Optical measurements of the condensate phase

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We propose a light-scattering scheme to measure the relative phase of atomic Bose-Einstein condensates and its diffusion rate. The proposal relies on the existence of two independent condensates coupled to a common excited state. To this end, we consider a two-well ground-state potential together with excited-state trap wave functions that extend over the whole region. When the first trap is driven by a weak monochromatic laser field, the light scattered from the second trap has a nonzero mean-electric field amplitude with a phase shift proportional to the difference of the condensate phases. When both condensates are driven, the phases of the two laser fields can be adjusted to cancel the scattering completely by a quantum interference. The particular value of the laser phase difference that gives zero scattering determines the relative phase of the two condensates. [S1050-2947(97)50102-8]

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The impressive progress in laser and evaporative cooling of atoms has recently resulted in the experimental realization of a Bose-Einstein condensate (BEC) of cold rubidium [1], lithium [2], and sodium [3] atoms. These experiments have established that a macroscopic number of atoms occupy the lowest-energy quantum state of an atom trap, without providing any further information about the condensate wave function. The majority of theoretical work on the BEC in a weakly interacting Bose gas utilizes mean-field theory [4,5], where the ground state of the condensate is predicted to be in a coherent state with a well-defined phase. Many interesting physical phenomena, such as superfluidity and the Josephson effect, are associated with the phase of the condensate. It is therefore interesting to measure and monitor the time evolution of the condensate phase in atomic BEC experiments using a nondestructive method.

We propose two related schemes for measuring the phase difference between two independent condensates. Both schemes rely on the fact that coherent light is generated by an absorption-emission cycle between initial- and final-state condensates. Our goal is to illustrate in principle how these generic schemes give information on the relative phase of two condensates. To be concrete we will present an analysis of what is probably the theoretically simplest configuration. The limitations that are necessary to impose along the way, such as a relatively stable excited state, suggest alternative configurations that may be more practical. However, as these are not different in principle, we choose the simplest setup as a basis for our discussion.

We envisage a two-well, spatially separated, ground-state potential along with a single-well excited-state potential that extends over the regions R_1 and R_2 of both ground-state potentials (Fig. 1). Non-overlapping probe laser fields are used to weakly excite the condensates in either trap. As described by Ketterle and co-workers, the use of a strong blue-detuned laser (coupled to an atomic state $|f_1\rangle$) would realize the described ground state potentials [3]. The excited state $|e\rangle$ of the probe transitions must be relatively stable so that the laser-excited atoms have enough time to travel from the region of one (ground-state) potential well to the other; this is the primary criterion that we must fulfill. How can this be arranged? The excited-state potential could be formed by using the dipole force from an additional laser that couples $|e\rangle$ to another excited level $|f_2\rangle$, with a red detuning. Excited atoms created in one trap would then be accelerated along the z axis that connects the two ground-state minima. The trap frequency is controlled by the intensity of the additional laser. We require that the resulting excited-state trap frequency ω_z ($\gg\omega_x,\omega_y$) exceed the effective linewidth of the excited state $|e\rangle$ and assume that the probe fields (near) resonantly couple the two condensates to excited trap states with the same q_z quantum number, where $\mathbf{q}=(q_x,q_y,q_z)$ labels the trap eigenstates. Clearly, if $|e\rangle$ is an electronically excited state, its natural linewidth is going to provide a lower bound on ω_z. Alternatively, if $|e\rangle$ were stable, for example, another hyperfine level of the ground state, this would be ideal. Such a state could be excited from the condensate by resonant two-photon Raman transitions, and this suggests an

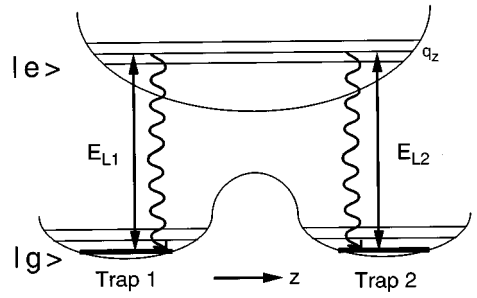

FIG. 1. The ground- and excited-state trap potentials that we envision. The ground-state trap minima are assumed to be separated by at least a few optical wavelengths, so that each trap may be accessed independently by two laser fields. To satisfy the long lifetime requirement for the excited state $|e\rangle$, it might in practice be necessary to use two-photon transitions.

alternative configuration in which the excitation and deexcitation stages of the cycle between the condensates are each made using Raman transitions. The intermediate propagation between condensates takes place in a stable hyperfine level with acceleration provided by dipole forces. In this case, we have a field tunable excited-state lifetime that eliminates the long (single-photon) lifetime requirement and enables us to keep the medium optically thin.

Returning to our basic model, we assume that only a single Zeeman sublevel is involved in both ground and excited levels. In general, two weak, resonant monochromatic light fields $\mathbf{E}_{L1}(\mathbf{r})$ and $\mathbf{E}_{L2}(\mathbf{r})$ excite the two wells, which we label 1 and 2, respectively; these classical fields do not overlap spatially. We start from the Hamiltonian

$$\hat{H}=\int d^3r\Bigg[\sum_{i=1,2}\hat{\psi}_{gi}^{\dagger}(\mathbf{r})H_{c.m.}^{g,i}(\mathbf{r})\hat{\psi}_{gi}(\mathbf{r})+\hat{\mathcal{H}}_{gg}+\hat{\psi}_e^{\dagger}(\mathbf{r})(H_{c.m.}^{e}(\mathbf{r})+\hbar\omega_0)\hat{\psi}_e(\mathbf{r})-\sum_{i=1,2}\mathbf{D}^{(+)}\cdot\hat{\mathcal{E}}(\mathbf{r})\hat{\psi}_e^{\dagger}(\mathbf{r})\hat{\psi}_{gi}(\mathbf{r})+\text{H.c.}\Bigg], \tag{1}$$

where $H_{c.m.}^{g}(\mathbf{r})$ and $H_{c.m.}^{e}(\mathbf{r})$ denote the center-of-mass (trap) single-particle Hamiltonians for the ground- and excited-state atoms, respectively. $\hat{\mathcal{H}}_{gg}$ is the Hamiltonian density for the interactions between ground-state atoms. $\mathbf{D}=\mathbf{D}^{(+)}+\mathbf{D}^{(-)}$ denotes the dipole moment and $\hat{\mathcal{E}}(\mathbf{r})$ is the electric-field operator. The atomic transition and laser frequencies are given by ω_0 and ω_L, respectively. $\hat{\psi}_{gi}(\mathbf{r})$ denotes the ground-state field annihilation operator for the ith potential well ($i=1,2$). For simplicity, we assumed a single-photon coupling between the ground- and excited-state atoms in Eq. (1); extension to the practically more important two-photon transitions (described above) is straightforward.

We introduce slowly varying field operators by setting $\hat{\psi}_g=\hat{\Psi}_g$, $\hat{\psi}_e=\hat{\Psi}_e e^{-i\omega_L t}$, and $\hat{\mathcal{E}}=\hat{\mathbf{E}}^{(+)}e^{-i\omega_L t}+\hat{\mathbf{E}}^{(-)}e^{i\omega_L t}$. The atomic field operators in the rotating frame obey the Heisenberg equations

$$\dot{\hat{\Psi}}_{gi}=-\frac{i}{\hbar}H_{c.m.}^{g}\hat{\Psi}_{gi}+\frac{i}{\hbar}\mathbf{D}^{-}\cdot\hat{\mathbf{E}}^{-}\hat{\Psi}_e-\frac{i}{\hbar}[\hat{\Psi}_{gi},\hat{\mathcal{H}}_{gg}]$$

$$\dot{\hat{\Psi}}_e=-i\left(\omega_0-\omega_L+\frac{H_{c.m.}^{e}}{\hbar}\right)\hat{\Psi}_e+\frac{i}{\hbar}\sum_{i=1,2}\mathbf{D}^{+}\cdot\hat{\mathbf{E}}^{+}\hat{\Psi}_{gi}, \tag{2}$$

where a rotating-wave approximation on the electric field has been made. Following Javanainen [6,7], we express the scattered field amplitude in terms of the atomic field operators

$$\hat{\mathbf{E}}_{sc}^{+}(\mathbf{r},t)=\int d^3r'\Bigg[\mathbf{K}\left(\mathbf{r},\mathbf{r}'-\frac{\mathbf{d}}{2}\right)\hat{\Psi}_{g1}^{\dagger}\left(\mathbf{r}'-\frac{\mathbf{d}}{2},t_{ret}\right)\times\hat{\Psi}_e\left(\mathbf{r}'-\frac{\mathbf{d}}{2},t_{ret}\right)+\mathbf{K}\left(\mathbf{r},\mathbf{r}'+\frac{\mathbf{d}}{2}\right)\times\hat{\Psi}_{g2}^{\dagger}\left(\mathbf{r}'+\frac{\mathbf{d}}{2},t_{ret}\right)\hat{\Psi}_e\left(\mathbf{r}'+\frac{\mathbf{d}}{2},t_{ret}\right)\Bigg], \tag{3}$$

where $\mathbf{K}(\mathbf{r},\mathbf{r}')$ is the appropriate propagation kernel for the configuration of two ground-state traps [7], t_{ret} is the retarded time, $\mathbf{d}$ is the trap separation, and the origin is chosen midway between the traps. The integration is taken over the volume of the respective traps. The next step is to obtain an expression for the excited-state operators that is correct to first order in the incident classical fields. As stated before, we are interested in resonant coupling to a set of quasimetastable excited states $|e,q_x,q_y,q_z\rangle$, all with the same q_z value; in this case, it is convenient to work with mode operators for the excited-state trap potential $\hat{\Psi}_e(\mathbf{r},t)=\Sigma_{\mathbf{q}}\phi_{\mathbf{q}}^{e}(\mathbf{r})\hat{a}_{\mathbf{q}}(t)$. Since we are interested in near-resonance conditions, the lifetime broadening of the excited state needs to be included in the effective detuning $\Delta\omega_{\mathbf{q}}=(\omega_0+\omega_{\mathbf{q}}-\omega_L)-i\Gamma_{\mathbf{q},eff}/2$, where $\Gamma_{\mathbf{q},eff}$ is the Bose-enhanced collective linewidth of the $\mathbf{q}$th trap eigenstate. We remark that $\Gamma_{\mathbf{q},eff}$ depends on the particular trap state under consideration via the overlap integral between $\phi_{\mathbf{q}}^{e}(\mathbf{r})$ and the condensed ground-state (trap) wave function. Equation (2) can then be integrated to give

$$\hat{a}_{\mathbf{q}}(t)\approx\frac{i}{\hbar}\int_0^t dt''e^{-i\Delta\omega_{\mathbf{q}}(t-t'')}\times\int d^3r''\mathbf{D}^{+}\cdot\Bigg[\phi_{\mathbf{q}}^{e*}\left(\mathbf{r}''-\frac{\mathbf{d}}{2}\right)\mathbf{E}_{L1}^{+}\left(\mathbf{r}''-\frac{\mathbf{d}}{2}\right)\times\hat{\Psi}_{g1}\left(\mathbf{r}''-\frac{\mathbf{d}}{2}\right)+\phi_{\mathbf{q}}^{e*}\left(\mathbf{r}''+\frac{\mathbf{d}}{2}\right)\mathbf{E}_{L2}^{+}\left(\mathbf{r}''+\frac{\mathbf{d}}{2}\right)\times\hat{\Psi}_{g2}\left(\mathbf{r}''+\frac{\mathbf{d}}{2}\right)\Bigg] \tag{4}$$

where we have ignored the contribution from the initial value of the mode operator (Langevin noise term). The expansion used to obtain Eq. (4) is valid as long as the medium remains optically thin; we will discuss the validity of this assumption shortly.

We now substitute Eq. (4) back into Eq. (3) to obtain our principal result,

$$\hat{\mathbf{E}}_{sc}^{+}(\mathbf{r},t)=\frac{i}{\hbar}\sum_{\mathbf{q}}\int d^3r'\int d^3r''\int_0^{t_{ret}}dt''e^{-i\Delta\omega_{\mathbf{q}}(t_{ret}-t'')}\times\Bigg\{\Bigg[\mathbf{k}_{\mathbf{q}}^{e}\left(\mathbf{r},\mathbf{r}'-\frac{\mathbf{d}}{2},\mathbf{r}''-\frac{\mathbf{d}}{2}\right)\times\hat{\Psi}_{g1}^{\dagger}\left(\mathbf{r}'-\frac{\mathbf{d}}{2},t_{ret}\right)\hat{\Psi}_{g1}\left(\mathbf{r}''-\frac{\mathbf{d}}{2},t''\right)+\mathbf{k}_{\mathbf{q}}^{e}\left(\mathbf{r},\mathbf{r}'+\frac{\mathbf{d}}{2},\mathbf{r}''-\frac{\mathbf{d}}{2}\right)\times\hat{\Psi}_{g2}^{\dagger}\left(\mathbf{r}'+\frac{\mathbf{d}}{2},t_{ret}\right)\hat{\Psi}_{g1}\left(\mathbf{r}''-\frac{\mathbf{d}}{2},t''\right)\Bigg]\times\mathbf{D}^{+}\cdot\mathbf{E}_{L1}^{+}\left(\mathbf{r}''-\frac{\mathbf{d}}{2}\right)+[1\leftrightarrow2,\mathbf{d}\leftrightarrow-\mathbf{d}]\Bigg\}, \tag{5}$$

where the last term in parentheses is obtained by interchanging the subscripts 1 and 2, and $\mathbf{d}\leftrightarrow-\mathbf{d}$ in the previous expression. For compactness we have defined

$$\mathbf{k}_{\mathbf{q}}^{e}(\mathbf{r},\mathbf{r}',\mathbf{r}'')\equiv\phi_{\mathbf{q}}^{e}(\mathbf{r}')\phi_{\mathbf{q}}^{e*}(\mathbf{r}'')\mathbf{K}(\mathbf{r},\mathbf{r}'). \tag{6}$$

The quantum interference and nonlocal correlations between the two condensates are apparent in Eq. (5). If retardation effects are negligible one may set $t_{ret}=t$ [7]. We note that the excited-state potential is likely to be strongly asymmetric with $\omega_x,\omega_y\ll\Gamma_{\mathbf{q},eff}$, which implies that many q_x,q_y states will contribute to the scattered field. The scattering therefore carries information about nonlocal atomic correlations along the z axis only.

When the phase diffusion rate of the condensate atoms is slow on the scale of the inverse effective detuning $\Delta\omega_{\mathbf{q}}^{-1}$ an adiabatic elimination of the excited-state atoms may be carried out. In this limit, $t''\rightarrow t_{ret}$ and $\int_0^{t_{ret}}dt''e^{-i\Delta\omega_{\mathbf{q}}(t_{ret}-t'')}\rightarrow 1/\Delta\omega_{\mathbf{q}}$ in Eq. (5). We will assume that such an adiabatic elimination is valid in order to describe the light-scattering schemes we propose:

(a) *Excitation of trap 1 only* ($\mathbf{E}_{L2}\equiv\mathbf{0}$). We assume that laser 1 is used to excite trap 1 only, and only the photons emitted from trap 2 are collected. In terms of Eq. (5) we may formally put terms that depend on creating ground-state atoms in trap 1 equal to zero. The part of the scattered field that is detected is thus given by

$$\hat{\mathbf{E}}_{sc,2}^{+}(\mathbf{r},t)=\int d^3r'\int d^3r''\,\overleftrightarrow{\mathbf{A}}\left(\mathbf{r},\mathbf{r}'+\frac{\mathbf{d}}{2},\mathbf{r}''-\frac{\mathbf{d}}{2}\right)\cdot\mathbf{E}_{L1}^{+}\left(\mathbf{r}''-\frac{\mathbf{d}}{2}\right)\hat{\Psi}_{g2}^{\dagger}\left(\mathbf{r}'+\frac{\mathbf{d}}{2},t\right)\hat{\Psi}_{g1}\left(\mathbf{r}''-\frac{\mathbf{d}}{2},t\right), \tag{7}$$

where the tensor

$$\overleftrightarrow{\mathbf{A}}(\mathbf{r},\mathbf{r}',\mathbf{r}'')\equiv\sum_{\mathbf{q}}\frac{\mathbf{K}(\mathbf{r},\mathbf{r}')}{\hbar\Delta\omega_{\mathbf{q}}}\mathbf{D}^{+}\phi_{\mathbf{q}}^{e}(\mathbf{r}')\phi_{\mathbf{q}}^{e*}(\mathbf{r}''). \tag{8}$$

In contrast to scattering from independent atoms, the detected scattered field $\langle\hat{\mathbf{E}}_{sc,2}^{+}(\mathbf{r},t)\rangle$ has a nonzero mean value that is determined by the phase difference of the two condensates. Using the exciting laser field as the *local oscillator*, either homodyne or heterodyne measurements may be used to measure this difference. For independent condensates nonlocal field scattering will realize a continuous quantum measurement on a single quantum system, rather than an ensemble of systems [8]. While this measurement will fix the mean value of the relative phase, it does not unambiguously determine that the condensates were prepared in coherent states [9].

We remark that there will be two contributions to the scattered field: Stimulated scattering of the excited-state atoms into the ground condensed trap state and spontaneous scattering into noncondensed excited trap states. In principle, by operating in the *boson-accumulation regime* [10], where the number of condensate atoms greatly exceeds the number of trap states into which spontaneous emission may occur, the noise due to spontaneous scattering may be effectively eliminated.

We have seen that in the absence of reabsorption or rescattering events, the light scattered from the nonexcited condensate has a coherent component due to stimulated scattering. Since the analysis was carried out to first order in the incident classical field, the predictions are valid as long as the atomic medium remains *optically thin*. In the case of optically thick media, a decorrelation approximation has been employed to discuss coherent light scattering in Ref. [11]. In our case this implies that the coherently scattered field depends on nonlocal correlations of the form $\langle\hat{\Psi}_{g2}^{\dagger}(\mathbf{r}'+\mathbf{d}/2,t_{ret})\hat{\Psi}_{g1}(\mathbf{r}''-\mathbf{d}/2,t)\rangle$, which are still sensitive to the relative phase of the two condensates.

(b) *Excitation of both traps*. Although both traps are excited to a common electronic excited state, for simplicity we will assume that only photons scattered from one of the traps, say trap 2, are detected. The scattered electric field is given in this case by

$$\begin{aligned}\hat{\mathbf{E}}_{sc,2}^{+}(\mathbf{r},t)=\int d^3r'\int d^3r''\hat{\Psi}_{g2}^{\dagger}\left(\mathbf{r}'+\frac{\mathbf{d}}{2},t\right)&\\ \times\left[\overleftrightarrow{\mathbf{A}}\left(\mathbf{r},\mathbf{r}'+\frac{\mathbf{d}}{2},\mathbf{r}''-\frac{\mathbf{d}}{2}\right)\cdot\mathbf{E}_{L1}^{+}\left(\mathbf{r}''-\frac{\mathbf{d}}{2}\right)\right.&\\ \times\hat{\Psi}_{g1}\left(\mathbf{r}''-\frac{\mathbf{d}}{2},t\right)&\\ \left.+\overleftrightarrow{\mathbf{A}}\left(\mathbf{r},\mathbf{r}'+\frac{\mathbf{d}}{2},\mathbf{r}''+\frac{\mathbf{d}}{2}\right)\cdot\mathbf{E}_{L2}^{+}\left(\mathbf{r}''+\frac{\mathbf{d}}{2}\right)\right.&\\ \left.\times\hat{\Psi}_{g2}\left(\mathbf{r}''+\frac{\mathbf{d}}{2},t\right)\right].&\end{aligned} \tag{9}$$

As the laser fields are c numbers their amplitude and relative phase may in principle be fixed, over times short compared to that for laser phase diffusion. We assume that they are derived from a common source so that diffusion of the relative phase can be ignored. Now, if we write $\mathbf{E}_{L1,2}^{+}(\mathbf{r})\equiv\mathbf{E}_{1,2}f_{L1,2}(\mathbf{r})$, then Eq. (9) gives $\langle\hat{\mathbf{E}}_{sc,2}^{+}(\mathbf{r},t)\rangle=(\mathbf{D}^{+}\cdot\mathbf{E}_1)\alpha_{nonlocal}(\mathbf{r},t)+(\mathbf{D}^{+}\cdot\mathbf{E}_2)\alpha_{local}(\mathbf{r},t)$. In the presence of the BEC in both traps, the mean scattered field from trap 2 is a superposition of contributions due to the excitation of the condensates in traps 1 and 2, respectively, and at any particular observation point, can be made to vanish by adjusting the amplitudes and relative phase of the laser fields. This is the general result, cancellation of the scattered field at the point of observation by a fixed pair of $\mathbf{E}_{1,2}$, the mean field being nonzero elsewhere.

We observe from Eqs. (3) and (4), that if $\langle\hat{\Psi}_e(\mathbf{r},t)\rangle=0$, or equivalently $\langle\hat{a}_{\mathbf{q}}\rangle=0$ for every $\mathbf{q}$ (with a fixed q_z), the

mean field vanishes everywhere, provided that the factorization $\langle \hat{\Psi}^{\dagger}_{g\mu}\hat{\Psi}_e\rangle = \langle \hat{\Psi}^{\dagger}_{g\mu}\rangle\langle\hat{\Psi}_e\rangle$, $\mu=1,2$, is valid in Eq. (3). Using Eq. (4) one can show that $\langle \hat{a}_{\mathbf{q}}\rangle = 0$, for all $\mathbf{q}$, is possible for a fixed pair of $\mathbf{E}_{1,2}$, under certain conditions. Here we simply quote sufficient conditions: (i) the lasers are plane waves (over the trap dimensions), with identical wave vectors directed perpendicular to the z axis, (ii) the two condensate wave functions are proportional $\langle \hat{\Psi}_{g\mu}(\mathbf{r}+(-)^{\mu}\mathbf{d}/2)\rangle \propto \Phi_g(\mathbf{r})$, $\mu=1,2$; and (iii) the excited-state wave functions and condensate wave function have a well-defined parity, i.e., $\phi^e_{\mathbf{q}}(x,y,z) = \pm\,\phi^e_{q_x}(x)\,\phi^e_{q_y}(y)\,\phi^e_{q_z}(-z)$ and $\Phi_g(x,y,z) = \pm\,\Phi_g(x,y,-z)$.

This scheme based on the quantum interference of the dipole moments generated by the two laser fields is the nonlocal analog of coherent population trapping (CPT) in a double lambda scheme, proposed earlier in the context of lasers without population inversion [12]. In the latter, two strong laser fields pre-fix the relative phase and magnitude of the atomic probability amplitudes in two lower-energy atomic states by transferring the atoms into the stable CPT state. Two weak laser fields then couple the two lower states to a common upper state where scattering may be observed unless a certain phase-amplitude relation exists between the laser fields. In the case of CPT systems, the cancellation of the atomic scattering remains valid to all orders in the laser field intensity. Even though our analysis is valid to first order, it is clear from the form of Eq. (4) that the excited-state amplitude remains zero at all times, provided that the laser fields are turned on adiabatically. Therefore, it is likely that even in an optically thick sample, the cancellation of the mean scattered field could be possible. Finally, we remark that this method provides a way to realize *optical Josephson coupling* between two independent condensates. The strength and phase of the coupling may be adjusted simply by changing the amplitude and relative phase of the laser fields, respectively.

A limitation of both methods is the assumed long lifetime for the excited atomic state. We believe that experimentally the most promising method is to use two-photon transition to a stable excited state, as described earlier. In contrast to earlier works [6,7], our proposal relies on spatially nonlocalized transitions, which in turn requires that we access one (or a few) of the excited-state trap eigenstates resonantly, either with one- or two-photon transitions. We have already mentioned that to select out a single q_z eigenstate, the excited-state trap frequency along the z axis (ω_z) must exceed Γ_{q_z}; using a strong red-detuned laser field (on the $|e\rangle - |f_2\rangle$ transition), one can obtain $\omega_z \simeq 10^5\ \mathrm{s}^{-1}$. Physically, ω_z is determined by the laser field intensity that is required to accelerate the excited atom so that it travels from one potential well to the other ($\sim 20\ \mu$m) in an effective upper-state lifetime. The corresponding lifetimes $\Gamma^{-1}_{q_z} \sim 10^{-1} - 10^{-4}$ s may be readily obtained (and tuned) if one uses the two-photon transition between hyperfine states.

In summary, we have discussed related coherent light-scattering schemes that may be used to measure the relative phase of two atomic condensates. Both schemes rely on the assumption that the phase of the condensates will not diffuse during the time of the measurement so that an adiabatic elimination of the excited state may be carried out. Conversely, the proposed measurements may be used to determine the phase diffusion rate of Bose condensates.

Note added in revision. Since the completion of this work we have become aware of a related preprint by J. Javanainen.

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