Spontaneous radiation of free electrons in a nonrelativistic collapse model

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One way to solve the long-standing ''measurement''problem in quantum mechanics is to modify Schrödinger's equation so that the collapse of a state vector is the result of some intrinsic physical dynamics. The introduction of such collapse dynamics often results in energy nonconservation. In this paper, we first derive a general expression for the rate of change in the expectation value of energy in a general collapse model that supports a linear evolution of the density matrix. In particular, we show, under certain plausible assumptions, that energy nonconservation is an inevitable consequence of collapse. We then work with a specific model, namely, the nonrelativistic continuous spontaneous localization (CSL) model, to derive further consequences of collapse. In CSL, in a nonunitary evolution, a particle interacts with a fluctuating scalar field, leading to the collapse of the state vector. One consequence of this interaction is that a free electron can radiate spontaneously. We calculate the spectrum of this radiation. The result is then compared with the observed upper bound on spontaneous radiation in Ge and a constraint on the parameters of CSL is obtained.
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I. INTRODUCTION

The ''measurement''problem has long been a difficult unresolved issue in the foundations of quantum mechanics. On one hand, the theory states that the state vector corresponding to a closed physical system undergoes a continuous unitary evolution governed by Schrödinger's equation; on the other hand, in order to translate the theory into measurable results, the theory prescribes a sudden jump motion (governed by the well-known probability rule) to the state of a physical system undergoing a measurement by an external device.

One way to solve the ''measurement'' problem, i.e., to make a consistent theory from the two aforementioned components of the standard quantum mechanics, is to introduce a new physical dynamics that naturally collapses a state vector. In a nonrelativistic collapse model developed by Ghirardi, Rimini, Weber [1,2] and Pearle [3,4], namely, the continuous spontaneous localization model (CSL), the state vector undergoes a nonunitary evolution in which particles interact with a fluctuating scalar field. Besides collapsing the state vector towards the particle number density eigenstates in position space, this interaction increases the expectation value of the particle's energy. For a free charged particle such as an electron, this implies electromagnetic radiation. Such spontaneous radiation by a free electron is a phenomenon predicted by CSL but not by the standard quantum mechanics.

In this paper, we investigate the spontaneous radiation by free electrons predicted by CSL and seek its observational consequences. This is of interest for several reasons. First, there have been other collapse models proposed to solve the ''measurement''problem of the standard quantum mechanics. An earlier proposal of a collapse model by Karolyhazy, for example, postulated that fluctuations in space-time cause a state vector to collapse [5]. However, Diosi and Lukacs recently calculated the spectrum of the spontaneous radiation by free charged particles in Karolyhazy's model and showed that the model produces an unreasonable amount of radiation in the x-ray range [6]. Thus, the plausibility of the CSL model depends critically on whether it gives rise to a similar unreasonable spectrum of spontaneous radiation by free charged particles. Second, such spontaneous radiation by free electrons is subject to an experimental test and may help verify or reject the CSL model. Third, even if present experiments are unable to provide conclusive evidence for or against the theory, the comparison between the theoretical predictions and the experimental results will provide helpful constraints on the parameters in the CSL model.

In Sec. II, we discuss the increase in the expectation value of the energy of a single particle in the setting of a general collapse model that supports a linear evolution of the density matrix. We show, under plausible assumptions, that production of energy is an unavoidable feature of collapse models.

In the third section, we investigate the radiation by a free electron predicted by the CSL model. In particular, we calculate the spectrum of such radiation. This raises the possibility of experimental tests for the theory.

In the fourth section, we use this result to calculate the strength of such radiation by the valence electrons in Ge. The result is compared with the observed value of the upper bound on spontaneous radiation in Ge [7]. We see that the CSL model, unlike Karolyhazy's model, does not produce an unreasonable amount of x-ray radiation by free electrons inconsistent with observations. We also note that the constraint on the parameters in the CSL model derived from this comparison is less stringent than a previous constraint derived from the consideration of the radiation due to spontaneous excitations of bound electrons in Ge [8], except if it is assumed that the fluctuating field is coupled to the particle mass density. If this assumption is made, then the spontaneous radiation of free electrons, while less than the current

*Deceased.

observed upper bound, is responsible for the major amount of spontaneous radiation in Ge.

II. ENERGY NONCONSERVATION IN A GENERAL COLLAPSE MODEL

In this section, we investigate energy nonconservation for a single nonrelativistic particle in a general collapse model that supports a linear evolution of the density matrix. We assume the collapse is towards the position basis of the particle. We shall see in particular that energy nonconservation is an unavoidable feature of such collapse models.

The most general linear evolution of the density matrix ρ (preserving its hermiticity, trace, and positivity) is given by the Lindblad equation [9]:

$$\dot{\rho}=i[\rho,H]-\lambda\sum_n (K_nK_n^\dagger\rho+\rho K_nK_n^\dagger-2K_n^\dagger\rho K_n)\quad(\lambda>0),\tag{2.1}$$

where H is the normal quantum mechanical Hamiltonian, the K_n's are arbitrary operators, and λ is a non-negative constant. The first term on the right-hand side of Eq. (2.1) is the usual Hamiltonian evolution; the additional term on the right, as we shall see, causes collapse behavior. For simplicity, we shall first consider the case where there is only one K and it is a function of the position operator X, i.e., $K=K(X)$. [This choice will be justified later. I shall use K_x to mean $K(x)$ in the following calculation.] Letting $H=0$ to concentrate on the effect of the additional term, we have

$$\langle x|\dot{\rho}|y\rangle=-\lambda(K_xK_x^*+K_yK_y^*-2K_x^*K_y)\rho_{xy}.$$

Since $K_xK_x^*+K_yK_y^*-2K_yK_x^*=|(K_x-K_y)|^2+K_xK_y^*-K_x^*K_y$ has a positive real part, the magnitudes of the off-diagonal elements in ρ are obviously shrinking. Thus the additional term in the Lindblad equation effectively collapses the off-diagonal elements of ρ in K's eigenstate representation. This also justifies our choice of K as a function of X, since we have stated earlier that the collapse should be toward the position basis of the particle.

Now, let us see the effect of this additional term on the expectation value of the particle's momentum.

$$\begin{aligned}\langle\dot{P}\rangle&=\int dx\langle x|P\dot{\rho}|x\rangle\\&=i\lambda\int dx\ dy\ \delta(x-y)\ \frac{\partial}{\partial x}[(K_xK_x^*+K_yK_y^*\\&\quad-2K_x^*K_y)\rho_{xy}]\\&=i\lambda\int dx(K_x^*\overleftrightarrow{\partial}K_x)\rho_{xx},\end{aligned}$$

where $A\overleftrightarrow{\partial}B\equiv A\partial_xB-B\partial_xA$. If we require, due to consideration of space-rotation symmetry (or space-reflection symmetry in one dimension), that the expectation value of momentum does not change, we should have $K_x^*\overleftrightarrow{\partial}K_x=0$. Hence $K_x^*\partial_xK_x$ is real for all x. This implies, letting $K=P+iQ$ where P, Q are Hermitian, that $P(x)=cQ(x)$ where c is a constant. But since the choice of K's phase factor in the Lindblad equation is free, we can eliminate the imaginary part of K and thus require K to be Hermitian.

The general form for the density matrix evolution (2.1) for the case of only one K is thus reduced to

$$\dot{\rho}=i[\rho,H]-\lambda[K,[K,\rho]].\tag{2.2}$$

If we generalize Eq. (2.2) to the case of multiple K's, we have instead

$$\dot{\rho}=i[\rho,H]-\lambda\sum_n [K_n,[K_n,\rho]].\tag{2.2$'$}$$

Accordingly, we have for the energy

$$\langle\dot{H}\rangle=\frac{1}{2m}\int dx\langle x|P^2\dot{\rho}|x\rangle=\frac{\lambda}{m}\sum_n\int dx\rho_{xx}[\partial_xK_n(x)]^2.\tag{2.3}$$

Thus, assuming the density matrix follows a linear evolution, we can conclude on general grounds that state-vector collapse to the position basis in nonrelativistic quantum mechanics entails a production of energy given by Eq. (2.3).

It is worth noting that Eq. (2.3) remains valid even when the particle is coupled with the electromagnetic potential, i.e., if we add $H_{em}=J^\mu A_\mu$, where J^μ is the four-current of the charged particle and A_μ is the electromagnetic potential. In this case the additional change of energy is

$$\begin{aligned}\langle\dot{H}_{em}\rangle&=\mathrm{tr}(J^\mu A_\mu\dot{\rho})=\mathrm{tr}\left(-\lambda A_\mu J^\mu\sum_n [K_n,[K_n,\rho]]\right)\\&=\mathrm{tr}\left(-\lambda A_\mu\rho\sum_n [K_n,[K_n,J^\mu]]\right).\end{aligned}\tag{2.4}$$

But $[K_n,[K_n,J^\mu]]=0$ since $[K_n,J^\mu]$ is a function of the position X. Thus, Eq. (2.4) vanishes.

Furthermore, for free particles, we can also argue from space-rotation symmetry that the rate of energy production (2.3) must be a constant. The argument is the following. We start by considering a particle at rest ($|\Psi,0\rangle=|\mathbf{p}=\mathbf{0}\rangle$). After a while, due to collapse, the particle will gain a distribution of momentum symmetric over all orientations. If we do measurements of energy at this moment, we will get a certain mean value of the change in energy given by $\Delta E=\langle\mathbf{p}^2/2m\rangle$. Now, suppose we start with a particle with momentum $\mathbf{p}_1\neq\mathbf{0}$. If we move with the particle, we will see exactly the same situation described earlier. But in the rest frame we will see the symmetric distribution of momentum not centered at $\mathbf{p}=\mathbf{0}$ but at $\mathbf{p}_1$, i.e., $\mathbf{P}'=\mathbf{P}+\mathbf{p}_1$, where $\mathbf{P}'$ is the momentum of the moving particle observed from the rest frame. Therefore

$$\Delta E'=\left\langle\frac{\mathbf{P}'^2}{2m}\right\rangle-\frac{\mathbf{p}_1^2}{2m}=\left\langle\frac{\mathbf{P}^2}{2m}\right\rangle=\Delta E.$$

Thus the change in the expectation value of the energy over a fixed interval of time is the same whether or not the particle is moving; i.e., it is a constant.

III. SPONTANEOUS RADIATION BY FREE ELECTRONS IN CSL

Ghirardi, Rimini, and Weber (GRW) originally proposed a collapse model in which the state vector is occasionally multiplied by a Gaussian centered at an arbitrary location with a prescribed probability [1,2]. From that, Pearle further developed a model (CSL) in which a non-Hermitian interaction between a fluctuating scalar field and the particle was introduced in the Hamiltonian to cause collapse [3]. In our calculations, we shall use the CSL model.

For now, we shall use the interaction picture in which operators evolve according to the conventional Hamiltonian (the free field Hamiltonian plus the normal quantum field interaction terms) and the state vector evolves according to the collapse term. In CSL, the state vector evolves according to the modified Schrödinger equation

$$\frac{d|\Psi,t\rangle}{dt}=-\frac{1}{4\lambda}\int d\mathbf{x}[W(\mathbf{x},t)-2\lambda K(\mathbf{x},t)]^2|\Psi,t\rangle, \tag{3.1}$$

where

$$K(\mathbf{x},t)=\int d\mathbf{z}(\pi a^2)^{-3/4}e^{-(1/2a^2)(\mathbf{z}-\mathbf{x})^2}N(\mathbf{z},t).$$

Here $W(\mathbf{x},t)$ is a fluctuating scalar field and $N(\mathbf{z},t)=\psi^{\dagger}(\mathbf{z},t)\psi(\mathbf{z},t)$ is the particle number operator of a nonrelativistic quantum field. The values for λ and a suggested by GRW are $\lambda=10^{-16}\ \mathrm{s}^{-1}$ and $a=10^{-7}$ m.

The probability density of $W(t)$ is given by

$$\mathrm{Prob}(W(\mathbf{x},t))={}_W\langle\Psi,t|\Psi,t\rangle_W \tag{3.2}$$

(the subscript W has been attached to the state vector for added emphasis). The solution of Eq. (3.1) is given by

$$|\Psi,t\rangle_W=\mathcal{T}e^{-(1/4\lambda)\int_0^t dt d\mathbf{x}[W(\mathbf{x},t)-2\lambda K(\mathbf{x},t)]^2}|\Psi,0\rangle, \tag{3.3}$$

where $\mathcal{T}$ is the time ordering operator. [See Ref. [3] for a discussion of how the evolution in Eq. (3.1) causes state vectors to collapse toward particle number operator eigenstates in position space.]

The density matrix corresponding to Eq. (3.1) is given by

$$\rho=\int DW\ \mathrm{Prob}(W(\mathbf{x},t))\ \frac{|\Psi,t\rangle_W\ {}_W\langle\Psi,t|}{{}_W\langle\Psi,t|\Psi,t\rangle_W}=\int DW\mathcal{T}e^{-(1/4\lambda)\int_0^t dt\, d\mathbf{x}[W(\mathbf{x},t)-2\lambda K(\mathbf{x},t)]^2}|\Psi,0\rangle\langle\Psi,0|e^{-(1/4\lambda)\int_0^t dt\, d\mathbf{x}[W(\mathbf{x},t)-2\lambda K(\mathbf{x},t)]^2}$$
$$=\mathcal{T}e^{-\lambda/2\int_0^t dt\ d\mathbf{x}\ d\mathbf{x}'[N(\mathbf{x},t)\otimes 1-1\otimes N(\mathbf{x},t)]e^{-(\mathbf{x}-\mathbf{x}')^2/4a^2}[N(\mathbf{x}',t)\otimes 1-1\otimes N(\mathbf{x}',t)]}\rho(0)\,, \tag{3.4}$$

where the functional integration element $DW=\Pi_{\mathbf{x},t}dW(\mathbf{x},t)/\sqrt{2\pi\lambda/d\mathbf{x}\,dt}$, $(A\otimes B)C\equiv ACB$, and it is understood that $\mathcal{T}$ implies time reversal for operators to the right of ρ. The last step in Eq. (3.4) involves the functional integration of $W(\mathbf{x},t)$.

From our earlier discussion, we know that a collapse process as in Eq. (3.4) inevitably produces energy. This means that a free electron can radiate spontaneously—a new phenomenon predicted by CSL. In the rest of this section we calculate the spectrum of this radiation.

To aid our calculation of the S matrix for the radiation by a free electron, we first notice an alternative equation for the state-vector evolution that produces the same evolution equation for the density matrix. In this alternative model, we assume the particle is under the influence of a fluctuating potential field $\eta(\mathbf{x},t)$ with Gaussian probability distribution, whose statistical property is given by $\langle\eta(\mathbf{x},t)\rangle=0$ and

$$\langle\eta(\mathbf{x},t)\eta(\mathbf{x}',t')\rangle=\frac{1}{4\lambda}e^{-(\mathbf{x}-\mathbf{x}')^2/4a^2}\delta(t-t'). \tag{3.5}$$

In momentum space, the latter becomes

$$\langle\eta(\mathbf{p},w)\eta^*(\mathbf{p}',w')\rangle=(4\pi a^2)^{3/2}e^{-\mathbf{p}^2a^2}\frac{1}{4\lambda}\delta(\mathbf{p}-\mathbf{p}')\times\delta(w-w'). \tag{3.6}$$

The state vector evolves according to

$$\frac{d|\Psi,t\rangle}{dt}=-i\int d\mathbf{x}2\lambda N(\mathbf{x},t)\eta(\mathbf{x},t)|\Psi,t\rangle. \tag{3.7}$$

The solution of Eq. (3.6) is

$$|\Psi,t\rangle=\mathcal{T}e^{-i2\lambda\int_0^t dt\,d\mathbf{x}\,\eta(\mathbf{x},t)N(\mathbf{x},t)}|\Psi,0\rangle. \tag{3.8}$$

The corresponding density matrix $\int D\eta|\Psi,t\rangle\langle\Psi,t|$ is readily found from Eqs. (3.4) and (3.7) to be the same as Eq. (3.4). We emphasize that Eq. (3.6) does *not* produce collapse for individual state vectors since it describes a unitary evolution. Thus, even though Eq. (3.7) results in the same evolution of the density matrix as Eq. (3.1), only Eq. (3.1) is a real collapse theory. We nevertheless may use Eq. (3.7) as a tool to calculate the S matrix of the radiation of an electron and derive from it the theoretical prediction of the real collapse model.

Since the CSL model is not relativistic, we shall for simplicity treat electrons as spinless Schrödinger particles and use the following form for the interaction between the electron's field and the photon's field:

$$\frac{e}{2mi}\int d\mathbf{x}\psi^{\dagger}(\mathbf{x},t)\overleftrightarrow{\nabla}\psi(\mathbf{x},t)\cdot A^T(\mathbf{x},t), \tag{3.9}$$

where A^T denotes the transverse mode of the electromagnetic potential. (We only care about radiation.) Thus the complete interaction Hamiltonian in our tool model becomes

$$H=\frac{e}{2mi}\int d\mathbf{x}\,\psi^{\dagger}(\mathbf{x},t)\overleftrightarrow{\nabla}\psi(\mathbf{x},t)\cdot A^{T}(\mathbf{x},t)$$
$$+2\lambda\int d\mathbf{x}\,\psi^{\dagger}(\mathbf{x},t)\,\psi(\mathbf{x},t)\,\eta(\mathbf{x},t), \qquad (3.10)$$

where

$$\psi(\mathbf{x},t)=\frac{1}{(2\pi)^{3/2}}\int d\mathbf{p}\,b_{\mathbf{p}}e^{i\mathbf{p}\cdot\mathbf{x}-i(p^2/2m)t}$$

and

$$A(\mathbf{x},t)^{T}=\frac{1}{(2\pi)^{3/2}}\sum_{\gamma=1,2}\int d\mathbf{k}\,\frac{\epsilon_{\mathbf{k}}^{\gamma}}{\sqrt{2k}}\,(a_{\mathbf{k}}^{\gamma}e^{i\mathbf{k}\cdot\mathbf{x}-ikt}$$
$$+a_{\mathbf{k}}^{\dagger\gamma}e^{-i\mathbf{k}\cdot\mathbf{x}+ikt}),$$

where $k=|\mathbf{k}|$ and $p=|\mathbf{p}|$. Here $b_{\mathbf{p}}$ is the annihilation operator for the Schrödinger particle of momentum $\mathbf{p}$ and $a_{\mathbf{k}}^{\gamma}$ and $a_{\mathbf{k}}^{\dagger\gamma}$ are the annihilation and creation operators for photons of momentum $\mathbf{k}$ and polarization γ. For the nonrelativistic Schrödinger field, the Feynman propagator is

$$D_F(\mathbf{x},t)=\frac{i}{(2\pi)^4}\int d\mathbf{p}\,dp^0\,\frac{e^{i\mathbf{p}\cdot\mathbf{x}-ip^0t}}{p^0-\mathbf{p}^2/2m+i\epsilon}.$$

The initial state is an electron at rest, $|\Psi,0\rangle=|\mathbf{p}_0=0\rangle$. (The normalization of this initial state vector will be carried out at the end of the calculation.) Also, in the natural unit system, we require the momentum of the electron in any stage be much less than $m=512\ \mathrm{keV}=2.59\times10^{12}\ \mathrm{m}^{-1}$, in order to apply our nonrelativistic calculation.

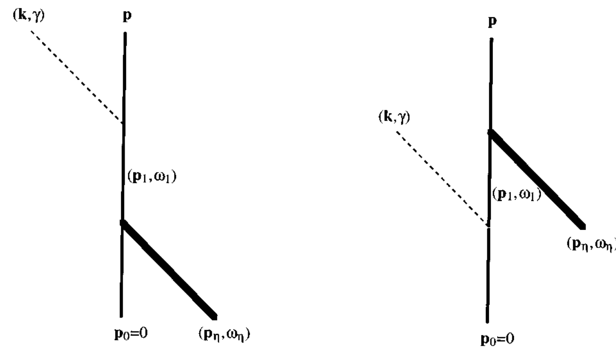

FIG. 1. Feynman diagrams for the lowest-order spontaneous radiative processes by a free electron.

The lowest order of approximation of the radiation process yields the two diagrams in Fig. 1. Here we denote $\mathbf{p}_0=\mathbf{0}$ as the electron's initial momentum, $\mathbf{p}$ as the electron's final momentum, $\mathbf{k}$ and γ as the radiated photon's momentum and polarization, $\mathbf{p}_1$ and w_1 as the off-mass-shell electron's momentum and energy, $\mathbf{p}_\eta$ and w_η as the η-field's momentum and energy. Notice that diagram (1b) gives no contribution to the S matrix since it involves the factor $\epsilon_{\mathbf{k}}^{\gamma}\cdot(\mathbf{p}_1+\mathbf{p}_0)=\epsilon_{\mathbf{k}}^{\gamma}\cdot(-\mathbf{k})=0$.

We have for diagram 1(a) the contribution, using the usual Feynman rules,

$$\langle\mathbf{k},\gamma,\mathbf{p}|S|\Psi,0\rangle=\left(\frac{1}{(2\pi)^{3/2}}\right)^3\frac{1}{(2\pi)^2}\frac{1}{\sqrt{2k}}\,(2\pi)^4(-i2\lambda)\,\eta(\mathbf{p}_\eta,w_\eta)(2\pi)^4\,\frac{(-ie)}{2m}\,(\mathbf{p}+\mathbf{p}+\mathbf{k})\cdot\epsilon_{\mathbf{k}}^{\gamma}$$
$$\times\frac{1}{(2\pi)^4}\,\frac{i}{k+p^2/2m-(\mathbf{k}+\mathbf{p})^2/2m+i\epsilon}\,\delta(\mathbf{p}+\mathbf{k}-\mathbf{p}_\eta)\,\delta\!\left(w_k+\frac{p^2}{2m}-w_\eta\right)$$
$$=-\frac{2e\lambda}{(2\pi)^{5/2}}\,\frac{1}{\sqrt{2k}}\,\frac{\mathbf{p}\cdot\epsilon_{\mathbf{k}}^{\gamma}}{m}\,\eta(\mathbf{p}_\eta,w_\eta)\,\frac{i}{k-k^2/2m-(\mathbf{k}\cdot\mathbf{p})/m+i\epsilon}\,\delta(\mathbf{p}+\mathbf{k}-\mathbf{p}_\eta)\,\delta\!\left(w_k+\frac{p^2}{2m}-w_\eta\right). \qquad (3.11)$$

Thus, we have for the rate $d\Gamma(\mathbf{k},\gamma,\mathbf{p})$ (i.e., probability per unit time) of radiation of a photon of momentum $\mathbf{k}$ and polarization γ while the electron leaves with momentum $\mathbf{p}$:

$$\frac{d\Gamma(\mathbf{k},\gamma,\mathbf{p})}{d\mathbf{k}\,d\mathbf{p}}=\frac{(4\pi a^2)^{3/2}}{(2\pi)^6}\,\frac{e^2\lambda}{m^2}\,\frac{1}{2k}\,\frac{(\mathbf{p}\cdot\epsilon_{\mathbf{k}}^{\gamma})^2}{(k-k^2/2m-(\mathbf{k}\cdot\mathbf{p})/m)^2}$$
$$\times e^{-(\mathbf{k}+\mathbf{p})^2a^2}, \qquad (3.12)$$

where the η-field ensemble has been integrated over with the help of Eq. (3.6) [the volume factor $\delta(\mathbf{0})$ has been gotten rid of by the normalization of the initial state vector].

Note that because of the Gaussian factor in Eq. (3.12), only when $|\mathbf{p}+\mathbf{k}|<1/a$ do we get a considerable contribution. For the radiation of a photon with much higher energy than $1/a$, this implies $\mathbf{p}\approx-\mathbf{k}$, i.e., the electron and the photon recoil ''back to back'' after the radiation. For our nonrelativistic calculation, k dominates the denominator $k-k^2/2m-(\mathbf{k}\cdot\mathbf{p})/m$. Upon integrating Eq. (3.12) over $\mathbf{p}$ and summing over γ, we have for the rate of radiation of photons of momentum $\mathbf{k}$:

$$\frac{d\Gamma(\mathbf{k})}{d\mathbf{k}}\approx2\times\frac{(4\pi a^2)^{3/2}}{(2\pi)^6}\,\frac{e^2\lambda}{m^2}\,\frac{1}{2k}\,\frac{1}{k^2}\,\frac{\pi^{3/2}}{2a^5}=\frac{e^2\lambda}{16\pi^3k^3a^2m^2}. \qquad (3.13)$$

Integrating over solid angle yields the rate of radiation of photons of energy k:

$$\frac{d\Gamma(k)}{dk}=4\pi k^2\,\frac{e^2\lambda}{16\pi^3k^3a^2m^2}=\frac{e^2\lambda}{4\pi^2a^2m^2k}. \quad (3.14)$$

The apparent infrared divergence of Eq. (3.14) can be treated in the standard way. Now, if we use the GRW values for λ and a, we have for the predicted number of photons radiated per second per keV per free electron:

$$R(k)=2.74\times10^{-31}\left(\frac{1}{k\ \text{keV}}\right)\text{counts/s keV} \quad (3.15)$$

or in terms of energy distribution,

$$E(k)=2.74\times10^{-31}\ \text{keV/s keV}. \quad (3.16)$$

IV. COMPARISON WITH EXPERIMENT

We now investigate an experimental consequence of this result. A measurement has been done on the radiation appearing in an isolated slab of Ge [7]. The experiment may be regarded as establishing an upper bound for the rate of spontaneous radiation in different energy ranges, as shown in Table I. (We emphasize that this is raw data, without subtraction of radiation appearing due to known processes. Future analysis of data from this and other experiments now being undertaken can be expected to improve these upper bounds considerably.) For a later comparison, we note that at $k=11.1$ keV, $R_{\text{expt}}=0.049$ counts/(keV kg d).

For the theoretical prediction of such radiation, we consider the outermost four electrons in a Ge atom. They are very weakly bound so that we can treat them as free electrons for the consideration of the radiation of energetic photons in the range of 1 keV or larger. According to our result (3.15) on the radiation rate per free electron, using 8.29×10^{24} atoms/kg as the particle density of Ge, the theoretical prediction of R at 11.1 keV is then

$$\begin{aligned}R_{\text{theory}}&=(2.74\times10^{-31})\times4\times(8.29\times10^{24})\times(8.6\times10^4)\\&\quad\times\frac{1}{11.1}\\&=0.071\ \ \text{counts/(keV kg d)}. \quad (4.1)\end{aligned}$$

The theoretical predictions of the rate of radiation for other energy ranges follow similar calculations and are shown in Table I. (Note that the theoretical predictions are only valid in the energy range much less than $m_e=512$ keV, due to nonrelativistic constraint of the CSL model. The theoretical values near 512 keV are shown nevertheless as rough projections.)

Several observations result from comparing the theoretical predictions and the experimental upperbounds. Although in the range between 100 and 500 keV it appears that the theory would seem to nicely account for the observations if we increase λ by roughly a factor of 10, such a conclusion is unwarrented. (The major portion of this radiation is most likely due to radiation arising in the lead shield.) Since, at 11 keV, the theory predicts 45% more radiation than the observed upper bound, λ is forced to be at least 45% lower than the GRW suggested value.

TABLE I. Experimental upper bounds and theoretical predictions of the spontaneous radiation by free electrons in Ge for a range of photon energy values.

Energy (keV)	Expt. upper bound (counts/keV/kg/day)	Theory (counts/keV/kg/day)
11	0.049	0.071
101	0.031	0.0073
201	0.030	0.0037
301	0.024	0.0028
401	0.017	0.0019
501	0.014	0.0015

A previous calculation has been done on the CSL rate of spontaneous radiation due to a 1s bound electron in Ge [8]. Such radiation occurs when a 1s electron in the atom is excited by collapse to an ionized state and then the atom decays back to the ground state. The bound state radiation rate expression is larger than our rate (3.13) for free electrons, mainly because our factor $e^2/\hbar c$ is missing and because our factor $[(\hbar/mc)/a]^2$ is replaced by $[(1s\ \text{radius})/a]^2$. It was shown that, with the GRW parameters, the rate of such radiation at the 1s electron ionization energy of 11.1 keV is about 5500 counts/keV kg d. (The predicted radiation rate falls off rapidly at higher energy.) This is 10^5 times larger than the observed upper bound R_{expt} quoted above. If we keep the GRW value of a, then λ_e (the collapse rate for an electron) has to be at least 10^5 times less than the GRW suggested value, making it less than approximately $10^{-21}\ \text{s}^{-1}$. Thus, at 11.1 keV, the discrepancy due to radiation by free electrons in Ge is considerably less severe than that due to excitation of the 1s electrons.

In Ref. [8], the authors argue that comparison of theory with experiment suggests that the fluctuating field is coupled to the particle mass density, so that if λ is the collapse rate for protons, then $\lambda_e=(m_e/m_p)^2\lambda$ (m_e and m_p are the electron and proton masses). In this case, it turns out that the bound state radiation rate vanishes identically to order a^{-2}, and the predicted rate is $\approx\lambda(m_e/m_p)^2[(1s\ \text{radius})/a]^4\approx10^{-10}$ counts/(keV kg d). However, the free-particle radiation rate at 11.1 keV is $(m_e/m_p)^2$ times smaller than Eq. (4.1), or 2.1×10^{-8} counts/(keV kg d). That is, the spontaneous radiation from free electrons is expected to be the dominant radiation. We may then express the experimental constraint at 11.1 keV as

$$2.3\times10^6>\frac{(\lambda/a^2)}{(\lambda/a^2)_{\text{GRW}}}. \quad (4.2)$$

We note that there is no *a priori* or experimental reason why, e.g., λ may not be much larger than λ_{GRW}, so Eq. (4.2) represents an interesting upper limit. Moreover, the experimental upper bound on the spontaneous radiation rate cited here is expected to be substantially improved in the future, resulting in a substantial improvement over (4.2)—or even, possibly, observation of spontaneous radiation by free electrons.

V. SUMMARY

In the preceding sections we have followed the CSL model of collapse, and investigated one of its significant consequences—a free charged particle radiates. We calculated the spectrum of this radiation and showed that the CSL model does not lead to an unreasonable level of ultraviolet radiation by free charged particles, unlike Karolyhazy's model. We then examined whether such radiation could be observed in the Ge experiment. The experiment provides some constraints on the CSL parameters.

Note added. Following the author's untimely death, this paper has been revised by P. Pearle in light of referee reports.

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