Interferometric measurement of an atomic wave function

Matthias Freyberger and Stefan H. Kienle
Abteilung für Quantenphysik, Universität Ulm, D-89069 Ulm, Germany

Valery P. Yakovlev
Moscow State Engineering Physics Institute, 115409 Moscow, Russia

(Received 14 January 1997)

We discuss a simple method to probe the wave function $\psi(x)$ describing the transverse center-of-mass motion of an atomic beam. We detect position distributions of the atoms after they have interacted with two counterpropagating laser beams. The resulting interference pattern depends crucially on the cross term $\psi^*(x)\psi(x-\Delta x)$, which finally allows us to reconstruct the phase of $\psi(x)$. [S1050-2947(97)07207-7]

PACS number(s): 03.65.Bz, 03.75.Dg

I. INTRODUCTION

The concept of a quantum state has always played a key role in discussions treating the foundations of quantum theory. On one hand Dirac's notion is very clear: each physical quantity can be represented by a Hermitian operator, which he calls an observable [1]. A measurement of this observable leaves the system in an eigenstate of the operator. This is a way of *preparing* a quantum system in a specific state. On the other hand for Heisenberg a quantum state represented a set of potentialities that was revealed by certain experimental conditions [2]. A single measurement performed on a quantum system reveals a certain aspect of its state, but it will not uncover this state completely. However, if we know how to determine the whole set of potentialities, the quantum state can be reconstructed. This is a way to *measure* the state of a quantum system.

The question about the measurability of a quantum state is actually an old one [3], which has led to a number of answers over the years [4]. Recently much work focused again on the reconstruction of quantum states [5]. This new interest grew out of the intriguing possibilities of quantum optics, atom optics, cavity QED, and the physics of trapped particles. There it became possible to design certain quantum states, for example, Schrödinger cats, which existed before on paper only [6,7]. Along with this *engineering* of states the measurability of a quantum state has experienced a great renaissance. Of course it is a basic assumption of quantum theory that an infinite ensemble of systems contains all the information about the wave function. But how can we unravel it?

Several methods have been proposed to measure quantum states of light as well as quantum states of matter. A central method that allows us to perform measurements on both kinds of wave functions is the so-called tomographic method [8,9]. Regarding light this method can be applied in combination with homodyne detection and it has been used in a wonderful set of experiments that reconstructed nonclassical states of the electromagnetic field [10]. However, important proposals [11] exist showing the reconstruction of quantum states of light without use of tomographic techniques. Regarding matter several proposals exist to reconstruct wave functions of an atomic beam [12] as well as quantum states of a trapped atom [13]. Experimentally the tomographic reconstruction has been applied to the vibrational state of a diatomic molecule [14].

A second set of methods that allow us to determine the quantum state of an electromagnetic field in a cavity is based on the fundamental interaction of atoms with the cavity field under investigation. The endoscopy method [15] reconstructs the quantum state from the excitation statistics of atoms that have interacted with the cavity field. The atomic deflection method [16] uses momentum distributions of atoms in order to reveal the quantum state of light inside the cavity. In both cases—endoscopy and atomic deflection—the atoms serve as a tool that probes a quantum state of light. For probing matter endoscopy turns out to be a very useful method as well [17]. In addition, the art of treating trapped particles has now reached a point where fascinating experiments [18] uncover quantum states of single atoms.

In the measurement scheme presented here we probe a beam of two-level atoms with the help of running light waves that form an atomic interferometer [19–21]. We will show that certain interference patterns of atomic waves that have interacted with the light beams of the interferometer contain enough information for the reconstruction of the initial wave function of the atoms. In contrast to the tomographic method mentioned before we would like to call this an *interferometric method* to probe matter waves [22]. The way to reconstruct the atomic state is completely different and its mathematics is much simpler. Furthermore the experimental prerequisites are easier to realize than in the case of a tomographic measurement. Therefore we are convinced that the probing of an atomic beam along the lines indicated in this paper should be possible in the near future.

In Sec. II we describe the setup of the interferometer and the elastic scattering of the initial atomic wave function. This leads to an expression for the measured spatial interference patterns of the atoms. A certain set of interferences will allow us to reconstruct the initial wave function. We will discuss the strategy for the reconstruction in Sec. III. In Sec. IV we conclude with an estimation of important experimental parameters for the presented method.

II. THE INTERFEROMETRIC SCHEME

In this section we analyze the setup, shown in Fig. 1, that allows us to determine the wave function $\psi(x)$ describing

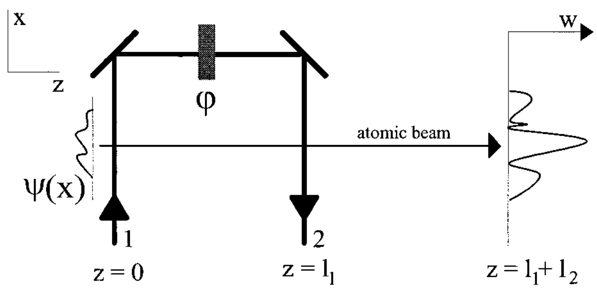

FIG. 1. Setup for the interferometric measurement of the wave function $\psi(x)$. Two counterpropagating lasers 1 and 2 interact resonantly with a beam of two-level atoms whose transverse center-of-mass motion is described by $\psi(x)$. We record atomic position distributions on a screen at $z=l_1+l_2$. Note that these distributions depend crucially on the adjustable phase φ between the two lasers.

the transverse center-of-mass motion of an atomic matter wave. The scheme consists of two counterpropagating lasers 1 and 2, which interact resonantly with a beam of two-level atoms. The lasers located at $z=0$ and $z=l_1$ respectively define interaction regions of length a. A phase shifter controls the relative phase φ between them.

The atom of mass m moving in the z direction with a velocity much higher than any velocity component in the x direction possesses the constant energy $E_0\equiv\hbar^2 p_0^2/(2m)$ [23]. In other words, the characteristic momentum $\hbar p_0$ is supposed to be much larger than any transverse momentum component in the x direction. After crossing both lasers the atom's position is recorded on a screen placed at $z=l_1+l_2$.

We start at $z=0$ with two-level atoms that are assumed to be in the ground state $|g\rangle$. Therefore, the stationary incident matter wave at $z=0$ reads

$$|\Psi(x,z=0,t)\rangle \equiv e^{-iE_0t/\hbar}\psi(x)|g\rangle \equiv e^{-iE_0t/\hbar}\int\frac{dp}{2\pi}\phi(p)e^{ipx}|g\rangle. \tag{1}$$

Here $\psi(x)$ and $\phi(p)$ represent the wave function of the transverse motion in its position and momentum representation, respectively. The aim of our considerations will be to demonstrate an interferometric method for the reconstruction of this wave function. For simplicity we restrict ourselves here to the case of a pure state and discuss later how the results can be generalized to mixed states described by density matrices.

The incident matter wave $|\Psi(x,z=0,t)\rangle$, Eq. (1), is scattered elastically at the two laser beams. In the rotating-wave approximation the Hamiltonian that governs the scattering process reads

$$\hat{H}=\frac{\hbar^2}{2m}(\hat{p}^2+\hat{p}_z^2)-\Theta(z)\Theta(a-z)\hbar\kappa[\sigma_-e^{-ikx}+\sigma_+e^{ikx}] -\Theta(z-l_1)\Theta(l_1+a-z)\hbar\kappa[\sigma_-e^{i(kx+\varphi)} +\sigma_+e^{-i(kx+\varphi)}], \tag{2}$$

when we deal with resonant interactions of the atom with the light fields. The first part of this Hamiltonian contains the transverse and longitudinal kinetic energy in terms of the momentum operators $\hbar\hat{p}$ and $\hbar\hat{p}_z$ respectively. As usual the Pauli spin matrices σ_+ and σ_- define the internal transitions of the two-level atom due to the resonant interaction with the counterpropagating laser beams with wave vectors k and $-k$. The coupling of atom and field is given by a coupling constant κ and it is turned on in the interaction regions $0\leqslant z\leqslant a$ and $l_1\leqslant z\leqslant l_1+a$ via the combination of two Heaviside functions. Between the laser beams 1 and 2 as well as between laser beam 2 and the detection screen the atoms behave as free particles.

In the next step we are going to answer the question of how the incident matter wave $|\Psi(x,z=0,t)\rangle$, Eq. (1), transforms due to the interaction with lasers 1 and 2. Hence our aim is to find the state $|\Psi(x,z=l_1+l_2,t)\rangle$ that determines the position distribution of atoms on the detection screen.

We present the details of the calculation in the Appendix and restrict ourselves here to a physical interpretation of the state

$$\begin{aligned}|\Psi(x,l_1+l_2,t)\rangle &\equiv e^{-iE_0t/\hbar}[\psi_g(x,l_1+l_2)|g\rangle+\psi_e(x,l_1+l_2)|e\rangle]\\ &=e^{-iE_0t/\hbar}\Bigg[C^2\int\frac{dp}{2\pi}\phi(p)e^{i[p_z(p)(l_1+l_2)+px]}|g\rangle\\ &-S^2e^{i\varphi}\int\frac{dp}{2\pi}\phi(p)e^{i[p_z(p+k)l_1+p_z(p+2k)l_2+(p+2k)x]}|g\rangle\\ &+iSC\int\frac{dp}{2\pi}\phi(p)e^{i[p_z(p+k)(l_1+l_2)+(p+k)x]}|e\rangle\\ &+iSCe^{-i\varphi}\int\frac{dp}{2\pi}\phi(p)\\ &\times e^{i[p_z(p)l_1+p_z(p-k)l_2+(p-k)x]}|e\rangle\Bigg].\end{aligned} \tag{3}$$

The longitudinal momentum is defined by the function $p_z(p)=\sqrt{p_0^2-p^2}$, which expresses energy conservation during the scattering process with scattering amplitudes $C\equiv\cos[\kappa m/(\hbar p_0)a]$ and $S\equiv\sin[\kappa m/(\hbar p_0)a]$.

The first term on the right-hand side of Eq. (3) describes atoms that have experienced no internal transition on their way from the first laser beam to the detection screen. Therefore, this part of the total quantum state accumulates the phase factor $\exp[ip_z(p)(l_1+l_2)]$ of free evolution. Atoms that undergo a $|g\rangle-|e\rangle$ transition during the first laser interaction and a further $|e\rangle-|g\rangle$ transition during the second laser interaction are represented by the second term. Those atoms absorb a photon from laser 1 giving rise to the phase factor $\exp[ip_z(p+k)l_1]$ and emit a photon in laser 2 resulting in the phase factor $\exp[ip_z(p+2k)l_2]$. Equivalently, the phases of the last two terms have a clear physical meaning. The third and fourth terms describe atoms undergoing a $|g\rangle-|e\rangle$ transition only in laser 1 and laser 2, respectively. Summing up, Eq. (3) is a superposition of four probability amplitudes: (i) no transition, (ii) two transitions due to laser 1 and laser 2, (iii) one transition due to laser 1, and (iv) one transition due to laser 2.

The state $|\Psi(x,l_1+l_2,t)\rangle$, Eq. (3), determines the atomic position distribution on the detection screen. Let us suppose that we record ground-state atoms only, which is no limitation. The interferometric method works as well if we measure the positions of atoms irrespective of their internal states, that is, if we take the trace over the internal states in Eq. (3). However, in this case the equations become more complicated and we would like to formulate the principle as simple as possible.

Ground-state atoms are distributed over the detection screen according to the probability amplitude $\psi_g(x,l_1+l_2)$ in Eq. (3). We can rewrite the exponents of ψ_g when we apply the expansion

$$p_z(p)=\sqrt{p_0^2-p^2}\approx p_0-\frac{p^2}{2p_0} \tag{4}$$

and analogously for $p_z(p+k)$ and $p_z(p+2k)$. This approximation is valid as long as the characteristic momentum p_0 is much larger than any momentum p in the x direction. Note that we need the same assumption in the Appendix in order to derive the final state, Eq. (3). In that case ψ_g simplifies and we arrive at

$$\psi_g(x,l_1+l_2)=e^{ip_0(l_1+l_2)}[C^2\psi(x,T)-S^2\exp(i\tilde{\varphi}+2ikx) \times\psi(x-\Delta x,T)] \tag{5}$$

with the freely propagated wave function

$$\psi(x,T)\equiv\exp\left(-i\frac{\hbar\hat{p}^2}{2m}T\right)\psi(x) =\int\frac{dp}{2\pi}\phi(p)\exp\left(ipx-i\frac{\hbar p^2}{2m}T\right) \tag{6}$$

and the definitions of a free propagation time

$$T\equiv\frac{m(l_1+l_2)}{\hbar p_0}, \tag{7}$$

of a spatial ruler

$$\Delta x\equiv\frac{k}{p_0}(l_1+2l_2) \tag{8}$$

and of a modified phase

$$\tilde{\varphi}\equiv\varphi-\frac{k^2}{2p_0}(l_1+4l_2). \tag{9}$$

Hence $\psi(x,T)$ is just the initial wave function $\psi(x)$ spread by a period of free evolution during the time T, which is the time a classical particle with kinetic energy E_0 would need to pass from laser 1 to the detection screen. In other words, if we know $\psi(x,T)$, we can easily reconstruct $\psi(x)$ just by inverting Eq. (6). In the next section we will show how the position distribution $|\psi_g|^2$ enables us to find the spread wave function $\psi(x,T)$.

III. MEASUREMENT AND RECONSTRUCTION

How can we extract modulus *and* phase of $\psi(x,T)$ from the measured data? It turns out that we have to identify a certain set of position distributions measured on the screen in order to reconstruct this wave function.

First, we can directly measure the modulus $|\psi(x,T)|$ by just switching off the laser beams, which is equivalent to setting $a=0$ or $C=1$ and $S=0$ in Eq. (5). Then the measured distribution on the screen at $z=l_1+l_2$ provides us $|\psi(x,T)|^2$. Subsequent to this premeasurement we are left with the crucial question how to extract the phase information contained in $\psi(x,T)\equiv|\psi(x,T)|\exp[i\theta(x,T)]$.

According to Eq. (5) the probability distribution for the positions of the ground-state atoms reads

$$w(x,\Delta x,\tilde{\varphi}) \equiv|\psi_g|^2=C^4|\psi(x,T)|^2+S^4|\psi(x-\Delta x,T)|^2 -2S^2C^2\mathrm{Re}\{e^{i(\tilde{\varphi}+2kx)}\psi^*(x,T)\psi(x-\Delta x,T)\}. \tag{10}$$

The phase difference of $\psi(x,T)$ and $\psi(x-\Delta x,T)$ is encoded in the interference term of this equation and it depends on the adjustable phase $\tilde{\varphi}$, Eq. (9), as well as on the spatial ruler Δx, Eq. (8). Hence, we proceed as follows. First, we choose a certain spatial ruler Δx by appropriately fixing the distances l_1 and l_2 of the interferometer. Note that we can modify Δx without changing the time T [Eq. (7)], that is, without changing the total length of the interferometer. Second, we measure a set of four position distributions $w(x,\Delta x,\tilde{\varphi})$ with $\tilde{\varphi}=0$, $\pi/2$, π, and $3\pi/2$. The variation of $\tilde{\varphi}$ is achieved with the help of the phase shifter between laser beams 1 and 2. Having this set of distributions at our disposal it is straightforward to show with the help of Eq. (10) that the interference term can be expressed as

$$\psi^*(x,T)\psi(x-\Delta x,T)\equiv|\psi(x,T)\psi(x-\Delta x,T)| \times\exp[-i(\theta(x,T)-\theta(x-\Delta x,T))] =\frac{e^{-i2kx}}{(2CS)^2}\left[w(x,\Delta x,\pi)-w(x,\Delta x,0) +iw\left(x,\Delta x,\frac{\pi}{2}\right)-iw\left(x,\Delta x,\frac{3\pi}{2}\right)\right]. \tag{11}$$

With the help of this relation we can experimentally determine [24] the set of phase differences $\{\theta_{\mathrm{expt}}(x,T)-\theta_{\mathrm{expt}}(x-\Delta x,T)\}$. Starting with the additional assumption $\theta_{\mathrm{expt}}(0,T)=0$ we recursively calculate $\theta_{\mathrm{expt}}(\Delta x,T),\theta_{\mathrm{expt}}(2\Delta x,T),\ldots$ on a grid defined by the spacing Δx. Note that the assumption $\theta_{\mathrm{expt}}(0,T)=0$ is not a limitation of the method. Instead of $\psi(x,T)$ we reconstruct the wave function [25]

$$\psi_{\mathrm{expt}}(x,T)\simeq\psi(x,T)e^{-i\theta(0,T)}, \tag{12}$$

that is, the original wave function up to a common phase factor, which is arbitrary and not observable. The reconstruction can be refined by choosing further values for Δx, which should define grid points lying between the previous ones.

To finish the reconstruction we finally apply Eq. (6), which leads to the function

$$\psi_{\mathrm{expt}}(x)=\exp\left(i\frac{\hbar\hat{p}^2}{2m}T\right)\psi_{\mathrm{expt}}(x,T)\simeq\psi(x)e^{-i\theta(0,T)}, \tag{13}$$

which in the limits of the experiment represents $\psi(x)$ up to a common phase factor.

Note at this point that we have determined $\psi_{\mathrm{expt}}(x,T)$ at discrete grid points only with a resolution given by the spatial ruler Δx. However, the application of the operator $\exp[i(\hbar\hat{p}^2)/(2m)T]$ requires in principle the knowledge of the continuous function $\psi(x,T)$. This places certain limitations on wave functions that can be reconstructed by using the discussed method. If, for example, the structures in $\psi(x,T)$ are much finer than the grid spacing Δx then the method will fail. We will discuss those limitations and the corresponding numerical problems elsewhere.

IV. CONCLUSIONS

We presented the principle of an interferometric method to probe matter waves. The method is applicable to a stationary beam of two-level atoms with large and sufficiently monochromatic longitudinal momenta $p_z\approx p_0\gg p,k$. It allows us to measure the wave function of the atomic center-of-mass motion in the transverse direction.

The most essential information, that is, the information on the phase of a wave function, is encoded in the interference term of Eq. (10), which obviously plays the prime role in the measurement scheme. The scheme is based on the induced recoil effects due to the interaction of the two-level atoms with resonant light fields. Changing the atomic transverse momentum by $\pm\hbar k$ leads to a small change of the atom's longitudinal momentum in accordance with energy conservation. Taking into account these deviations provides the correct description of the spatial interference patterns that lie at the heart of the presented method.

The phase $\tilde{\varphi}$ and the dimensionless spatial ruler $k\Delta x$ in the interference term are determined by the parameter $\omega_R T$, where $\omega_R=\hbar k^2/(2m)$ is the recoil frequency and $T=(l_1+l_2)/v_0$ is the characteristic time of an atom passing from the first laser to the detection screen with characteristic velocity $v_0\equiv\hbar p_0/m$. For typical values $\omega_R\approx10^5$ s^{-1}, $l_1+l_2\approx10$ cm and $v_0\approx10^6$ cm/s the parameter is $\omega_R T\approx1$.

The main limitations of the method are connected with nonmonochromaticity in the velocity of the atomic beam and with spontaneous relaxation. We must properly control the phase $\tilde{\varphi}$ and the ruler $k\Delta x$ in order to have a chance in evaluating the interference pattern. Therefore, the deviation of the parameter $\omega_R T$ due to nonmonochromaticity of the atomic beam should be small:

$$\delta(\omega_R T)=\omega_R T\,\delta v_0/v_0\ll1. \tag{14}$$

Second, spontaneous relaxation of the excited state can be neglected in the case $\gamma T\ll1$, when γ describes the relaxation rate of the atoms in the beam.

For example, let us consider ^{40}Ca atoms on the weak transition $4s^2-4s4p$ with $k=10^5$ cm^{-1} and $\gamma=2.6\times10^3$ s^{-1}. If the size of the interferometer is given by $l_1+l_2=10$ cm and the average velocity by $v_0=10^6$ cm/s we have $\omega_R T\approx1$ and $\gamma T<0.1$. Then the nonmonochromaticity $\delta v_0/v_0$ should be less than 10%, which is a realistic value.

For simplicity we only discussed the interference patterns of ground-state atoms. One can use as well interferences of excited atoms or even both internal states, which obviously provides us with additional phase information. However, the principle of the presented method remains the same and only the mathematics for the reconstruction becomes more sophisticated. Note finally that the suggested method allows us, in principle, to reconstruct the density matrix $\hat{\rho}$ of the transverse center-of-mass motion of the atomic beam. In this case the interference term will be proportional to $\rho(x,x-\Delta x)$, that is, by changing the ruler Δx we directly measure off-diagonal elements of the density matrix in position representation.

ACKNOWLEDGMENTS

V.P.Y. acknowledges support from the Heraeus Foundation and the Russian Foundation for Basic Research. S.H.K. acknowledges support by the Studienstiftung des Deutschen Volkes.

APPENDIX: SCATTERING OF THE ATOMIC BEAM

In this Appendix we solve the scattering problem for a stationary incident matter wave at $z=z_0$:

$$|\Psi(x,z_0,t)\rangle=e^{-iE_0t/\hbar}\left[\int\frac{dp}{2\pi}\Phi_g(p,z_0)e^{ipx}|g\rangle+\int\frac{dp}{2\pi}\Phi_e(p,z_0)e^{ipx}|e\rangle\right], \tag{A1}$$

whose interaction with a laser of wave vector k in the region $z_0\leqslant z\leqslant z_0+a$ is described by the Hamiltonian

$$\hat{H}=\frac{\hbar^2}{2m}(\hat{p}^2+\hat{p}_z^2)-\Theta(z-z_0)\Theta(z_0+a-z)\hbar\kappa(\sigma_-e^{-i(kx-\alpha_0)}+\sigma_+e^{i(kx-\alpha_0)}). \tag{A2}$$

Note that the momentum operator for the x degree of freedom reads $\hat{p}=-i\partial/\partial x$ and the one for the z degree of freedom is given by $\hat{p}_z=-i\partial/\partial z$. Two Heaviside functions Θ switch the interaction on and off and κ determines the coupling of atom and field. As usual σ_- and σ_+ are the ladder operators of the atomic two-level system and α_0 is an arbitrary phase.

In the interaction region the matter wave will have the general form

$$|\Psi(x,z,t)\rangle = e^{-iE_0t/\hbar}\left[\int \frac{dp}{2\pi}\Phi_g(p,z)e^{ip_z(p)(z-z_0)}e^{ipx}|g\rangle + \int \frac{dp}{2\pi}\Phi_e(p,z)e^{ip_z(p)(z-z_0)}e^{ipx}|e\rangle\right], \tag{A3}$$

where we have separated the z dependence into the slowly varying amplitudes $\Phi_g(p,z)$ and $\Phi_e(p,z)$ and the fast oscillating function $\exp\{ip_z(p)(z-z_0)\}$. Note that the scattering process is elastic, that is, the total energy $E_0 \equiv \hbar^2 p_0^2/2m = (\hbar^2/2m)(p^2+p_z^2)$ will remain constant, defining the longitudinal momentum $p_z = p_z(p)$.

In the next step we investigate the evolution of the amplitudes Φ_g and Φ_e in order to find them after the interaction region at $z = z_0 + a$. Hence we use Eq. (A3) as an ansatz for the solution of the time-dependent Schrödinger equation $i\hbar(\partial/\partial t)|\Psi\rangle = \hat{H}|\Psi\rangle$ with the Hamiltonian Eq. (A2). In the interaction region $z_0 \leqslant z \leqslant z_0 + a$ we get the coupled equations

$$\left(E_0 - \frac{\hbar^2 p^2}{2m} - \frac{\hbar^2 p_z^2(p)}{2m}\right)\Phi_g(p,z) = -\frac{i\hbar^2}{m}p_z(p)\frac{\partial}{\partial z}\Phi_g(p,z) - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}\Phi_g(p,z) - \hbar\kappa e^{i\alpha_0}e^{i[p_z(p+k)-p_z(p)](z-z_0)}\Phi_e(p+k,z) \tag{A4}$$

and

$$\left(E_0 - \frac{\hbar^2 p^2}{2m} - \frac{\hbar^2 p_z^2(p)}{2m}\right)\Phi_e(p,z) = -\frac{i\hbar^2}{m}p_z(p)\frac{\partial}{\partial z}\Phi_e(p,z) - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2}\Phi_e(p,z) - \hbar\kappa e^{-i\alpha_0}e^{i[p_z(p-k)-p_z(p)](z-z_0)}\Phi_g(p-k,z). \tag{A5}$$

The equations simplify considerably when we take $E_0 = (\hbar^2/2m)(p^2+p_z^2)$ and two approximations into account. These approximations are as follows. (i) We neglect the second derivatives of the slowly varying amplitudes Φ_g and Φ_e and (ii) we approximate the momentum in the z direction by the characteristic momentum, that is, $p_z(p+k) \approx p_z(p-k) \approx p_z(p) \approx p_0$. The second approximation means that phase changes of the matter wave in the scattering area $z_0 \leqslant z \leqslant z_0 + a$ are small compared to unity, that is,

$$[p_0 - p_z(p)]a \approx \frac{p^2}{2p_0}a \ll 1 \tag{A6}$$

for all momenta p in the x direction.

Under these conditions Eqs. (A4) and (A5) read

$$iv_0\frac{\partial}{\partial z}\Phi_g(p,z) = -\kappa e^{i\alpha_0}\Phi_e(p+k,z) \tag{A7}$$

and

$$iv_0\frac{\partial}{\partial z}\Phi_e(p,z) = -\kappa e^{-i\alpha_0}\Phi_g(p-k,z), \tag{A8}$$

where we have introduced the characteristic velocity $v_0 \equiv \hbar p_0/m$. Now their solutions can be found easily and we arrive at

$$\Phi_g(p,z) = \Phi_g(p,z_0)\cos\left(\frac{\kappa}{v_0}(z-z_0)\right) + ie^{i\alpha_0}\Phi_e(p+k,z_0)\sin\left(\frac{\kappa}{v_0}(z-z_0)\right) \tag{A9}$$

and

$$\Phi_e(p,z) = \Phi_e(p,z_0)\cos\left(\frac{\kappa}{v_0}(z-z_0)\right) + ie^{-i\alpha_0}\Phi_g(p-k,z_0)\sin\left(\frac{\kappa}{v_0}(z-z_0)\right). \tag{A10}$$

The amplitudes $\Phi_g(p,z_0)$ and $\Phi_e(p,z_0)$ are determined by the initial conditions at $z = z_0$, that is, by Eq. (A1). Hence we find the scattered wave at $z = z_0 + a$ when we insert Eqs. (A9) and (A10) into Eq. (A3). From there on the wave undergoes free evolution over a distance l_0, which leads to a further phase factor $\exp\{ip_z(p)l_0\}$. Combining these results we arrive at

$$\begin{aligned}|\Psi(x,z_0+a+l_0,t)\rangle = e^{-iE_0t/\hbar}\Bigg[&\cos\left(\frac{\kappa}{v_0}a\right)\int\frac{dp}{2\pi}\Phi_g(p,z_0)e^{i[p_z(p)(a+l_0)+px]}|g\rangle + ie^{i\alpha_0}\sin\left(\frac{\kappa}{v_0}a\right)\\ &\times\int\frac{dp}{2\pi}\Phi_e(p+k,z_0)e^{i[p_z(p)(a+l_0)+px]}|g\rangle + \cos\left(\frac{\kappa}{v_0}a\right)\int\frac{dp}{2\pi}\Phi_e(p,z_0)e^{i[p_z(p)(a+l_0)+px]}|e\rangle\\ &+ ie^{-i\alpha_0}\sin\left(\frac{\kappa}{v_0}a\right)\int\frac{dp}{2\pi}\Phi_g(p-k,z_0)e^{i[p_z(p)(a+l_0)+px]}|e\rangle\Bigg],\end{aligned} \tag{A11}$$

which represents the scattered wave at $z=z_0+a+l_0$.

Let us finally specialize this solution to our interferometer (see Fig. 1) where the atomic beam interacts with the first laser beam in the region $0\leqslant z\leqslant a$. This interaction is described by the Hamiltonian Eq. (2), which has the structure of Eq. (A2) with $z_0=0$ and a relative phase $\alpha_0=0$. The incident matter wave at $z_0=0$ reads

$$|\Psi(x,0,t)\rangle=e^{-iE_0t/\hbar}\int\frac{dp}{2\pi}\phi(p)e^{ipx}|g\rangle \tag{A12}$$

and the initial conditions are $\Phi_g(p,0)=\phi(p)$ and $\Phi_e(p,0)=0$. After free evolution over the distance $l_0\equiv l_1-a$ we find for the scattered wave at $z=l_1$

$$|\Psi(x,l_1,t)\rangle=e^{-iE_0t/\hbar}\left[C\int\frac{dp}{2\pi}\phi(p)e^{i[p_z(p)l_1+px]}|g\rangle+iS\int\frac{dp}{2\pi}\phi(p-k)e^{i[p_z(p)l_1+px]}|e\rangle\right], \tag{A13}$$

in accordance with the general solution Eq. (A11). Here we have used the simpler notations $C\equiv\cos[(\kappa/v_0)a]$ and $S\equiv\sin[(\kappa/v_0)a]$.

The same calculation has to be repeated for the interaction with the second counterpropagating laser beam in the region $l_1\leqslant z\leqslant l_1+a$. This interaction is governed by the third term of the Hamiltonian Eq. (2). Therefore, we can again apply our general solution Eq. (A11) when we set $\alpha_0=\varphi$ and replace k by $-k$. The initial conditions $\Phi_g(p,l_1)=C\phi(p)e^{ip_z(p)l_1}$ and $\Phi_e(p,l_1)=iS\phi(p-k)e^{ip_z(p)l_1}$ at $z_0=l_1$ follow from Eq. (A13). Using these expressions in Eq. (A11) we arrive at

$$\begin{aligned}|\Psi(x,l_1+l_2,t)\rangle\\ =e^{-iE_0t/\hbar}\Bigg[&C^2\int\frac{dp}{2\pi}\phi(p)e^{i[p_z(p)(l_1+l_2)+px]}|g\rangle\\ &-S^2e^{i\varphi}\int\frac{dp}{2\pi}\phi(p)e^{i[p_z(p+k)l_1+p_z(p+2k)l_2+(p+2k)x]}|g\rangle\\ &+iSC\int\frac{dp}{2\pi}\phi(p)e^{i[p_z(p+k)(l_1+l_2)+(p+k)x]}|e\rangle\\ &+iSCe^{-i\varphi}\int\frac{dp}{2\pi}\phi(p)e^{i[p_z(p)l_1+p_z(p-k)l_2+(p-k)x]}|e\rangle\Bigg],\end{aligned} \tag{A14}$$

where we have taken into account the free evolution over the distance $l_0\equiv l_2-a$.

[1] P. A. M. Dirac, *The Principles of Quantum Mechanics* (Clarendon Press, Oxford, 1958).

[2] W. Heisenberg, *The Physical Principles of the Quantum Theory* (Dover Publications, New York, 1949).

[3] See the reprint W. Pauli, *General Principles of Quantum Mechanics* (Springer, Berlin, 1980).

[4] W. Gale, E. Guth, and G. T. Trammell, Phys. Rev. **165**, 1434 (1968); J. Bertrand and P. Bertrand, Found. Phys. **17**, 397 (1987); A. Royer, *ibid.* **19**, 3 (1989); Phys. Rev. Lett. **55**, 2745 (1985); **73**, 913 (1994); J. Bohn, *ibid.* **66**, 1547 (1991); K. R. W. Jones, Phys. Rev. A **50**, 3682 (1994); M. G. Raymer, M. Beck, and D. F. McAlister, Phys. Rev. Lett. **72**, 1137 (1994); S. Massar and S. Popescu, *ibid.* **74**, 1259 (1995); S. Weigert, Phys. Rev. A **53**, 2078 (1996).

[5] See *Quantum State Preparation and Measurement*, edited by M. G. Raymer and W. P. Schleich, special issue of J. Mod. Opt. (to be published).

[6] M. Brune, S. Haroche, J. M. Raimond, L. Davidovich, and N. Zagury, Phys. Rev. A **45**, 5193 (1992); K. Vogel, V. M. Akulin, and W. P. Schleich, Phys. Rev. Lett. **71**, 1816 (1993); A. S. Parkins, P. Marte, P. Zoller, and H. J. Kimble, *ibid.* **71**, 3095 (1993); B. M. Garraway, B. Sherman, H. Moya-Cessa, P. L. Knight, and G. Kurizki, Phys. Rev. A **49**, 535 (1994); G. Harel, G. Kurizki, J. K. McIver, and E. Coutsias, *ibid.* **53**, 4534 (1996); C. K. Law and J. H. Eberly, Phys. Rev. Lett. **76**, 1055 (1996).

[7] J. I. Cirac, R. Blatt, A. S. Parkins, and P. Zoller, Phys. Rev. Lett. **70**, 762 (1993); H. Zeng and F. Lin, Phys. Rev. A **50**, 3589 (1994); **52**, 809 (1995); J. I. Cirac, R. Blatt, and P. Zoller, Phys. Rev. A **49**, R3174 (1994); D. M. Meekhof, C. Monroe, B. E. King, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. **76**, 1796 (1996); C. Monroe, D. M. Meekhof, B. E. King, W. M. Itano, and D. J. Wineland, Science **272**, 1131 (1996).

[8] K. Vogel and H. Risken, Phys. Rev. A **40**, 2847 (1989).

[9] G. M. D'Ariano, U. Leonhardt, and H. Paul, Phys. Rev. A **52**, R1801 (1995); U. Leonhardt, H. Paul, and G. M. D'Ariano, *ibid.* **52**, 4899 (1995).

[10] D. T. Smithey, M. Beck, M. G. Raymer, and A. Faridani, Phys. Rev. Lett. **70**, 1244 (1993); M. Munroe, D. Boggavarapu, M. E. Anderson, and M. G. Raymer, *ibid.* **52**, R924 (1995); S. Schiller, G. Breitenbach, S. F. Pereira, T. Müller, and J. Mlynek, *ibid.* **77**, 2933 (1996).

[11] H. Paul, P. Törmä, T. Kiss, and I. Jex, Phys. Rev. Lett. **76**, 2464 (1996); O. Steuernagel and J. A. Vaccaro, *ibid.* **75**, 3201 (1995); M. G. A. Paris, Phys. Rev. A **53**, 2658 (1996); S. Wallentowitz and W. Vogel, *ibid.* **53**, 4528 (1996); K. Banaszek and K. Wódkiewicz, Phys. Rev. Lett. **76**, 4344 (1996); A. Zucchetti, W. Vogel, and D.-G. Welsch, Phys. Rev. A **54**, 856 (1996).

[12] U. Janicke and M. Wilkens, J. Mod. Opt. **42**, 2183 (1995); S. H. Kienle, M. Freyberger, W. P. Schleich, and M. G. Raymer, in *Experimental Metaphysics: Quantum Mechanical Studies for Abner Shimony*, edited by R. S. Cohen, M. A. Horne, and J. Stachel (Kluwer Academic Publishers, Dordrecht, 1997); M. Tegmark, Phys. Rev. A **54**, 2703 (1996).

[13] S. Wallentowitz and W. Vogel, Phys. Rev. Lett. **75**, 2932 (1995); C. D'Helon and G. J. Milburn, Phys. Rev. A **54**, R25 (1996); J. F. Poyatos *et al.*, *ibid.* **53**, R1966 (1996).

[14] T. J. Dunn, J. N. Sweetser, I. A. Walmsley, and C. Radzewicz, Phys. Rev. Lett. **70**, 3388 (1993); T. J. Dunn, I. A. Walmsley, and S. Mukamel, *ibid.* **74**, 884 (1995).

[15] P. J. Bardroff, E. Mayr, and W. P. Schleich, Phys. Rev. A **51**, 4963 (1995); P. J. Bardroff, E. Mayr, W. P. Schleich, P. Domokos, M. Brune, J. M. Raimond, and S. Haroche, Phys. Rev. A **53**, 2736 (1996).

[16] A. M. Herkommer, V. M. Akulin, and W. P. Schleich, Phys. Rev. Lett. **69**, 3298 (1992); M. Freyberger and A. M. Herkommer, *ibid.* **72**, 1952 (1994).

[17] P. J. Bardroff, C. Leichtle, G. Schrade, and W. P. Schleich, Phys. Rev. Lett. **77**, 2198 (1996).

[18] D. Leibfried, D. M. Meekhof, B. E. King, C. Monroe, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. **77**, 4281 (1996).

[19] V. P. Chebotayev, B. Y. Dubetsky, A. P. Kasantsev, and V. P. Yakovlev, J. Opt. Soc. Am. B **2**, 1791 (1985).

[20] T. Riehle, T. Kisters, A. Witte, J. Helmcke, and C. J. Bordé, Phys. Rev. Lett. **67**, 177 (1991); C. J. Bordé, Phys. Lett. A **140**, 10 (1989); E. M. Rasel, M. K. Oberthaler, H. Batelaan, J. Schmiedmayer, and A. Zeilinger, Phys. Rev. Lett. **75**, 2633 (1995); M. Kasevitch and S. Chu, *ibid.* **67**, 181 (1991); O. Carnal and J. Mlynek, *ibid.* **66**, 2689 (1991); D. W. Keith, C. R. Ekstrom, Q. A. Turchette, and D. E. Pritchard, *ibid.* **66**, 2693 (1991).

[21] See also *Optics and Interferometry with Atoms*, edited by J. Mlynek, V. Balykin, and P. Meystre, special issue of Appl. Phys. B **54**, (1992).

[22] C. Iaconis and I. A. Walmsley, Opt. Lett. **21**, 1783 (1996).

[23] Note that we use the symbol p for the wave vector of the atomic matter wave whereas k denotes the wave vector of the laser.

[24] We use the subscript expt for the reconstructed quantities.

[25] The sign $\simeq$ means equality within the basic limitations of the method: $\psi_{\rm expt}(x,T)$ is known at certain grid points only.