding of the interactions of light with matter in quantum degenerate systems has become especially topical after the first evidence for a Bose-Einstein condensate (BEC) of an atomic gas has appeared [1]. Recently, Andrews *et al.* [2] have also reported on nondestructive optical detection of a Bose-Einstein condensate.

In the limit of large detuning of the driving light from atomic resonance, the dynamics of the light and matter fields may be decoupled. It then turns out that the spectrum of the scattered light conveys direct signatures of atom statistics [3,4] and that under various conditions even the phase of the macroscopic wave function may be observed optically [5]. On the other hand, for a dense enough sample and small enough atom-field detuning, the analysis of the response of matter requires a concurrent treatment of light with its own dynamics. The result is that nearby atoms alter the optical response of each other. A large body of work in this direction [6–12] seems to have brought about the general notion that atom statistics should have only a minor effect on absorption, dispersion, or diffraction of light [3,13]. Nonetheless, even for the Maxwell-Boltzmann gas and for the simplest optical properties such as refractive index, there still are no proven solutions for microscopic theories regarding a dense, near-resonance sample.

In this paper we continue our rigorous quantum field-theoretical analysis of light-matter interactions [3,12]. We argue that the contact interaction terms between different atoms that arise in the *length* gauge do not have any effect on light-matter dynamics. As a result, for a model BEC we find precisely the Lorentz-Lorenz (LL) shift familiar from classical electrodynamics [14–16].

In the many-particle formalism of light-matter interactions, the Hamiltonian is often transformed into the length gauge using the Power-Zienau-Woolley transformation [17]. The electric displacement is then the basic dynamical degree of freedom, instead of the electric field, and the Hamiltonian picks up a polarization self-energy term

$$H_P = \frac{1}{2\epsilon_0} \int d^3r \mathbf{P}(\mathbf{r}) \cdot \mathbf{P}(\mathbf{r}) = \frac{1}{2\epsilon_0} \sum_{i \neq j} \mathbf{d}_i \cdot \mathbf{d}_j \delta(\mathbf{r}_i - \mathbf{r}_j), \quad (1)$$

where  $\mathbf{P}(\mathbf{r}) = \sum_i \mathbf{d}_i \delta(\mathbf{r}_i - \mathbf{r})$  is the electric polarization in the dipole approximation and  $\mathbf{d}_i$  and  $\mathbf{r}_i$  are the dipole operator and the center-of-mass (c.m.) position operator for the  $i$ th atom. On the right-hand side of Eq. (1) we have ignored the

divergent self-energies with  $i=j$ . Polarization self-energy describes a contact interaction between polarized atoms. One might argue that for a quantum degenerate Bose gas this term is important, as the de Broglie wavelengths of the atoms are of the order of the interatomic distances and the wave functions of different atoms overlap strongly. The polarization energy has been duly included in many theoretical approaches to light-atom interactions in the quantum degenerate regime [9,11,12,18].

In our previous paper [12] we have derived a hierarchy of equations of motion for correlation functions that contain one excited-atom field and one, three, five, etc., ground-state atom fields, for the limit of low light intensity. All contact interactions, such as those produced by the polarization self-energy (1), were included. For the total electric field  $\mathbf{E}^+$  we found the following monochromatic expression in the presence of scattering by atomic dipole moments:

$$\epsilon_0 \mathbf{E}^+(\mathbf{r}) = \mathbf{D}_F^+(\mathbf{r}) + \frac{1}{i\kappa} \int d^3r' \mathbf{G}(\mathbf{r} - \mathbf{r}') \mathbf{P}^+(\mathbf{r}'), \quad (2)$$

where  $\mathbf{D}_F^+$  is the positive-frequency component of the driving electric displacement, the scalar constant  $\kappa$  is defined in terms of the reduced dipole moment  $\mathcal{D}$  as  $\kappa = \mathcal{D}^2 / \hbar \epsilon_0$ , and the positive-frequency part of the polarization operator  $\mathbf{P}^+$  is given in second quantization by

$$\mathbf{P}^+(\mathbf{r}) = \mathbf{d}_{ge} \psi_g^\dagger(\mathbf{r}) \psi_e(\mathbf{r}). \quad (3)$$

Here  $\psi_g$  and  $\psi_e$  are the ground-state and the excited-state field operators in the Heisenberg picture and  $\mathbf{d}_{ge}$  is the dipole matrix element for the transition  $g \rightarrow e$ . In our notation the same level index appearing twice in a product indicates a summation over the magnetic substates of the level. The monochromatic version of the  $3 \times 3$  tensor propagator  $\mathbf{G}(\mathbf{r})$  is

$$\mathbf{G}_{ij}(\mathbf{r}) = i\kappa \left\{ \left[ \frac{\partial}{\partial r_i} \frac{\partial}{\partial r_j} - \delta_{ij} \nabla^2 \right] \frac{e^{ikr}}{4\pi r} - \delta_{ij} \delta(\mathbf{r}) \right\}. \quad (4)$$

The expression  $\mathbf{G}(\mathbf{r} - \mathbf{r}') \mathcal{D}$  is equal to the positive-frequency component of the electric field from a monochromatic dipole with the complex amplitude  $\mathcal{D}$ , given that the dipole resides at  $\mathbf{r}'$  and the field is observed at  $\mathbf{r}$  [16]. The explicit expression is

$$\mathbf{G}(\mathbf{r})\mathcal{D} = \frac{i\kappa}{4\pi} \left\{ k^2 (\hat{\mathbf{n}} \times \mathcal{D}) \times \hat{\mathbf{n}} \frac{e^{ikr}}{r} \right. \\ \left. + [3\hat{\mathbf{n}}(\hat{\mathbf{n}} \cdot \mathcal{D}) - \mathcal{D}] \left( \frac{1}{r^3} - \frac{ik}{r^2} \right) e^{ikr} \right\} - \frac{i\kappa}{3} \mathcal{D} \delta(\mathbf{r}), \quad (5)$$

with  $\hat{\mathbf{n}} = \mathbf{r}/r$  and  $k = \Omega/c$ ,  $\Omega$  being the dominant frequency of the incident light field. The volume integral over  $1/r^3$  in Eq. (5) is not absolutely convergent in the neighborhood of the origin. Expression (5) should be understood in such a way that the integral of the term inside the curly brackets over an infinitesimal volume enclosing the origin vanishes [12]. Equation (5) is precisely the classical expression of the dipolar field, including the peculiar delta function divergence at the origin [16].

With the definitions

$$\mathbf{P}_l(\mathbf{r}_1, \dots, \mathbf{r}_{l-1}; \mathbf{r}_l) \\ \equiv \langle \psi_g^\dagger(\mathbf{r}_1) \cdots \psi_g^\dagger(\mathbf{r}_{l-1}) \mathbf{P}^+(\mathbf{r}_l) \psi_g(\mathbf{r}_{l-1}) \cdots \psi_g(\mathbf{r}_1) \rangle, \quad (6a) \\ \rho_l(\mathbf{r}_1, \dots, \mathbf{r}_l) \equiv \langle \psi_g^\dagger(\mathbf{r}_1) \cdots \psi_g^\dagger(\mathbf{r}_l) \psi_g(\mathbf{r}_l) \cdots \psi_g(\mathbf{r}_1) \rangle \quad (6b)$$

and considering a  $J_g = 0 \rightarrow J_e = 1$  transition for simplicity, we were able to cast the hierarchy of equations for atomic correlations functions into the form [12]

$$\dot{\mathbf{P}}_l(\mathbf{r}_1, \dots, \mathbf{r}_{l-1}; \mathbf{r}_l) \\ = (i\delta - \gamma) \mathbf{P}_l(\mathbf{r}_1, \dots, \mathbf{r}_{l-1}; \mathbf{r}_l) \\ + \sum_{k=1}^{l-1} \mathbf{G}(\mathbf{r}_l - \mathbf{r}_k) \mathbf{P}_l(\mathbf{r}_1, \dots, \mathbf{r}_{k-1}, \mathbf{r}_{k+1}, \dots, \mathbf{r}_l; \mathbf{r}_k) \\ + i\kappa \rho_l(\mathbf{r}_1, \dots, \mathbf{r}_l) \mathbf{D}_F^+(\mathbf{r}_l) \\ + \int d^3 r_{l+1} \mathbf{G}(\mathbf{r}_l - \mathbf{r}_{l+1}) \mathbf{P}_{l+1}(\mathbf{r}_1, \dots, \mathbf{r}_l; \mathbf{r}_{l+1}). \quad (7)$$

The quantity  $\mathbf{P}_l$  reflects correlations between the dipole moment of one atom and the positions of  $l-1$  other atoms and  $\rho_l$  is simply the density correlation function for  $l$  ground-state atoms.

In Eq. (7) the atoms are regarded as point dipoles. As we have emphasized already in Ref. [12], this assumption must fail at short distances. Real atomic interaction potentials are thought to have a hard core, which prevents the atoms from overlapping. Mathematically, one expects that all atomic correlation functions such as  $\mathbf{P}_l$  and  $\rho_l$  vanish if any two position arguments are the same. Evidently, the contact interaction should be omitted as inconsequential in Eq. (7). In view of Eq. (5), this is done by removing the  $\delta$ -function contributions from the field propagator  $\mathbf{G}$  by the substitution  $\mathbf{G}_{ij} \rightarrow \mathbf{G}'_{ij}$ , with

$$\mathbf{G}'_{ij}(\mathbf{r}) = \mathbf{G}_{ij}(\mathbf{r}) + i\kappa \delta_{ij} \delta(\mathbf{r})/3. \quad (8)$$

The key mathematical insight of this paper is that the same substitution applies even for point dipoles. In other words, the resulting light-matter dynamics is exactly the

same whether we use the field propagator  $\mathbf{G}$  or  $\mathbf{G}'$  in Eq. (7). To demonstrate this contention, we consider the steady-state solutions of Eqs. (7). For each  $l$  we have a system of  $l$  coupled equations for  $\mathbf{P}_l(\dots; \mathbf{r}_1)$ ,  $\mathbf{P}_l(\dots; \mathbf{r}_2)$ ,  $\dots$ ,  $\mathbf{P}_l(\dots; \mathbf{r}_l)$ . The coupling arises from the collective line shifts and linewidths, those  $\mathbf{G}$  terms in Eq. (7) that appear outside the integral. The idea is that if we solve this system of equations for a fixed  $l$ , in  $\mathbf{P}_l(\dots; \mathbf{r}_l)$  and indeed even in the seemingly more divergent expression  $\mathbf{G}(\mathbf{r}_k - \mathbf{r}_l) \mathbf{P}_l(\dots; \mathbf{r}_l)$  with  $k \neq l$  the  $\delta$  functions of the form  $\delta(\mathbf{r}_j - \mathbf{r}_s)$  ( $j \neq s$ ) cancel. The  $\delta$  functions are inconsequential in integrals of the type  $\int d^3 r_{l-1} \mathbf{G}(\mathbf{r}_{l-1} - \mathbf{r}_l) \mathbf{P}_l(\dots; \mathbf{r}_l)$ , and hence in the solution of the hierarchy (7) for  $\mathbf{P}_1$ .

We begin the detailed discussion by noting that in the derivation of Eqs. (2) and (7) in Ref. [12] we used a cutoff in the wave numbers  $q$  of the photons. Here we again invoke the high-frequency cutoff. The  $\delta$  function is then regarded an ordinary function, albeit one that is sharply peaked over a small distance scale  $\alpha$ . Seemingly unorthodox operations such as divisions by a  $\delta$  function are therefore well defined. After the calculations we set  $\alpha \rightarrow 0$ .

The steady-state solutions for  $\mathbf{P}_l(\dots; \mathbf{r}_k)$ ,  $k = 1, \dots, l$ , of Eq. (7) are obtained from a system of linear equations  $A\mathbf{x} = \mathbf{b}$ , where the  $l \times l$  matrix  $A$  and the  $l \times 1$  vectors  $\mathbf{x}$  and  $\mathbf{b}$  are defined by

$$A_{kq} = \delta_{kq} - (1 - \delta_{kq}) \frac{\alpha}{i\kappa} \mathbf{G}(\mathbf{r}_k - \mathbf{r}_q), \quad x_k = \mathbf{P}_l(\dots; \mathbf{r}_k), \\ b_k = \alpha \rho_l(\mathbf{r}_1, \dots, \mathbf{r}_l) \mathbf{D}_F^+(\mathbf{r}_k) \\ + \frac{\alpha}{i\kappa} \int d^3 r_{l+1} \mathbf{G}(\mathbf{r}_k - \mathbf{r}_{l+1}) \mathbf{P}_{l+1}(\mathbf{r}_1, \dots, \mathbf{r}_l; \mathbf{r}_{l+1}).$$

Here  $\alpha = -\kappa/(\delta + i\gamma)$  is the polarizability of a single atom. The solution for this linear system may be expressed using Cramer's rule [19]. In particular,  $x_l = \mathbf{P}_l(\dots; \mathbf{r}_l) = \det[\bar{A}^{(l)}] / \det(A)$ , where  $\bar{A}_{kq}^{(l)} = A_{kq}$ , for  $q \neq l$ , and  $\bar{A}_{kl}^{(l)} = b_l$ . For any two indices  $j \neq s$ , the matrix  $A$  contains two entries  $\mathbf{G}(\mathbf{r}_j - \mathbf{r}_s)$ , one on the row  $j$  and column  $s$  and the other on row  $s$ , column  $j$ . The determinant  $\det(A)$  therefore is a quadratic polynomial of the propagator  $\mathbf{G}(\mathbf{r}_j - \mathbf{r}_s)$ . The coefficient of  $\mathbf{G}(\mathbf{r}_j - \mathbf{r}_s)^2$  is a function of the position variables  $\{\mathbf{r}_i\}$  and may technically be zero for some sets of coordinates  $\{\mathbf{r}_i\}$ . In our argument we assume that such accidental zeros, if any, are inconsequential in the physics. We thus posit that  $\det(A)$  contains a term proportional to  $\mathbf{G}(\mathbf{r}_j - \mathbf{r}_s)^2$  and hence is a proper quadratic polynomial of  $\delta(\mathbf{r}_j - \mathbf{r}_s)$ . By the same token,  $\det[\bar{A}^{(l)}]$  is a second-order polynomial of  $\delta(\mathbf{r}_j - \mathbf{r}_s)$  for  $j \neq s$ ,  $j \neq l$ ,  $s \neq l$  and a first-order polynomial in  $\delta(\mathbf{r}_j - \mathbf{r}_l)$ ,  $j \neq l$ .

Let us first consider the case  $l = N$ , where  $N$  is the total number of atoms. All the expectation values in Eq. (6) vanish for  $l > N$ . Thus  $\mathbf{P}_{N+1} = 0$  and  $b_j = \alpha \rho_N \mathbf{D}_F^+(\mathbf{r}_j)$  for  $j = 1, \dots, N$ . By the preceding argument,  $\mathbf{P}_N = \det[\bar{A}^{(N)}] / \det(A)$  is a rational expression whose denominator is a quadratic polynomial in each  $\delta(\mathbf{r}_j - \mathbf{r}_s)$ ,  $j \neq s$ , whereas the numerator is a quadratic expression of  $\delta(\mathbf{r}_j - \mathbf{r}_s)$  with  $j \neq s$ ,  $j \neq N$ ,  $s \neq N$  and a first-order polynomial in  $\delta(\mathbf{r}_j - \mathbf{r}_N)$ ,  $j \neq N$ . In the expression  $\mathbf{G}(\mathbf{r}_N - \mathbf{r}_{N-1}) \mathbf{P}_N(\mathbf{r}_1, \dots, \mathbf{r}_{N-1}; \mathbf{r}_N)$  all the  $\delta$ -function diver-

gences in the numerator are therefore canceled by equal (or higher) powers of the same  $\delta$  functions in the denominator.

Next consider the case  $l=N-1$ . Because the expression  $\mathbf{G}(\mathbf{r}_N-\mathbf{r}_{N-1})P_N(\dots;\mathbf{r}_N)$  does not contain uncanceled  $\delta$  functions, its integral over  $\mathbf{r}_N$  needed to calculate the  $(N-1)\times 1$  vector  $\mathbf{b}$  for  $l=N-1$  does not contain any  $\delta$ -function divergences either. With similar arguments to before we then conclude that also  $\mathbf{G}(\mathbf{r}_{N-2}-\mathbf{r}_{N-1})\mathbf{P}_{N-1}(\dots;\mathbf{r}_{N-1})$  is free from uncanceled  $\delta$  functions. By repeating the procedure for all  $l$  we confirm that the expressions  $\mathbf{G}(\mathbf{r}_{l-1}-\mathbf{r}_l)\mathbf{P}_l(\dots;\mathbf{r}_l)$ , for any  $l$ , do not contain  $\delta$  functions that could affect the integrations.

Because our derivation is very technical, we give a simple example for the case of two atoms. Then,  $\mathbf{P}_3=\mathbf{0}$ . The ground-state density and the polarization at  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are coupled by the collective linewidths and line shifts leading to the resonant dipole-dipole interaction [12]. For  $l=2$ ,  $A_{11}=A_{22}=1$ ,  $A_{12}=A_{21}=-\alpha\mathbf{G}(\mathbf{r}_1-\mathbf{r}_2)/i\kappa$ , and  $\mathbf{P}_2(\mathbf{r}_1;\mathbf{r}_2)$  may be expressed using Cramer's rule. In particular,

$$\begin{aligned} & \int d^3r_2 \mathbf{G}(\mathbf{r}_1-\mathbf{r}_2)\mathbf{P}_2(\mathbf{r}_1;\mathbf{r}_2) \\ &= \int d^3r_2 \mathbf{G}(\mathbf{r}_1-\mathbf{r}_2)\alpha\rho_2(\mathbf{r}_1,\mathbf{r}_2) \\ & \quad \times \frac{\mathbf{D}_F^+(\mathbf{r}_2)+\alpha\mathbf{G}(\mathbf{r}_1-\mathbf{r}_2)\mathbf{D}_F^+(\mathbf{r}_1)/i\kappa}{1-[\alpha\mathbf{G}(\mathbf{r}_1-\mathbf{r}_2)/i\kappa]^2}. \end{aligned} \quad (9)$$

With  $\mathbf{r}_2\neq\mathbf{r}_1$ , the  $\delta$  functions in  $\mathbf{G}(\mathbf{r}_1-\mathbf{r}_2)$  vanish, and at  $\mathbf{r}_2=\mathbf{r}_1$  we may cancel  $[\delta(\mathbf{r}_1-\mathbf{r}_2)]^2$ . The  $\delta$  functions do not affect the integral because the highest power of  $\delta(\mathbf{r}_1-\mathbf{r}_2)$  is 2, in both the denominator and the numerator. Thus the contact interactions do not affect the polarization  $\mathbf{P}_1(\mathbf{r}_1)$ , which is obtained by inserting expression (9) into Eq. (7).

We have shown that all  $\delta$  functions contained in  $\mathbf{G}$  cancel in Eq. (7). A corollary of the derivation is that the polarization self-energy term (1) does not have any effect on the light-matter dynamics for point dipoles and may be ignored.

There is a concurring physical explanation for our result. A comparison of Eqs. (5) and (7) shows that the  $\delta$  function lumps with the detuning  $\delta$  and gives a divergent frequency shift. As the dipole at  $\mathbf{r}_k$  draws closer to the dipole at  $\mathbf{r}_q$  and the electric fields of the dipoles on each other grow stronger, at the same time the dipoles are also shifted further and further away from resonance with one another. From our present viewpoint it is the level shift due to resonant dipole-dipole interactions that dynamically curbs the interaction energy between the dipoles.

After arguing that the  $\delta$  functions may be ignored in Eq. (7), we now rewrite the example presented in Ref. [12] for the optical response of a homogeneous condensate with the contact interactions omitted. The condensate is assumed to fill the half space  $z\geq 0$  with the constant density  $\rho$ ; for an ideal condensate the density  $\rho$  immediately implies the other correlation functions as  $\rho_l=\rho^l$ . The optical response is solved in an approximation that ignores the collective line shifts and linewidths. Specifically, we replace  $\mathbf{G}$  in our earlier argument by  $\mathbf{G}'$  and then ignore all tensors  $\mathbf{G}'$  that reside outside integrals in Eq. (7). This approximation is only guaranteed to be valid for dilute condensates  $\rho\lambda^3\ll 1$ , where

$\lambda$  is the wavelength of light [20]. What the exact limit of validity is and what types of corrections emerge at high densities is at the moment largely unknown.

We consider the steady-state solution of Eq. (7) with  $l=1,\dots$ . The initial free field is written  $\mathbf{D}_F^+(\mathbf{r})=D_F\hat{\mathbf{e}}\exp(ikz)$ . The following set of damped plane waves solves Eq. (7):

$$\mathbf{P}_l(\mathbf{r}_1,\dots;\mathbf{r}_l)=\begin{cases} P\rho^{l-1}\hat{\mathbf{e}}e^{ik'z_l}, & z_1,\dots,z_l\geq 0 \\ 0 & \text{otherwise,} \end{cases} \quad (10)$$

with  $k'$  [ $\text{Im}(k')>0$ ] and  $P$  being the variable parameters. This may be seen by evaluating the integral for  $z_1>0$ ,

$$\begin{aligned} & \int_{z_2\geq 0} d^3r_2 \hat{\mathbf{e}}^* \cdot \mathbf{G}'(\mathbf{r}_1-\mathbf{r}_2) \cdot \hat{\mathbf{e}} e^{ik'z_2} \\ &= i\kappa \left[ \frac{2k^2+k'^2}{3(k'^2-k^2)} e^{ik'z_1} + \frac{k'^2}{2k(k-k')} e^{ikz_1} \right]. \end{aligned} \quad (11)$$

The integrals on the right-hand side of Eq. (7) produce sums of two exponentials, the vacuum component proportional to  $e^{ikz}$ , which should cancel the free-field terms, and a remaining  $e^{ik'z}$  term, which pairs up with the correlation functions  $\mathbf{P}_l$ . Equation (7) then reduces to the two conditions

$$\left( i\delta - \gamma + \frac{i\rho\kappa(2k^2+k'^2)}{3(k'^2-k^2)} \right) P = 0, \quad (12a)$$

$$D_F + \frac{k'^2}{2k(k-k')} P = 0, \quad (12b)$$

which give the wave number  $k'$  and the polarization amplitude  $P$ .

On the other hand, Eq. (2) for the electric field refers to a one-atom quantity  $\mathbf{P}^+\equiv\mathbf{P}_1$ . There is no evident reason to drop the  $\delta$  function contribution in  $\mathbf{G}$ , so we use Eq. (2) as it is. The total electric field is assumed to be of the form  $E\hat{\mathbf{e}}e^{ik'z}$ . With the choice (12b), the vacuum type contributions  $e^{ikz}$  indeed cancel and the polarization amplitude is related to the amplitude of electric field by  $\epsilon_0 E = k^2 P / (k'^2 - k^2)$ . With the help of Eq. (12a) we find

$$n^2 - 1 = \frac{\rho\alpha}{1 - \rho\alpha/3}, \quad (13)$$

where the refractive index  $n$  is defined in a familiar manner  $k'=nk$ . Because we are dealing with a linear theory, the electric field and the polarization are naturally related by  $\mathbf{P}^+=\epsilon_0\chi\mathbf{E}^+$ , but in addition we now find the familiar relation  $\chi=n^2-1$  between the susceptibility and refractive index.

It should be noted that keeping or ignoring the  $\delta$  function of  $\mathbf{G}$  in Eq. (2) simply makes a difference between two different definitions of the electric field inside a dielectric medium. This is a thorny and often an inconsequential issue. The final measurements are usually carried out outside the dielectric, whereupon the  $\delta$  function does not contribute any-

way. Our choice has the advantage that it aligns with the standard conventions of electrodynamics.

While we earlier [12] obtained the column density result  $n^2 - 1 = \chi = \rho\alpha$ , Eq. (13) is the classic column density result with the added LL local-field correction [14]. As far as our microscopic argument is concerned, the correction emerges as a result of divergent level shifts of the atoms due to dipole-dipole interactions. Within the present approach the main approximations for the LL shift are two: collective linewidths and line shifts are ignored and correlation functions for ground-state atoms as appropriate for a BEC are assumed. Atom-atom correlation functions, though trivial, are formally taken into account to infinite order. In comparison, Ref. [11] strives at taking into account collective linewidths and line shifts, but in exchange makes approximations concerning atomic correlation functions.

Apart from heralding the physics insights that originally lead to the Lorentz-Lorenz formula, our results should have practical consequences in the ongoing studies of the condensates. The LL shift could, and possibly should, be studied as the first correction to the column density arguments, which

so far have been the exclusive tool in the analysis of the experimental results. As another example, we conjectured in [12] that the entire linear hierarchy of Eq. (7) for  $l=1,2,\dots,N$  may be solved by solving numerically the classical equations for the coupled system of electromagnetic fields and  $N$  charged harmonic oscillators. The position correlation functions  $\rho_l$  could be taken into account by repeating the solution for a number of initial configurations chosen from the proper stochastic ensemble and averaging the results. This type of a Monte Carlo approach would seem difficult in the presence of  $\delta$  functions because they dwell in a set of measure zero in phase space of the positions of the atoms and yet may substantially affect the results. However, as the troubling  $\delta$  functions have now proven inconsequential, the door is ajar for an essentially exact numerical solution of the optical response of a dense, near-resonance BEC.

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