Self-induced transparency soliton laser via coherent mode locking

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The properties of coherent nonlinear gain are utilized for generating stable pulses with durations shorter than the inverse spectral width of the gain profile. A passive cw mode-locking technique making use of the coherent gain and the coherent absorber is proposed as the basis for a self-induced transparency soliton laser. The basic features of pulse evolution including the pulse shortening effect are described. [S1050-2947(97)06408-1]

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I. INTRODUCTION

The generation of ultrashort pulses is based on the confinement of the energy in the cavity in a small spatial region. A laser operation in this fashion is said to be mode locked (see, e.g., Ref. [1]). Essentially the techniques for mode-locking lasers require some form of amplitude modulation applied to the laser radiation, which has a period equal to the cavity round-trip time. This modulation may be derived externally as in the case of active mode-locking, or it may be derived passively from the radiation itself via an intensity-dependent loss mechanism.

The traditional active-locking technique involves the use of an intracavity loss or phase modulator driven by a sinusoidal RF signal whose period is closely matched to the round-trip transit time of the system. In the case of passive mode locking, mode-locked operation is achieved by inserting in the cavity an absorbing element that exhibits significant saturation at the power levels reached during the laser action. New generation of mode-locked lasers was ushered in by the demonstration of the soliton laser by Mollenauer and Stolen [2], which showed that nonresonant mode-locking techniques based on the optical Kerr effect could be used to produce femtosecond pulses.

In general, a steady state is reached in a mode-locked laser for which the pulse compression mechanisms are balanced by pulse-broadening mechanisms. In any case it is believed that a pulse will have a finite duration that is ultimately limited by the gain bandwidth of the laser gain. This may correspond to the spectral width of the gain profile of the laser or to the spectral width of any intracavity frequency-selective elements. However, unlike passive frequency-selective elements the dispersion of the amplifying medium presents a nonlinear function of intense field that allows it to support pulses with durations beyond the ordinary limit. As a pulse becomes shorter than the phase memory time of a medium, the field-matter interaction is exhibiting coherent features. In this paper the properties of coherent nonlinear gain are utilized as a basis of a new mode-locking technique for generating pulses with durations shorter than the inverse spectral width of the gain profile.

The benefits of coherent medium-field interaction are twofold: the gain serves as an active medium and as a modulator for pulse formation. While propagating through the amplifier the leading edge of the pulse extracts the energy stored in it while the trailing edge vanishes, experiencing smaller gain. Throughout the formation process a pulse in an amplifying two-level system evolves toward a π pulse (see Ref. [3]) allowing the full release of the stored energy and moving with the velocity of light.

Fox and Smith [4] were the first to suggest that the pulses traveling back and forth in a self-mode-locked laser oscillator were π pulses. However, the amplifier placed into the cavity does not sustain π-pulse shaping. It is well known that for certain choices of laser parameters there exists an intrinsic instability in the laser equations (see Ref. [5]). This leads to a pulse buildup from the unstable cw (continuous wave) solution under the influence of a small disturbance as shown by Risken and Nummedal [6]. Hence the final pulse parameters essentially depend on initial noise characteristics, and the pulses are not simply related to π pulses [6]. In contrast to the fluctuation nature of the self-pulsing regime (see also Ref. [7]), in this paper we are dealing with *deterministic* pulses.

In order to achieve a stable mode locking the net gain for low-intensity cw radiation should be negative. The stability condition may be fulfilled by placing a cell with a passive medium into a cavity. The following requirements are made on the absorber: its optical transition frequency should be coincident with resonant frequency of the amplifier, its concentration should be enough to provide a high damping rate for low-intensity radiation and at the same time provide as little as possible attenuation for pulse regime. A more plausible choice may be realized by using the *same* medium for the absorber as for the amplifier, but without external pumping. The coherent character of interaction between the pulse and the absorber may lead to a severe decrease in energy damping rate compared to that for cw radiation.

This model is similar to that proposed by Fox, Schwarz, and Smith [8] and Frova *et al.* [9]. They reported that a neon-discharge absorption cell can very effectively lock the modes of a He-Ne laser. But as pointed out in [9] the experimental data appeared to be only a *continuation* into the high-power region of the results for the self-locked laser (without absorbing cell) obtained by Uchida and Ueki [10] and Smith [11]. Moreover, it is important to note that the pulse spectral width achieved in these experiments did not exceed the spectral

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width of the amplifying medium. Pulses that are substantially wider than the gain bandwidth have been reported in [12] from an additive-pulse mode-locked erbium fiber laser. Possibly extreme pulse shortening can be the result of the change of the pulse width by an order of magnitude within the cavity.

A special feature of passive mode locking with the coherent absorber is that the self-induced transparency effect in the absorber discovered by McCall and Hahn [13] can serve as an additional pulse shaping mechanism. The special design of the cavity configuration, namely, fitting of the beam radius in the gain to twice that as in the absorber, gives rise to 2π-pulse soliton formation in the absorbing medium. So, the field in the cavity constitutes a π pulse for the amplifier and a 2π pulse for the absorber with the same pulse shape of sech-hyperbolic form. Generality of the model is sustained by the *independence* of pulse shape and pulse area on any material parameters except dipole moments. The main objective of this paper is to present the basis for a mechanism of passive cw mode locking utilizing the coherent properties of both amplifying and absorbing media—''coherent mode locking.''

II. BASIC EQUATIONS AND SOLITARY WAVE SOLUTION

For the purpose of simplifying the laser equations, a two-level ring laser system with homogeneously broadened lines for both active and passive media is treated. Also, let us assume that the pulse suffers only a slight change during a single transit through any of the elements of the cavity. In the ring laser under consideration, we assume one allowed direction of propagation; hence,

$$\mathcal{E}(t,z)=(\hbar/d_g)E(t,z)\exp(i\phi)+\text{c.c.},$$

$$\mathcal{P}_i(t,z)=P_i(t,z)\exp(i\phi)+\text{c.c.},$$

where $\phi=-(\omega t-kz)$; $i=a,g$; c.c. means the complex conjugate; the indices a and g are associated with the absorber and the gain, respectively. $E(t,z)$ and $P_i(t,z)$ are slowly varying functions in z and in t. The boundary conditions of a ring laser require that the variables $E(t,z)$ and $P_i(t,z)$ are periodic in z with period L: $E(t,z+L)=E(t,z)$, $P_i(t,z+L)=P_i(t,z)$, and $N_i(t,z+L)=N_i(t,z)$, where L is the laser cavity length.

So, an analysis in this paper starts from the laser matter equations and wave equation in the form

$$c\,\partial E/\partial z=-\partial E/\partial t-lE+(a/T_a)P_a+(g/T_g)P_g, \tag{1}$$

$$\partial P_i/\partial t=-P_i/T_i+\sqrt{\mu_i}EN_i, \tag{2}$$

$$\partial N_i/\partial t=-(\sqrt{\mu_i}/2)(EP_i^*+E^*P_i), \tag{3}$$

where $i=a,g$. Following the assumption of identical material for the laser and absorbing media we put equal values for the relaxation times of the polarizations and equal values for the dipole moments: $T_a=T_g$, $d_a=d_g$. The parameters exhibit a distinction in the ratio of the beam area in the gain to that in the absorber $\mu_a=S_g/S_a$ (while $\mu_g=1$). In Eqs. (1)–(3), $a=2\pi\omega d_a^2 n_a T_a/\hbar$ is the linear optical-absorption coefficient, $g=2\pi\omega d_g^2 n_g T_g/\hbar$ is the linear gain, and l describes the losses of the cavity and of the host material. The loss terms are omitted in Eq. (3) considering the pulse duration shorter than the relaxation time of population difference.

The steady-state solution is obtained by requiring that the pulse repeat itself after one transit, therefore the final pulse variables have a space and time dependence of the form $f(t-z/v)$. A sufficiently great number of laser modes is considered to be involved in laser operation and thus the periodic boundary conditions can be changed for conditions in infinity: $E(t,z\to\infty)\to 0$, $P_i(t,z\to\infty)\to 0$, $N_a(t,z\to\infty)\to -1$, and $N_g(t,z\to\infty)\to +1$. This simplification is equal to the requirement that the length of the pulse $v\tau$ is small comparable to the cavity length L. The steady-state pulse generation becomes possible only for a laser configuration that can maintain the same value of gain and the same value of absorption at each point of the cavity after every transit. Thus, a gain is to be restored in a cavity transit time. Keeping in mind the above arguments a particular solution of Eqs. (1)–(3) is given by

$$E=A\,\text{sech}[(t-z/v)/\tau], \tag{4}$$

which requires a special configuration of the laser cavity so that the μ_a parameter is equal to 4. The field is taken in the form (4) as observed from the active medium location. Substituting Eq. (4) into (1)–(3) yields relationships for the pulse square $\Theta=\int E\,dt$ and for the pulse velocity v,

$$\Theta=A\tau\pi=\pi, \tag{5}$$

$$\frac{c}{v}=1+\frac{6(\tau/T_a)}{3+4(\tau/T_a)+(\tau/T_a)^2}a\tau. \tag{6}$$

Also the energy-balance equation takes place,

$$3(\eta-1)(JT_g)^2+2(4\eta-1)(JT_g)+4(\eta-2\rho)=0, \tag{7}$$

where $J=\int E^2dt=2A^2\tau=2A$ describes the pulse energy. $\eta=(g-l)/g$ (pump parameter) and $\rho=a/g$ are the normalized gain minus cavity losses and normalized absorption.

Stability of the mode-locking operation requires negative net linear gain in order to prevent the initiation of the cw regime: $\eta-\rho<0$. This condition together with Eq. (7) specify the stability domain on the (ρ,η) plane for solitary pulse generation; see Fig. 1. Figure 2 illustrates the dependence of pulse duration on absorber density. Minimum values of pulse durations correspond to the bottom boundary of the stability domain. While the sharp termination of the maximum values is traced to the property of constant area under pulse envelope, enhancement of absorber concentration leads to amplitude reduction with concurrent increasing in duration. Thus, the energy-loss coefficient incorporates not only the linear dependence on absorber concentration, but also the additional contribution from increasing a fraction of pulse energy left in the medium. Besides, the pulses with greater durations extract the energy stored in a gain with less efficiency. Plots of unstable solutions that have energy increasing with the rise in the absorber concentration are shown by dashed curves on Fig. 2.

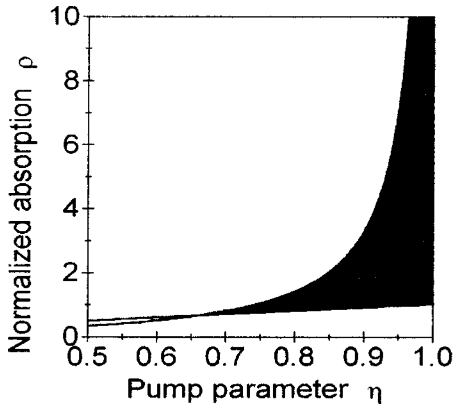

FIG. 1. Stability domain (the shaded region) for the solitary pulse solution (4) on (ρ,η) plane.

The solution (4) has the form of a π pulse [see Eq. (5)] in the amplifier, which proves its coherent nature. During a cycle of interaction the energy is pumped into the electric field, and after the passage of the pulse the inversion *always* became negative; see Fig. 3. The repopulation of inversion is full in the limit of $(\tau/T_g)\rightarrow 0$. At the same time the absorber sees the field in the form of a 2π pulse. Thus the field dynamics comprises the stage of exciting of resonant dipoles into the inverted state by the leading edge of the pulse, and the stage of the coherent recirculation of the stored energy into the remaining portion of the pulse. The lossless propagation is attained only for $\tau/T_a\rightarrow 0$, otherwise the pulse leaves a small fraction of energy in the medium; see Fig. 3.

Equation (6) shows no contribution of a gain to the pulse velocity. This fact correlates well with the results of the previous studies on π-pulse propagation (see, e.g., [3,14]). On the other hand, pulse delay in an absorber comprises an essential property of the self-induced transparency effect. Equation (6) extends this feature to the case of coherent pulse propagation with losses. As directly follows from the latter considerations the coherent effects in both media play a crucial role in the process of pulse formation. In this sense, the laser based on this mode-locking technique might be recognized as a *self-induced transparency soliton laser*.

It is believed that for *all* ultrashort pulse lasers, there will be a balance between pulse compression, achieved through mode-locking, and spectral compression, which is a consequence of the finite gain bandwidth. As the pulse spectral width approaches the bandwidth of the gain, the pulse starts to lose energy to this spectral filter and so a steady state is reached for which the pulse energy and its spectral and temporal widths stay constant. Undoubtedly this picture of pulse evolution is correct and at the same time there is no discrepancy with coherent mode-locking picture proposed above. A special feature of the solution (4) for the pulse envelope lies in its existence *only* for a range of durations limited by the inequality $\tau<T_a,T_g$. Hence the stability domain of coherent pulses (4) never overlaps with that of pulses produced by traditional mode-locking techniques. This ''pure'' coherent character of the solution (4) proves that such pulses could be observed only in the experiments where the pulse spectra broader than gain spectral width were detected at the laser output. On the other hand, the same peculiarity has given rise to some problems of self-starting of the coherent mode-locking operation because of difficulties for the pulse to overcome the ''energy barrier'' between two distinct types of optical nonlinearities—coherent and incoherent.

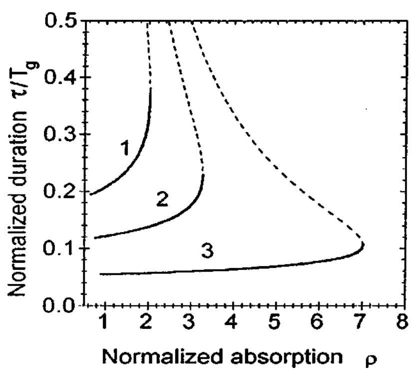

FIG. 2. Typical curves for dependence of pulse duration on absorber density for three values of pump parameter η: (1) 0.85; (2) 0.90; (3) 0.95. The dashed curves correspond to unstable solutions.

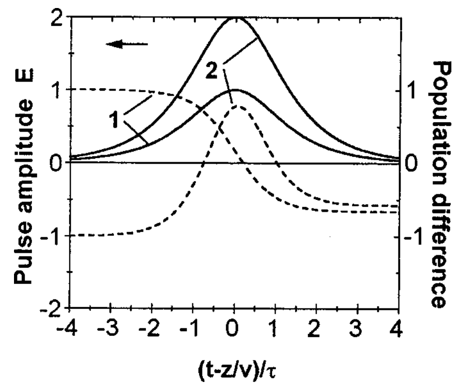

FIG. 3. Pulse shapes (solid curves) and population difference dynamics (dashed curves): (1) for the gain; (2) for the absorber; $\tau=0.2T_g$ is taken. Direction of pulse propagation is indicated by an arrow.

III. MODE-LOCKING DYNAMICS AND PROPERTIES OF SOLITARY-WAVE SOLUTION

The solution of the form (4) refers to the final stage of the mode-locking process when the pulse parameters have reached their steady-state values. In order to achieve a profound understanding of coherent mode-locking phenomenon there is a need to analyze how a seeding pulse being initially broad experiences shortening.

For the sake of simplicity the incoherent losses in both media are suggested negligibly small $(T_a,T_g\rightarrow\infty)$. Then Eqs. (1)–(3) can be reduced to a single equation for the Bloch angle $\theta=\int_{-\infty}^{t}E\,dt$:

$$\frac{\partial^2\theta}{\partial t^2}+c\,\frac{\partial^2\theta}{\partial t\,\partial z}=\left(\frac{g}{T_g}\right)\sin\theta-\left(\frac{a}{T_a}\right)\sin 2\theta-l\,\frac{\partial\theta}{\partial t}. \qquad (8)$$

A pulse evolution in a lossless two-component media $(l\rightarrow 0)$ was described in Ref. [15]. A threshold value of

$$\Theta_{\rm th}=\arccos(g/2a),$$

where

$$\Theta=\theta(z,t=+\infty)$$

differentiates between two distinct ways of pulse evolution: a seeding pulse with $\Theta < \Theta_{\text{th}}$ is absorbed, and a pulse with $\Theta_{\text{th}} < \Theta < 2\pi - \Theta_{\text{th}}$ evolves to a π pulse. For the π-pulse shaping scenario the propagation of the pulse is accompanied by its *infinite* amplification with corresponding shortening in duration keeping the square under envelope equal to π.

The loss term ($l > 0$) does not change the value of Θ_{th} and dynamics of the Bloch angle at the first stage of evolution, and also has no effect on the final pulse square. It can be found from Eq. (8) that if there is a perturbation in E such that the total area Θ is greater than π, then

$$\frac{\partial E}{\partial z} \approx \left\{ \left(\frac{g}{T_g} \right) \sin(\Theta + \varepsilon) - \left(\frac{a}{T_a} \right) \sin[2(\Theta + \varepsilon)] \right\} < 0.$$

The field at the trailing edge therefore tends to decrease to recover a total area of π. On the other hand, if the perturbation is such that Θ is less than π, then $\partial E / \partial z > 0$. Therefore, small deviations from the stationary value $\Theta = \pi$ are damped out, which proves the stability of pulses with square equal to π.

The cavity losses dramatically change the evolution of pulse energy:

$$\frac{\partial J}{\partial z} = 2 \frac{g}{T_g}(1 - \cos\Theta) - \frac{a}{T_a}(1 - \cos 2\Theta) - 2lJ, \tag{9}$$

resulting in a stationary value for J:$J_{\text{st}} = 2g/(T_g l)$, which coincides with that found from Eq. (7).

The above considerations refer only to area and energy stability and leave open the possibility of perturbations in which the total area and energy remain unchanged. For the stability consideration of the pulse shape given in Eq. (4) one may linearize Eq. (8) near the steady-state solution. The first-order perturbation analysis of Eq. (8) can be performed analytically for the following range of parameters: $l > 0$ and $\rho > 1$, and leads to the ''Schrödinger'' equation for the particle motion in the potential well. The zero eigenvalue is a single one of a discrete spectrum for the problem. It corresponds to the eigenfunction in the form of localized perturbation. Other eigenvalues are all negative [16]. This result is similar to the case of a pure absorber; see Ref. [14]. So the pulse shape is stable but not asymptotically stable, i.e., perturbations remain finite.

For completeness of the analysis it needs to be ascertained if the pulse solution (4) is actually unique in that it is the only solitary wave solution. As previously derived a pulse launched into a cavity evolves to a solitary wave with a stationary square value and with energy equal to J_{st}. Apart from this transient process and keeping in mind only the final stage of evolution rewrite Eq. (8) in terms of a single variable $t - z/v$:

$$\left(\frac{c}{v} - 1 \right) \frac{dE}{d(t - z/v)} = lE - \left(\frac{g}{T_g} \right) \sin\theta + \left(\frac{a}{T_a} \right) \sin 2\theta. \tag{10}$$

From the right-hand side of Eq. (10) one sees that a pulse solution should have extremum points at $\theta_m = \pi/2 + k\pi$, $k = 0,1,2,\ldots$. Consequently, one obtains for the pulse amplitude $A = (g/T_g)/l$. In terms of the new independent variable θ, Eq. (10) becomes

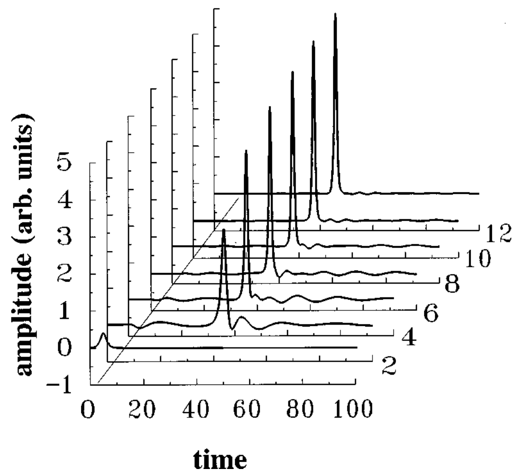

FIG. 4. Computer plots of evolution of input $\Theta = 0.4$ hyperbolic-secant pulse with distance and time for $\eta = 0.8$, $\rho = 2$, and $\tau \ll T_a, T_g$. The distance z is marked off in units of $(g/cT_g)^{-1}$.

$$\rho \frac{d}{d\theta} \left[\left(\frac{E}{A} \right)^2 - \sin^2\theta \right] = \frac{E}{A} - \sin\theta, \tag{11}$$

where we use the relationship

$$\left(\frac{c}{v} - 1 \right) = 2 \frac{a}{T_a} \frac{1}{A^2},$$

which directly follows from the condition of max$|E|$ at $\theta = \theta_m$.

Equation (11) is invariant under $\theta \to -\theta$ and $E \to -E$. Also one can see that $E(\theta)$ is periodic with a period of 2π. For one period of θ: $0-2\pi$, E is antisymmetric about the π point. Consequently, we have $E(\pi) = 0$, which after referring to Eq. (10) denotes simultaneous vanishing of the field and its first derivative at $\theta = \pi$. Thus $E(\theta)$ is localized between 0 and π, that in its turn is proof that the class of π pulses is the only class of solitary wave solutions of Eq. (8).

The discussion leaves open the question about the variety of π solutions. In order to solve the problem one looks more closely at the symmetrical properties of Eq. (11). $E(\theta)$ should be symmetric about the $\pi/2$ point. Hence for arbitrary pair $(\theta, \pi - \theta)$ the field derivative must be of the same magnitude but with the opposite sign. There is no way to satisfy this requirement except by the choice of the solution of the specific form:

$$E = A \sin\theta \quad \text{or} \quad \frac{dE}{du} = E\sqrt{A^2 - E^2}. \tag{12}$$

Equation (12) has a unique solitary wave solution, namely (4).

Numerical simulations were performed to prove and to supplement the above analytical speculations. For the initial sech-shape profile pulse with square equal to 0.4π one can see in Fig. 4 the dramatic increase in the pulse energy, by a factor of 12, with the corresponding shortening in duration by a factor of 5. After a short transient, for $z > 15(g/cT_g)^{-1}$, the pulse takes its steady-state form. When

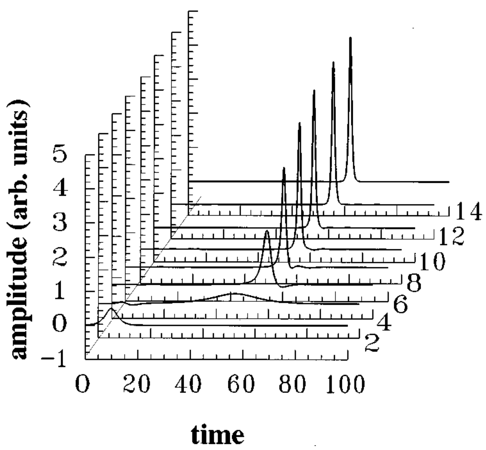

FIG. 5. Computer plots of evolution of input $\Theta = 0.5$ hyperbolic-secant pulse with distance and time for $\eta = 0.8$, $\rho = 2$, and $\tau = T_a/3 = T_g/3$. The distance z is marked off in units of $(g/cT_g)^{-1}$.

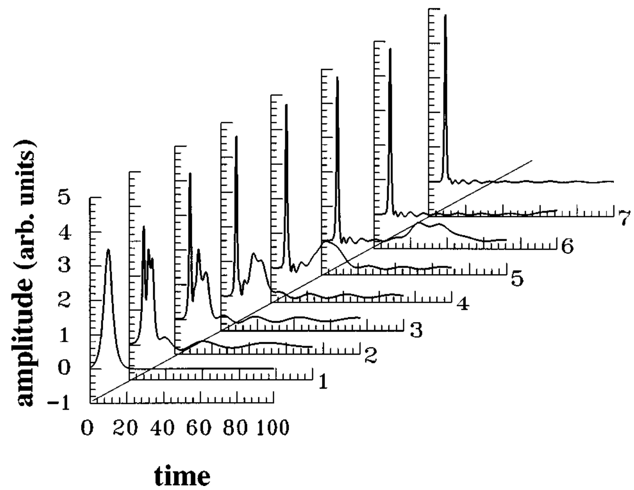

FIG. 6. Computer plots of evolution of input $\Theta = 3.5$ hyperbolic-secant pulse with distance and time for $\eta = 0.8$, $\rho = 2$, and $\tau \ll T_a, T_g$. The distance z is marked off in units of $(g/cT_g)^{-1}$.

this pulse shape is compared in detail with that of Eq. (4), it is apparent that they are equivalent. In the transition region, the self-consistent interaction of the field and the amplifier component of laser medium gives rise to ringing that is less pronounced as compared to the case of a pure amplifier. The effect of ringing suppression arises from nonvanishing intracavity losses that have been demonstrated with our computer simulations with different values of the η parameter (not presented here).

Up to this point the assumption has been made that the relaxation processes are negligible for the pulse dynamics ($\tau \ll T_a, T_g$). One can expect that the inclusion of them leads to the enhancement of the threshold value for the pulse square, Θ_{th}, and changes only some details in mode-locking dynamics; see Fig. 5. Thus, the effect of ringing practically dies out.

In addition to the π point Eq. (8) has an infinite number of stationary points. It is reasonable to assume that they are related to $(2k+1)\pi$ pulses. For the classical self-induced transparency effect it has been observed, both experimentally and from machine computations, that the combination of field strength and magnitude of dipole moment sufficient to induce two and more inversions in the population of the two-level system does not propagate as a single pulse but rather separates into a sequence of fundamental pulses. Pulse decomposition is a natural by-product of the alternate amplification and attenuation of a pulse that accompanies the coherent oscillations in population and induced polarization of the two-level systems. Figure 6 shows the process of 3.5π pulse decomposition into two pulses with subsequent survival of a single one only. For the stage of practically full separation Fig. 7 illustrates typical field dynamics together with behavior of population differences of absorber and amplifier. Thus one may conclude that the formation of a 3π pulse involving the π pulse and the 2π pulse takes place. The generated 2π pulse outdistances the π pulse and then exhibits spreading and quick scattering. Later on only the π pulse propagates back and forth inside laser cavity. The distortion of the 2π pulse happens because it does not extract energy from amplifier (see Fig. 7) but at the same time experiences linear losses inside the cavity.

The above consideration shows that an arbitrary pulse with square greater than a threshold value being seeded into a laser evolves towards a solitary wave solution of the form (4).

IV. DISCUSSION

A type of mode locking is demonstrated in a laser with a coherent gain and a coherent absorber. The pulse shortening effect occurs as a result of a self-induced transparency effect in both (gain and absorbing) media. An initially weak pulse ($\Theta_{\mathrm{th}} < \Theta_{\mathrm{in}} < 2\pi - \Theta_{\mathrm{th}}, \tau_{\mathrm{in}} < T_a, T_g, J_{\mathrm{in}} < J_{\mathrm{st}}$) launched into a cavity transforms into a π pulse for a gain and a 2π pulse for an absorber at the first stage of evolution. At the second stage the pulse exhibits amplification as long as its energy

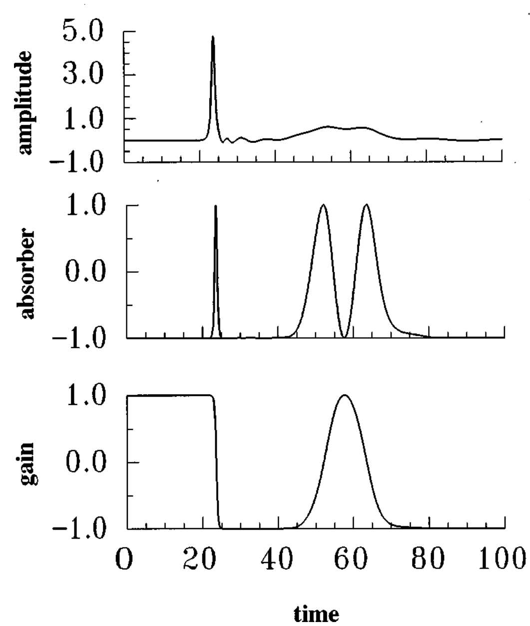

FIG. 7. Plots of pulse envelope and population differences of absorber and amplifier for $z = 4.5(g/cT_g)^{-1}$. The input pulse and laser parameters are the same as in Fig. 6.

reaches the stationary value J_{st}. This process is accompanied by the corresponding decreasing in duration in order to keep the constant value of the pulse square. The extent of pulse compression can be estimated by a ratio $(J_{st}/J_{in})(\Theta_{in}/\pi)$. After the transient process the pulse takes the stable soliton-like form; see Eq. (4). Moreover, the two-component coherent laser medium does not sustain multisoliton operation, and an initial pulse with $\Theta_{in} > 2\pi - \Theta_{th}$ also tends to take the unique π-pulse form, see Eq. (4).

It may appear that the conception of coherent mode-locking may be applied only to the case when the beam radius in the gain is twice as that in the absorber. What actually happens is that *mode-locking effect manifests itself for a large range of variations of* μ *parameter* [16].

In conclusion, the technique of coherent passive mode locking may be successfully applied to the whole class of media with narrow gain linewidth. In general, this mode-locking technique imposes a single vital demand on laser configuration. It refers to the high-Q cavity in order to provide a considerable gain excess over linear intracavity loss ($g > 2.28l$, following the notations of this paper). A significant advantage of the mode-locking technique lies in its simplicity: it can be realized without precise cavity length tuning or special matching of media.

ACKNOWLEDGMENTS

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[1] For the last review, see P. M. W. French, Rep. Prog. Phys. **58**, 169 (1995).
[2] L. F. Mollenauer and R. H. Stolen, Opt. Lett. **9**, 13 (1984).
[3] J. P. Wittke and P. J. Warter, J. Appl. Phys. **35**, 1668 (1964); F. T. Arecchi and R. Bonifacio, IEEE J. Quantum Electron. **1**, 169 (1965).
[4] A. G. Fox and P. W. Smith, Phys. Rev. Lett. **18**, 826 (1967).
[5] H. Statz and C. L. Tang, J. Appl. Phys. **36**, 3923 (1965).
[6] H. Risken and K. Nummedal, J. Appl. Phys. **39**, 4662 (1968).
[7] J. M. Dudley, J. D. Harvey, and K. Leonhardt, J. Opt. Soc. Am. B **10**, 840 (1993).
[8] A. G. Fox, S. E. Schwarz, and P. W. Smith, Appl. Phys. Lett. **12**, 371 (1968).
[9] A. Frova, M. A. Duguay, C. G. B. Garret, and S. L. McCall, J. Appl. Phys. **40**, 3969 (1969).
[10] T. Uchida and Q. Ueki, IEEE J. Quantum Electron. **3**, 17 (1967).
[11] P. W. Smith, IEEE J. Quantum Electron. **3**, 627 (1967).
[12] K. Tamura, E. P. Ippen, H. A. Haus, and L. E. Nelson, Opt. Lett. **18**, 1080 (1993).
[13] S. L. McCall and E. L. Hahn, Phys. Rev. Lett. **18**, 908 (1967); S. L. McCall and E. L. Hahn, Phys. Rev. **183**, 457 (1969).
[14] G. L. Lamb, Jr., Rev. Mod. Phys. **43**, 99 (1971).
[15] A. V. Andreev and P. V. Polevoi, Zh. Éksp. Teor. Fiz. **106**, 1343 (1994) [Sov. Phys. JETP **79**, 727 (1994)].
[16] More detailed study is planned to be published elsewhere.