

Aharonov-Bohm interference in the presence of metallic mesoscopic cylinders

Silviu Olariu

Institute of Physics and Nuclear Engineering, Sectia a III-a, 76900 Magurele, Casuta Postala MG-6, Bucharest, Romania

(Received 3 June 1996)

This work studies the interference of electrons in the presence of a line of magnetic flux surrounded by a normal-conducting mesoscopic cylinder at low temperature. It is found that, while there is a supplementary phase contribution from each electron of the mesoscopic cylinder, the sum of these individual supplementary phases is equal to zero, so that the presence of a normal-conducting mesoscopic ring at low temperature does not change the Aharonov-Bohm interference pattern of the incident electron. It is shown that it is not possible to ascertain by experimental observation that the shielding electrons have responded to the field of an incident electron, and at the same time to preserve the interference pattern of the incident electron. It is also shown that the measuring of the transient magnetic field in the region between the two paths of an electron interference experiment with an accuracy at least equal to the magnetic field of the incident electron generates a phase uncertainty which destroys the interference pattern. [S1050-2947(97)03501-4]

PACS number(s): 03.65.Bz

I. INTRODUCTION

In an Aharonov-Bohm (AB) experiment [1], the incident electrons are prevented from entering the region of the magnetic flux by certain shields. If these shields are superconducting cylinders or mesoscopic cylinders at low temperature, the shielding electrons occupy states which possess phase coherence around the line of magnetic flux. These shielding electrons could in principle bring an additional phase contribution to the conventional AB phase shift of the incident electrons. Experiments carried out by Lischke [2] and Tonomura *et al.* [3] have demonstrated, however, the persistence of the conventional AB shift in the presence of metallic shields.

The absence of a supplementary phase shift due to the shielding electrons has been explained in the case of a metallic shield at normal temperature by Peshkin [4], who pointed out that one could imagine the conductor as being cut, so that the vector potential of the enclosed flux which acts on the *shielding* electrons can be gauged away. Recently, Peshkin [5] has shown that a macroscopic normal conductor can experience no AB effect, by assuming only that a normal conductor lacks off-diagonal long-range order, which means that the electrons lack long-range phase coherence. Moreover, Goldhaber and Kivelson [6,7] have shown that there are no additional phase shifts due to the electrons of a superconducting shield, because of the $2e$ charge of the electron pairs and of the quantization of the magnetic flux.

The case when the shield is a metallic mesoscopic cylinder at low temperature is studied in the present work. In a mesoscopic ring, the phase coherence length may be comparable to or larger than the circumference of the ring, as demonstrated by the oscillations of the magnetoconductance, with a magnetic flux period $h/2e$ [8–11]. It is found in this work that, although there is a supplementary phase contribution from each electron of the mesoscopic cylinder, the sum of these individual supplementary phases is equal to zero, so that the presence of a normal-conducting mesoscopic cylinder at low temperature does not change the AB interference pattern of the incident electron.

The shields in the AB experiments are supposed to prevent the overlap between the incident electrons and the line of magnetic flux *and* to screen the electromagnetic fields of the incident electrons. It is shown that it is not possible to ascertain by experimental observation that the shielding electrons have responded to the field of an incident electron, and at the same time to preserve the interference pattern of the incident electron. It is also shown that the measuring of the transient magnetic field in the region between the two paths of an electron interference experiment with an accuracy at least equal to the magnetic field of the incident electron generates a phase uncertainty which destroys the interference pattern.

The discussion in Sec. II of the classical interaction of an incident electron with a charged rotator and a line of magnetic flux serves as basis for the determination in Sec. III of the supplementary quantum-mechanical phase shift due to the interaction of an electron with the line of magnetic flux and in the presence of the charged rotator. The interaction of an incident electron with a line of magnetic flux in the presence of a circular metallic string at 0 K is discussed in Sec. IV. In Sec. V it is shown that the supplementary phase shift for a metallic circular string decreases exponentially with temperature above 0 K. In Sec. VI it is found that the supplementary contribution averages to zero in the case of a metallic mesoscopic hollow cylinder of nonzero thickness and height. The limitations inherent to the process of observation of weak transient magnetic fields are discussed in Sec. VII.

II. CLASSICAL INTERACTION OF AN INCIDENT ELECTRON WITH A LINE OF MAGNETIC FLUX AND A CHARGED ROTATOR

The interaction of an incident electron with a line of magnetic flux in the presence of a mesoscopic ring is schematically represented in Fig. 1. The electric field of an incident electron exerts an action on the electrons of the mesoscopic cylinder, so that the motion of these shielding electrons is correlated, or coherent with the motion of the incident elec-

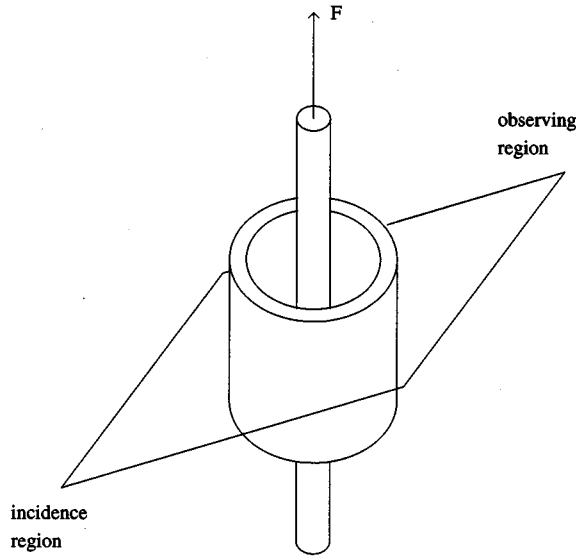


FIG. 1. Electron interference experiment in the presence of a tube of magnetic flux and of a normal-conducting mesoscopic cylinder.

tron on one or the other side of the line of enclosed flux. Supplementary flux-dependent phase shifts could then be expected in principle from these shielding electrons, in addition to the conventional AB phase shift due to the interaction of the incident electron with the line of magnetic flux. The simpler case which will be analyzed in this section is the classical interaction of an incident particle of charge q and velocity $v > 0$ which moves along a straight line passing at a distance d from the axis of the enclosed magnetic flux F , while a charge Q uniformly distributed and rigidly attached to a ring of radius R can freely rotate with angular velocity Ω round the axis of the magnetic flux, as shown in Fig. 2. For $d \gg R$, the Lagrange function of this system is, in Système International (SI) units,

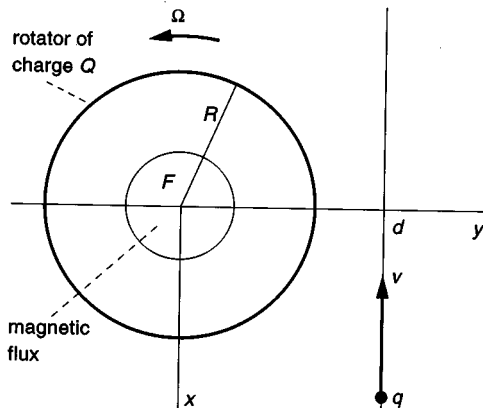


FIG. 2. Incident particle of charge q moving with velocity v along a straight line which interacts with an enclosed magnetic flux F and with a ring of uniformly distributed charge Q , rotating with angular velocity Ω .

$$L(v, \Omega) = \frac{1}{2}mv^2 + \frac{1}{2}MR^2\Omega^2 + \frac{qQR^2d\Omega v}{8\pi\epsilon_0c^2(x^2+d^2)^{3/2}} + \frac{qFdv}{2\pi(x^2+d^2)} + \frac{1}{2\pi}QF\Omega, \quad (1)$$

where m is the mass of the incident particle and M the mass of the charged rotator. The last three terms in Eq. (1) represent, respectively, the interaction between the charged ring and the incident particle; between the magnetic flux and the incident particle; and between the magnetic flux and the charged ring. These contributions are obtained as the product of a charge, of a vector potential, and of a velocity, the vector potential of the charged ring being calculated in the magnetic dipole approximation, valid as long as $d \gg R$. It can be shown from the Lagrange equations that

$$\Omega + \frac{a(x)v}{MR^2} = \Omega_0, \quad (2)$$

$$v^2 \left[1 - \frac{a^2(x)}{mMR^2} \right] = v_0^2, \quad (3)$$

where $a = a(x)$ is given by

$$a(x) = \frac{qQR^2d}{8\pi\epsilon_0c^2(x^2+d^2)^{3/2}}, \quad (4)$$

and where Ω_0 and v_0 are, respectively, the angular velocity of the charged ring and the velocity of the incident particle when the distance between the ring and the particle is very large. If q and Q are equal to the electron charge $-e < 0$ and the masses m, M are equal to the electron mass, it results from Eqs. (2)–(4) that the parameter $a(x)$ is proportional to the classical electron radius $r_0 = 2.8 \times 10^{-15}$ m, so that the variation $\Omega - \Omega_0$ of the angular velocity of the ring is proportional to r_0 , while the variation $v - v_0$ of the velocity of the incident electron is proportional to r_0^2 . Therefore, in a first-order approximation with respect to r_0 , we can consider that the velocity of the incident electron is constant. The enclosed magnetic flux F does not appear in Eqs. (2)–(4), which means that there are no observable classical effects of an enclosed magnetic flux.

III. SUPPLEMENTARY FLUX-DEPENDENT PHASE SHIFT IN THE PRESENCE OF A CHARGED ROTATOR

In Fig. 3 is represented a system composed of an incident particle of charge q moving along a straight line, of a tube of magnetic flux F , and of a particle of charge Q and mass M which can move on a circle of radius R . The nonrelativistic, quantum-mechanical evolution of this system will be analyzed by assuming that the incident particle is moving with constant velocity v along its straight path. The phase shift will be determined by considering that the magnetic field of the incident particle and the vector potential of the enclosed flux are given functions of space and time, and shall neglect the irrotational component of the electric field of the incident particle.

The Schrödinger equation for the wave function $\Psi(\theta, t)$

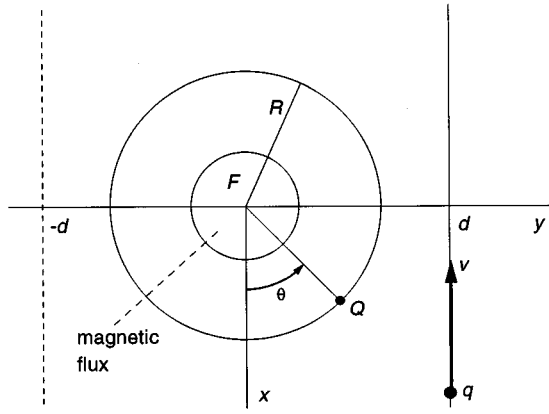


FIG. 3. Incident particle of charge q and velocity v interacting with an enclosed magnetic flux F , and a particle of charge Q which can move on a circle of radius R . The dashed line shows the second possible path of the charge q from the incidence region to the observing region.

of the particle of charge Q is, in a first-order approximation with respect to the parameter $qQ/(4\pi\epsilon_0Mc^2)$,

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{1}{2MR^2} \left(-i\hbar \frac{\partial}{\partial \theta} - \frac{QF}{2\pi} \right)^2 - \frac{qQdv}{8\pi\epsilon_0Mc^2(v^2t^2 + d^2)^{3/2}} \left(-i\hbar \frac{\partial}{\partial \theta} - \frac{QF}{2\pi} \right). \quad (5)$$

The quantity $(-i\hbar \partial/\partial \theta - QF/2\pi)/MR^2$ is the operator for the angular velocity of the particle of charge Q , and the last term in Eq. (5) corresponds to the energy of interaction between the magnetic moment of the rotating charge Q and the magnetic field of the incident particle, written as a function of time. The solutions of Eq. (5) are of the form

$$\Psi_n(\theta, t) = e^{in\theta - i\Phi_n(t)}, \quad (6)$$

where the phase $\Phi_n(t)$ is given by

$$\Phi_n(t) = \frac{\hbar}{2MR^2} \left(n - \frac{QF}{2\pi\hbar} \right)^2 t - \frac{qQ(n - QF/2\pi\hbar)}{8\pi\epsilon_0Mc^2d} \frac{vt}{(v^2t^2 + d^2)^{1/2}}, \quad (7)$$

n being an integer. Thus, as the particle of charge q passes from the incidence region ($t = -\infty$) to the observing region ($t = \infty$), Eq. (7) shows that there is a supplementary phase shift δ_n given by

$$\delta_n = - \frac{qQ}{4\pi\epsilon_0Mc^2d} \left(n - \frac{QF}{2\pi\hbar} \right). \quad (8)$$

If the incident particle of charge q is moving along a straight line situated to the left of the enclosed flux, a path represented by the dashed line of Fig. 3, then the supplementary phase shift due to the action of this particle on the charge Q , which is rotating in the presence of the enclosed

flux F , will be $-\delta_n$. If both q and Q are electrons, then the interference pattern of the incident electron will be shifted by

$$\Delta_n = \Delta_{AB}^{(e)} - \frac{2r_0}{d} \left(n - \frac{QF}{2\pi\hbar} \right), \quad (9)$$

where $\Delta_{AB}^{(e)} = -eF/\hbar$ is the conventional AB shift for the incident electron. The shift in Eq. (9) is independent of the radius R of the string. An alternative way to obtain this result would be to regard the incident electron as moving in the *applied* vector potential of the enclosed flux F and of the charged rotator.

IV. SUPPLEMENTARY FLUX-DEPENDENT PHASE SHIFT IN THE PRESENCE OF A CIRCULAR METALLIC STRING AT 0 K

A circular chain of atoms having the property that each atom contributes one electron which can move freely along the chain will be referred to as a circular metallic string. The analysis in this section of the interaction of an incident electron with a tube of magnetic flux in the presence of the one-dimensional circular metallic string will be used in Sec. VI to study the AB interactions in the presence of real metallic rings.

The state of the electrons in the circular metallic string is described by a multielectron antisymmetric wave function $\Psi_{\text{meso}}(\theta_1, \theta_2, \dots, \theta_{N_0}, t)$ depending on the angular variables $\theta_1, \theta_2, \dots, \theta_{N_0}$ which give the positions of the N_0 electrons. The wave function Ψ_{meso} is a solution of the Schrödinger equation

$$i\hbar \frac{\partial \Psi_{\text{meso}}}{\partial t} = \frac{1}{2m_e R^2} \sum_{j=1}^{N_0} \left(-i\hbar \frac{\partial}{\partial \theta_j} + \frac{eF}{2\pi} \right)^2 \Psi_{\text{meso}} - \frac{e^2 dv}{8\pi\epsilon_0 m_e c^2 (v^2 t^2 + d^2)^{3/2}} \times \sum_{j=1}^{N_0} \left(-i\hbar \frac{\partial}{\partial \theta_j} + \frac{eF}{2\pi} \right) \Psi_{\text{meso}}, \quad (10)$$

where the charge and the mass of the electron are $-e < 0$ and m_e . The wave function Ψ_{meso} can be written as a Slater determinant involving the single-electron states of angular momenta m_1, m_2, \dots, m_{N_0} . Since the single-electron states are the charged-rotator states of Sec. III, the phase shift in the interference pattern of the incident electron in the presence of the circular metallic string can be obtained from Eq. (9) as

$$\Delta_N = \Delta_{AB}^{(e)} - \frac{2N_0 r_0}{d} S_N, \quad (11)$$

where

$$S_N = \frac{1}{N_0} \sum_{j=1}^{N_0} \left(m_j + \frac{eF}{2\pi\hbar} \right). \quad (12)$$

At 0 K the single-electron states are occupied in the order of increasing energy, and the states above the Fermi level are

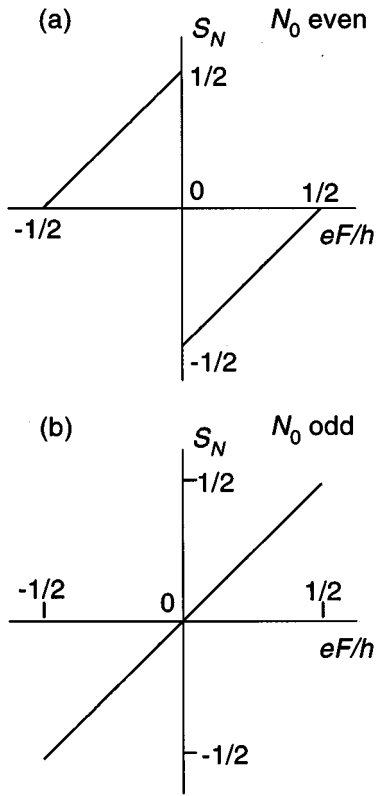


FIG. 4. Phase function S_N for a metallic mesoscopic string at $T=0$ K, for (a) even values of the number of electrons N_0 and (b) odd values of N_0 , for magnetic fluxes $|eF/2\pi\hbar| < 1/2$. S_N is a periodic function of F , of period h/e .

empty. The supplementary phase shift $\Delta_N - \Delta_{AB}^{(e)}$ written in Eqs. (11) and (12) is a *periodic* function of the enclosed magnetic flux F , the period being h/e . If the number of electrons N_0 is odd and $e|F/2\pi\hbar| < 1/2$, the occupied states have at 0 K the angular momentum numbers $0, \pm 1, \dots, \pm[N_0/2]$, where $[N_0/2]$ is the integer part of $N_0/2$, so that

$$S_N = eF/2\pi\hbar \text{ for } N_0 \text{ odd and } e|F/2\pi\hbar| < 1/2. \quad (13)$$

If N_0 is even and $0 < eF/2\pi\hbar < 1/2$, the occupied states have the quantum numbers $0, \pm 1, \dots, \pm(N_0/2 - 1), -N_0/2$, so that

$$S_N = -1/2 + eF/2\pi\hbar \text{ for } N_0 \text{ even and } 0 < eF/2\pi\hbar < 1/2, \quad (14)$$

and if N_0 is even and $-1/2 < eF/2\pi\hbar < 0$, the occupied states have the quantum numbers $0, \pm 1, \dots, \pm(N_0/2 - 1), N_0/2$, so that

$$S_N = 1/2 + eF/2\pi\hbar \text{ for } N_0 \text{ even and } -1/2 < eF/2\pi\hbar < 0. \quad (15)$$

The function S_N is represented in Fig. 4 as a function of the magnetic flux F for a circular string at 0 K, assuming that the number of electrons appearing in Eq. (12) is very large. It can be seen from Fig. 4 that S_N , and with it the phase shift in Eq. (11), is different for circular strings containing an even number of electrons or an odd number of electrons.

The maximum value of the supplementary phase shift is, from Eq. (11), r_0/d per electron of the circular string. For $d=1 \mu\text{m}$, this maximum value is 2.8×10^{-9} rad per electron.

V. TEMPERATURE DEPENDENCE OF THE SUPPLEMENTARY PHASE SHIFT FOR A CIRCULAR METALLIC STRING AT 0 K

The expression of the phase shift given in Eqs. (11) and (12) is valid provided that the occupation number for the electron states changes abruptly from 1 below the Fermi level to 0 above the Fermi level. For temperatures of the metallic string above 0 K, the occupation numbers are given by the Fermi-Dirac distribution, and the phase shift in the interference pattern of the incident electron will be

$$\Delta_T = \Delta_{AB}^{(e)} - \frac{2r_0}{d} C_T, \quad (16)$$

where

$$C_T = \sum_{n=-\infty}^{\infty} \frac{n + eF/2\pi\hbar}{\exp\left[\frac{(\hbar^2/2m_e R^2)(n + eF/2\pi\hbar)^2 - E_0}{kT}\right] + 1}, \quad (17)$$

and E_0 is the energy of the Fermi level for the circular string containing N_0 electrons. If the temperature of the string is such that

$$\frac{N_0 \hbar^2}{4\pi^2 k m_e R^2} \ll T \ll \frac{N_0^2 \hbar^2}{8\pi k m_e R^2}, \quad (18)$$

where k is the Boltzmann constant, the sum in Eq. (17) can be evaluated with the aid of the Poisson sum formula [12] as

$$C_T = -\frac{4\pi m_e R^2 k T}{\hbar^2} \sin(2eF/\hbar) \exp\left(-\frac{4\pi^2 m_e R^2 k T}{N_0 \hbar^2}\right). \quad (19)$$

Due to the condition (18), the factor C_T is exponentially small, and unlike the function S_N , Eq. (12), C_T is not proportional to the number of electrons of the string. Thus the supplementary phase shift due to the global interaction of the incident electron, magnetic flux, and metallic string vanishes rapidly with increasing temperature of the string. The AB shift is, of course, independent of temperature. If we consider that $N_0 = 2\pi R/a_0$, where a_0 is the interatomic distance, then the lower and upper limits in Eq. (18) are, for $a_0 = 2.5 \times 10^{-10}$ m and $R = 10^{-6}$ m, $N_0 \hbar^2 / (4\pi^2 k m_e R^2) = 0.56$ K and $N_0^2 \hbar^2 / (8\pi k m_e R^2) = 2.2 \times 10^4$ K [13].

VI. CANCELLATION OF THE SUPPLEMENTARY PHASE SHIFT FOR A REAL METALLIC MESOSCOPIC CYLINDER

In the case of a mesoscopic cylinder of nonzero thickness and height having the axis along the z direction there are, in addition to the angular momentum quantum number, two quantum numbers resulting from boundary conditions on the motion in the radial and the z directions. For each pair of these radial and z quantum numbers, the number of substates

depending on the angular momentum quantum number may be even or odd, as discussed in Sec. IV. The phase shift in the interference pattern of the incident electron will be

$$\Delta_I = \Delta_{AB}^{(e)} - \frac{2Nr_0}{d} S_I, \quad (20)$$

where N is the number of free electrons of the mesoscopic ring, and S is the average of S_N with the weights $e|F|/\pi\hbar$ for an even number of substates and $(1 - e|F|/\pi\hbar)$ for an odd number of substates, where we have assumed that $e|F|/\pi\hbar < 1/2$. The weight $e|F|/\pi\hbar$ is obtained as the ratio of the energy separation between the two sublevels with the same m , which is proportional to $2em|F|/\pi\hbar$, and the energy separation between the substates of magnetic quantum numbers $m-1$ and m , this separation being proportional to $2m$. These weights and Eqs. (13)–(15) then give

$$S = 0, \quad (21)$$

which means that, due to averaging, the AB interference pattern of the incident electron is not changed by the presence of the metallic mesoscopic cylinder.

VII. MEASURABILITY OF WEAK TRANSIENT MAGNETIC FIELDS IN ELECTRON INTERFERENCE EXPERIMENTS

The primary function of a shield in an AB experiment is of separating the region of space accessible to the incident electrons from the region of the magnetic flux. At the same time, a metallic cylinder of sufficient thickness is expected to prevent the electric and magnetic fields of an incident electron from entering the region of the magnetic flux. In a classical picture, this screening action is explained by the motion of the electrons of the metallic shield produced by the transient electric field of the incident electron.

The part of the electric field of the incident electron having a nonzero circulation is of the order of $ev^2/(4\pi\epsilon_0c^2d^2)$, for a velocity v and at a distance d . If the distance between the incident electron and a shielding electron is of the order of d , and the velocity of the incident electron is v , the displacement of the shielding electron under the action of the aforementioned part of the incident electric field is, for a time $2d/v$, of the order of $2r_0$, the classical electron radius, thus being extremely small.

If N electrons are taking place in the shielding process, then in order to be able to ascertain that they have indeed responded to the field of the incident electron, their average position must be known with a precision better than $2r_0$. The uncertainty in the total momentum of the N electrons will be $\hbar/4r_0$, and the uncertainty in the magnetic field generated by these electrons orbiting on circles of radius R will be of the order of $e\hbar/(16\pi\epsilon_0c^2m_e r_0 R^2)$, so that the uncertainty of the magnetic flux associated with the shielding electrons over an area πR^2 will be of the order of $\pi\hbar/4e$, which produces a phase uncertainty of $\pi/4$, which is sufficient to wipe out the interference pattern of the incident electron. Thus it is not possible to ascertain by experimental observation that the shielding electrons have indeed responded to the field of an incident electron, and at the same time to preserve the interference pattern of the incident electrons. This analysis is

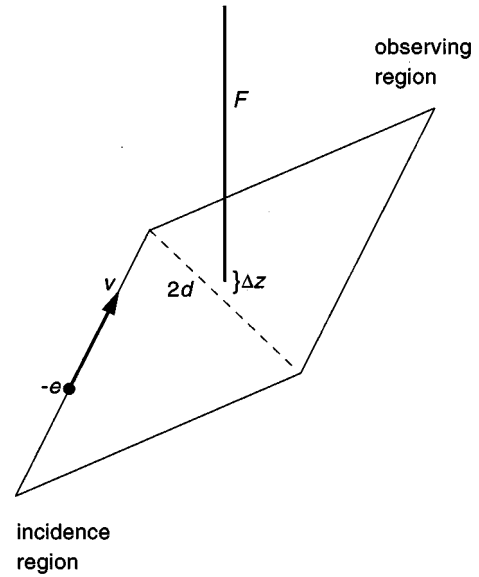


FIG. 5. Measurement of the transient magnetic field extant in the region between the arms of an electron interference experiment with the aid of a magnetic semistring carrying the flux F . The semistring can move freely along the z direction, and the uncertainty in the momentum of the string and the phase difference for the two paths are complementary.

close to that of Furry and Ramsey [13], who have discussed the relation between the AB effect and Bohr's complementarity principle, and is related to the more general problem of measurability of fields in quantum mechanics [14].

A similar limitation exists when we try to measure the transient magnetic field extant in the region between the two paths of an electron interference experiment in the absence of any shielding. These measurements could be conducted for example with the aid of a magnetic semistring carrying the flux F , which can move freely along the z direction, as shown in Fig. 5. If a magnetic field B acts for a time $2d/v$ on the test semistring, the z momentum transferred to the semistring is $2\epsilon_0c^2FBd/v$. At the same time, the flux uncertainty due to the uncertainty in the z position of the extremity of the semistring is $F\Delta z/\pi d$, which entails a phase uncertainty $\Delta\Phi = eF\Delta z/\pi\hbar d$. As it is necessary that $2\epsilon_0c^2FBd/v > \hbar/2\Delta z$, it follows that $B\Delta\Phi > B_e$, where $B_e = ev/(4\pi\epsilon_0c^2d^2)$. Thus a measurement of the magnetic field in the region between the interference paths with an accuracy better than B_e destroys the interference pattern of the incident electron. This shows that the AB effect is not a vector potential versus magnetic field case, but rather it is an example of a global phase effect in quantum mechanics.

VIII. CONCLUSIONS

In quantum mechanics, the state of a system composed of several parts is described by a wave function having a *single* phase. Contributions to this phase arising from the interaction of various parts of the system may become observable in interference experiments involving only a *part* of the system. Thus details of the interaction mechanisms which are relevant from the viewpoint of the classical description, such as a transient magnetic field in a certain region or a transient

change in the velocity of an electron, are not always observable in quantum mechanics.

From this perspective, the Aharonov-Bohm effect appears to be relevant not so much for the problem of the description of the electromagnetic continuum by field strengths or electromagnetic potentials, but rather it demonstrates the global character of the states in quantum mechanics.

ACKNOWLEDGMENTS

This work has been supported by a research grant from the Romanian Academy of Sciences. The author thanks Dr. Murray Peshkin for a correspondence which contributed much to the clarification of problems related to the subject of this paper.

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