Semiclassical four-level single-atom laser

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The recent single-atom microlaser experiment [K. An *et al.*, Phys. Rev. Lett. **73**, 3375 (1994)], in which a stream of inverted two-level atoms is injected into an ultrahigh Q cavity, can be understood in the context of a semiclassical four-level laser model. Transit time broadening due to short atom-cavity interaction time effectively introduces nonradiative decay of the two levels in the model. The steady-state solution of semiclassical photon and atom rate equations for the intracavity mean photon number versus the intracavity atom number provides a good fit to the experimental data. [S1050-2947(97)03208-3]

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The single-atom microlaser (SAM) [1] is a laser device in which two-level atoms (^{138}Ba) in a beam, inverted outside a cavity, are injected into the cavity one by one at random intervals, induce laser oscillation, and then exit. For an ultrahigh Q cavity (2×10^9), one atom on average can sustain a buildup of a few laser photons inside the cavity.

Previous analyses of the SAM have been based on quantized field theory. The micromaser theory of Filipowicz *et al.* [2] is based on a master equation approach. Recently, the quantum trajectory simulation technique of Carmichael [3] has been used to analyze the thresholdlike transition in the SAM [4]. The present paper presents a semiclassical analysis; we show that the SAM, which uses *two*-level atoms, can be analyzed in the context of the standard semiclassical *four*-level laser model. Because it is semiclassical, the present work cannot provide an accurate picture of the enhanced Rabi interaction processes associated with the thresholdlike transition, nor can it address photon statistics. Nevertheless, as shown below, it provides an alternative description of the recent SAM experiments, which investigated the steady-state photon number.

The SAM can be treated as a four-level system for the following reasons. The inverted atoms injected into the cavity at random intervals can be viewed as being pumped from a reservoir level (level 3 in Fig. 1) to the upper laser level (level 2), via a metastable state (level 4), at the rate of injection R_p. Laser transition is from level 2 to level 1. When atoms exit the cavity, they are lost forever as far as the gain of the SAM is concerned. We can view this as atoms decaying back to the reservoir level. Since atoms in both levels 1 and 2 equally exit the cavity in the transit time, we can make the approximation that both levels decay nonradiatively with the same rate, γ.

In the experiment the cavity mode waist is 43 μm, and hence the atomic transit time through the cavity, $t_{\rm int}$ (also called the atom-cavity interaction time), is about 0.2 μs for the most probable velocity of atoms 360 m/s. This corresponds to a transit time broadening $\Delta\omega_t$ of 3.1 MHz in the fluorescence line shape (see below). Atoms in both levels 1 and 2 then decay nonradiatively to level 4 at $\gamma=\Delta\omega_t/2$. In Fig. 1 the radiative decay rate of level 2 to level 1, $\Gamma_{\rm rad}$, is 50 kHz. The upper level also decays to the $^3D_2$ state at a rate $\Gamma'_{\rm rad}$ ($\sim$ 50 kHz). However, this decay is not important in the present analysis since its only effect is in Γ_2, the total decay rate of level 2, with $\Gamma_2=\Gamma_{\rm rad}+\Gamma'_{\rm rad}+\gamma\approx\gamma$. The cavity decay rate Γ_c is 150 kHz. The atoms in level 1 decay to the reservoir level at a rate $\Gamma_1(=\Delta\omega_t/2)$.

In order to complete the four-level description of the SAM, we need to specify laser emission coefficient K, which appears in semiclassical photon and atom rate equations [5]. We derive the expression for K from the quantized field description of the SAM, in which atoms undergo vacuum Rabi oscillations in an empty cavity at an oscillation frequency, which is twice the atom-cavity coupling constant g [6] with

$$g=\frac{\mu}{\hbar}\sqrt{\frac{2\pi\hbar\omega_a}{V_c}}, \tag{1}$$

with μ the dipole moment, ω_a the transition frequency, and V_c the cavity mode volume. If the cavity contains n photons, the Rabi oscillation is enhanced by a factor $\sqrt{n+1}$. For the SAM, the product of g and $t_{\rm int}$ is much smaller than π. The probability that each atom traversing the empty cavity emits a photon is then $\sin^2 gt_{\rm int}\simeq g^2t_{\rm int}^2$ (assuming constant coupling throughout the cavity). The photon emission rate, i.e., the laser emission coefficient, is then $K=g^2t_{\rm int}$.

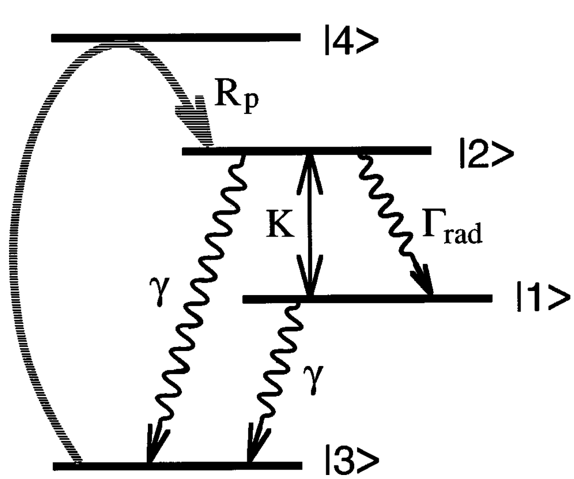

FIG. 1. A four-level laser model: atomic-level structure and pumping process.

We denote N_1 and N_2 as the number of atoms in levels 1 and 2, respectively, and n the mean number of photons stored in the resonator. Following the semiclassical treatment of the four-level laser [5], we set up coupled rate equations describing the SAM:

$$\frac{dn}{dt}=K(n+1)N_2-KnN_1-\Gamma_c n, \tag{2}$$

$$\frac{dN_2}{dt}=R_p-\Gamma_2 N_2-KnN_2+KnN_1, \tag{3}$$

$$\frac{dN_1}{dt}=-\Gamma_1 N_1+\Gamma_{rad}N_2+KnN_2-KnN_1, \tag{4}$$

We are interested in the steady-state solution, for which $dn/dt=0=dN_1/dt=dN_2/dt$. Eliminating N_1 and N_2 in Eqs. (2)–(4), we obtain a quadratic equation for the steady-state photon number $n_{\rm ss}$. The solution is

$$n_{\rm ss}=\frac{p'}{2}\left[(r-1)+\sqrt{(r-1)^2+\frac{4rn_0}{p'}}\right], \tag{5}$$

where

$$p'\equiv\frac{p}{\epsilon_1(1-\epsilon_2)+\epsilon_2}, \tag{6}$$

with $p\equiv\Gamma_{\rm rad}/K$ and $\epsilon_{1,2}\equiv\Gamma_{\rm rad}/\Gamma_{1,2}$, respectively. The pumping parameter r and the characteristic photon number n_0 are defined as

$$n_0\equiv\frac{1}{1-\epsilon_1(1-p^{-1})}, \tag{7}$$

$$r\equiv\frac{\epsilon_2}{pn_0}\left(\frac{R_p}{\Gamma_c}\right). \tag{8}$$

Before applying the result to the SAM of Ref. [1], we examine the physical meaning of the parameter p. For the present model, with $\Delta\omega_a$ the fluorescence linewidth of the transition, $\Delta\omega_a=\Gamma_1+\Gamma_2=\Delta\omega_t+\Gamma_{\rm rad}+\Gamma'_{\rm rad}$. In the SAM, the transit time broadening due to the short atom-cavity interaction time is much larger than the radiative decay rates of level 2, $\Gamma_{\rm rad}$ and $\Gamma'_{\rm rad}$, and hence $\Delta\omega_a\simeq\Delta\omega_t$. Note that the cavity has a Gaussian transverse mode profile with a mode waist w_0. The transit time associated with this mode is

$$t_{\rm int}=\int_{-\infty}^{\infty}\exp[-(vt/w_0)^2]dt=\sqrt{\pi}w_0/v. \tag{9}$$

From the Fourier transform of the exponential function in the above integral we obtain a transit time broadening (full width at half maximum):

$$\Delta\omega_t=2\sqrt{2\ln 2}v/w_0=2\sqrt{2\pi\ln 2}/t_{\rm int}\approx 4/t_{\rm int} \tag{10}$$

Hence, the laser emission coefficient becomes

$$K=g^2t_{\rm int}\simeq\frac{(2g)^2}{\Delta\omega_t}\simeq\frac{(2g)^2}{\Delta\omega_a}, \tag{11}$$

and therefore

$$p=\frac{\Gamma_{\rm rad}\Delta\omega_a}{(2g)^2}. \tag{12}$$

Using the expression for g [Eq. (1)] and $\Gamma_{\rm rad}$ [7]

$$\Gamma_{\rm rad}=\frac{4\mu^2}{3\hbar}\left(\frac{\omega_a}{c}\right)^3, \tag{13}$$

we obtain

$$p=\frac{V_c\omega_a^2\Delta\omega_a}{6\pi c^3}=\frac{4\pi^2}{3}\frac{\Delta\omega_a V_c}{\omega_a\lambda^3}, \tag{14}$$

which can be rewritten as

$$p=\frac{\pi}{6}\rho_c(\omega_a)\Delta\omega_a, \tag{15}$$

where $\rho_c(\omega)$ is the usual density of modes of a cavity with a volume V_c [8]:

$$\rho_c(\omega)=\frac{V_c\omega^2}{\pi^2c^3}. \tag{16}$$

Hence, p can be thought of as the number of cavity modes, including not only longitudinal but also any transverse modes, within the atomic fluorescence linewidth $\Delta\omega_a$ [5]. Such interpretation is meaningful for conventional lasers, for which there are so many cavity modes within the atomic line shape that the mode density is a smooth continuous function across the line shape. This is not the case for the SAM, in which the atomic transition couples to at most only one longitudinal cavity mode. In this case Eq. (12) provides an alternative interpretation: $1/p$ is a measure of how strong the coupling constant g is, relative to $\Gamma_{\rm rad}$ and the fluorescence linewidth $\Delta\omega_a$.

Our model can also be applied to conventional lasers without modification. For these lasers, $\Gamma_2\sim\Gamma_{\rm rad}\ll\Gamma_1$ (and hence $p'\sim p$) and $p\gg 1$ (typically 10^5-10^{12}). In this case, depending on the magnitude of the pumping parameter r, the solution can be further simplified by expanding the square root part of the solution in a power series:

$$\begin{aligned} n_{\rm ss}&=\sqrt{pn_0} \quad \text{for } r=1 \text{ (at threshold)}\\ &\simeq n_0\left(\frac{r}{1-r}\right) \quad \text{for } r<1 \text{ (below threshold)}\\ &\simeq p(r-1) \quad \text{for } r>1 \text{ (above threshold)}. \end{aligned} \tag{17}$$

The laser oscillation condition is then simply $r\geqslant 1$.

In the SAM, on the other hand, $p\sim 1$. This strikingly contrasts with its value in a conventional laser. Also note that with a small value of the p the sudden threshold behavior,

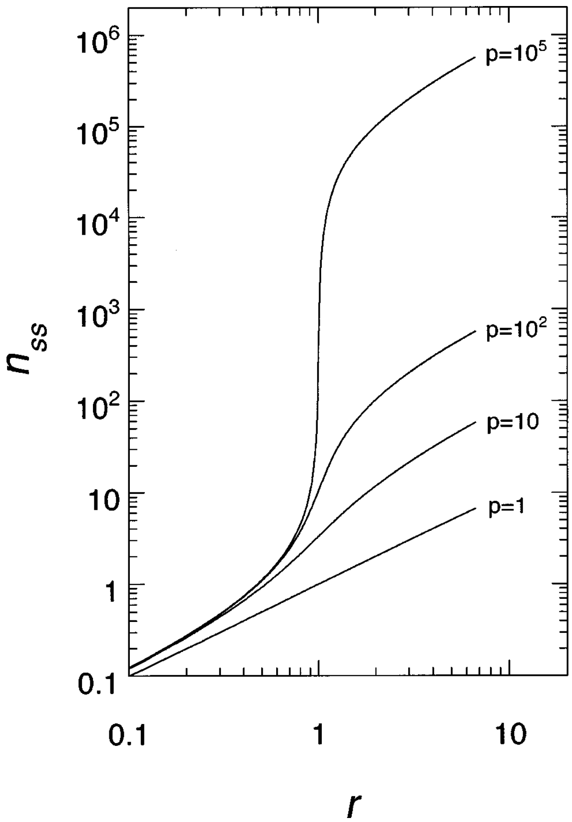

FIG. 2. n_{ss} as a function of a pumping parameter r as defined in the text.

which results from the large size of p parameter, would be absent (see Fig. 2). A similar trend has been observed in the SAM [4] (Fig. 3).

The laser threshold behavior has been studied in the literature in terms of the so-called ''β'' parameter. For a laser with a very small solid angle associated with the cavity mode, this parameter is related to p parameter as follows:

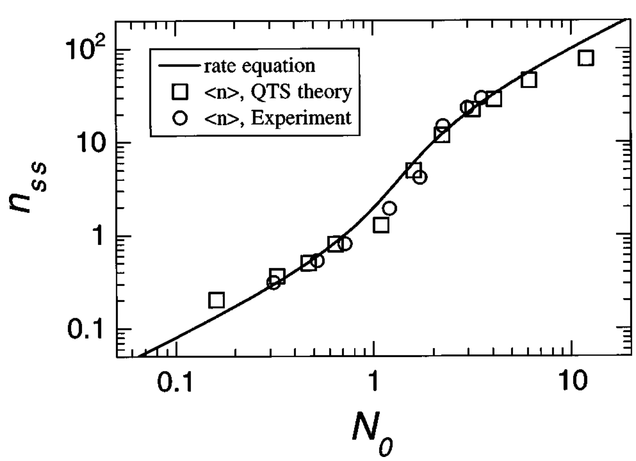

FIG. 3. Mean intracavity photon number, measured as a function of mean intracavity atom number in the SAM experiment of Ref. [1], is compared with the present semiclassical analysis. Also shown is the quantum trajectory simulation results reported in Ref. [4], in which systematic errors in detector calibration were corrected by fitting the data to the quantum trajectory simulation results. After this correction experimental error bars are smaller than the point size.

$$\beta=\frac{\text{(emission rate into the cavity mode)}}{\text{(total emission rate)}}$$

$$=\frac{4g^2/\Gamma_c}{\Gamma_{\rm rad}+4g^2/\Gamma_c}=\frac{\Delta\omega_a/\Gamma_c}{p+\Delta\omega_a/\Gamma_c}, \qquad (18)$$

from which we can see that $\beta\approx\Delta\omega_a/(p\Gamma_c)$ for $p\gg\Delta\omega_a/\Gamma_c$, and $\beta\approx 1$ for $p\ll\Delta\omega_a/\Gamma_c$. For example, in a typical He-Ne laser $\beta\sim 10^{-6}$, whereas for the SAM $\beta=0.96$.

We now apply our model to the SAM. For this, we need to introduce the mean intracavity atom number $N_0=\rho_0 V_c$, with ρ_0 the density of atoms in the cavity. We can express the density in terms of the pumping rate R_p in the following way. As mentioned before, the cavity has a Gaussian mode profile in the transverse directions. It also has a standing wave mode profile along the cavity axis. In the experiment an atomic beam with a beam diameter l traverses the cavity mode. Because of the Gaussian transverse profile, only atoms confined in a cross-sectional area of $2w_0 l$ will significantly interact with the cavity. The total number of atoms injected into the cavity across the above area during $t_{\rm int}$ divided by a volume $2w_0 l v t_{\rm int}$ is the required density:

$$\rho_0=\frac{R_p}{2w_0 l v}. \qquad (19)$$

Hence, the mean intracavity atom number reduces to

$$N_0=\frac{R_p}{2w_0 l v}\,\frac{1}{4}\pi w_0^2 l=\frac{\sqrt{\pi}}{8}R_p t_{\rm int}. \qquad (20)$$

The K value introduced above must now be averaged over the standing wave and transverse Gaussian profiles. We define an averaged $\overline{K}$:

$$\overline{K}=\frac{g^2 t_{\rm int}}{2w_0 l}\int_{-w_0}^{w_0}dy\int_{-l/2}^{l/2}dz\,\exp[-2(y/w_0)^2]\sin^2[2\pi z/\lambda]$$

$$\simeq\frac{\sqrt{\pi/2}}{4}g^2 t_{\rm int}, \qquad (21)$$

and an averaged $\overline{p}$:

$$\overline{p}=\frac{\Gamma_{\rm rad}}{\overline{K}}=4\sqrt{2/\pi}\frac{\Gamma_{\rm rad}\Delta\omega_a}{(2g)^2}. \qquad (22)$$

The quantities p and K in Eqs. (5)–(8) are then replaced with $\overline{p}$ and $\overline{K}$. The resulting parameter values are $\overline{p}=0.95$, $p'=15$, $\overline{K}=3.2\times10^5\ \mathrm{s}^{-1}$, $R_p=2.3\times10^7\ \mathrm{s}^{-1}$, $n_0=1.0$, and $N_0=1.4r$.

We plot n_{ss} as a function of N_0 (not r) in Fig. 3. Also plotted is the corresponding curve of the SAM experiment and the results of the analysis based on the quantum trajectory simulation [4]. In that work the atoms were treated strictly as a two-level system, quantum mechanically interacting with the cavity mode with coupling constant g. The Schrödinger equation was numerically integrated. Atomic and cavity damping processes were handled by means of a stochastic wave function. Note that the present semiclassical analysis is in excellent agreement with the experiment (and with the QTS results). This is because the SAM operates in a *semiclassical* regime in which atoms are injected into the cavity with random arrival times, and gt_{int} is much smaller than π and averaged over the cavity mode profile, so that the atom-cavity interaction can be adequately characterized by a constant rate K. In this regime, nonclassical photon statistics are not expected. In order to see nonclassical photon statistics, either atoms need to be injected regularly [9], or the product gt_{int} has to be much larger than π [10] and/or fairly constant for all of the atoms. The later situation is being pursued experimentally in our group.

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