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Teleportation with identity interchange of quantum states

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We show that, beyond teleportation, it is possible to realize the interchange of states between two quantum systems once their states are simultaneously teleported from one to the other. The experimentally feasible scheme presented here, employing cavity QED phenomena, can be used to realize this ''identity interchange'' process between entangled particles as well, and even to teleport an entanglement from one particle system to another. [S1050-2947(97)50704-9]

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Recently, the phenomenon of quantum nonlocality has received considerable attention (i) partly due to a variety of new suggested tests of nonlocality and a number of successfully realized experimental proofs, and (ii) partly due to the extension of its implications.

(i) Some intriguing proofs of nonlocality have been given using [1] or not using [2,3] Bell-type inequalities [4]. These proofs demonstrate the incompatibility of quantum mechanics with local realism, even considering a single-photon field as predicted by Tan, Walls, and Collett [1], or employing more than two particles as in the Greenberger, Horne, and Zeilinger (GHZ) *gedanken experiment* [2]. Similarly to GHZ's proof, Hardy [3] has demonstrated nonlocality without using inequalities. However, beyond GHZ's proof but like in Bell's, Hardy has considered only a two-particle state to formulate his proof of nonlocality *for almost all entangled states*. The construction of sources of polarization-entangled photon pairs with increasing momentum definition has allowed, simultaneously to the theoretical achievement, the demonstration of the violation of Bell's inequalities with increasingly high fringe visibility and particle collection efficiency [5].

(ii) Quantum nonlocality, a key process for understanding fundamental quantum physics, has been considered in a variety of striking subjects. The possibility of quantum cryptography [6], quantum computers [7], and teleportation of an unknown quantum state [8] has recently been suggested. By teleportation Bennett *et al.* [8] refer to the process by which an unknown quantum state $|\Psi\rangle_A$ of a particle A is exactly replicated into another particle B far away from A. The principle of teleportation is outlined by combining the possibility of entanglement between two separated systems and the projection postulate. Furthermore, dual classical and Einstein-Podolsky-Rosen (EPR) [9] channels are required.

Let us consider that particle A has been given to a sender, Alice, who shares an EPR state, the quantum channel, with a receiver, Bob. This EPR pair, $|\Psi\rangle_{BC}$, consists of the above-mentioned particle B, which has been given to Bob, and a third particle C, which has also been given to Alice. By performing a joint measurement of the von Neumann type on particles A and C, Alice couples the particle to be teleported with the EPR state. As a result of this measurement, particle B is automatically projected into a pure state that differs from $|\Psi\rangle_A$ just by an irrelevant phase factor or a rotation around the x, y, or z axes. Through a classical channel Alice communicates the outcome of her measurement to Bob, who finally performs a unitary transformation on his previously entangled particle that brings it to the original state of Alice's particle A. Obeying the no-cloning theorem [10] Alice's original state is destroyed in the teleportation process. Experimental schemes have been proposed for the veri-

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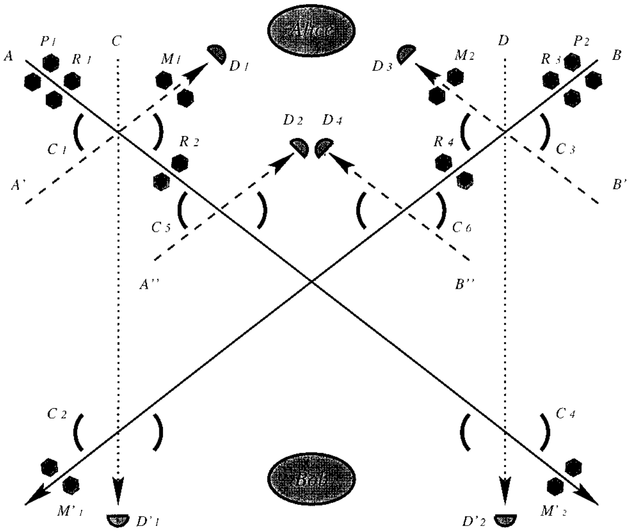

FIG. 1. Sketch of the experimental setup for the identity interchange process.

fication of teleportation which rely on recent advances, both in microwave and optical regimes of cavity QED phenomena [11,12]. Through these new techniques, coherent atom-field interaction can be made to dominate over dissipative processes due to cavity losses and atomic spontaneous emission.

In the present work we show that through the principles leading to teleportation it is possible to go beyond this process. We present an experimentally feasible scheme to realize the interchange of unknown states characterizing two quantum systems. The setup is based on the two-cavity teleportation experiment proposed by Davidovich *et al.* [11]. Basically, nonclassical coherent superpositions of states of the electromagnetic field are required to set up the quantum channel, while Ramsey-type arrangements [13] permit us to perform the required joint measurements. Two particles are considered, A and B, whose states $|\Psi\rangle_A$ and $|\Psi\rangle_B$ are teleported one to the other. This ''identity interchange'' process illustrates that, despite the fact that quantum information cannot be cloned, it cannot only be swapped from one system to another, as in teleportation, but also be interchanged between systems.

A sketch of the ''identity interchange'' experiment is displayed in Fig. 1. The setup consists of identical two-level atoms, identical initially empty and high-Q cavities, state-selective ionization detectors, and auxiliary microwave fields, whose roles in the experiment are discussed below. Atomic Rydberg states with adjacent principal quantum numbers are considered, and the transition from the excited to the ground state is tuned to resonance with the cavity mode frequency.

Quantum channel. First, two nonlocal field states, each occupying simultaneously two cavities [15], are required to set up the quantum channel. The cavity pairs C_1-C_2 and C_3-C_4 are considered for this proposal. Atoms C and D, initially prepared in their excited states $|e\rangle_C$ and $|e\rangle_D$ are thus sent across the cavity pairs C_1-C_2 and C_3-C_4, respectively, as indicated in Fig. 1. These atoms are made resonant with their respective cavities and undergo, on the $|e\rangle\rightarrow|g\rangle$ transition, a $\pi/2$ pulse in C_1 and C_3 and a π pulse in C_2 and C_4. Thus, atoms C and D exit their respective cavities in their ground states $|g\rangle_C$ and $|g\rangle_D$, while each of the cavity pairs C_1-C_2 and C_3-C_4 is left in the entangled state

$$|\Psi^{(+)}\rangle_{ij}=\frac{1}{\sqrt{2}}(|0\rangle_i|1\rangle_j+|1\rangle_i|0\rangle_j), \tag{1}$$

where i,j refer to the field states in cavities 1,2 (3,4), respectively. These nonlocal correlations established on systems C_1+C_2 and C_3+C_4 constitute two EPR states that set up the required quantum channel as soon as atoms C and D are detected in D_1' and D_2', respectively [16].

States to be interchanged. After the preparation of the quantum channel, atoms A and B are prepared by the microwave zones P_1 and P_2, respectively, in the arbitrary superpositions $|\Psi\rangle_A=c_A|g\rangle_A+c_A'|e\rangle_A$ and $|\Psi\rangle_B=c_B|g\rangle_B+c_B'|e\rangle_B$. These atoms are thus sent across cavities C_1 and C_3, respectively, where Alice has to perform two joint measurements on systems $A+C_1$ and $B+C_3$. Labeling the atomic states of beam A (B) by α, while i,j refer to the field states in cavities 1,2 (3,4), respectively, the combined ''atom α + field i,j'' state, composing the state vector of the entire system $|\Psi\rangle_A|\Psi\rangle_B|\Psi\rangle_{12}|\Psi\rangle_{34}=|\Psi\rangle_{A12}|\Psi\rangle_{B34}$, can be expanded as

$$\begin{aligned}|\Psi\rangle_{\alpha ij}=\tfrac{1}{2}[&|\Psi^{(+)}\rangle_{\alpha i}(c_\alpha|1\rangle_j+c_\alpha'|0\rangle_j)+|\Psi^{(-)}\rangle_{\alpha i}(c_\alpha|1\rangle_j\\&-c_\alpha'|0\rangle_j)+|\Phi^{(+)}\rangle_{\alpha i}(c_\alpha|0\rangle_j+c_\alpha'|1\rangle_j)\\&+|\Phi^{(-)}\rangle_{\alpha i}(c_\alpha|0\rangle_j-c_\alpha'|1\rangle_j)],\end{aligned} \tag{2}$$

where we have introduced the Bell operator basis,

$$|\Psi^{(\pm)}\rangle_{\alpha i}=\frac{1}{\sqrt{2}}(|e\rangle_\alpha|0\rangle_i\pm|g\rangle_\alpha|1\rangle_i), \tag{3a}$$

$$|\Phi^{(\pm)}\rangle_{\alpha i}=\frac{1}{\sqrt{2}}(|e\rangle_\alpha|1\rangle_i\pm|g\rangle_\alpha|0\rangle_i). \tag{3b}$$

From Eq. (2) we observe that, by performing two joint measurements, on systems $A+C_1$ and $B+C_3$, Alice induces each of these systems to collapse into one of the Bell operator states in Eqs. (3a) and (3b). Thus, each of the cavity fields in C_2 and C_4 is automatically projected into one of the four superpositions of one- and zero-photon field states appearing in Eq. (2), which contain information on the states to be interchanged between atoms A and B. These states, $|\Psi\rangle_A$ and $|\Psi\rangle_B$, have been replicated on cavities C_2 and C_4, respectively, through known unitary transformations which replace the atomic states $|g\rangle$ and $|e\rangle$ into the field states $|0\rangle$ and $|1\rangle$.

Joint measurements. The joint measurements on systems $A+C_1$ and $B+C_3$ are performed through Ramsey-type arrangements, where the atoms are made to cross two separated resonant microwave fields with their respective cavities placed between them. As shown in Fig. 1, atoms A and B are sent across the arrangements $R_1-C_1-R_2$ and $R_3-C_3-R_4$, respectively. By using the above-mentioned labels, first of all, atom α is tuned to have a dispersive interaction with the field in cavity C_i. The effect of such an interaction is probed by two separated oscillatory fields applied in microwave zones R_i and R_j which are sandwiching C_i. Equivalently to a recently demonstrated Ramsey atomic interferometry [14], the probability for atom α to undergo an $|e\rangle_\alpha \to |g\rangle_\alpha$ transition exhibits a fringe pattern which is characteristic of the photon number in C_i. Such a transition probability depends on a given setting of the microwave fields in R_i and R_j, besides depending on the atom-cavity interaction. For the present purpose we assume that the microwave zones R_i and R_j are set so that atom α undergoes exactly a $\pi/2$ pulse on the $|e\rangle_\alpha \to |g\rangle_\alpha$ transition in each zone. The cavity detuning is also set so that the atom undergoes a phase shift per photon exactly equal to π. In this way, as obtained in Ref. [13], the $|e\rangle_\alpha \to |g\rangle_\alpha$ transfer probability is unity when C_i is empty and, therefore, zero when C_i contains one photon. In a dispersive atom-field interaction the photon number in the cavity always remains unchanged and the system $\alpha + C_i$ undergoes the transformations: $|e\rangle_\alpha|0\rangle_i \to -|g\rangle_\alpha|0\rangle_i$, $|e\rangle_\alpha|1\rangle_i \to -|e\rangle_\alpha|1\rangle_i$, $|g\rangle_\alpha|0\rangle_i \to |e\rangle_\alpha|0\rangle_i$, and $|g\rangle_\alpha|1\rangle_i \to |g\rangle_\alpha|1\rangle_i$. Through these transformations the ‘‘atom-field’’ Bell states turn out to be

$$|\Psi^{(\pm)}\rangle_{\alpha i} = -\frac{1}{\sqrt{2}}|g\rangle_\alpha(|0\rangle_i \mp |1\rangle_i), \tag{4a}$$

$$|\Phi^{(\pm)}\rangle_{\alpha i} = -\frac{1}{\sqrt{2}}|e\rangle_\alpha(|1\rangle_i \mp |0\rangle_i). \tag{4b}$$

Once atom α has crossed the Ramsey-type arrangement $R_i - C_i - R_j$, we proceed to the first step in the realization of the joint measurement on system $\alpha + C_i$ by sending a reference atom α' across the arrangement $C_i - M_i$. This reference atom, initially prepared in the ground state $|g\rangle_{\alpha'}$, is tuned to interact resonantly with C_i, undergoing a π pulse and leaving the cavity empty if it initially contains one photon. The system $\alpha + C_i$ thus evolves as

$$\frac{1}{\sqrt{2}}|g\rangle_{\alpha'}(|0\rangle_i \pm |1\rangle_i) \to \frac{1}{\sqrt{2}}(|g\rangle_{\alpha'} \pm |e\rangle_{\alpha'})|0\rangle_i. \tag{5a}$$

After crossing cavity C_i atom α' undergoes a $\pi/2$ pulse in microwave zone M_i, so that from the evolution (5a) we get the result

$$\frac{1}{\sqrt{2}}(|g\rangle_{\alpha'} \pm |e\rangle_{\alpha'})|0\rangle_i \to \begin{cases} |e\rangle_{\alpha'}|0\rangle_i \\ |g\rangle_{\alpha'}|0\rangle_i. \end{cases} \tag{5b}$$

Therefore, by considering the transformations (4) and (5), the combined system ‘‘α + C_i’’ evolves from the state (2) into

$$\begin{aligned} |g\rangle_{\alpha'}|\Psi\rangle_{\alpha ij} = -\frac{1}{2\sqrt{2}}[&|g\rangle_{\alpha'}|g\rangle_\alpha(c_\alpha|1\rangle_j + c'_\alpha|0\rangle_j) \\ &+|e\rangle_{\alpha'}|g\rangle_\alpha(c_\alpha|1\rangle_j - c'_\alpha|0\rangle_j) \\ &+|g\rangle_{\alpha'}|e\rangle_\alpha(c_\alpha|0\rangle_j + c'_\alpha|1\rangle_j) \\ &+|e\rangle_{\alpha'}|e\rangle_\alpha(c_\alpha|0\rangle_j - c'_\alpha|1\rangle_j)]|0\rangle_i. \end{aligned} \tag{6}$$

We thus see from the result (6) that state-selective detection of atom α' permits us to discern between the phases + and −, since they correspond to find α' in the states $|g\rangle_{\alpha'}$ and $|e\rangle_{\alpha'}$, respectively.

To completely determine the state $\alpha + C_i$ we finally have to discern between the Bell states $|\Psi\rangle$ and $|\Phi\rangle$, which is achieved by sending atom α across the initially empty cavity C_k, with $k = 5,6$, depending on $\alpha = A,B$, respectively. Similarly to the interaction between α' and C_i, α is tuned to interact resonantly with C_k, undergoing a π pulse and leaving a photon in this cavity if it initially contains one. So, after the interaction, atom α exits cavity C_k in the ground state $|g\rangle_\alpha$, while the cavity is left in a superposition of the one- and zero-photon state. Next, a second reference atom α'', initially prepared in the ground state $|g\rangle_{\alpha''}$, is sent across cavity C_k, tuned to interact resonantly with this cavity, undergoing a π pulse. Through the operations required by this second step in the realization of the joint measurements the combined state (6) gets transformed into

$$\begin{aligned} |g\rangle_{\alpha''}|g\rangle_{\alpha'}|\Psi\rangle_{\alpha ij}|0\rangle_k = -\frac{1}{2\sqrt{2}}[&|g\rangle_{\alpha''}|g\rangle_{\alpha'}(c_\alpha|1\rangle_j + c'_\alpha|0\rangle_j) \\ &+|g\rangle_{\alpha''}|e\rangle_{\alpha'}(c_\alpha|1\rangle_j - c'_\alpha|0\rangle_j) \\ &-|e\rangle_{\alpha''}|g\rangle_{\alpha'}(c_\alpha|0\rangle_j + c'_\alpha|1\rangle_j) \\ &-|e\rangle_{\alpha''}|e\rangle_{\alpha'}(c_\alpha|0\rangle_j \\ &-c'_\alpha|1\rangle_j)]|g\rangle_\alpha|0\rangle_i|0\rangle_k. \end{aligned} \tag{7}$$

In summary, complete information on the Bell states describing the system $\alpha + C_i$ is obtained through the measurement results on atoms α' and α'', with the correspondences $|g\rangle_{\alpha'}|g\rangle_{\alpha''} \to |\Psi^{(+)}\rangle$, $|e\rangle_{\alpha'}|g\rangle_{\alpha''} \to |\Psi^{(-)}\rangle$, $|g\rangle_{\alpha'}|e\rangle_{\alpha''} \to |\Phi^{(+)}\rangle$, and $|e\rangle_{\alpha'}|e\rangle_{\alpha''} \to |\Phi^{(-)}\rangle$.

It is worth stressing that cavities C_5 and C_6 play two important roles in the present scheme: (i) The second reference atom A'' (B'') permits us to complete the joint measurement on system $A + C_1$ ($B + C_3$) by transporting a photon away from cavity C_5 (C_6), whenever there exists one. (ii) Moreover, cavities C_5 and C_6 leave atoms A and B in their ground states, preparing them for the final stage of the identity interchange process through cavities C_4 and C_2, respectively.

Identity interchange. After the realization of the joint measurements, cavities C_2 and C_4 are left in superposition states which contain information about the original states of atoms A and B, respectively. This information is inversely replicated on atoms B and A once they have been prepared in their ground states and have been made to cross cavities C_2 and C_4, respectively. By tuning atoms B and A to resonance with cavities C_2 and C_4, after the interactions the information stored in these cavities is completely transferred to their respective atoms in the following way:

$$(a|1\rangle_i + b|0\rangle_i)|g\rangle_\alpha \to (a|e\rangle_\alpha + b|g\rangle_\alpha)|0\rangle_i, \tag{8}$$

with i referring to cavity C_2 (C_4), α referring to atom B (A), and $a,b = \pm c_\alpha, \pm c'_\alpha$. Cavities C_2 and C_4 are thus left in their vacuum state while atoms A and B are left in superposition states which differ from $|\Psi\rangle_B$ and $|\Psi\rangle_A$, respectively, by known unitary transformations. Finally, by having Alice tell Bob the classical outcomes of her joint measurements, Bob has to apply the inverse transformations independently to atoms A and B. These transformations, accomplished through the microwave zones M_1' and M_2', convert the states of atoms A and B to the original states of atoms B and A, respectively, concluding the identity interchange process.

Identity interchange with entangled states. All of what we have described above can be considered for the identity interchange process of the most general entangled state between two two-level atoms, A and B,

$$|\Psi\rangle_{AB}=c_1|e\rangle_A|e\rangle_B+c_2|e\rangle_A|g\rangle_B+c_3|g\rangle_A|e\rangle_B+c_4|g\rangle_A|g\rangle_B. \tag{9}$$

For this proposal, the setup sketched in Fig. 1 needs to be complemented by additional apparatuses considered for the preparation of such an entangled state [17]. Once the entangled state in Eq. (9) has been prepared, atoms A and B are sent across cavities C_1 and C_3, respectively, and we follow the same steps pursued for the identity interchange process in the case where these atoms are not correlated. After crossing cavities C_2 and C_4, and the microwave zones M_1' and M_2', where appropriate rotations have to be applied on atoms B and A, respectively, these atoms will be found in the superposition $c_1|e\rangle_B|e\rangle_A+c_2|e\rangle_B|g\rangle_A+c_3|g\rangle_B|e\rangle_A+c_4|g\rangle_B|g\rangle_A$, which corresponds to an identity interchange process between entangled systems. It is easy to verify that we can also consider the setup in Fig. 1 to realize the teleportation of the entangled state in Eq. (9) from atoms A and B to atoms E and F.

Finally, it is worth accounting for some sensitive points in the present experimental scheme, such as the dissipative processes due to cavity losses and atomic spontaneous emission, the dispersion in the atomic velocity, and the efficiency of atomic detection. Due to the strong coupling between the considered circular Rydberg states and microwave fields, and due to their long radiative decay times, the atom-field interactions are supposed to dominate the dissipative processes. For high-Q superconducting cavities the cavity damping times, in the 10^{-2}-s range [13], are about three orders of magnitude longer than typical atom-cavity interaction times. For Rydberg atoms in circular states $l=n-1$, atomic excited-state lifetimes are also of the order of 10^{-2} s [13]. Current experiments involving the interaction of circularRydberg atoms with microwave fields can reach the parameters necessary in order for the velocity dispersion not to cause the required atom-field entangled states to deviate appreciably from the expected one. Such parameters correspond to a coupling strength between atoms and quantized cavity fields around 2 x 10^{-5} s^{-1} and atomic velocities around 10^3 m/s [12]. As regards the efficiency of atomic detection, it can be estimated for an average success of the identity interchange process. The dispersion in the atomic velocities can also be assimilated to an effective efficiency of detection [11].

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[16] In fact, in a real experiment instead of atoms C and D we have atomic beams C and D (as well as atomic beams A, A', A'' and B, B', B''), and the triggering atoms from beams C and D must be the first to cross the initially empty cavity pairs C_1-C_2 and C_3-C_4, respectively. If D_1' and D_2' fail to detect their respective atoms, the experiment is triggered by subsequent atoms and fields having more than one photon are generated in the cavity pairs C_1-C_2 and C_3-C_4, which reduce the quantum correlation required for the identity interchange process. See Ref. [11].

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