Photon generation by time-dependent dielectric: A soluble model

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A soluble model of electromagnetic field interacting with time-dependent dielectric medium is given. The main assumption is that the dielectric constant is switched rapidly from the initial to the final value. Generation of electromagnetic field and photon statistics are found. [S1050-2947(96)07712-8]

I. INTRODUCTION

The notion of the photon is the most basic concept in quantum electrodynamics. However, the quantum nature of the electromagnetic field is well understood in the case of electromagnetic field in empty space only. In the case of fields interacting with external currents and other sources the notion of photons is not so clear. Of particular interest has become the case of electromagnetic field in cavities. If the cavity consists of perfect conductors proper boundary conditions for the field mimic the interaction of the field with charges in the mirrors. The case of field quantization in the presence of dielectrics without dispersion was studied in some detail [1]. Dispersion inevitably leads to damping, which makes the field quantization and hence the notion of photons rather complicated and not yet fully understood [2]. The even more complicated case of photons in a cavity with oscillating perfect mirrors has attracted a lot of interest recently. The corresponding classical problem of electromagnetic field in a cavity with oscillating mirrors does not have a solution in the closed form, but many approximate solutions have been found and discussed.

Quantum effects in the electromagnetic field confined to a cavity with moving mirrors has attracted a lot of interest [3]. This problem is also interesting from the point of view of plasma physics [4]. The quantization of the field in such a situation with explicit time dependence does not follow in any natural way from the standard quantization procedures. Some purely quantum effects such as photon noise reduction have been discussed [5].

Yet another aspect of quantum electrodynamics in rapidly changing geometry was discussed recently in Ref. [6]. The fully inhibited spontaneous emission of the atom placed in a node of the resonant cavity mode is assumed. Then, suddenly one of the cavity mirrors is removed. Now the atom is free to emit a photon. The paper predicts an instantaneous response of the detector exposed by the removed mirror.

Quantization of the electromagnetic field in the case of time-dependent boundaries is thus needed. Deeper understanding of the quantization, the idea of photons, will provide a tool for the study of these effects.

The present paper is devoted to a detailed study of a simple system with an explicitly time-dependent environment. Instead of discussing a cavity with oscillating mirrors, which is a rather difficult problem, we will discuss a much simpler, yet nontrivial case of a cavity from which a dielectric slab is rapidly removed. This system contains most of the ingredients of much more complicated ones, especially with time-dependent components. Our model also contains an external time-dependent element, the dielectric slab. Its time dependence leads to a variety of phenomena — photon creation, squeezing of electromagnetic field, etc. On the other hand, the model is simple enough to be solved exactly, as opposed to much more complicated ones. Our discussion of the model sheds some light on cavity QED processes with explicit time dependence.

II. THE SYSTEM

The system under consideration consists of a cavity with perfectly reflecting metallic mirrors. The mirrors are of the shape of two parallel planes separated by a distance denoted by $2L + d$. Because of the planar symmetry we will consider electromagnetic waves propagating in the direction perpendicular to the mirrors, in this way the problem is reduced to one dimension. The dielectric medium is present between the mirrors forming the cavity. It forms a slab parallel to the cavity mirrors, has width d, and is characterized by dielectric constant ϵ, which for simplicity is assumed to be independent of the frequency. The geometry of the system is shown in Fig. 1. We will consider a case when the electromagnetic field is in the state of vacuum (i.e., the lowest-energy eigenstate) for times $t < 0$. At time $t = 0$ the dielectric slab is removed. This is done instantaneously, i.e., faster than any other characteristic time scale in the problem. The question we will ask and solve in the paper is what is the state of the electromagnetic field for $t > 0$.
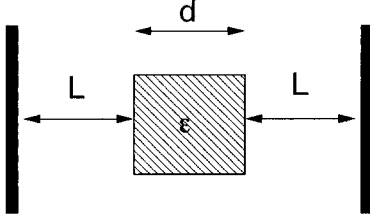
FIG. 1. The system consists of a cavity with perfectly reflecting metallic mirrors. The mirrors are of the shape of two parallel planes separated by a distance $2L+d$.

III. ELECTROMAGNETIC FIELD IN THE PRESENCE OF THE DIELECTRIC

Let us consider now the electromagnetic field in the cavity with the dielectric slab present, i.e., for times $t<0$. Quantum theory of electromagnetic field in the presence of a dispersionless dielectric was given in [1]. The first step in field quantization is to find the classical eigenmodes of the field, i.e., monochromatic solutions to the Maxwell equations with proper boundary conditions. We assume that the mirrors are perfectly reflecting, thus the electric field, parallel to the mirror, vanishes at the boundaries. Apart from coefficients that will be specified later, the vector potential for each mode can be written as

$$A_n(x,t)=f_n(x)\exp(-i\omega_n t)+\text{c.c.}, \tag{1}$$

where $f_n(x)$ satisfies the equation

$$\frac{\partial^2}{\partial x^2}f_n(x)+\frac{\omega_n^2\epsilon(x)}{c^2}f_n(x)=0. \tag{2}$$

The fields are then

$$E_n(x,t)=i\omega_n/cf_n(x)\exp(-i\omega_n t)+\text{c.c.}, \tag{3}$$

$$B_n(x,t)=\frac{\partial}{\partial x}f_n(x)\exp(-i\omega_n t)+\text{c.c.} \tag{4}$$

In the previous equations, ω_n is the frequency of the mode and c denotes the speed of light in the vacuum. The values of the allowed wave vector k_n, and hence of the frequencies ω_n, follow from the boundary conditions for the field at the cavity mirrors. The electric field must vanish at the mirrors, and both the electric field and magnetic field must be continuous at the surfaces between the slab and the vacuum. When these conditions are imposed, the values of k_n are solutions of the transcendental equation

$$\exp[ik_n(2L+nd)]\frac{1-r\exp(-2ik_nL)}{1-r\exp(2ik_nL)}=\pm 1, \tag{5}$$

where n is the index of refraction $n=\epsilon^{1/2}$ and

$$r=\frac{\epsilon^{1/2}-1}{\epsilon^{1/2}+1}. \tag{6}$$

The complex equation (5) defines the values of k_n uniquely.

The explicit form of the functions $f_n(x)$ will not be given here. It suffices to say that they form a complete, orthonormal set of functions.

Every solution of the Maxwell equations in the cavity can be expanded into the eigenmodes $f_n(x)$:

$$E(x,t)=i\sum_n (2\pi\hbar\omega_n)^{1/2}[\beta_n f_n(x)\exp(-i\omega_n t)-\text{c.c.}], \tag{7}$$

$$B(x,t)=c\sum_n (2\pi\hbar/\omega_n)^{1/2}\left[\beta_n\frac{\partial}{\partial x}f_n(x)\exp(-i\omega_n t)+\text{c.c.}\right], \tag{8}$$

where β_n are expansion coefficients. Planck's constant $\hbar$ has been introduced into this classical formula in order to simplify the quantization.

Quantization of the electromagnetic field consists in replacing the β_n coefficients by operators $\hat{b}_n$ satisfying canonical commutation relations:

$$[\hat{b}_n,\hat{b}_{n'}^\dagger]=\delta_{n,n'}. \tag{9}$$

After quantization the fields E,B become operators:

$$\hat{E}(x,t)=i\sum_n (2\pi\hbar\omega_n)^{1/2}[\hat{b}_n f_n(x)\exp(-i\omega_n t)-\text{H.c.}], \tag{10}$$

$$\hat{B}(x,t)=c\sum_n (2\pi\hbar/\omega_n)^{1/2}\left[\hat{b}_n\frac{\partial}{\partial x}f_n(x)\exp(-i\omega_n t)+\text{H.c.}\right]. \tag{11}$$

The states of the electromagnetic field are constructed from the vacuum Ω, the state for which $\hat{b}_n\Omega=0$, by applying creation operators $\hat{b}_{n'}^\dagger$. In what follows we will assume that the electromagnetic field is in the vacuum state for $t<0$.

IV. ELECTROMAGNETIC FIELD WITHOUT THE DIELECTRIC

At time $t=0$ the dielectric slab is suddenly removed. Thus the cavity between the mirrors becomes empty and the electromagnetic field can be characterized by free field. We can repeat the quantization procedure for the case of an empty cavity. For an empty cavity the solutions of the Maxwell equations can be written as

$$E(x,t)=i\omega_n/cg_j(x)\exp(-i\omega_j t)+\text{c.c.}, \tag{12}$$

$$B(x,t)=\frac{\partial}{\partial x}g_j(x)\exp(-i\omega_j t)+\text{c.c.}, \tag{13}$$

where the values of the frequencies ω_j can be found from the condition that the electric field vanishes at the walls. Hence we get the equation

$$k_j(2L+d)=\pi j \tag{14}$$

and $\omega_j = ck_j$. This time the equation can be solved to give $k_j = \pi j/(2L+d)$, where $j = \pm 1, \pm 2, \ldots$. The exact form of the $g_j(x)$ functions will not be given here; when we combine together the functions $g_j(x)$ and $g_{-j}(x)$ we obtain the correct modes, which are just standing waves. We shall indicate these modes with $p_j(x)$, where $j = 1,2,\ldots$. Every solution of the Maxwell equations can be expressed as a linear combination of the monochromatic solutions:

$$E(x,t) = i \sum_j p_j(x)(2\pi\hbar\omega_n)^{1/2}\alpha_j \exp(-i\omega_j t) + \text{c.c.}, \quad (15)$$

$$B(x,t) = c \sum_j \frac{\partial}{\partial x} p_j(x)(2\pi\hbar/\omega_n)^{1/2}\alpha_j \exp(-i\omega_j t) + \text{c.c.}$$
$$(16)$$

Replacing the coefficients α_j by operators $\hat{a}_j$, $\hat{a}_j^\dagger$, satisfying canonical commutation relations,

$$[\hat{a}_j, \hat{a}_{j'}^\dagger] = \delta_{j,j'}, \quad (17)$$

we get the quantum theory of the electromagnetic field in the free cavity. The states of the electromagnetic field are characterized by amplitudes of finding a given photon number in each mode.

V. SUDDEN REMOVAL OF THE SLAB

We will use the Heisenberg picture and Heisenberg equations of motion to discuss the evolution of the electromagnetic field. For $t < 0$ the evolution is very simple; the operators $\hat{b}_j, \hat{b}_{j'}^\dagger$ satisfy the free equations of motion. It is only the phase of the operators that changes with time:

$$\hat{b}_n(t) = \hat{b}_n(t_0)\exp[-i\omega_n(t-t_0)]. \quad (18)$$

Both t and t_0 should be negative and ω_n is the frequency of the nth mode.

For positive times the time evolution of the electromagnetic field is also free in the cavity without the dielectric slab. Since $\hat{a}_j$ operators correspond to frequency eigenmodes we have

$$\hat{a}_j(t) = \hat{a}_j(t_1)\exp[-i\omega_j(t-t_1)], \quad (19)$$

where both t and t_1 should be positive. It is only the relation between the field just before and just after the removal of the dielectric that leads to the nontrivial part of the time evolution.

Sudden removal of the slab results in matching conditions:

$$D(x,0^-) = D(x,0^+), \quad (20)$$

$$B(x,0^-) = B(x,0^+), \quad (21)$$

valid for all x.

The continuity of the magnetic field B is natural; the medium is nonmagnetic both for $t < 0$ as well as for $t > 0$. The continuity of the D field requires a comment since continuity of D leads to a discontinuity of the E field. A look at the Maxwell equations clearly shows that it should be the D field

that changes in a continuous way when the dielectric is removed. Maxwell's equations contain the time derivative of the D field, not the E field. Discontinuous dependence of D on time would mean that Maxwell's equation could not be satisfied.

Relations (20) and (21) can be expressed in terms of the $\hat{b}_n$, $\hat{b}_n^\dagger$ $\hat{a}_j$, $\hat{a}_j^\dagger$ operators. They should be considered as a set of equations for $\hat{a}_j$, $\hat{a}_j^\dagger$ as unknowns. These operators, expressed in terms of $\hat{b}_n$, $\hat{b}_n^\dagger$ operators, will provide the solution of the Heisenberg equations of motion.

The solution can be found by making use of the completeness relations for the $f_n(x)$. These relations provide the solution to our dynamical problem: creation and annihilation operators $\hat{a}_j$, $\hat{a}_j^\dagger$ in the future are expressed as linear functions of the creation and annihilation operators $\hat{b}_n$, $\hat{b}_n^\dagger$ in the past:

$$\hat{a}_j = \sum_n (h_{j,n}\hat{b}_n + g_{j,n}\hat{b}_n^\dagger), \quad (22)$$

$$\hat{a}_j^\dagger = \sum_n (h_{j,n}^*\hat{b}_n^\dagger + g_{j,n}^*\hat{b}_n). \quad (23)$$

Such relations between creation and annihilation operators are also found in, e.g., parametric processes and are known as the Bogoliubov transformation. The exact form of $h_{j,n}$ and $g_{j,n}$, although simple in principle, will not be reproduced here. We will only note that they are related to overlap integrals of the mode functions with and without the dielectric slab.

This is the main result of our paper. We have found exact solution of the dynamical equations of motion for quantized electromagnetic field in the presence of the dielectric with a time-dependent dielectric constant. This has been possible because of our assumption of rapid change of the dielectric constant. A similar approach is often used in quantum theory and is called sudden approximation. Here we have applied it to electrodynamics.

VI. PHOTON CREATION

Relations (20) and (21) lead to spontaneous photon creation. Removal of the slab and thus a change in the spatial shape of the eigenmodes as well as a change in the spectrum of the wave vectors provide the mechanism of creation of real photons. In this section we will find the average number of photons and the spectrum of photons created by sudden removal of the slab.

The average photon number in the mode j for times $t > 0$ is given by the expectation value of the photon number operator $\hat{N}_j = \hat{a}_j^\dagger \hat{a}_j$. The state of the field is the vacuum, i.e., the state that is annihilated by all the $\hat{b}_n$.

In Fig. 2 we have plotted the dependence of the photon number N in jth mode as a function of the mode number j. Since the frequency of the field is given by $\omega_j = c\pi j/(2L+d)$, the number j can be considered as a measure of the frequency. Note that the number of photons N approaches a constant for large frequencies. This rather unphysical limit is due to our assumption of sudden switching of the dielectric constant. If the dielectric constant was
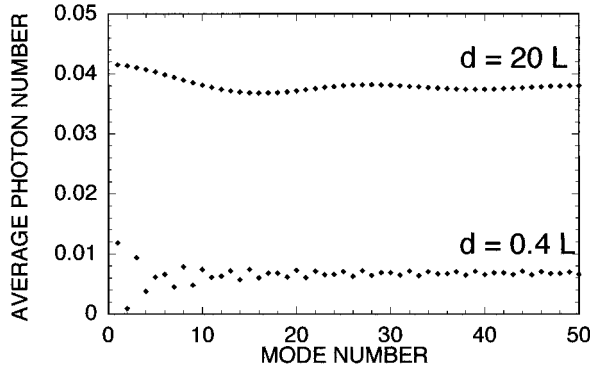
FIG. 2. Photon number N in jth mode as a function of the mode number j for $n=1.5$, $d=0.4L$, and $d=20L$.

FIG. 3. Dispersion of $\hat{x}_j$ as a function of mode number j for $n=1.5$, $d=0.4L$, and $d=20L$.

changed smoothly from an initial value to the final value, high-frequency photons would not be emitted.

We can also consider the opposite case. For $t<0$ the cavity is empty and the electromagnetic field is in the vacuum state (empty vacuum); at $t=0$ the dielectric slab is suddenly inserted. In this case we need the relations that express the $\hat{b}_j$ operators in terms of the $\hat{a}_n$ and $\hat{a}_n^\dagger$ operators. Such relations have the same form as Eqs. (22) and (23). Then we find spontaneous photon creation also in this case.

VII. SQUEEZING OF THE FIELD

The electromagnetic field created in the cavity has distinctly quantum features. One such feature is squeezing. Squeezing is a reduction of field fluctuations below their vacuum value at the expense of the fluctuations of the conjugate variable. Let us define a Hermitian operator related to the $\hat{a}_j$ field:

$$\hat{x}_j(\phi) = \exp(i\phi)\hat{a}_j + \exp(-i\phi)\hat{a}_j^\dagger, \tag{24}$$

where ϕ is a parameter. Fluctutation of the $\hat{x}_j(\phi)$ operator may be reduced below their vacuum value equal to 1 for some ϕ. If this is the case we say that the field is in a quadrature squeezed state. Our solution allows one to calculate directly the fluctuations of $\hat{x}_j(\phi)$. We have found that squeezing indeed occurs in our system. This is quite natural since the solution for $\hat{a}_j$ gives a linear combination of $\hat{b}_n$ and $\hat{b}_n^\dagger$. This is just the squeezing transformation.

The largest fluctuation reduction occurs for $\phi=\pi/2$ but it extends for ϕ from about 40° to about 140° for all cases considered. In Fig. 3 we have shown the dispersion $\sigma^2 = \langle \hat{x}^2 - \langle \hat{x} \rangle^2 \rangle$ for $\phi=90°$. Squeezing is evident in all modes; it is rather small for small values of d and assumes larger values for larger values of d. Squeezing is stronger for modes with small frequencies.

VIII. CONCLUSIONS

We have considered the quantum theory of electromagnetic field in cavity with a time-dependent dielectric. A very simple time dependence, sudden removal of the dielectric slab, leads to a soluble model. We have shown that explicit time dependence leads to a variety of effects. The most interesting one is the creation of photons. It should be noted
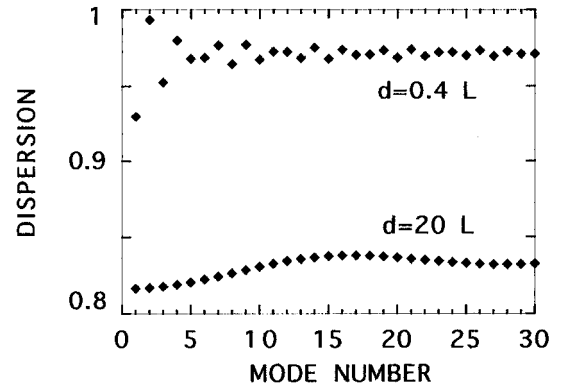
that there is no classical counterpart of this effect. In classical electrodynamics a sudden change of the dielectric constant is not equivalent to a current radiating electromagnetic field. Thus there is no radiation in the framework of classical theory. Quantum theory, on the other hand, allows for radiation, and in fact predicts it. The expectation value of the radiated field is equal to zero, as could be expected from the correspondence principle. The intensity of the radiated field, however, is different from zero. This is characteristic for quantum theory. In fact, as we have shown, the state of the radiated field is a squeezed state, which has many nonclassical features. As an example let us mention that the flux of radiated energy goes to zero in the limit of the Planck constant going to zero. In this way we have shown that the time dependence of the dielectric leads to a nonclassical effect—creation of photons from vacuum.

Nonlinear optical effects, such as, e.g., the parametric effect (see, e.g., [7]) are often believed to be needed to obtain squeezed states of light. In fact nonlinear effects are not essential. We have shown here that an explicitly time-dependent dielectric constant can also lead to squeezing. This fact was also mentioned in [4].

We have considered a time-dependent problem; the dielectric constant exhibits explicit time dependence. In the remote past and remote future the system becomes stationary and it is therefore possible to quantize the electromagnetic field using standard methods. In this way our system is much simpler than systems with oscillating walls, or oscillating dielectric constant, where quantization is a conceptual problem. Thus we have been able to give a natural quantum theory and discuss such quantum effects as, e.g., squeezing.

After writing this paper we learned that Artoni, Bulatov, and Birman [8] studied a problem similar to ours. They consider the removal of a lossless dielectric slab that fills *all* the space between the mirrors. They obtain relations similar to Eqs. (22) and (23), which lead to squeezing and photon creation.

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