Self-trapping and self-focusing of a coherent atomic beam

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Light-induced dipole-dipole interaction in a coherent atomic-field results in an effective nonlinearity for atoms. This nonlinearity can induce self-trapping and self-focusing of a coherent atomic beam undergoing propagation through a traveling-wave laser beam; we show how such a scheme could be realized and evaluate the critical density required for atomic self-trapping and self-focusing. An analogy to optical self-trapping and self-focusing is discussed. [S1050-2947(97)07208-9]

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I. INTRODUCTION

The study of atoms in and beyond the ultracold regime has experienced rapid advances recently. For example, Bose-Einstein condensation in dilute atomic gases has been realized for Rubidium-87, for Lithium-7, and for Sodium-23 [1]. Further improvements in forming atom condensates bring experiments closer to the possibility of producing coherent atomic beams [2] consisting of large numbers of bosonic atoms condensed into a single momentum state. Coherent atomic beams offer a variety of applications in atom optics including, but not limited to, atomic solitons [3,4] and atomic pulse compression [5].

Here we are concerned with the exploitation of the nonlinearity to perform the atomic field analog of the self-trapping and self-focusing of optical beams. This nonlinearity, under appropriate conditions, shares some similarity with the optical Kerr nonlinearity, which is responsible for the self-trapping and self-focusing of optical beams [6,7]. Acting opposite to the transverse beam diffraction, the nonlinearity negates beam expansion by causing the beam to trap or focus into itself, thereby resulting in a decreasing waist size and an increasing density during propagation.

In the ultracold regime, it is best to think of atoms as quantum fields and employ a vector quantum field theory to describe the system. The analogy between matter fields of ultracold atoms and the electromagnetic fields of conventional optics has extended atom optics to study the quantum statistics of ultracold atoms [8–12], as well as nonlinear atom optics [3–5,13].

Attaining nonlinear atom-optical effects generally needs two conditions: (1) the existence of a nonlinear medium for the atomic beam and (2) preparation of the atomic beam with a high bosonic degeneracy, thereby ensuring a large nonlinearity. The nonlinear medium is provided by a light wave which mediates nonlinear atomic wave interactions [3,5,13]. High bosonic degeneracy requires a coherent atomic source analogous to the conventional laser beam. Here we assume a high bosonic degeneracy for the coherent atomic beam in the ultracold regime and study its propagation in a traveling-wave laser beam. Depicted in Fig. 1 is a scheme where the atomic beam passes through a hole in a mirror which reflects a laser beam. The atomic beam then propagates down into the laser beam which provides significant nonlinear atom-atom coupling. Nonlinear propagation of an atomic wave packet composed of a finite-size Bose-Einstein condensate in a traveling-wave laser beam has been studied [4], but here we extend the work to treat a continuous-wave coherent atomic beam in the longitudinal direction.

The paper is organized as follows. We begin, in Sec. II, by reviewing the general formalism of nonlinear atom optics and deriving the nonlinear hydrodynamic equation for the propagation of a coherent atomic beam in a laser beam. In Sec. III, the nonlinear hydrodynamic equation is solved under the appropriate conditions and applied to the case of self-trapping and self-focusing of a coherent atomic beam in a laser beam. The critical density required for the form of self-trapping and self-focusing are evaluated. The differences of atomic self-trapping and self-focusing from optical self-trapping and self-focusing are discussed. Particularly, the nature of the focal point for atomic self-focusing is analyzed. The conclusions are in Sec. IV.

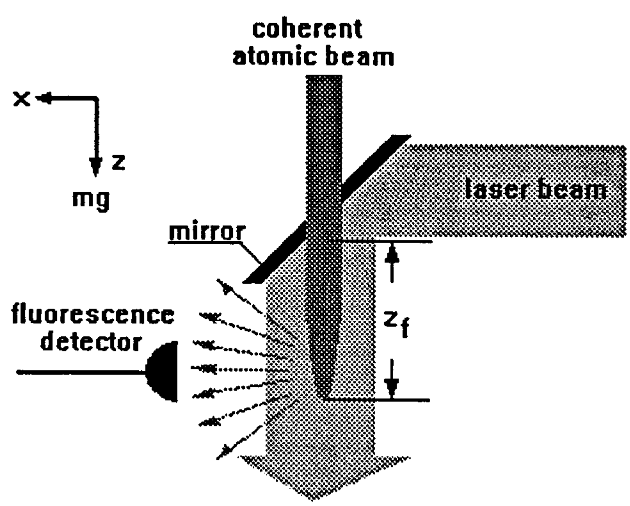

FIG. 1. Injection of the coherent atomic beam into a laser beam. The laser beam is reflected from a mirror with a hole where that atomic beam passes through. A detector receives fluorescent light.

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II. HYDRODYNAMIC APPROACH FOR COHERENT ATOMIC BEAM

The quantum field theory of ultracold atoms interacting with a light wave [3–5,13] permits the ultracold atomic ensemble to be treated as an N-component vector quantum field, with each component corresponding to one of the internal electronic states of the atoms. When the laser field is linearly polarized and the near-resonance transition of atoms corresponds to $J_g=0\rightarrow J_e=1$, the atoms can be approximated by a two-level system with the two-component field

$$\psi(\vec{r})=\psi_1(\vec{r})|1\rangle+\psi_2(\vec{r})|2\rangle, \tag{2.1}$$

where $|1\rangle$ and $|2\rangle$ denote the internal ground state and excited state, and ψ_1 and ψ_2 are the corresponding atomic-field operators. When the ultracold atomic ensemble interacts with a laser beam, the dynamic evolution of the ensemble can be described by a nonlinear stochastic Schrödinger equation [13].

The interaction time of a propagating atomic beam with a laser beam can be significantly longer than the characteristic time for the dipole interaction with light, namely, the inverse spontaneous emission rate of the excited state, the inverse atom-field detunings and the inverse Rabi frequencies of the various transitions under consideration. Spontaneous emission by excited atoms in the laser beam is the main dissipative mechanism of the atomic field leading to loss and decoherence of atoms by inelastic scattering of atoms into other incoherent channels. To avoid the dissipation due to spontaneous emission we choose a laser detuning sufficiently large to allow adiabatic elimination of the excited state. In this case, the propagation of the atomic beam in a laser beam can be described by a reduced nonlinear Schrödinger equation for the ground-state atomic quantum field which can transport the atomic coherence over a large distance. On the other hand, for a coherent atomic beam with a high bosonic degeneracy, a large number of bosonic atoms is expected to condense in a single momentum mode.

Under these circumstances, the ground-state atomic quantum field ψ_1 may be replaced by a macroscopic atomic wave function ϕ [5,13]. Gravitational acceleration of the atoms is ignored; hence, the atoms are assumed to move with constant center-of-mass velocity $v_g=\hbar K_0/m$, for K_0 the wave number of the atomic beam and m the mass of the atom. With these approximations, the nonlinear Schrödinger equation adopts the general form

$$i\hbar\left(\frac{\partial}{\partial t}+v_g\frac{\partial}{\partial z}\right)\phi=-\frac{\hbar^2\nabla_T^2}{2m}\phi+\frac{\hbar|\Omega^{(+)}(\vec{r})|^2}{4(\delta+i\gamma/2)}\phi+\int d^3\vec{r}\,'Q(\vec{r},\vec{r}\,')|\phi(\vec{r}\,')|^2\phi(\vec{r}), \tag{2.2}$$

$$\frac{\partial\Omega^{(+)}}{\partial z}=-\left(\frac{1}{2}+i\delta/\gamma\right)\sigma|\phi(\vec{r})|^2\Omega^{(+)}, \tag{2.3}$$

where γ is the spontaneous decay rate of the atom and

$$\delta=\omega_L-\omega_0-k_Lv_g-\hbar k_L^2/2m \tag{2.4}$$

is the effective detuning between the atomic field with internal transition frequency ω_0 and the laser field with frequency ω_L and wave number $k_L=\omega_L/c=2\pi/\lambda_L$. The transverse Laplacian operator $\nabla_T^2\equiv\partial^2/\partial x^2+\partial^2/\partial y^2$ has been used in the above expression. Equation (2.3) describes the effect of photon absorption on the Rabi frequency $\Omega^{(+)}(\vec{r})=2\vec{\mu}\cdot\vec{E}^{(+)}/\hbar$ with the assumption that the laser beam width W_L is larger than the atomic beam width W_0. The absorption cross section $\sigma=\sigma_{\rm peak}\gamma^2/(4\delta^2+\gamma^2)$ with $\sigma_{\rm peak}=3\lambda_L^2/2\pi$. The laser-induced nonlinear term has the definition

$$\begin{aligned}Q(\vec{r},\vec{r}\,')&\equiv\hbar(Q_R+iQ_I)\\&\approx|\Omega^{(+)}(\vec{r})|^2\frac{\hbar\gamma\delta}{4(\delta^2+\gamma^2/4)^2}(\delta-i\gamma/2)\\&\quad\times[2W(\vec{r}-\vec{r}\,')\cos k_L(z-z')-K(\vec{r}-\vec{r}\,')\\&\quad\times\sin k_L(z-z')]f_c(\vec{r}-\vec{r}\,'),\end{aligned} \tag{2.5}$$

where

$$\begin{aligned}&\tfrac{1}{2}K(\vec{r}-\vec{r}\,')-iW(\vec{r}-\vec{r}\,')\\&\quad=\tfrac{3}{4}[i\xi^2\sin^2\theta+(1-3\cos^2\theta)(\xi-i)]\frac{e^{-i\xi}}{\xi^3},\end{aligned} \tag{2.6}$$

with $\xi=k_L|\vec{r}-\vec{r}\,'|$ and θ the angle between the dipole moment and the relative coordinate [9]. In Eq. (2.5), the real part corresponds to the light-induced dipole-dipole interaction potential between atoms, and the imaginary part corresponds to the nonlinear dissipation due to the many-atom spontaneous emission. The function

$$f_c(\vec{r}-\vec{r}\,')=\exp(-\pi|\vec{r}-\vec{r}\,'|^2/\lambda_{\rm dB}^2) \tag{2.7}$$

appearing in Eq. (2.5) is the result of the Doppler effect due to the random thermal motion of atoms in the atomic beam. Previously we used the pseudopotential approximation [14] by setting the function $f_c(\vec{r}-\vec{r}\,')=1$ which corresponds to the zero-temperature limit of the atomic ensemble [4]. The thermal de Broglie wavelength $\lambda_{\rm dB}=\sqrt{(2\pi\hbar^2/mk_BT)}$ gives the coherent length of the atomic beam. Hence the function $f_c(\vec{r}-\vec{r}\,')$ determines the degree of the coherent overlapping of the individual atomic wave packet in the atomic beam or the coherence of the atomic beam. For a high temperature, the thermal de Broglie wavelength or the coherent length is short and $f_c(\vec{r}-\vec{r}\,')\rightarrow 0$ rapidly with increasing interatomic distance. As a result, the dipole-dipole interaction does not effectively contribute a nonlinearity to an incoherent atomic beam. Therefore, for a thermal atomic beam, the nonlinear Schrödinger equation reduces to the single-atom Schrödinger equation. The dipole-dipole interaction only produces effects on higher-order collisions which can be described by the methods of density operators. In terms of the above analysis, to observe coherent nonlinear effects from light-induced dipole-dipole interaction, one must employ a coherent atomic beam with a high bosonic degeneracy in a coherent length.

On the other hand, in addition to there being a complex nonlinear potential $Q(\vec{r},\vec{r}\,')$, the nonlinear Schrödinger equation (2.2) also includes a complex linear potential caused by single-atom spontaneous emission. The imaginary part of the linear potential describes the loss of atoms in the laser beam leading to the decoherence of the atomic waves. To solve the nonlinear Schrödinger equation (2.2) in the large detuning limit, we employ the hydrodynamic approach of separating the phase and amplitude of the coherent atomic beam

$$\phi(\vec{r})=\sqrt{\rho(\vec{r})}e^{i\Theta(\vec{r})}, \tag{2.8}$$

where $\rho(\vec{r})$ is the atomic density and Θ the phase of the atomic beam. Here we assume that the atomic beam is prepared with a predetermined phase. The stationary solution for the equation of propagation of the coherent atomic beam within the laser beam, obtained by replacing $\partial/\partial t$ by zero, yields

$$v_g\frac{\partial\rho}{\partial z}+\nabla_T(\rho\vec{v}_T)=-\frac{I\gamma}{4\delta^2+\gamma^2}\rho(\vec{r})+2\chi_I\rho^2(\vec{r}), \tag{2.9}$$

$$\hbar v_g\frac{\partial\Theta}{\partial z}=-\frac{1}{2}m(\vec{v}_T)^2-\frac{\hbar\,\delta I}{4\delta^2+\gamma^2}-\hbar\chi_R\rho(\vec{r})+\frac{\hbar^2}{2m}\frac{1}{\sqrt{\rho}}\nabla_T^2\sqrt{\rho}, \tag{2.10}$$

$$\frac{\partial I}{\partial z}=-\sigma\rho(\vec{r})I, \tag{2.11}$$

with laser intensity $I=|\Omega^{(+)}|^2$ and transverse velocity $\vec{v}_T=\hbar\nabla_T\Theta/m$, and the nonlinear coefficients are given by

$$\chi_R+i\chi_I=\frac{\delta I\gamma}{4(\delta^2+\gamma^2/4)^2}(-\delta+i\gamma/2)\lambda_L^3V_d, \tag{2.12}$$

$$V_d=\left|\frac{1}{(2\pi)^3}\int d^3\xi[2W(\vec{\xi})\cos\xi_z-K(\vec{\xi})\sin\xi_z]\times\exp\left(-\frac{\pi\xi^2}{(k_L\lambda_{\rm dB})^2}\right)\right|. \tag{2.13}$$

In the derivation of Eqs. (2.9) and (2.10), we have assumed that the density of the atomic beam varies slowly over the coherence length. Numerical solutions of the integral (2.13) demonstrate convergence to a factor quantifying the strength of coherent atomic dipole-dipole interaction over a coherence length. Equations (2.9)–(2.13) determine the dynamics of transportation of coherent atomic flow along the traveling-wave laser beam. Coherent atomic flow is not a conserved quantity in Eq. (2.9) due to the loss caused by inelastic photon scattering during spontaneous emission. This leads to decoherence of the atomic beam. The exact dynamics of the propagation of the coherent atomic beam is studied in Sec. III by solving Eqs. (2.9)–(2.11).

III. ATOMIC SELF-TRAPPING AND SELF-FOCUSING

The nonlinear equations for atomic flow (2.10)–(2.11) are difficult to solve exactly, in general, but can, however, be solved analytically in the paraxial regime, where the atomic wave-front distortion, due to the nonlinear phase change, is small. If the initial incident atomic beam has cylindrical spatial symmetry with a Gaussian distribution

$$\rho(r,0)=\rho_0e^{-r^2/W_0^2}, \tag{3.1}$$

with ρ_0 the peak density and W_0 the transverse width, we can make the paraxial transformation [15]

$$\Theta=\frac{r^2}{2}\alpha(z)+\beta(z)$$

$$\rho(r,z)=\rho_0\frac{Y(z)}{f^2(z)}\exp\left(-\frac{r^2}{W_0^2f^2(z)}\right), \tag{3.2}$$

with initial conditions

$$\alpha(0)=\beta(0)=0,\quad f(0)=Y(0)=1. \tag{3.3}$$

Substituting Eq. (3.2) into Eqs. (2.9)–(2.11), we have the five coupled differential equations

$$\begin{aligned}
\frac{1}{f}\frac{\partial f}{\partial z}&=\frac{\hbar}{mv_g}\alpha(z)-\frac{\chi_I\rho_0Y(z)}{v_gf^2(z)},\\
\frac{\partial\alpha}{\partial z}&=-\frac{\hbar}{mv_g}\alpha^2(z)+\left(\frac{2\chi_R\rho_0Y(z)}{v_gW_0^2}+\frac{1}{L_qW_0^2}\right)f^{-4},\\
\frac{\partial\beta}{\partial z}&=-\frac{I\delta}{(4\delta^2+\gamma^2)v_g}-\left(\frac{\chi_R\rho_0}{v_g}Y(z)+\frac{1}{L_q}\right)f^{-2},\\
\frac{\partial I}{\partial z}&=-\sigma\rho_0\frac{Y(z)I(z)}{f^2(z)},\\
\frac{\partial Y}{\partial z}&=-\frac{\gamma}{(4\delta^2+\gamma^2)v_g}I(z)Y(z),
\end{aligned} \tag{3.4}$$

where $L_q=mv_gW_0^2/\hbar$ is the quantum diffraction length. Equations (3.4) have analytical solutions if the loss of atoms due to incoherent spontaneous emission and decreasing laser intensity, due to photon absorption, are negligible over the scale of several quantum diffraction lengths. Consequently, the spontaneous decay γ and absorption cross section σ satisfy the conditions

$$\frac{\gamma}{(4\delta^2+\gamma^2)v_g}L_q\ll1,\quad \sigma\rho_0L_q\ll1 \tag{3.5}$$

and Eqs. (3.4) reduce to

$$\frac{\partial^2f}{\partial z^2}=-\left(\frac{1}{L_d^2}-\frac{1}{L_q^2}\right)f^{-3} \tag{3.6}$$

with an effective dipole-dipole interaction length

$$L_d=\frac{4\delta^2+\gamma^2}{2\delta\sqrt{I}}\left(\frac{mv_g^2/2}{\hbar\gamma}\right)^{1/2}\frac{W_0}{\sqrt{\rho_0\lambda_L^3V_d}}. \tag{3.7}$$

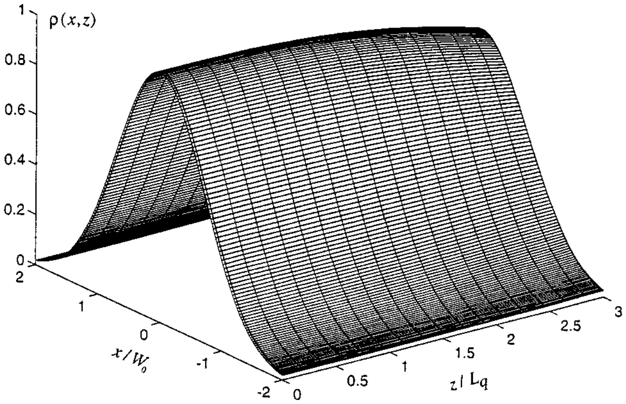

FIG. 2. The spatial distribution of atomic density for self-trapping as a function of the scaled propagation distance z/L_q with respect to the quantum diffusion length, and the scaled transverse spread of the atomic beam x/W_0 with respect to the initial width of the injected atomic beam.

The solution to Eq. (3.6), given the initial conditions (3.3), is

$$f^2=1-\left(\frac{1}{L_d^2}-\frac{1}{L_q^2}\right)z^2. \tag{3.8}$$

Equation (3.8) determines the dependence of the radius of the atomic beam on the propagation distance z. Evidently when the quantum diffraction effect dominates over the nonlinear effect, i.e., $1/L_q^2>1/L_d^2$, the beam radius increases with the propagation distance z. As a result the atomic beam diverges in the transverse direction due to quantum diffusion.

To obtain atomic self-trapping and self-focusing, the nonlinear term should play a dominant role in the propagation of the atomic beam. This leads to a requirement for a critical density of the incident atomic beam for the self-focusing. The critical density can be determined by the condition

$$L_d=L_q, \tag{3.9}$$

for which the nonlinear effect exactly cancels the quantum diffraction effect thereby causing the atomic beam to propagate with a constant radius. This case corresponds to self-trapping of the atomic beam. From condition (3.9) the critical density is

$$\rho_c=\frac{(\delta^2+\gamma^2/4)^2}{\delta^2 I}\frac{E_r}{E_\gamma}\frac{1}{(\pi W_0)^2\lambda_L V_d}, \tag{3.10}$$

where $E_r=\hbar^2k_L^2/2m$ is the single-photon recoil energy, and $E_\gamma=\hbar\gamma$ is the energy associated with the single-atom spontaneous emission decay rate. For the critical density ρ_c, atomic self-focusing is achieved for

$$\rho_0>\rho_c. \tag{3.11}$$

Given condition (3.11), the density of the atomic beam on the beam axis, $\rho(0,z)=\rho_0/f(z)^2$ will increase. After a propagation distance $z=z_f$, satisfying $f(z_f)\to 0$, the density along the beam axis $\rho(0,z)\to\infty$. The corresponding point z_f on the z axis is a focus and z_f gives the self-focusing length, which has the expression

$$z_f=L_d(1-\rho_c/\rho_0)^{-1/2}. \tag{3.12}$$

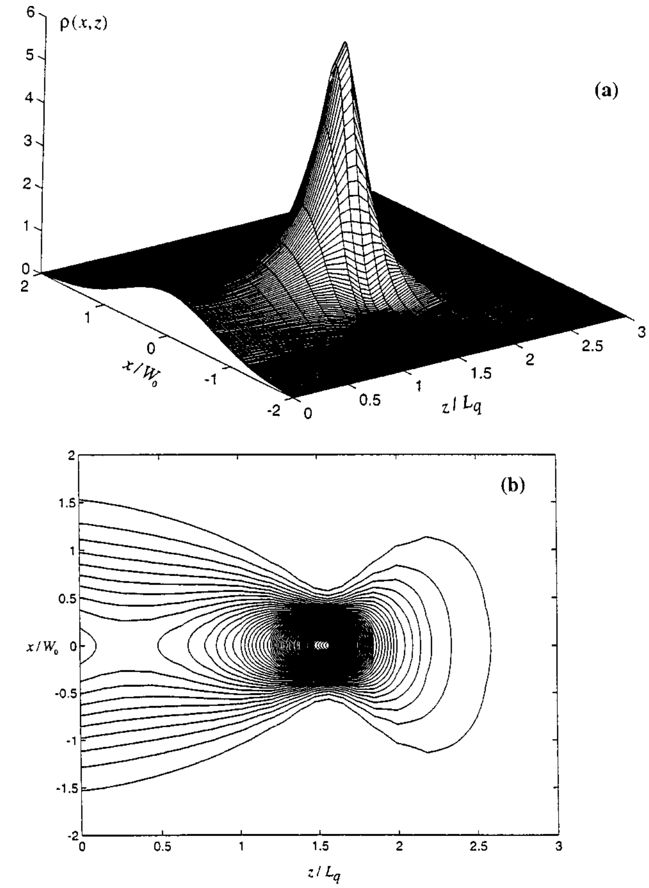

FIG. 3. The spatial distribution of atomic density for self-focusing as a function of the scaled propagation distance z/L_q with respect to the quantum diffusion length, and the scaled transverse spread of the atomic beam x/W_0 with respect to the initial width of the injected atomic beam (a) as a surface plot and (b) as a contour plot version of (a).

In terms of Eq. (3.10), the critical density for atomic self-focusing is proportional to the single-photon recoil energy. Physically, the photon recoil results in diffusion of momentum for the atomic beam; hence, the recoil acts opposite to the self-focusing process. Evidently, the heavier atoms require a lower critical density for self-focusing than lighter atoms do, as photon recoil becomes less significant for heavier atoms. Hydrogen atoms require the highest critical density. In addition, the critical density for self-focusing depends on laser detuning, laser intensity, and the incident beam width W_0. From Eq. (3.10), we see that for a large detuning, a high critical density is required to achieve the large nonlinearity required for self-focusing. An increasing laser intensity can increase the atomic nonlinearity which leads to a lower critical density being required. Quantum diffraction depends on the width W_0 of the incident beam. For a wide atomic beam, quantum diffraction will be small, and hence, a wide beam can exhibit self-focusing with a low critical density.

The above conclusions are obtained by assuming the conditions (3.5). Of particular interest is the focal point for the self-focusing atomic beam. At the focal point, the theory becomes singular with the density increasing without bound. The singularity is caused by neglecting the loss of atoms due to spontaneous emission and photon absorption of atoms which become very important when $f(z)\to 0$ as shown by Eqs. (3.4). Hence the exact description of self-focusing dynamics, particularly that near the focal point, requires the complete solution of Eqs. (3.4) including the loss of atoms and photon absorption. To further observe the dynamics of the atomic density evolution in the propagation of the beam with self-trapping and self-focusing, we solve the density distribution numerically for the atomic beam in terms of the coupled nonlinear hydrodynamic equations (3.4). Figure 2 shows that the self-trapping can be obtained over a length scale greater than three quantum diffraction lengths L_q. In the numerical simulation, a large laser detuning $\delta = 100\gamma$ and a low peak laser intensity $I_0 = \gamma/250$ for an atomic beam with width $W_0 = 5\,\mu$m is chosen to reduce the loss of atoms due to incoherent spontaneous emission. The self-trapping is achieved for an appropriate density $\rho_0 \lambda_L^3 V_d = 1$. The numerical calculation shows that the parameter V_d varies approximately between 0 and 100 depending on the thermal de Broglie wavelength λ_{dB}. The self-focusing occurs at a higher density and requires a higher laser intensity to increase the atomic nonlinearity. We simulate the self-focusing by using the same parameters for the self-trapping except the laser intensity chosen $I_0 = \gamma/5$ and the atomic density satisfying $\rho_0 \lambda_L^3 V_d = 2$. The result is shown in Fig. 3.

We find that the density singularity in the approximate analytical solution is removed in the exact numerical simulation. This is because in the realistic self-focusing dynamics, the photon absorption will sharply increase at the focal point due to the sharp rise in the atomic density. In terms of Eqs. (3.4), the strong photon absorption near the focal point results in the decrease of the laser intensity in the regime. As a result, the light-induced dipole-dipole interaction which leads to atomic self-focusing will decrease with decreasing laser intensity. The dynamic processes last until self-focusing stops. Then the quantum diffraction process will be important and atomic defocusing occurs as shown in the contour plot 3(b).

IV. CONCLUSION

Light-induced dipole-dipole interactions yields an atomic wave nonlinearity which can lead to self-trapping and self-focusing of an atomic beam propagating along the axis of a traveling-wave laser beam. This effect is analogous to the self-trapping and self-focusing of light beams due to the optical Kerr effect.

Certain criteria must be met for this self-trapping and self-focusing phenomenon to occur. The atomic beam must be coherent and have a high bosonic degeneracy in a single momentum mode. Moreover, the atomic beam must be above the critical density which itself depends on the atomic mass.

In contrast to optical self-trapping and self-focusing, atomic self-trapping and self-focusing exhibits more complicated focal dynamics due to incoherent loss channels arising from the inelastic photon scattering during spontaneous emission and density-dependent photon absorption. The latter results in a self-modified decrease of the nonlinearity response for self-focusing. Both analytically and numerically, we study the dependence of atomic self-trapping and self-focusing on laser and atomic parameters. The experimental observation of the atomic self-trapping and atomic self-focusing requires a coherent source for atoms such as an ‘‘atom laser.’’

ACKNOWLEDGMENTS

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