Role of collisions in creation of overlapping Bose condensates

S. J. J. M. F. Kokkelmans, H. M. J. M. Boesten, and B. J. Verhaar
Eindhoven University of Technology, P.O. Box 513, 5600 MB Eindhoven, The Netherlands
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We study the elastic scattering length and inelastic decay rate associated with collisions of ^{23}Na and ^{87}Rb atoms in different hyperfine states: one with atoms in the $|f=2,m_f=2\rangle$ and one with atoms in the $|f=1,m_f=-1\rangle$ state. For Na the real part of the $a_{(22)+(1-1)}$ scattering length for $|22\rangle+|1-1\rangle$ collisions is predicted to be positive and equal to 65 ± 5 Bohr length units a_0 for small B. The zero-temperature low-field decay rate is of the order of a typical exchange rate: $G_{(22)+(1-1)}=(1.5\pm0.7)10^{-11}$ cm^3 s^{-1}, showing that a two-condensate experiment is not feasible for a Na gas sample in a static magnetic trap. For the case of Rb atoms the present knowledge of the singlet interaction does not allow a similar calculation. The extreme suppression of $G_{(22)+(1-1)}$ demonstrated by a recent experiment is shown to be very restrictive for the value of the singlet accumulated phase. [S1050-2947(97)50403-3]

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The successful realization of a Bose condensate in dilute ultracold gas samples of ^{87}Rb [1], ^{23}Na [2], and ^{7}Li [3] atoms has opened a rapidly expanding field of studies of condensate properties, starting with the study of its collective modes [4,5]. A very remarkable new result [6] is the creation of overlapping ^{87}Rb condensates in two different ground-state hyperfine levels $|f=2,m_f=2\rangle$ and $|f=1,m_f=-1\rangle$, thus realizing a fascinating system that has been studied theoretically long ago in the case of spin-polarized atomic hydrogen [7] and very recently also for binary mixtures of alkali Bose condensates [8]. The most remarkable aspect of this experiment is the slow decay due to mixed collisions of pairs of atoms in the two different hyperfine states: due to the presence of decay channels with the same total $m_F=+1$ value one would expect the much faster decay for a typical exchange collision ($G_{exch}\approx10^{-11}$ cm^3 s^{-1} [9]) instead of that experimentally observed: $G_{(22)+(1-1)}=2.2(9)\times10^{-14}$ cm^3 s^{-1}.

In this paper we consider the question of whether this can be understood on the basis of the theory of cold collisions. We study the rates of decay due to mixed collisions to the three available exchange decay channels (21)+(10), (20)+(11), and (11)+(10), in the low-field range of experimental interest. Another mixed collision property of interest is the elastic (22)+(1−1) scattering length, its sign implying an effectively repulsive or attractive interaction of the condensates [10] and its magnitude determining the efficiency of the sympathetic cooling of the two interpenetrating gas samples. In view of the interest in similar experiments for a Na gas sample, an obvious second item to be considered is the analogous decay rate and elastic scattering length for Na. We start with this system, which has the advantage that the singlet and triplet interaction properties are rather well known, and find that for weak fields the mixed collisional decay rate has the full strength expected for exchange relaxation, while the scattering length is positive. Analogous results for lithium atoms are included in a separate paper [11] that is mainly devoted to a mixed system of a different kind: a combined boson-fermion system of ^{7}Li and ^{6}Li atoms. Note that the rates and scattering lengths that we calculate have implications for both condensate and noncondensate atoms. In the case of the inelastic rates, one needs to take into account the well-known reduction by a factor of 2 for processes inside a condensate [12]. This reduction is not yet included in the following rate equation (1).

For the Na system the singlet and triplet interaction properties in cold collisions have been predicted rather accurately, both on the basis of the accumulated phases determined from interlevel spacings between highly excited rovibrational singlet and triplet Na_2 states [13] and from cold-atom photoassociation [14]. To compare the predicted singlet and triplet interaction properties obtained with these two methods, we present in Fig. 1 the triplet and singlet scattering lengths as a function of the accumulated phases of the decoupled triplet and singlet radial wave functions. The actual abscissa is the modification $\Delta\phi_T(\Delta\phi_S)$ relative to a reference value calculated for the IPA (inverse-perturbation-approach) singlet potential obtained by Moerdijk, Verhaar, and Axelsson [13] and the Rydberg-Klein-Rees triplet potential obtained by Zemke and Stwalley [15]. The dispersion coefficients characterizing the long-range interaction have been taken from Marinescu, Sadeghpour and Dalgarno [16]. The a_T and a_S intervals determined in Ref. [14] correspond

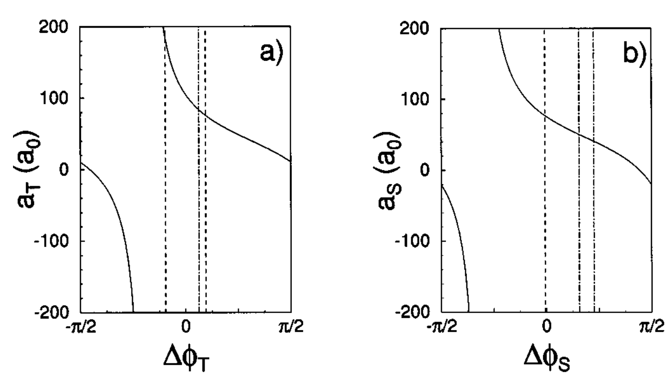

FIG. 1. (a) Triplet scattering length, as a function of the triplet accumulated phase. (b) Singlet scattering length, as a function of the singlet accumulated phase. The phase ranges of Ref. [13] (dashed lines) and those derived from Ref. [14] (dash-dotted lines) are indicated. Small intervals are given by one single line.

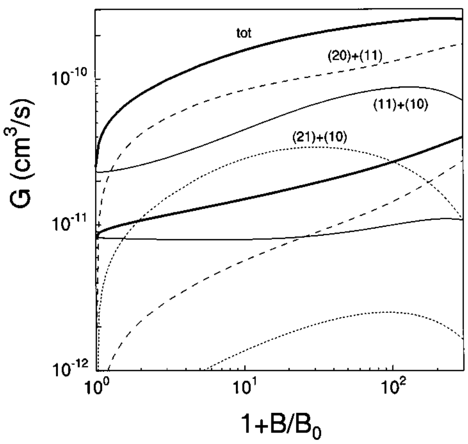

FIG. 2. Zero-temperature partial and total decay rate constants G due to mixed collisions of Na atoms as a function of magnetic field for our choice of singlet and triplet accumulated phases. To combine the advantages of a linear field scale for small B and a logarithmic scale for large B we plot $1+B/B_0$ logarithmically with $B_0=1$ G. Pairs of lines of the same line type indicate results for two extreme choices of the phases corresponding to our error bars.

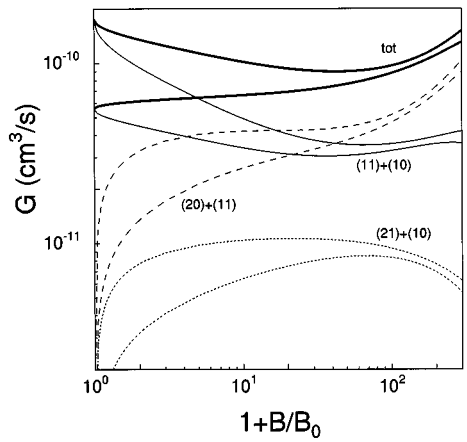

FIG. 3. Same as Fig. 2 for the phase parameters derived from Ref. [14].

to ranges $0.16<\Delta\phi_T<0.23$, $0.49<\Delta\phi_S<0.71$. While the range for $\Delta\phi_T$ is in agreement with that of Ref. [13] ($-0.3<\Delta\phi_T<0.3$), we find a considerable discrepancy in the case of $\Delta\phi_S$ ($-0.04<\Delta\phi_S<0.00$ from Ref. [13]). A strong point in favor of a $\Delta\phi_S$ interval close to 0 is its preference for the rather reliable IPA potential, analogous to the case of Li atoms [17,18]. In the following we will present the predictions for the $(22)+(1-1)$ collision properties for both choices of the $\Delta\phi_S$ interval.

The decay rate due to $(22)+(1-1)$ mixed atomic collisions is described by the equation

$$\frac{dn_{22}}{dt}=\frac{dn_{1-1}}{dt}=-G_{(22)+(1-1)}n_{22}n_{1-1}, \tag{1}$$

with n_{22} and n_{1-1} the respective number densities. The decay channels available for exchange relaxation are $(21)+(10)$, $(20)+(11)$, and $(11)+(10)$. Since the former two channels become closed for $B\rightarrow 0$, one would expect the $(11)+(10)$ decay channel to dominate at the very low B values of primary experimental interest. In this field range F is almost a good quantum number. For s waves, only spin channels $\{f_1f_2\}Fm_F$ that are symmetric under exchange of the two atoms are allowed because of Bose symmetry. As a consequence, for $f_1=f_2=1$ only $F=0$ and 2 contribute. Since the $m_F=+1$ value in the initial channel is conserved, only $F=2$ remains. If this part would be purely triplet or singlet for either the initial or final channel, an exchange transition would be forbidden. Some straightforward Clebsch-Gordan algebra shows that this is not the case: the initial channel spin state is 87.5% triplet and 12.5% singlet, and the final channel spin state is 81.25% triplet and 18.75% singlet. Of course, this does not exclude the fact that the exchange decay vanishes due to destructive interference of triplet and singlet amplitudes. The measured slow decay in the case of ^{87}Rb suggests that we are close to this accidental situation for this atom.

In Fig. 2 we present the zero-temperature value of $G_{(22)+(1-1)}$, as well as its contributions from the three separate decay channels for our choice of singlet and triplet accumulated phases, as a function of B. Results are given for two extreme choices of the phases corresponding to our error bars. The partial decay rates to the $(21)+(10)$ and $(20)+(11)$ channels increase proportional to $\sqrt{k_f}\sim\sqrt{B}$ in the ratio $1{:}3\sqrt{2}$ (k_f denotes the final wave number), as one should expect from the Clebsch-Gordan coefficients involved and from the ratio of final wave numbers: The decay rate to the $(11)+(10)$ channel starts from a nonvanishing value. At small B the total rate constant is of the order of 10^{-11} cm^3 s^{-1}, showing that a two-condensate experiment is not feasible for a Na gas sample in a static magnetic trap. Figure 3 shows similar results for the above-mentioned phase intervals derived from Ref. [14]. Clearly, the low-B range of predicted G values is almost one order of magnitude larger than our prediction, so that the two predictions may be distinguishable experimentally at the lower fields. Figure 4 shows the corresponding real parts of the complex scattering lengths $a_{(22)+(1-1)}$ (solid line, results for our phase values; dashed line, results for phase values derived from Ref. [14]). Clearly, the predicted mixed scattering length is large and positive, indicating an effectively repulsive interaction.

We note that the present results do not exclude a suppres-

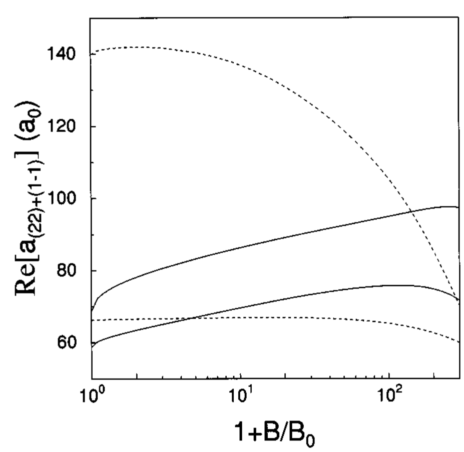

FIG. 4. Elastic scattering length for mixed collisions of Na atoms for two extreme values of the phase parameters. Solid line, our phase values; dashed line, phase values derived from Ref. [14].

sion of the two-condensate decay rate for fields in the vicinity of a Feshbach resonance in the (22)+(1−1) channel. This would open the possibility of a two-condensate experiment by appropriate tuning of the magnetic field. A detailed theoretical search, however, did not lead to such resonances in the field range in which both condensates can be magnetically trapped.

In the case of ^{87}Rb the long-range triplet interaction is known rather accurately from photoassociation work [20,21]. The situation with respect to the singlet interaction is much less certain. In the following we discuss qualitatively what behavior of the mixed scattering length and decay rate can be expected, by applying the DIS (degenerate internal states) approximation [19]. This approximation neglects the atomic hyperfine splitting, i.e., classically speaking the hyperfine precession, during the collision. It reduces the multichannel collision problem to a potential scattering problem of separate singlet and triplet waves in which the initial and final channel spin states can be expressed. The resulting expressions are

$$a_{(22)+(1-1)}=(\sin^2\theta_{-1}+\tfrac{1}{2}\cos^2\theta_{-1})a_T+\tfrac{1}{2}(\cos^2\theta_{-1})a_S, \tag{2}$$

$$G_{(22)+(1-1)\to(21)+(10)}=\frac{2\pi\hbar k_f}{m}(\sin\theta_1\sin\theta_0\cos\theta_{-1})^2\times(a_T-a_S)^2, \tag{3}$$

$$G_{(22)+(1-1)\to(20)+(11)}=\frac{2\pi\hbar k_f}{m}(\cos\theta_1\cos\theta_0\cos\theta_{-1})^2\times(a_T-a_S)^2, \tag{4}$$

$$G_{(22)+(1-1)\to(11)+(10)}=\frac{2\pi\hbar k_f}{m}(\cos\theta_1\sin\theta_0\cos\theta_{-1})^2\times(a_T-a_S)^2. \tag{5}$$

The parameters $\theta_1,\theta_0,\theta_{-1}$ are functions of the magnetic field B defined by

$$\tan 2\theta_{(\pm 1)}=\frac{a_{\mathrm{hf}}\sqrt{3}}{\pm a_{\mathrm{hf}}+\hbar B(\gamma_e+\gamma_N)},$$

$$\tan 2\theta_0=\frac{2a_{\mathrm{hf}}}{\hbar B(\gamma_e+\gamma_N)}, \tag{6}$$

where a_{hf} is the hyperfine constant, and γ_e and γ_N are the electronic and nuclear gyromagnetic ratios.

The triplet scattering length is taken from Ref. [20]. We treat the singlet accumulated phase as a variable parameter. The singlet scattering length can be expressed in this phase and in C_6. The value of the latter is taken from Ref. [21].

Figure 5 shows a three-dimensional diagram for the total decay rate constant $G_{(22)+(1-1)}$ as a function of both B and $\Delta\phi_S(\bmod \pi)$. Unlike the case of Na, the zero of the latter scale does not have a special significance. We indeed see a strong suppression of the total decay rate in a rather narrow phase interval. We therefore expect that the above-mentioned extremely small experimental value of the decay rate, $G_{(22)+(1-1)}=2.2(9)\times10^{-14}\ \mathrm{cm}^3\ \mathrm{s}^{-1}$, almost three orders of magnitude smaller than for Na, will impose a strong constraint on the value of the singlet accumulated phase in a future more complete analysis [22]: Fig. 5 suggests that it will be possible to derive for the singlet phase a value with an error bar $\pm0.003\pi$. Also, one would expect that the near equality of a_S and a_T following from the above DIS picture, would continue to hold in a more rigorous coupled-channels treatment.

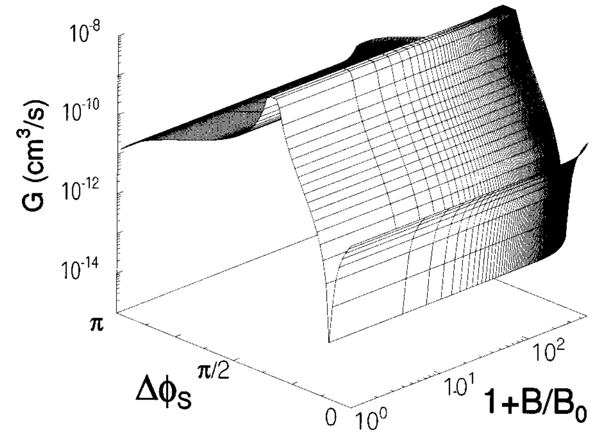

FIG. 5. Zero-temperature total decay rate constant $G_{(22)+(1-1)}$ for ^{87}Rb in the DIS approximation, as a function of B and $\Delta\phi_S$. We plot $1+B/B_0$ logarithmically with $B_0=1$ G.

Figure 6 shows the real part of the mixed $a_{(22)+(1-1)}$ scattering length at $B=0$ as a function of $\Delta\phi_S(\bmod \pi)$, again in the DIS approximation. The excursion through $\pm\infty$ is due to the same feature in the $\Delta\phi_S$ dependence of a_S. It will probably be smoothed to a wiggle with a much reduced amplitude when deviations from the DIS approximation are taken into account. In any case one would expect Re$[a_{(22)+(1-1)}]$ to be positive in the narrow interval of phase values corresponding to the experimental $G_{(22)+(1-1)}$ decay rate constant, in accordance with experiment [6] (see dashed line).

We conclude that the suppression of the ^{87}Rb decay due to mixed atomic collisions can be understood on the basis of the theory of cold atomic collisions: Destructive interference

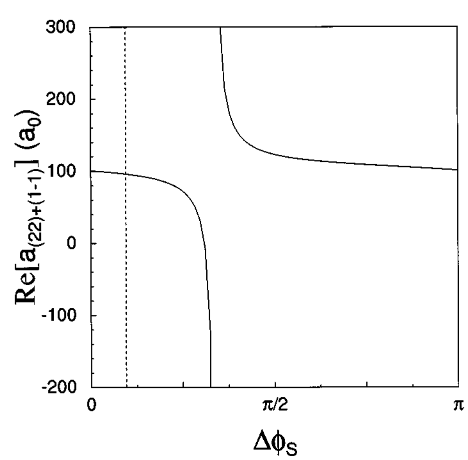

FIG. 6. The $B=0$ $a_{(22)+(1-1)}$ scattering length (DIS) for ^{87}Rb, as a function of $\Delta\phi_S$. The dashed line indicates the $\Delta\phi_S$ value corresponding to the experimental decay rate constant $G_{(22)+(1-1)}$ (see Fig. 5).

between triplet and singlet transition amplitudes between the (22)+(1−1) and (11)+(10) channels leads to a strong suppression relative to typical rates for exchange collisions by almost three orders of magnitude. For Na the decay rate is not suppressed, and a similar two-condensate experiment does not seem possible. In the case of ^{87}Rb the suppression can be used as a strong constraint in the determination of the boundary condition on the s-wave radial wave function at the boundary of the inner range of interatomic distances where the WKB approximation is valid.

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