Effects of inelastic scattering on tunneling time based on the generalized diffusion process approach

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We analyze the effects of inelastic scattering on the tunneling time theoretically, using generalized Nelson's quantum mechanics. This generalization enables us to describe a quantum system with optical potential and channel couplings in a real-time stochastic approach, which seems to give us a new insight into quantum mechanics beyond Copenhagen interpretation. [S1050-2947(97)01408-X]

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I. INTRODUCTION

An issue of the tunneling time, i.e., the time associated with the passage of a particle through a tunneling barrier, has been discussed in many theoretical studies [1–17], and is not settled yet. This difficulty arises mainly from the fact that time is not an observable represented by a self-adjoint operator, but is just a parameter in quantum mechanics.

In our previous paper [20], we proposed a method to evaluate the tunneling time, using Nelson's approach of quantum-mechanics [18]. Our aim then was to treat tunneling effects in a detailed time-dependent and fully quantum-mechanical way, as any theoretical expression of the tunneling time must be tested by experiments which are feasible at present and in the near future.

As discussed in Ref. [20], Nelson's approach to quantum mechanics has several advantages to study the tunneling time, a few of which are listed below. First of all, this approach, using the real-time stochastic process, enables us to describe individual experimental runs of a quantum system in terminology of the ''analog'' of classical mechanics. This is true even in the tunnel region where a classical path is forbidden. From sample paths generated by the stochastic process, we obtain information on the time parameter, in particular, the tunneling time.

As a matter of course, the whole ensemble of sample paths gives us the same results as quantum mechanics in the ordinary approach, e.g., expectation values of the observable, transmission and reflection probabilities in scattering problem, and so on. It is important for us to note that in scattering phenomena (those without bound states) the transmission and reflection ensembles are defined unambiguously, that is, each sample path is classified distinctively into either a transmission ensemble or reflection one.

We need to accumulate a sufficient number of sample paths in numerical simulations. In thick or/and high potential cases the transmission probability is low, and consequently we have the difficulty that a number of sample paths belonging to the transmission ensemble are also low when each sample path is followed in the forward time direction. However, in Nelson's approach there is not only the forward Langevin equation but also the backward Langevin equation [see Eq. (2) below], both being equivalent to each other in physical results. The difficulty above is avoided when the backward Langevin equation is employed.

Taking account of these advantages, we developed a theoretical of time-dependent description of tunneling phenomena based on Nelson's stochastic approach in Ref. [20]. Numerical simulations for a one-dimensional square-well potential barrier model were demonstrated. An important result about the tunneling time then is that there are three characteristic times, i.e., *the passing time and the hesitating time*, and their sum, *the interacting time*. The probability distribution of these three times were calculated numerically.

Our previous study treated only a quantum system of a single particle under a simple potential. But realistic experimental situations are more complicated. Naturally we are tempted to extend our previous formulation to more general scattering phenomena. In this paper we consider cases in which transition processes into other channels or absorptive processes takes place during scattering processes, and look into these effects on the tunneling time.

Processes of transition into other channels and absorption are described by channel coupling and optical potential (complex potential), respectively, in ordinary quantum mechanics using the Schrödinger equation. So far it is known that Nelson's formulation is equivalent to the Schrödinger equation *only for a one-body problem with a single channel and a real potential*. The purpose of this paper is to generalize Nelson's stochastic quantization so that it can deal with multichannel coupling and/or optical potential problems. As will be shown below, one can construct such generalized formulations of Nelson's approach with additional stochastic jumping processes. These theoretical formulations allow us to perform numerical simulations of stochastic processes as before [20]. This way we can investigate the effects of transition into other channels, or absorption on the tunneling time.

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The paper is organized as follows: In Sec. II the original Nelson's quantum mechanics is reviewed briefly for later relevance. We propose a formulation of the Nelson's approach, generalized to a quantum system with channel coupling, in Sec. III. The formulation of Sec. III hints at how to develop a formulation for optical potential, which is shown in Sec. IV. In Sec. V a numerical simulation for a square-well potential model, using the formulations in Secs. III and IV, are demonstrated, and physical implications of these results are analyzed. Section VI is devoted to summary and some comments.

II. BRIEF REVIEW OF NELSON'S QUANTUM MECHANICS

We start with a brief review of the original Nelson's quantum mechanics, which consists of two basic conditions, i.e., the kinematical condition and the dynamical one. The kinematical condition is given by the Ito-type stochastic differential equation: There are two ways to express it, depending on the forward or backward time direction. Explicitly we have, for forward time evolution,

$$dx(t)=b(x(t),t)dt+dw(t), \tag{1}$$

and, for backward time evolution,

$$dx(t)=b_*(x(t),t)dt+dw_*(t). \tag{2}$$

The $dw(t)$ is the Gaussian white noise (representing the quantum fluctuation) with the statistical properties of

$$\langle dw(t)\rangle=0 \quad \text{and} \quad \langle dw(t)dw(t)\rangle=\frac{\hbar}{m}dt, \tag{3}$$

and the same properties for $dw_*(t)$ as in Eq. (3). Here $\langle\ \rangle$ means a sample average. It is easy to show that for these two Langevin equations hold the following Fokker-Planck equations for the distribution function $P(x,t)$ of the random variables $x(t)$,

$$\frac{\partial P(x,t)}{\partial t}=\left[-\frac{\partial}{\partial x}b(x,t)+\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2}\right]P(x,t) \quad \text{(forward in } t\text{)}, \tag{4}$$

$$-\frac{\partial P(x,t)}{\partial t}=\left[\frac{\partial}{\partial x}b_*(x,t)+\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2}\right]P(x,t) \quad \text{(backward in } t\text{)}. \tag{5}$$

Thus a pair of equations (1) and (2) is mathematically equivalent to a pair of equations (4) and (5). We obtain an osmotic velocity u from the sum of Eqs. (4) and (5) as

$$u=\frac{b-b_*}{2}=\frac{\hbar}{2m}\frac{1}{P}\frac{\partial P}{\partial x} \tag{6}$$

under the boundary condition of

$$P(x\to\infty,t)\to 0. \tag{7}$$

Subtraction of Eq. (5) from Eq. (4) gives

$$\frac{\partial P}{\partial t}=-\frac{\partial}{\partial x}(vP), \tag{8}$$

where v is a current velocity,

$$v=\frac{b+b_*}{2}. \tag{9}$$

The elimination of $P(x,t)$ from Eqs. (6) and (8) leads to an equation called the kinematical equation,

$$\frac{\partial u}{\partial t}=-\frac{\hbar}{2m}\frac{\partial^2 v}{\partial x^2}-\frac{\partial}{\partial x}(uv). \tag{10}$$

The dynamical condition is expressed through the ''mean time derivatives'' introduced as follows: The ''mean forward time derivative'' $Df(t)$ is defined as

$$Df(t)\equiv\lim_{\Delta t\to+0}\left\langle\frac{f(t+\Delta t)-f(t)}{\Delta t}\middle| f(s)(s\leqslant t)\ \text{ fixed}\right\rangle, \tag{11}$$

and the ''mean backward time derivative'' $D_*f(t)$ is defined as

$$D_*f(t)\equiv\lim_{\Delta t\to+0}\left\langle\frac{f(t)-f(t-\Delta t)}{\Delta t}\middle| f(s)(s\geqslant t)\ \text{ fixed}\right\rangle. \tag{12}$$

The ''mean balanced acceleration'' is introduced through the definitions of Eqs. (11) and (12) as

$$a(x(t),t)\equiv\frac{DD_*+D_*D}{2}x(t). \tag{13}$$

Note that this definition can be rewritten as

$$a(x,t)=-\frac{\hbar}{2m}\frac{\partial^2 u}{\partial x^2}+\frac{1}{2}\frac{\partial}{\partial x}(v^2-u^2)+\frac{\partial v}{\partial t} \tag{14}$$

from Eqs. (1) and (2) with Eqs. (6) and (9). The dynamical condition is nothing but the classical Newton equation to this ''mean balanced acceleration'' $a(x(t),t)$, that is,

$$ma(x,t)=-\frac{\partial V}{\partial x}, \tag{15}$$

from which we derive the ''Newton-Nelson equation''

$$\frac{\partial v}{\partial t}=\frac{\hbar}{2m}\frac{\partial^2 u}{\partial x^2}-v\frac{\partial v}{\partial x}+u\frac{\partial u}{\partial x}-\frac{1}{m}\frac{\partial V}{\partial x} \tag{16}$$

because of Eq. (14).

Next we summarize the mathematical structure of Nelson's quantum mechanics. The two basic equations, Eq. (10) from the kinematical condition, and Eq. (16) from the dynamical condition, form a set of simultaneous equations for two unknown functions $u(x,t)$ and $v(x,t)$, or equivalently, $b(x,t)$ and $b_*(x,t)$. Then we can determine the ensemble of sample paths or the distribution function $P(x,t)$. Although it is practically very difficult to solve these equations directly due to their nonlinearity, one can easily show the equiva-

lence between this approach and the ordinary approach of the Schrödinger equation. This fact tells us that one can solve the problem by means of the wave function much more easily. The equation

$$\frac{\partial}{\partial x}\left[i\frac{\hbar}{m}\frac{1}{\psi'}\frac{\partial\psi'}{\partial t}+\frac{1}{2}\left(\frac{\hbar}{m}\right)^2\frac{1}{\psi'}\frac{\partial^2\psi'}{\partial x^2}-\frac{1}{m}V\right]=0 \quad (17)$$

follows from the combination of Eq. (10) and the imaginary version of Eq. (16), where

$$u+iv=\frac{\hbar}{m}\frac{1}{\psi'}\frac{\partial\psi'}{\partial x}. \quad (18)$$

Equation (17) clearly shows the relationship between ψ' and the wave function ψ as the solution of Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t}=\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V\right)\psi, \quad (19)$$

that is,

$$\psi(x,t)=\psi'(x,t)\exp\left(-\frac{im}{\hbar}\int^t\eta(s)ds\right), \quad (20)$$

with an arbitrary function of t, $\eta(t)$, which has no physical relevance. It is easily seen from this proof of the equivalence that one has expressions for $b(x,t)$, $b_*(x,t)$, and $P(x,t)$ in terms of $\psi(x,t)$,

$$b(x,t)=\frac{\hbar}{m}(\mathrm{Im}+\mathrm{Re})\frac{\partial}{\partial x}\ln\psi(x,t), \quad (21)$$

$$b_*(x,t)=\frac{\hbar}{m}(\mathrm{Im}-\mathrm{Re})\frac{\partial}{\partial x}\ln\psi(x,t), \quad (22)$$

$$P(x,t)=|\psi(x,t)|^2. \quad (23)$$

III. STOCHASTIC FORMULATION FOR QUANTUM SYSTEM WITH CHANNEL COUPLING

We now generalize the above Nelson's approach to a system with a channel coupling. For simplicity, consider the two-channel Schrödinger equations ($\{i,j\}=\{1,2\}$)

$$i\hbar\frac{\partial}{\partial t}\psi_i(x,t)=\left(-\frac{\hbar^2}{2m_i}\frac{\partial^2}{\partial x^2}+V_{ii}(x,t)\right)\psi_i(x,t)+V_{ij}(x,t)\psi_j(x,t), \quad (24)$$

with

$$V_{ij}=V_{ji}^*. \quad (25)$$

Here and below the dummy index does not imply taking a sum. As will be seen, the generalization of the formulation in this section to the N-channel case ($N>2$) is straightforward.

Consider the Fokker-Planck equations in the stochastic formulation, corresponding to Eq. (24). First we require a natural extension of Eq. (23) to the present case,

$$P_i(x,t)=|\psi_i(x,t)|^2. \quad (26)$$

The diagonal parts (the kinetic energy and V_{ii} terms) in Eq. (24) are expected to be dealt with as in Sec. II. The Schrödinger equations (24) and their complex conjugates suggest the following equations for $P_i(x,t)$:

$$\frac{\partial P_i(x,t)}{\partial t}=\left[-\frac{\partial}{\partial x}b_i(x,t)+\frac{\hbar}{2m_i}\frac{\partial^2}{\partial x^2}-W_{(i\to j)}(x,t)\right]P_i(x,t)$$

(forward in time), (27)

$$-\frac{\partial P_i(x,t)}{\partial t}=\left[\frac{\partial}{\partial x}b_{*i}(x,t)+\frac{\hbar}{2m_i}\frac{\partial^2}{\partial x^2}+W_{(i\to j)}(x,t)\right]P_i(x,t)$$

(backward in time) (28)

as $P_i(x,t)$ increases or decreases, due to the potential V_{ij} causing transitions between i and j, at the rate of the absolute value of

$$W_{(i\to j)}P_i=-W_{(j\to i)}P_j=\frac{2}{\hbar}\mathrm{Im}\psi_j^*V_{ji}\psi_i. \quad (29)$$

Although the sum of Eq. (27) and (28) leads to Eq. (6) with the index i,

$$u_i=\frac{b_i-b_{*i}}{2}=\frac{\hbar}{2m_i}\frac{1}{P_i}\frac{\partial P_i}{\partial x}, \quad (30)$$

their difference provides us with

$$\frac{\partial P_i}{\partial t}=-\frac{\partial}{\partial x}(v_iP_i)-W_{(i\to j)}P_i \quad (31)$$

instead of Eq. (8), where

$$v_i=\frac{b_i+b_{*i}}{2}. \quad (32)$$

As a result, eliminating $P_i(x,t)$ from Eqs. (30) and (31), one derives the kinematical equation

$$\frac{\partial u_i}{\partial t}=-\frac{\hbar}{2m_i}\frac{\partial^2 v_i}{\partial x^2}-\frac{\partial}{\partial x}(u_iv_i)-\frac{\hbar}{2m_i}\frac{\partial}{\partial x}W_{(i\to j)} \quad (33)$$

instead of Eq. (10).

Here arises a natural question what are the stochastic differential equations corresponding to the Fokker-Planck equations in Eqs. (27) and (28), just as Eqs. (1) and (2) correspond to Eqs. (4) and (5). Apparently we need two random variables $x_i(t)$ ($i=1$ and 2), which are assumed to be subject to the stochastic differential equations, similar to Eqs. (1) and (2),

$$dx_i(t)=b_i(x_i(t),t)dt+dw_i(t) \quad \text{(forward in time)}, \quad (34)$$

$$dx_i(t)=b_{*i}(x_i(t),t)dt+dw_{*i}(t) \quad \text{(backward in time)}, \quad (35)$$

with the properties for $dw_i(t)$ and $dw_{*i}(t)$,

$$\langle dw_i(t)\rangle = 0, \quad \langle dw_i(t)dw_j(t)\rangle = \frac{\hbar}{m_i}\delta_{ij}dt, \tag{36}$$

$$\langle dw_{*i}(t)\rangle = 0, \quad \langle dw_{*i}(t)dw_{*j}(t)\rangle = \frac{\hbar}{m_i}\delta_{ij}dt.$$

As is easily seen, a naive interpretation of these independent stochastic differential equations leads only to the Fokker-Planck equations in Eqs. (27) and (28) *without the terms proportional to* $W_{(i\to j)}$. An additional mechanism to take account of the quantum jump between i and j represented by the terms involving $W_{(i\to j)}$ is necessary. For this purpose we supplement Eqs. (34) and (35) with a stochastic jumping process between i and j. Thus below we attempt the formulation of two random variables $x_i(t)$, subject to the stochastic differential equations (34) and (35) combined with a stochastic jumping process in the following way.

The ''dynamical'' rule to determine how each sample path $x_i(t)$ changes its index ($i=1\to 2$, or vice versa) during passage of time is described by the following random jumping process (Fig. 1): At each time a dice is cast, *independently of the stochastic equations* (34) and (35), and each sample path either keeps or changes its index at a certain rate. For the forward time direction, we have the rule in case of $W_{(i\to j)}>0$ $(i\neq j)$,

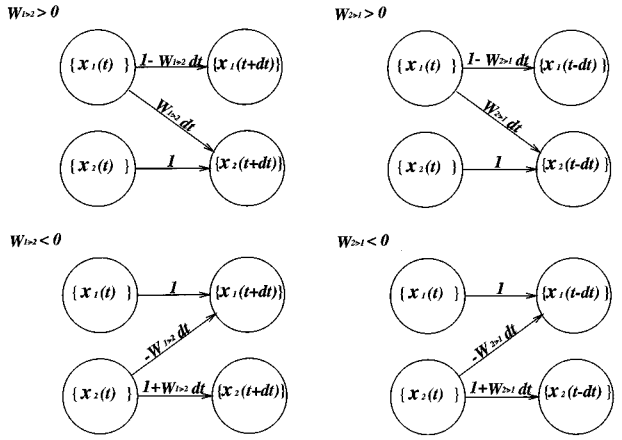

FIG. 1. Schematical illustration of the ''dynamical'' rule for the stochastic jumping process between two channels.

$$x_i(t)\to\begin{cases} x_j(t+dt) & \text{with the probability } W_{(i\to j)}(x_i(t),t)dt,\\ x_i(t+dt) & \text{with the probability of } 1-W_{(i\to j)}(x_i(t),t)dt,\end{cases}$$
$$x_j(t)\to x_j(t+dt) \quad \text{with the probability 1,} \tag{37}$$

and the rule in case of $W_{(i\to j)}<0$,

$$x_j(t)\to\begin{cases} x_i(t+dt) & \text{with the probability } -W_{(i\to j)}(x_j(t),t)dt,\\ x_j(t+dt) & \text{with the probability } 1+W_{(i\to j)}(x_j(t),t)dt,\end{cases}$$
$$x_i(t)\to x_i(t+dt) \quad \text{with the probability 1.} \tag{38}$$

Likewise, the rules for backward time direction state that, in the case of $W_{(i\to j)}>0$,

$$x_j(t)\to\begin{cases} x_i(t-dt) & \text{with the probability } W_{(i\to j)}(x_j(t),t)dt,\\ x_j(t-dt) & \text{with the probability } 1-W_{(i\to j)}(x_j(t),t)dt,\end{cases}$$
$$x_i(t)\to x_i(t-dt) \quad \text{with the probability 1,} \tag{39}$$

and, in the case of $W_{(i\to j)}<0$,

$$x_i(t)\to\begin{cases} x_j(t-dt) & \text{with the probability } -W_{(i\to j)}(x_i(t),t)dt,\\ x_i(t-dt) & \text{with the probability } 1+W_{(i\to j)}(x_i(t),t)dt,\end{cases}$$
$$x_j(t)\to x_j(t-dt) \quad \text{with the probability 1.} \tag{40}$$

According to the rules of the random jumping process above, the behavior of each sample path is illustrated as follows: For the forward time direction, a sample path starts from $x_i(t_I)$, develops according to Eq. (34) with i for a while, and, when a chance comes, it changes its index from i to j and follows Eq. (34) with j until the next jumping process takes place. The jumping process from x_i to x_j is allowed, and the reverse process is forbidden, where $W_{(i\to j)}>0$, and vice versa where $W_{(i\to j)}<0$. The jumping processes may be repeated or may not occur, depending on the sign and magnitude of $W_{(i\to j)}$. Sample paths show a similar behavior for the backward time direction.

It is remarked that $x_i(t)$ is generally a functional of both $dw_1(s)$ and $dw_2(s)$ $(s<t)$ [or $dw_{*1}(s)$ or $dw_{*2}(s)$

$(s>t)]$, as it may repeat jumps between $i=1$ and 2 in the past (in the future). Due to changes in the index for each sample path, there are several types of averages which are distinguished from each other carefully. It is convenient to introduce notations for conditional averages. The simple average $\langle\ \rangle$ should be taken over both of $dw_1(s)$ and $dw_2(s)$ $(s<t)$. To represent a physical average of the i state at t, we introduce a notation of

$$\langle\langle f(x(t))\rangle\rangle_{\{x_i(t)\}} \equiv \langle f(x_i(t))\rangle, \tag{41}$$

where the average on the left-hand side implies a conditional average only over sample paths, labeled by i at t. This average should be expressed in terms of the probability distribution $P_i(x,t)$ as

$$\langle\langle f(x(t))\rangle\rangle_{\{x_i(t)\}} = \int dx\ f(x) P_i(x,t). \tag{42}$$

The notation $\langle\langle f(x(t))\rangle\rangle_{\{x_1(t)\}\cup\{x_2(t)\}}$ has trivial interpretations,

$$\langle\langle f(x(t))\rangle\rangle_{\{x_1(t)\}\cup\{x_2(t)\}} = \langle f(x(t))\rangle. \tag{43}$$

Furthermore, conditional averages with different times such as $\langle\langle f(x(t))\rangle\rangle_{\{x_i(t+dt)\}\cap\{x_j(t)\}}$ can be introduced: This example represents the average only over sample paths which have the index j at t and i at $t+dt$.

Let us now evaluate the time derivative of the physical average $\langle\langle f(x(t))\rangle\rangle_{\{x_i(t)\}}$. For the forward time direction, using appropriate conditional averages, we write

$$\begin{aligned}
&\frac{d}{dt}\langle\langle f(x(t))\rangle\rangle_{\{x_i(t)\}} \\
&= \frac{1}{dt}[\langle\langle f(x(t+dt))\rangle\rangle_{\{x_i(t+dt)\}} - \langle\langle f(x(t))\rangle\rangle_{\{x_i(t)\}}] \\
&= \frac{1}{dt}[\langle\langle f(x(t+dt)) - f(x(t))\rangle\rangle_{\{x_i(t+dt)\}\cap\{x_i(t)\}} \\
&+ \langle\langle f(x(t+dt))\rangle\rangle_{\{x_i(t+dt)\}\cap\{x_j(t)\}} \\
&- \langle\langle f(x(t))\rangle\rangle_{\{x_j(t+dt)\}\cap\{x_i(t)\}}].
\end{aligned} \tag{44}$$

The three terms here are manipulated as

$$\begin{aligned}
\langle\langle f(x(t+dt)) - f(x(t))\rangle\rangle_{\{x_i(t+dt)\}\cap\{x_i(t)\}} &= \left\langle\left\langle \left.\frac{df(x)}{dx}\right|_{x=x(t)} dx(t) + \frac{1}{2}\left.\frac{d^2 f(x)}{dx^2}\right|_{x=x(t)} (dx(t))^2 + o(dt^{3/2}) \right\rangle\right\rangle_{\{x_i(t+dt)\}\cap\{x_i(t)\}} \\
&= \left\langle\left\langle \frac{df(x)}{dx} b_i(x(t),t)dt + \frac{d^2 f(x)}{dx^2}\frac{\hbar}{2m_i}dt \right\rangle\right\rangle_{\{x_i(t+dt)\}\cap\{x_i(t)\}} \\
&= \left\langle\left\langle \frac{df(x)}{dx} b_i(x(t),t)dt + \frac{d^2 f(x)}{dx^2}\frac{\hbar}{2m_i}dt \right\rangle\right\rangle_{\{x_i(t)\}} + o(dt^2) \\
&= dt\int dx \left(\frac{df(x)}{dx} b_i(x,t) + \frac{d^2 f(x)}{dx^2}\frac{\hbar}{2m_i}\right) P_i(x,t) + o(dt^2) \\
&= dt\int dx\ f(x)\left(-\frac{\partial}{\partial x} b_i(x,t) + \frac{\hbar}{2m_i}\frac{\partial^2}{\partial x^2}\right) P_i(x,t) + o(dt^2),
\end{aligned} \tag{45}$$

$$\langle\langle f(x(t+dt))\rangle\rangle_{\{x_i(t+dt)\}\cap\{x_j(t)\}} = -dt\int dx\ f(x) W_{(i\to j)}(x,t) P_i(x,t)\,\theta(-W_{(i\to j)}(x,t)) + o(dt^2), \tag{46}$$

and

$$\begin{aligned}
&\langle\langle f(x(t))\rangle\rangle_{\{x_j(t+dt)\}\cap\{x_i(t)\}} \\
&= dt\int dx\ f(x) W_{(i\to j)}(x,t) P_i(x,t)\,\theta(W_{(i\to j)}(x,t)) \\
&\quad + o(dt^2),
\end{aligned} \tag{47}$$

respectively, from Eqs. (34), (36), (37), (38), and (42). Collecting Eqs. (44)–(47), we obtain the correct time evolution of Eq. (27). This shows the equivalence between Eq. (27) and the stochastic equation (34) supplemented with the stochastic jumping processes (37) and (38). Likewise one can show the equivalence between Eq. (28) and the stochastic equation (35) supplemented with the stochastic jumping processes (39) and (40).

We need some careful treatment on the dynamical condition in the present case. For the equivalence between Nelson's and Schrödinger approaches, the dynamical condition should have the form

$$\frac{\partial v_i}{\partial t} = \frac{\hbar}{2m_i}\frac{\partial^2 u_i}{\partial x^2} - v_i\frac{\partial v_i}{\partial x} + u_i\frac{\partial u_i}{\partial x} - \frac{1}{m_i}\frac{\partial \widetilde{V}_{ii}}{\partial x}. \tag{48}$$

Here we introduce a ''quantum potential'' $\widetilde{V}_{ii}$ which is to include the effect of channel coupling as well as the usual

potential V_{ii}. The simplest way to achieve this equation is to define the ''mean balanced acceleration'' a_i through the ''mean (forward and backward) time derivative'' as usual, but for the stochastic process without any jumping process. We simply consider a stochastic process governed by Eq. (34) all the time, and denote X_i instead of x_i to distinguish them from each other. There is no mixing of dw_i and dw_j in X_i, contrary to x_i. For each $X_i(t)$ we define the ''mean balanced acceleration'' $a_i(X_i(t),t)$, and the ''Newton'' equations

$$m_i a_i(X_i(t),t) = -\frac{\partial \widetilde{V}_{ii}}{\partial X_i} \tag{49}$$

becomes Eq. (48).

The combination of equations $(33)+i(48)$ derives

$$\frac{\partial}{\partial x}\left[i\frac{\hbar}{m}\frac{1}{\psi_i'}\frac{\partial \psi_i'}{\partial t} + \frac{1}{2}\left(\frac{\hbar}{m_i}\right)^2 \frac{1}{\psi_i'}\frac{\partial^2 \psi_i'}{\partial x^2} - \frac{1}{m_i}\left\{ \widetilde{V}_{ii} - \frac{i\hbar}{2} W_{(i\to j)} \right\} \right] = 0, \tag{50}$$

where the relation

$$u_i + i v_j = \frac{\hbar}{m_i}\frac{1}{\psi_i'}\frac{\partial \psi_i'}{\partial x} \tag{51}$$

is used. If we shift the function ψ_i' to

$$\psi_i(x,t) = \psi_i'(x,t)\exp\left(-\frac{i m_i}{\hbar}\int^t \eta(s)\,ds \right), \tag{52}$$

choose the ''quantum potential'' as

$$\widetilde{V}_{ii} = V_{ii} + \mathrm{Re}\frac{\psi_i^* V_{ij}\psi_j}{|\psi_i|^2}, \tag{53}$$

and use the relation

$$W_{(i\to j)} = -\frac{2}{\hbar}\mathrm{Im}\frac{\psi_i^* V_{ij}\psi_j}{|\psi_i|^2}, \tag{54}$$

we can reproduce the Schrödinger equations (24). By the use of Eqs. (51) and (52), the relations

$$b_i(x,t) = \frac{\hbar}{m_i}(\mathrm{Im}+\mathrm{Re})\frac{\partial}{\partial x}\ln\psi_i(x,t), \tag{55}$$

$$b_{*i}(x,t) = \frac{\hbar}{m_i}(\mathrm{Im}-\mathrm{Re})\frac{\partial}{\partial x}\ln\psi_i(x,t), \tag{56}$$

and Eq. (26) are established again.

IV. STOCHASTIC FORMULATION FOR QUANTUM SYSTEM OF OPTICAL POTENTIAL

In this section, let us formulate Nelson's stochastic approach to a system of a single degree of freedom described by an optical potential. Then the Schrödinger equation with an imaginary part of the potential, denoted by iU (a physically relevant situation, i.e., an absorptive process corresponds to $U<0$), is written as

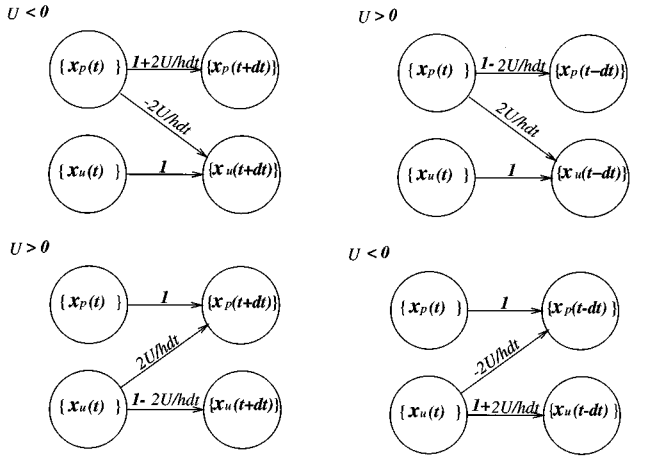

FIG. 2. Schematical illustration of the ''dynamical'' rule for the stochastic jumping process between physical and unphysical sector.

$$i\hbar\frac{\partial \psi(x,t)}{\partial t} = \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t) + iU(x,t) \right)\psi(x,t). \tag{57}$$

The formulation in Sec. III suggests a method to establish a stochastic formulation for this Schrödinger equation. The analogy between the channel-coupling model and the present model becomes apparent when we attempt the Fokker-Planck equation corresponding to Eq. (57) in the forms

$$\frac{\partial P(x,t)}{\partial t} = \left[-\frac{\partial}{\partial x}b + \frac{\hbar}{2m}\frac{\partial^2}{\partial x^2} + \frac{2U}{\hbar} \right]P(x,t) \quad (\text{forward in } t), \tag{58}$$

$$-\frac{\partial P(x,t)}{\partial t} = \left[\frac{\partial}{\partial x}b_* + \frac{\hbar}{2m}\frac{\partial^2}{\partial x^2} - \frac{2U}{\hbar} \right]P(x,t) \quad (\text{backward in } t). \tag{59}$$

Equations (58) and (59) are compared with Eqs. (27) and (28); both are quite similar to each other with the correspondence between $2U/\hbar$ and $-W_{(i\to j)}$.

While the sum of Eqs. (58) and (59) is given by Eq. (6), their difference leads to

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial x}(vP) + \frac{2U}{\hbar}P \tag{60}$$

instead of Eq. (8). From Eqs. (6) and (60) follows the kinematical equation

$$\frac{\partial u}{\partial t} = -\frac{\hbar}{2m}\frac{\partial^2 v}{\partial x^2} - \frac{\partial}{\partial x}(uv) + \frac{1}{m}\frac{\partial}{\partial x}U, \tag{61}$$

instead of Eq. (10).

The additional term in Eq. (58) simply describes production (absorption) effects for $U>0$ ($U<0$), which one may put in such a way that the production (absorption) process is a transition from an ''unphysical'' sector to a ''physical'' one (from a ''physical'' sector to an ''unphysical'' one). At this point the analogy between Sec. III and this section is

helpful to find stochastic processes equivalent to the Fokker-Planck equations in Eqs. (58) and (59): We consider the two random variables $x_p(t)$ and $x_u(t)$ for ''physical'' and ''unphysical'' sectors, respectively, and stochastic jumping between them occurs according to certain rules, which will be specified below. In contrast with the channel-coupling case with the index i, the stochastic differential equations for both of $x_p(t)$ and $x_u(t)$ can be common. Introducing a notation of a random variable $x(t)$ standing for both of $x_p(t)$ and $x_u(t)$, we require the same form of stochastic differential equations for this $x(t)$ as Eqs. (1) and (2) *all the time*,

$$\begin{aligned} dx(t) &= b(x(t),t)dt + dw(t) \quad \text{forward in time}, \\ dx(t) &= b_*(x(t),t)dt + dw_*(t) \quad \text{backward in time}, \end{aligned} \tag{62, 63}$$

with the same properties for $dw(t)$ as in Eq. (3), and so on. Each sample path is described by $x(t)$ as a whole, but has to be classified into either $x_p(t)$ or $x_u(t)$ at each t. Typically a sample path changes as, for example, $x_p(t_1) \to x_u(t_2) \to \cdots \to x_p(t_n)$ as a result of repeated jumping processes. A sample path is said to be physically relevant at t if the sample is represented by $x_p(t)$, while it is not so if it is represented by $x_u(t)$. In other words, the physical average at t is given by the average over ensemble of not all sample paths but only physically relevant sample paths at t. The notation $\langle\langle f(x(t))\rangle\rangle_{\{x_p(t)\}}$ is introduced to represent this conditional average for $f(x(t))$. Similarly the notations of other conditional averages such as $\langle\langle f(x(t))\rangle\rangle_{\{x_u(t)\}}$ and $\langle\langle f(x(t))\rangle\rangle_{\{x_p(t)\}\cup\{x_u(t)\}}$ are clear, in particular

$$\langle\langle f(x(t))\rangle\rangle_{\{x_p(t)\}\cup\{x_u(t)\}} = \langle f(x(t))\rangle. \tag{64}$$

Again, conditional averages related to many times can be introduced, e.g., $\langle\langle f(x(t))\rangle\rangle_{\{x_p(t+dt)\}\cap\{x_u(t)\}}$ is supposed to represent the average over all the sample paths which are described by x_u at t and x_p at $t+dt$.

Let us summarize the ''dynamical'' rule for stochastic jumping processes between p and u. The rules are given as follows (Fig. 2): (i) For the forward time direction, in the case of $U<0$,

$$\begin{aligned} x_p(t) &\to \begin{cases} x_u(t+dt) & \text{with the probability } -2U(x_p(t),t)/\hbar\ dt \\ x_p(t+dt) & \text{with the probability } 1+2U(x_p(t),t)/\hbar\ dt, \end{cases} \\ x_u(t) &\to x_u(t+dt) \quad \text{with the probability 1}, \end{aligned} \tag{65}$$

and, in the case of $U>0$,

$$\begin{aligned} x_u(t) &\to \begin{cases} x_p(t+dt) & \text{with the probability } 2U(x_u(t),t)/\hbar\ dt \\ x_u(t+dt) & \text{with the probability } 1-2U(x_u(t),t)/\hbar\ dt, \end{cases} \\ x_p(t) &\to x_p(t+dt) \quad \text{with the probability 1}. \end{aligned} \tag{66}$$

(ii) For the backward time direction, in the case of $U<0$,

$$\begin{aligned} x_u(t) &\to \begin{cases} x_p(t-dt) & \text{with the probability } -2U(x_u(t),t)/\hbar\ dt \\ x_u(t-dt) & \text{with the probability } 1+2U(x_u(t),t)/\hbar\ dt, \end{cases} \\ x_p(t) &\to x_p(t-dt) \quad \text{with the probability 1}, \end{aligned} \tag{67}$$

and, in the case of $U>0$,

$$\begin{aligned} x_p(t) &\to \begin{cases} x_u(t-dt) & \text{with the probability } 2U(x_p(t),t)/\hbar\ dt \\ x_p(t-dt) & \text{with the probability } 1-2U(x_p(t),t)/\hbar\ dt, \end{cases} \\ x_u(t) &\to x_u(t-dt) \quad \text{with the probability 1}. \end{aligned} \tag{68}$$

Note that for the forward time direction a jumping process from x_p to x_u is allowed, and the reverse process is forbidden where $U<0$, and vice versa where $U>0$, and that when U is nonpositive everywhere, the number of sample paths described $x_p(t)$ decreases, and that in $x_u(t)$ increases as t goes, the total number being conserved. Regardless of the indices of p and u, each sample path is a stochastic process described by Eq. (62) [or Eq. (63)].

To prove the equivalence between the Fokker-Planck equation (58) and the stochastic differential equation (62) with the jumping rules (65) and (66), we calculate, for example,

$$\frac{d\langle\langle f(x(t))\rangle\rangle_{\{x_p(t)\}}}{dt}=\frac{1}{dt}[\langle\langle f(x(t+dt))\rangle\rangle_{\{x_p(t+dt)\}}-\langle\langle f(x(t))\rangle\rangle_{\{x_p(t)\}}]$$
$$=\frac{1}{dt}[\langle\langle f(x(t+dt))-f(x(t))\rangle\rangle_{\{x_p(t+dt)\}\cap\{x_p(t)\}}+\langle\langle f(x(t+dt))\rangle\rangle_{\{x_p(t+dt)\}\cap\{x_u(t)\}}-\langle\langle f(x(t))\rangle\rangle_{\{x_u(t+dt)\}\cap\{x_p(t)\}}], \quad (69)$$

with

$$\langle\langle f(x(t+dt))-f(x(t))\rangle\rangle_{\{x_p(t+dt)\}\cap\{x_p(t)\}}=dt\int dx\ f(x)\left(-\frac{\partial}{\partial x}b(x,t)+\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2}\right)P(x,t)+o(dt^2), \quad (70)$$

$$\langle\langle f(x(t+dt))\rangle\rangle_{\{x_p(t+dt)\}\cap\{x_u(t)\}}=dt\int dx\ f(x)\frac{2U(x,t)}{\hbar}P(x,t)\theta(U(x,t))+o(dt^2), \quad (71)$$

and

$$\langle\langle f(x(t))\rangle\rangle_{\{x_u(t+dt)\}\cap\{x_p(t)\}}=-dt\int dx\ f(x)\frac{2U(x,t)}{\hbar}P(x,t)\theta(-U(x,t))+o(dt^2). \quad (72)$$

These equations (69)–(72) follow Eq. (58). The equivalence between the Fokker-Planck approach and the approach of the stochastic differential equation (62) with the stochastic jumping processes (65) and (66) has been shown for the forward direction. Similarly the equivalence between the two approaches can be proven for the backward time direction.

As for the dynamical condition, we do not modify the original Nelson's formulation. When the mean time derivatives $Df(t)$ and $D_*f(t)$ are concerned, there may be some ambiguity with respect to the taking expectation. Here we will follow the argument given above, Eq. (49). We define the ''mean balanced acceleration'' through the ''mean time derivatives'' as usual, but for the stochastic process without any jumping process. We simply consider a stochastic process governed by Eqs. (62) and (63) at all times. This leads to the ''Newton-Nelson equation'' in Eq. (16) in the present case.

The combination of the equations (61)$+i$(16) leads to

$$\frac{\partial}{\partial x}\left[i\frac{\hbar}{m}\frac{1}{\psi'}\frac{\partial\psi'}{\partial t}+\frac{1}{2}\left(\frac{\hbar}{m}\right)^2\frac{1}{\psi'}\frac{\partial^2\psi'}{\partial x^2}-\frac{1}{m}(V+iU)\right]=0, \quad (73)$$

where relation (18) is used. Again the relation between ψ' and the solution of Eq. (57) ψ is given as

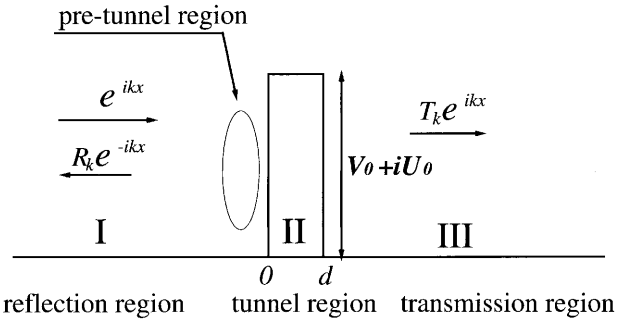

FIG. 3. Schematical illustration of one-dimensional optical barrier tunneling.

$$\psi=\psi'\exp\left(-\frac{im}{\hbar}\int^t\eta(s)ds\right) \quad (74)$$

and

$$b(x,t)=\frac{\hbar}{m}(\mathrm{Im}+\mathrm{Re})\frac{\partial}{\partial x}\ln\psi(x,t), \quad (75)$$

$$b_*(x,t)=\frac{\hbar}{m}(\mathrm{Im}-\mathrm{Re})\frac{\partial}{\partial x}\ln\psi(x,t), \quad (76)$$

$$P(x,t)=|\psi(x,t)|^2 \quad (77)$$

are satisfied.

V. NUMERICAL ANALYSIS

Now we can perform a numerical analysis of the effects of the optical potential and channel coupling on the tunneling time, using above generalized Nelson's approach. First, we discuss one-dimensional system with a static square-well optical potential,

$$V(x)=\begin{cases}0 & \text{in I} \quad (x<0)\\ V_0-iU_0 & \text{in II} \quad (0<x<d)\\ 0 & \text{in III} \quad (d<x)\end{cases} \quad (78)$$

(Fig. 3). We set the solution of the Schrödinger equation,

$$i\hbar\frac{\partial}{\partial t}\psi(x,t)=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+V(x)\right]\psi(x,t), \quad (79)$$

as

$$\psi(x,t)=\int_{-\infty}^{\infty}A(k)\varphi_k(x)e^{-i(E/\hbar)t}dk, \quad (80)$$

with a coefficient function $A(k)$ and $E=\hbar^2k^2/2m$. It is well known that $\varphi_k(x)$ is written as

$$\varphi_k(x)=\begin{cases}e^{ikx}+R_ke^{-ikx} & \text{in I}\\ C_ke^{\kappa x}+D_ke^{-\kappa x} & \text{in II}\\ T_ke^{ikx} & \text{in III,}\end{cases} \quad (81)$$

where

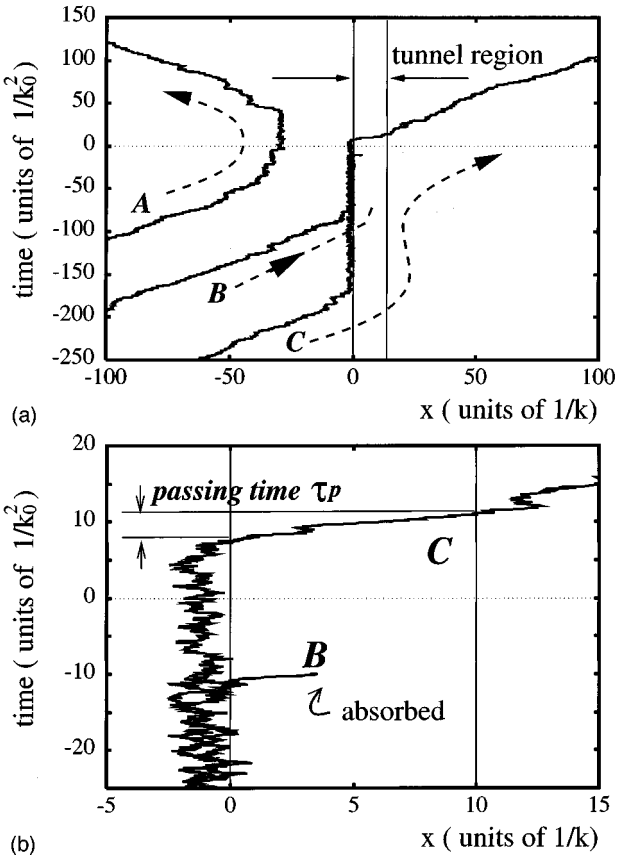

FIG. 4. (a) and (b) Three typical sample paths in the optical potential case. (b) is an enlarged version of (a) ($m=\hbar=1$).

$$\kappa=\frac{\sqrt{2m(V_0-iU_0-E)}}{\hbar}=\kappa_R-i\kappa_I \quad (\kappa_I>0), \tag{82}$$

$$\left(\kappa_0=\frac{\sqrt{2m(V_0-iU_0-E_0)}}{\hbar}=\kappa_{R0}-i\kappa_{I0} \quad (\kappa_{I0}>0)\right), \tag{83}$$

and R_k, T_k, C_k, and D_k are given as

$$\begin{Bmatrix} R_k \\ T_k \\ C_k \\ D_k \end{Bmatrix} = \mathcal{B}\times \begin{Bmatrix} -i(\kappa^2+k^2)\sinh\kappa d \\ 2k\kappa e^{-ikd} \\ k(\kappa+ik)e^{-\kappa d} \\ k(\kappa-ik)e^{\kappa d} \end{Bmatrix} \tag{84}$$

$$\mathcal{B}=\frac{1}{2k\kappa\cosh\kappa d+i(\kappa^2-k^2)\sinh\kappa d}. \tag{85}$$

We take a Gaussian form with its center at $k=k_0$, or

$$A(k)=A_{k_0}(k)=C\exp\left\{-\frac{(k_0-k)^2}{4\sigma^2}\right\}, \tag{86}$$

with a normalization constant C. Here we put $\sigma=k_0/100$ and $V_0=5E_0=(\hbar k_0/2m)^2$. Using this solution, we numerically calculate Eqs. (62), (63) and (65)–(68) .

Figure 4 shows the three typical sample paths calculated by Eqs. (62), (65), and (66). There is a sample path $x(t)$ which changes its property from ''physical'' to ''unphysical'' in the tunnel region.

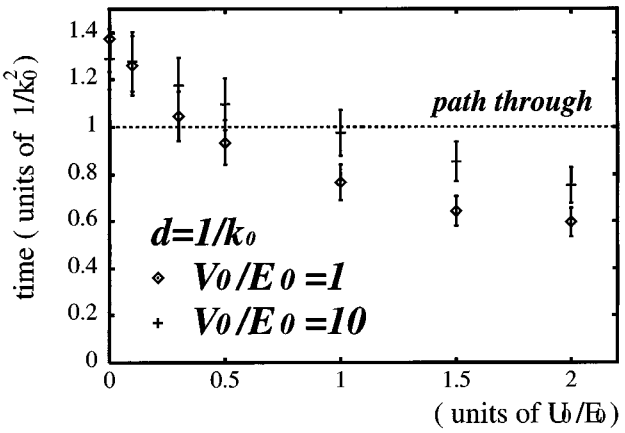

FIG. 5. The mean value of τ_p vs U_0/E_0 (thin potential cases).

Figures 5 and 6 show the parameter U_0/E_0 vs the average of passing time τ_p, calculated by Eqs. (63), (67), and (68). See the details of this ''backward time evolution method'' in our previous work [20]. Generally, τ_p decrease as U_0/E_0 become larger. Let us estimate τ_p analytically on the WKB-like approximation. If we can write the wave function in the tunnel region II as

$$\psi(x,t)\sim\psi(x)\sim C'\exp(-\kappa_0 x)=\exp\{-(\kappa_{R0}-i\kappa_{I0})x\}, \tag{87}$$

the drift of Eq. (63) becomes

$$b_*=\frac{\hbar}{m}(\kappa_{I0}+\kappa_{R0})\sim\frac{\hbar\overline{\kappa_0}}{m}\left[1+\frac{\kappa_{I0}}{\kappa_{R0}}+o^2\left(\frac{\kappa_{I0}}{\kappa_{R0}}\right)\right],$$

$$\overline{\kappa_0}=\frac{\sqrt{2m(V_0-E_0)}}{\hbar} \tag{88}$$

from Eq. (76). In these cases, the ''backward'' time evolution of the distribution function $P_T(x,t)$, which has an ''initial'' distribution $\delta(x-d)$, is written as

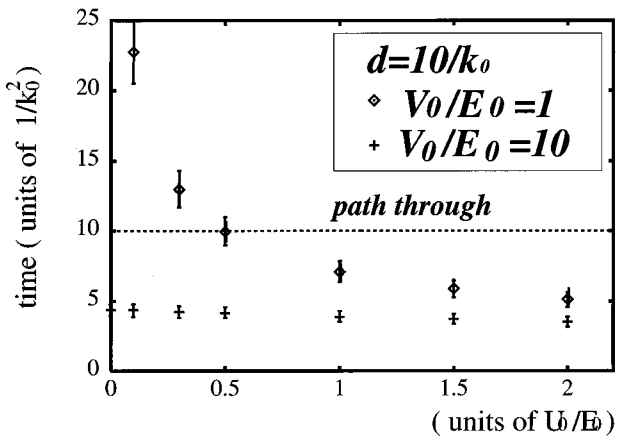

FIG. 6. The mean value of τ_p vs U_0/E_0 (thick potential cases).

$$\frac{\partial P_T(x,t)}{\partial t} = -\left[\frac{\hbar}{m}\overline{\kappa_0}\left(1+\frac{\kappa_{I0}}{\kappa_{R0}}\right)\frac{\partial}{\partial x}+\frac{\hbar}{2m}\frac{\partial^2}{\partial x^2} - \frac{2}{\hbar}U_0\right]P_T(x,t), \tag{89}$$

and we can obtain the solution of Eq. (89) easily:

$$P_T(x,t)=\left(\frac{2m\pi}{-\hbar t}\right)^{1/2}\exp\left\{\frac{\left(x-d+\dfrac{\hbar\overline{\kappa_0}\left(1+\dfrac{\kappa_{I0}}{\kappa_{R0}}\right)}{m}t\right)^2}{\dfrac{\hbar t}{2m}} - \frac{2U_0}{\hbar}t\right\} \quad (t<0). \tag{90}$$

There are two characteristic time intervals in this solution. One is the diffusion time $t_d \sim md^2/\hbar$ for which the distribution sizes up to the potential width d. The other is the current time

$$t_c \sim \frac{md}{\hbar\overline{\kappa_0}\left(1+\dfrac{\kappa_{I0}}{\kappa_{R0}}\right)} \sim \frac{md}{\hbar\overline{\kappa_0}}\left(1-\frac{\kappa_{I0}}{\kappa_{R0}}\right), \tag{91}$$

for which the peak of the distribution moves from $x=d$ to $x=0$. Of course, the approximation of Eq. (87) is justified when $\overline{\kappa_0}d$ is much larger than 1, and this leads us to the relation

$$t_d \gg t_c, \tag{92}$$

and the time interval t_c becomes the passing time in this extreme case. Note that this t_c has the tendency of decreasing as U_0/E_0 becomes larger.

Second, we discuss a one-dimensional system with a static square-well potential and two-channel coupling, or the case of the Schrödinger equation for this problem written as

$$i\frac{\partial}{\partial t}\begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix} = \begin{bmatrix} -\dfrac{1}{2m}\dfrac{\partial^2}{\partial x^2}+V & U \\ U & -\dfrac{1}{2m}\dfrac{\partial^2}{\partial x^2}+V\end{bmatrix}\begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix}. \tag{93}$$

V and U are supposed to be

$$V(x)=\begin{cases} 0 & \text{in I} \quad (x<0)\\ V_0 & \text{in II} \quad (0<x<d)\\ 0 & \text{in III} \quad (d<x).\end{cases} \tag{94}$$

and

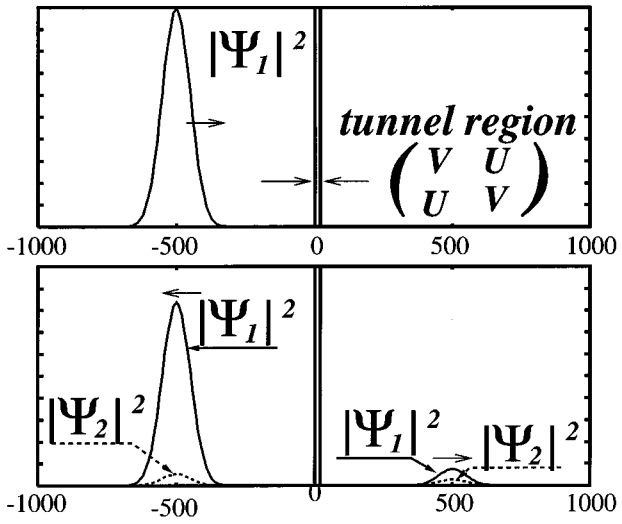

FIG. 7. Schematical illustration of one-dimensional scattering with channel coupling.

$$U(x)=\begin{cases} 0 & \text{in I} \quad (x<0)\\ U_0 & \text{in II} \quad (0<x<d)\\ 0 & \text{in III} \quad (d<x).\end{cases} \tag{95}$$

Figure 7 shows the schematical illustration of our simulation. We can diagonalize Eq. (93) as

$$i\frac{\partial}{\partial t}\begin{bmatrix}\psi_+\\ \psi_-\end{bmatrix} = \begin{bmatrix} -\dfrac{1}{2m}\dfrac{\partial^2}{\partial x^2}+V+U & 0 \\ 0 & -\dfrac{1}{2m}\dfrac{\partial^2}{\partial x^2}+V-U\end{bmatrix}\begin{bmatrix}\psi_+\\ \psi_-\end{bmatrix}, \tag{96}$$

where

$$\psi_+=\frac{1}{\sqrt{2}}(\psi_1+\psi_2) \quad \text{and} \quad \psi_-=\frac{1}{\sqrt{2}}(\psi_1-\psi_2), \tag{97}$$

and write down the time-dependent solution of ψ_+ and ψ_- easily as the same as Eq. (80), or

$$\psi_\pm(x,t)=\int_{-\infty}^{\infty}A(k)\varphi_{\pm k}(x)e^{-i(E/\hbar)t}dk, \tag{98}$$

with a Gaussian coefficient function $A(k)$ and $E=\hbar^2k^2/2m$. $\varphi_{\pm k}(x)$ is Eq. (81), substituting κ with

$$\kappa_\pm=\frac{\sqrt{2m(V_0\pm U_0-E)}}{\hbar}. \tag{99}$$

Figure 8 shows the same typical sample paths calculated by Eqs. (34), (37), and (38). There is a path which changes its index from 1 to 2 in the passage through the tunneling region, $t_{i=1,2}$. Figures 9 and 10 are the averages of the passing times over the sample paths which belong to $\{x_i(t)\}$ at

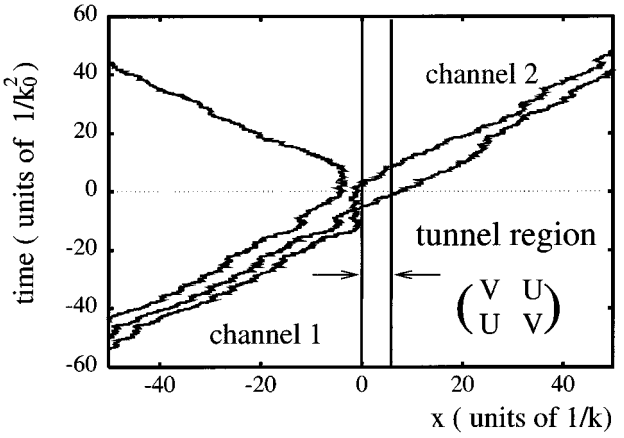

FIG. 8. Three typical sample paths with channel coupling.

$t\to\infty$. We can see in Fig. 10 that there is a critical parameter value of $(V_0-U_0)/E_0=1$ in the behavior of t_1 and t_2. This is understood as following: In the case of $(V_0-U_0)/E_0>1$, the ''$-$'' channel, which is dominant in the tunnel region II in comparison with ''$+$'' one, is not the tunneling channel, and it describes a particle which goes over the potential. Regardless of $x_1(t)$ and $x_2(t)$, the time spent in the ''potential region'' is expected to agree with the one which is expected from classical mechanics or

$$\frac{md}{\hbar k_{-0}} \quad \text{where} \quad k_{-0}=\frac{\sqrt{2m(E_0-V_0+U_0)}}{\hbar}. \tag{100}$$

This is also seen in Fig. 9. On the other hand, in the case of $(V_0-U_0)/E_0<1$, we can approximate the wave functions ψ_+ and ψ_- in the thick tunnel region

$$\begin{bmatrix}\psi_+\\ \psi_-\end{bmatrix}\sim\begin{bmatrix}0\\ C_-\exp(-\kappa_{-0}\ x)\end{bmatrix}, \tag{101}$$

and ψ_1 and ψ_2 as

$$\begin{bmatrix}\psi_1\\ \psi_2\end{bmatrix}\sim\frac{1}{\sqrt{2}}\begin{bmatrix}C_-\exp(-\kappa_{-0}\ x)\\ -C_-\exp(-\kappa_{-0}\ x)\end{bmatrix}, \tag{102}$$

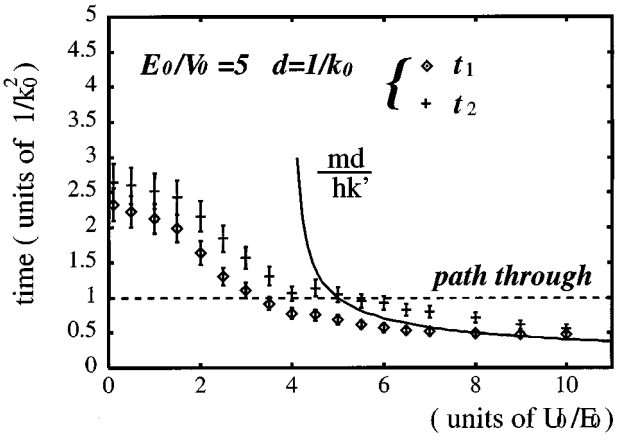

FIG. 9. The mean values of t_1 and t_2 vs U_0/E_0 (thin potential cases).

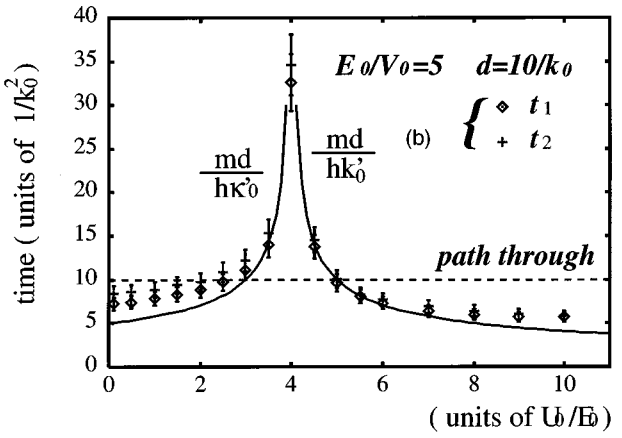

FIG. 10. The mean values of t_1 and t_2 vs U_0/E_0 (thick potential cases).

where

$$\kappa_{-0}=\frac{\sqrt{2m(V_0-U_0-E_0)}}{\hbar}. \tag{103}$$

So we can estimate ''passing time'' of both channels (1 and 2) at

$$\frac{md}{\hbar\kappa_{-0}}, \tag{104}$$

and likewise Eq. (91).

VI. SUMMARY AND COMMENTS

In this paper, we have analyzed the effects of inelastic scattering on the tunneling time theoretically, using generalized Nelson's quantum mechanics. This generalization enabled us to describe quantum system with optical potential and channel couplings in a real-time stochastic approach. In this formalism, the space-time development of dynamical variable, e.g., the coordinate of the particle, is described by a definite path determined stochastically. Each sample path has a definite form of trajectory in the space-time diagram, while a physical quantity averaged over the ensemble of these sample paths recovers the effect of quantum coherence. This is true even in Young's double slits interference experiment. Nelson's quantum mechanics gives each definite trajectory, and the ensemble of it, but it does not predict which path is selected when one wants to measure the position of a particle. In this sense, this ''real-time stochastic process approach'' seems to give us a new insight into quantum mechanics beyond the Copenhagen interpretation. On the other hand, the effects of more general cases (many-body systems, environment, temperature, and so on) are subjects for the future, and this work would be the first step to such a study. Recent experimental data of tunneling time using the neutron spin-echo shift through the magnetic films [19] seem to agree with the simulation based on our approach [21], and this study will be reported in near future.

ACKNOWLEDGMENTS

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[1] E. P. Wigner, Phys. Rev. **98**, 145 (1955).
[2] M. Büttiker and R. Landauer, Phys. Rev. Lett. **49**, 1739 (1982).
[3] H. W. Lee and M. O. Scully, Found. Phys. **13**, 61 (1983).
[4] M. Büttiker, Phys. Rev. B **27**, 6178 (1983).
[5] D. Sokolvski and L. M. Baskin, Phys. Rev. A **36**, 4604 (1987).
[6] H. A. Fertig, Phys. Rev. Lett. **65**, 2321 (1990).
[7] Ming-Quey Chen and M. S. Wang, Phys. Lett. A **149**, 441 (1990).
[8] D. Sokolovski and J. N. L. Connor, Phys. Rev. A **47**, 4677 (1993).
[9] V. S. Olkhovsky and E. Recami, Phys. Rep. **214**, 339 (1992).
[10] C. R. Leavens and G. C. Aers, in *Scanning Tunneling Microscopy III*, edited by R. Wiesendanger and H.-J. Güntherodt (Springer-Verlag, Berlin, 1993).
[11] T. Martin and R. Landauer, Phys. Rev. A **47**, 2023 (1993).
[12] C. R. Leavens and W. R. McKinnon, Phys. Lett. A **194**, 12 (1994).
[13] S. Brouard, R. Sala, and J. G. Muga, Phys. Rev. A **49**, 4312 (1994).
[14] A. M. Steinberg, Phys. Rev. Lett. **74**, 2405 (1995).
[15] S. Collins, D. Lowe, and J. R. Barker, J. Phys. C **20**, 6213 (1987).
[16] E. H. Hauge and J. A. Støvneng, Rev. Mod. Phys. **61**, 917 (1989).
[17] R. Landauer and Th. Martin, Rev. Mod. Phys. **66**, 217 (1994).
[18] E. Nelson, Phys. Rev. **150**, 1079 (1966).
[19] M. Hino, N. Achiwa, S. Tsaki, T. Ebisawa, T. Akiyoshi, and T. Kawai, J. Phys. Soc. Jpn., Suppl. A, **65**, 203 (1996).
[20] K. Imafuku, I. Ohba, and Y. Yamanaka, Phys. Lett. A **204**, 329 (1995).
[21] I. Ohba, K. Imafuku and Y. Yamanaka, J. Phys. Soc. Jpn., Suppl. A **65**, 41 (1996).