# Zeeman splitting in the Maxwell-Bloch theory of collisionally pumped lasers

D. Benredjem, A. Sureau, and C. Möller
*Laboratoire de Spectroscopie Atomique et Ionique, URA 775 du CNRS, Université Paris-Sud, Bâtiment 350, 91405 Orsay Cedex, France*
(Received 30 October 1996)

The radiation generated in the soft-x-ray domain by spontaneous emission and amplified while propagating in a plasma column is investigated in the Maxwell-Bloch (MB) formalism. The MB theory is especially appropriate to describe the interaction between the medium and the lasing radiation in the saturation regime, and asymptotically yields the commonly used theory of the small-signal gain coefficient. We have examined the effect of spontaneously created magnetic fields on gain coefficients, integrated intensity, and steady-state population of quantum states. Our calculations are applied to the 0-1 and 2-1 radiations at 196 and 236 Å, respectively, in collisionally pumped Ne-like germanium lasers. For these radiations and in our range of plasma parameters the Voigt function—accounting for the Zeeman splitting of the various sublevels associated with a given multiplet—provides accurate line shapes. [S1063-651X(97)08705-9]

PACS number(s): 42.55.Vc, 32.60.+i, 32.80.−t, 52.20.−j

## I. INTRODUCTION

Although the demonstration of soft-x-ray amplification and even saturation in collisionally pumped lasers has become almost a routine since the first observations in 1985 [1–4], the development of a complete theory of this scheme has progressed somewhat more slowly than experimental advances. Following a recent approach on the amplification of spontaneous emission (ASE) using the Maxwell-Bloch (MB) formalism [5], we intend to investigate the effect of self-generated magnetic fields on intensities and ion level populations in the saturation regime of collisionally pumped lasers.

One is essentially faced with the resolution of the radiative-transfer equation. In many problems involving nonpolarized radiation and relevant to x-ray laser modeling as well as to astrophysics, the purpose is to find the solution of the following one-dimensional (1D) equation:

$$\partial_z I(\nu,z) = G(\nu,z)[I(\nu,z) + S(\nu,z)], \tag{1}$$

where $I(\nu,z)$ is the intensity at the frequency $\nu$ and coordinate $z$. $G$ and $S$ designate the gain and the source functions, respectively. In x-ray lasers $G$ involves absorption and induced-emission processes. The ASE theory, with a constant gain coefficient (the gain coefficient being the opposite of the absorption function), is not adequate to model the history of radiation when the intensity nears saturation, as the populations and thus $G$ and $S$ depend on $z$. In this regime the problem is generally overcome by using a steady-state saturation intensity [6–8] that enables the effect of stimulated transitions on the populations to be modeled indirectly. However, this treatment fails to directly model changes in the level populations—potentially significant in a system that may have several coupled lasing transitions—and, moreover, neglects the modeling of certain quantum-mechanical effects arising in a linearly propagating field. The combination of the Maxwell wave equation and the Bloch equations may overcome this limitation [5]. The other effects arising in collisional lasers such as spontaneous radiative decay, collisional excitation, and deexcitation, ionization, and recombination need to be considered in the population rate equations.

Theoretical investigation of collisionally pumped systems is generally through the use of a hydrodynamics–atomic physics package generating a time-dependent description of the laser-produced plasma that is postprocessed by either a ray-tracing [9] or a wave optics [10–12] treatment. Furthermore, the $2J+1$ quantum states that form a given lasing level are assumed to react as a whole to the radiation field, and the rate equations that describe populations refer to levels and not specifically to states. Within such approximations, the role of saturation in determining the intensity of the laser output and in describing the gain narrowing for arbitrary inhomogeneous line broadening are now well understood [7,8].

A number of works have already used the MB formalism in specific investigations such as buildup of radiation [13], gain [14], transverse coherence [15,16], superfluorescence [17], or superradiance [18] theory. In x-ray laser modeling, the degeneracy of the lasing levels has been considered in the treatment of the interaction between the ASE electric field and the amplifying medium [5].

The aim of the present work is to investigate the effect of magnetic fields, which spontaneously build up in the plasma due to the large density and temperature gradients, on the population of ionic levels and on the amplified intensity. In this case the Zeeman sublevels, associated with each multiplet, are no longer degenerate. For such systems we can derive the MB equations that govern the evolution of the quantum-state populations, rather than the level populations, as well as the evolution of the nonpolarized spontaneous-emission intensity, which is amplified while propagating through the medium. The populations themselves depend on the intensity of the radiation through the population rate equations. The Maxwell wave equation is considered using the customary paraxial approximation (see Ref. [13]), and the density-matrix coherences are assumed to be in steady state with respect to their production and decay processes. The spatial evolution of the electric-field phase as a function of the free-electron density and population inversions, and the spatial evolution of the electric-field amplitude as a function of the population inversions are obtained. The gain coefficient can then be deduced directly, and is expressed in a form that reduces to the small-signal gain coefficient at low intensity.

At high intensity we are not able to associate a rate coefficient of stimulated emission, or absorption, to the multiplets involved in lasing. Due to the nature of the linearly propagating radiation (all electric-field oscillations occur in the plane perpendicular to the direction of propagation), the interactions between the radiation and the various quantum states are not identical. The $2J+1$ states $|JM\rangle$ associated with a given multiplet $J$ are no longer degenerate in the presence of a magnetic field, and inelastic electron-ion collisions of the type $|JM\rangle\otimes|\mathbf{p}l\rangle\rightarrow|JM'\rangle\otimes|\mathbf{p}'l'\rangle$, where $|\mathbf{p}l\rangle$ is a quantum state of an incident electron, must also be considered, as they tend to restore equilibrium populations among the states of the considered multiplet. In particular, these collisions lessen or even eliminate the polarization of the medium by the x-ray beam.

The irradiation of a target along a prescribed line focus is such that the produced plasma expands nearly cylindrically. The generated x-ray pencil then propagates along the axis of a cylindrical plasma column, which forms the amplifying medium. If the intensity of the beam is large enough to affect populations, the plasma becomes inhomogeneous in the direction of propagation.

The intensity satisfies the radiative-transfer equation given above, and is coupled to the quantum-states population densities, which are governed by the following rate equations:

$$\partial_t n_i(z) = -n_i(z)\Gamma_i(z) + r_i(z), \tag{2}$$

where $i$ designates a quantum state—upper or lower—of the considered ion. $\Gamma$ and $r$ are, respectively, the total decay rate of the considered state and the sum of all processes populating it.

## II. ZEEMAN SPLITTING AND VOIGT PROFILE FUNCTIONS

In the laser-produced-plasma conditions that prevail in x-ray lasers, magnetic fields may be spontaneously created owing to the existence of nonparallel gradients of the electron temperature and density. An estimate of such fields is obtained from the well-known relation [19]

$$\mathbf{B} = \frac{1}{eN_e}\frac{\nabla(kT_e)\times\nabla N_e}{|\nabla u|}, \tag{3}$$

where $u$ is the macroscopic flow velocity of the plasma. The $z$ axis is conventionally taken to be the plasma column axis or, what amounts to the same thing, the direction of propagation of the x-ray pencil (see Fig. 1), and is thus perpendicular to the direction of the heating beam. The electron-density gradient is large in the direction perpendicular to the target surface while the electron-temperature gradient is important in the lateral direction. The resulting magnetic field is then parallel to the plasma-column axis. Within the assumption of nearly cylindrical expansion, which is satisfied in most cases, it is easy to estimate the amplitude of the magnetic field in the gain domain. The following values are representative of collisionally pumped germanium lasers: $N_e \approx 7\times10^{20}$ cm$^{-3}$, $|\nabla N_e| \approx 2\times10^{23}$ cm$^{-4}$, $|\nabla T_e| \approx 10^4$ eV cm$^{-1}$, and $|\nabla u| \approx 3\times10^9$ s$^{-1}$ (see Ref. [20]), yielding $B\approx10$ T.

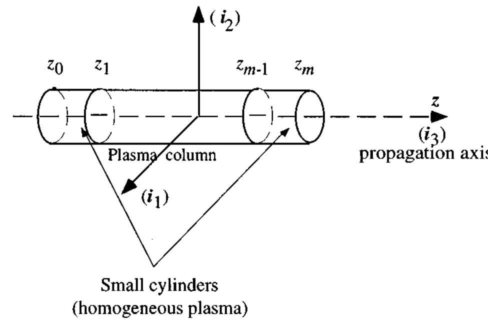



FIG. 1. Geometry of the amplifying plasma. The $z$ axis is taken to be the plasma-column axis, or equivalently the propagation axis. The nonparallel gradients of the electron density and electron temperature are contained in a transverse plane, and the resulting magnetic field is thus along the propagation axis. The plasma is represented as a succession of adjacent cylinders having a common axis, $z$ axis, and lengths $z_p - z_{p-1}$ $(p=1,\ldots,m)$. The lengths are taken sufficiently small, so that the plasma is assumed to be homogeneous in each fraction of the plasma column. The lasing radiations are amplified over the length $z_m - z_0$.

The inhomogeneous broadening, which is attributed to the Stark interaction with the slowly moving ions, is negligible for the Ne-like germanium radiations at 196 and 236 Å, at the density and temperature conditions that prevail in collisional lasers. We can thus assume that the Voigt function gives a good description of the line shape. The Voigt profile accounts for thermal Doppler broadening and electron-impact broadening, on the one hand, and electron-impact shift, on the other hand. The Zeeman interaction removes the spherical degeneracy, so that the $2J+1$ states $|JM\rangle$, associated with the $J$ multiplet, are symmetrically split. If we assume a Maxwellian distribution for ion velocities, with a most probable value $v_0$, the Voigt profile of a given transition $|JM\rangle\rightarrow|J'M'\rangle$, having a central angular frequency $\omega_0$ for a free ion, is

$$P_{JM,J'M'}(\omega) = \frac{\gamma}{2}\frac{1}{\pi^{3/2}v_0} \times \int_0^\infty dv\, \frac{\exp[-(v/v_0)^2]}{(\omega-\omega_0-\omega_0 v/c-\Delta\omega_e-\Delta\omega_B)^2+(\gamma/2)^2}, \tag{4}$$

where $\gamma$ and $\Delta\omega_e$ are the angular-frequency width and shift, due to electron collisions with the lasing ion, and $\Delta\omega_B$ is the Zeeman shift,

$$\Delta\omega_B = \hbar^{-1}(gM - g'M')\mu_B B, \tag{5}$$

in which $\mu_B[=e\hbar/(2mc)]$ designates the Bohr magneton and the $g$'s are the Landé factors. We have in the $j$-$j$ coupling scheme $g(1/2,1/2)=2/3$ for $J=0$ and $J=1$, and $g(1/2,3/2)=3/2$ for $J=2$.

Let $t=v/v_0$, $x=(\omega-\omega_0-\Delta\omega_e-\Delta\omega_B)/\Delta_D$, and $y=\gamma/(2\Delta_D)$, where $\Delta_D=\omega_0 v_0/c$ is the thermal Doppler width, which is related to the Doppler full width at half maximum (FWHM) $\delta\omega_D$ through $\delta\omega_D=2\sqrt{\ln 2}\,\Delta_D$. It is then easy to get (see, e.g., Ref. [21])

$$P_{JM,J'M'}(\omega)=\frac{1}{\pi^{3/2}}\frac{y}{\Delta_D}\int_0^\infty dt\,\frac{\exp(-t^2)}{(x-t)^2+y^2}$$
$$=\frac{1}{\pi^{1/2}}\frac{1}{\Delta_D}\,\mathrm{Re}\,w(x+iy). \quad (6)$$

The function $w(x+iy)$ is then expanded in terms of trigonometric and hyperbolic functions:

$$w(x+iy)=\exp[-(x+iy)^2]\{1-\mathrm{erf}[-i(x+iy)]\}$$
$$=\exp[-(x^2-y^2+2ixy)][1-\mathrm{erf}(y-ix)]$$
$$=\exp[-(x^2-y^2+2ixy)]$$
$$\times\Bigg[1-\Bigg\{\mathrm{erf}\,y+\frac{\exp(-y^2)}{2\pi y}$$
$$\times[1-\cos(2xy)-i\,\sin(2xy)]$$
$$+\frac{2}{\pi}\exp(-y^2)\sum_{m=1}^{\infty}\frac{\exp(-m^2/4)}{m^2+4y^2}$$
$$\times[f_m(y,-x)+ig_m(y,-x)]\Bigg\}\Bigg], \quad (7)$$

where

$$f_m(y,-x)=2y-2y\,\cosh(mx)\cos(2xy)$$
$$+m\,\sinh(mx)\sin(2xy) \quad (8a)$$

and

$$g_m(y,-x)=-2y\,\cosh(mx)\sin(2xy)$$
$$-m\,\sinh(mx)\cos(2xy). \quad (8b)$$

After a straightforward derivation the Voigt profile becomes

$$P(x)=\frac{1}{\pi^{1/2}\Delta_D}\exp(-x^2)\Bigg\{\frac{\sin^2(xy)}{\pi y}+\exp(y^2)\cos(2xy)$$
$$\times(1-\mathrm{erf}\,y)+\frac{4y}{\pi}\sum_{m=1}^{\infty}\frac{\exp(-m^2/4)}{m^2+4y^2}$$
$$\times[\cosh(mx)-\cos(2xy)]\Bigg\}. \quad (9)$$

Such a profile is symmetrical with respect to $x=0$, which means that the central angular frequency is $\omega_0+\Delta\omega_e+\Delta\omega_B$. The corresponding value of $P$ is

$$P(0)=\frac{1}{\pi^{1/2}\Delta_D}\exp(y^2)(1-\mathrm{erf}y). \quad (10)$$

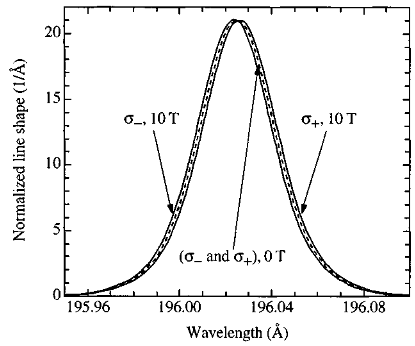



FIG. 2. Normalized Voigt profiles of the circularly polarized radiations associated with the $|(2p^5_{1/2}3p_{1/2})00\rangle\rightarrow|(2p^5_{1/2}3s)1\pm1\rangle$ transitions in Ne-like Ge, for two values of the spontaneously generated magnetic field. For finite values of $B$ the profiles of the $\sigma_+$ and $\sigma_-$ transitions are symmetrically split with respect to the unperturbed line shape (associated with $B=0$), which is then common to the two transitions.

The Voigt FWHM is obtained by setting $P(x)=P(0)/2$, which yields the following relation:

$$\exp(-x^2)\Bigg\{\frac{\sin^2(xy)}{\pi y}+\cos(2xy)C(y)$$
$$+\frac{4y}{\pi}\sum_{m=1}^{\infty}\frac{\exp(-m^2/4)}{m^2+4y^2}[\cosh(mx)$$
$$-\cos(2xy)]\Bigg\}-\frac{C(y)}{2}=0, \quad (11)$$

where $C(y)=\exp(y^2)(1-\mathrm{erf}y)$. It has been checked that the $m>5$ terms do not contribute significantly to the solution.

We now focus our attention on germanium slab targets, which have been the subject of recent theoretical [4] and experimental [22] progress. More specifically we consider the Ne-like transitions at 196 Å, i.e., between $(2p^5_{1/2}3p_{1/2})_{J=0}$ and $(2p^5_{1/2}3s)_{J=1}$, and at 236 Å, i.e., between $(2p^5_{1/2}3p_{3/2})_{J=2}$ and $(2p^5_{1/2}3s)_{J=1}$ (henceforth referred to as 0-1 and 2-1 radiations). It is worth noting that lasing around 200 Å in Ne-like ions constitutes the most efficient collisionally pumped lasers with large gain-length products [23,24]. Figure 2 shows the individual Voigt profiles for spontaneous emission in the presence of a magnetic field. To each $\sigma_+$ transition between an upper state with magnetic quantum number $M$ and a lower state with magnetic quantum number $M'$ such that $q=M-M'=1$, a symmetrical $\sigma_-$ transition ($-M\rightarrow-M'$) characterized by $q=-1$ is associated. As expected the profiles of these circularly polarized radiations are symmetrically split with respect to the profile of the $\pi$ transition $|J0\rangle\rightarrow|J'0\rangle$, and the separation increases linearly with the magnitude of the magnetic field. The shift of the various components is small compared to the linewidth.

## III. RADIATIVE-TRANSFER PROBLEM

Due to the splitting of the $\sigma_+$ and $\sigma_-$ radiations, the intensity of each component at a given frequency must be considered separately. The radiative-transfer equation for $\sigma_q$ photons ($q=\pm 1,0$) propagating in the $z$ direction may also be written as

$$\partial_z I^{(q)}_{J,J'}(\nu,z)=j^{(q)}_{J,J'}(\nu,z)+G^{(q)}_{J,J'}(\nu,z)I^{(q)}_{J,J'}(\nu,z), \quad (12)$$

where $j^{(q)}_{J,J'}$ and $G^{(q)}_{J,J'}$ designate the monochromatic emissivity and gain, respectively, both of which are $z$-dependent in the saturation regime. The emissivity of the medium corresponds to the radiation generated in a solid angle $\Theta$ ($\approx 10^{-4}$) centered on the $z$ axis by all the transitions from the Zeeman sublevels represented by $J$ to those represented by $J'$ such that $M-M'=q$, and is given by

$$j^{(q)}_{J,J'}(\nu,z)=\frac{3\Theta}{8\pi}h\nu\sum_M n_{JM}(z)A_{JM,J'M-q}(\nu), \quad (13)$$

where $A_{JM,J'M-q}(\nu)$ is the probability of spontaneous emission from $|JM\rangle$ to $|J'M-q\rangle$, at the frequency $\nu$. The index $q$ takes only the values 1 and $-1$ as a radiation propagating along the $z$ axis cannot include any $\pi$-polarized radiation (polarization component parallel to $z$, corresponding to $q=0$), and we have to consider only circularly polarized radiation ($q=\pm 1$). The spectral gain is then

$$G^{(q)}_{J,J'}(\nu,z)=\frac{k}{2\epsilon_0\hbar}\sum_M \langle JM|d_q|J'M-q\rangle^2 P_{JM,J'M-q}(\nu) \times[n_{JM}(z)-n_{J'M-q}(z)], \quad (14)$$

where $k=\omega/c$ is the wave vector of light. In order to calculate $A_{JM,J'M-q}(\nu)$, let us consider the spontaneous emission in the solid angle $\Theta$. The corresponding probability per unit time is (see, e.g., Ref. [25])

$$dA_{JM,J'M-q}(\nu)=\frac{1}{2\pi\hbar}\left(\frac{2\pi\nu}{c}\right)^3|\langle JM|d_q|J'M-q\rangle|^2 \times P_{JM,J'M-q}(\nu)\Theta, \quad (15)$$

where $d_q$ is the relevant tensorial component of the electric-dipole operator $\mathbf{d}$.

We thus obtain

$$\begin{aligned} dA_{JM,J'M-q}(\nu) &= \frac{1}{2\pi\hbar}\left(\frac{2\pi\nu}{c}\right)^3 (J\|d\|J')^2\begin{pmatrix} J & 1 & J' \\ -M & q & M-q\end{pmatrix}^2 \times P_{JM,J'M-q}(\nu)\Theta \\ &= \frac{1}{2\pi\hbar}\left(\frac{2\pi\nu}{c}\right)^3\frac{3\hbar}{4}\left(\frac{c}{2\pi\nu}\right)^3(2J+1) \times A_{J,J'}\begin{pmatrix} J & 1 & J' \\ -M & q & M-q\end{pmatrix}^2 P_{JM,J'M-q}(\nu)\Theta \\ &= \frac{3}{8\pi}(2J+1)A_{J,J'}\begin{pmatrix} J & 1 & J' \\ -M & q & M-q\end{pmatrix}^2 P_{JM,J'M-q}(\nu)\Theta. \end{aligned}$$

Owing to the fact that the emission in a given polarization is not isotropic (only two of the three orthogonal directions are allowed) the integration over angles introduces a factor $8\pi/3$. The total rate of spontaneous emission in term of the Einstein coefficient $A_{J,J'}$ is then

$$A_{JM,J'M-q}(\nu)=(2J+1)A_{J,J'}\begin{pmatrix} J & 1 & J' \\ -M & q & M-q\end{pmatrix}^2 \times P_{JM,J'M-q}(\nu). \quad (16)$$

We numerically solve the radiative-transfer problem by treating the plasma column as an assembly of $m$ adjacent cylinders with a common axis, $z$, and lengths $z_p-z_{p-1}$ ($p=1,\ldots,m$). $z_m-z_0$ is obviously the plasma length over which the radiation propagates (see Fig. 1). The intervals are chosen sufficiently small, so that populations, temperatures, and densities are independent of $z$ in each cylinder. In a given interval $[z_{p-1},z_p]$ the equation of transfer is then easily integrated, yielding

$$I^{(q)}(\nu,z)=\frac{j^{(q)}(\nu,z_p)}{G^{(q)}(\nu,z_p)}\{\exp[G^{(q)}(\nu,z_p)(z-z_{p-1})]-1\} +\exp[G^{(q)}(\nu,z_p)(z-z_{p-1})]I^{(q)}(\nu,z_{p-1}). \quad (17)$$

The first contribution in the right-hand side of the above relation is due to the emission spontaneously generated and amplified in the interval considered, while the second one describes the amplification, in the same interval, of the radiation coming from the preceding segment $[z_{p-2},z_{p-1}]$. Assuming $I^{(q)}(\nu,z_0)=0$ we easily obtain the intensity in terms of the source function $S^{(q)}(\nu,z)=j^{(q)}(\nu,z)/G^{(q)}(\nu,z)$ and of the spectral gain, at the various abscissa $z_p$:

$$I^{(q)}(\nu,z_1)=S^{(q)}(\nu,z_1)\{\exp[G^{(q)}(\nu,z_1)(z_1-z_0)]-1\}, \quad (18a)$$

$$I^{(q)}(\nu,z_2)=S^{(q)}(\nu,z_2)\{\exp[G^{(q)}(\nu,z_2)(z_2-z_1)]-1\} +S^{(q)}(\nu,z_1)\{\exp[G^{(q)}(\nu,z_1)(z_1-z_0)]-1\} \times\exp[G^{(q)}(\nu,z_2)(z_2-z_1)], \quad (18b)$$

$$I^{(q)}(\nu,z_m) =\sum_{p=1}^{m}S^{(q)}(\nu,z_p)\{\exp[G^{(q)}(\nu,z_p)(z_p-z_{p-1})]-1\} \times\exp[G^{(q)}(\nu,z_{p+1})(z_{p+1}-z_p)]\times\cdots \times\exp[G^{(q)}(\nu,z_m)(z_m-z_{m-1})], \quad (18c)$$

where all the indexes are $\leqslant m$. $S^{(q)}(\nu,z_p)$ and $G^{(q)}(\nu,z_p)$ are the source function and the spectral gain in $[z_{p-1},z_p]$. Both functions satisfy $\partial_z=0$ in this segment. The relation (18c) may be written in the more compact form:

$$I^{(q)}(\nu,z_m)=\sum_{p=1}^{m}[S^{(q)}(\nu,z_p)-S^{(q)}(\nu,z_{p-1})] \times\exp\sum_{l=p}^{m}[G^{(q)}(\nu,z_l)(z_l-z_{l-1})]-S^{(q)}(\nu,z_m), \quad (18d)$$

with the condition $S^{(q)}(\nu,z_0)=0$.

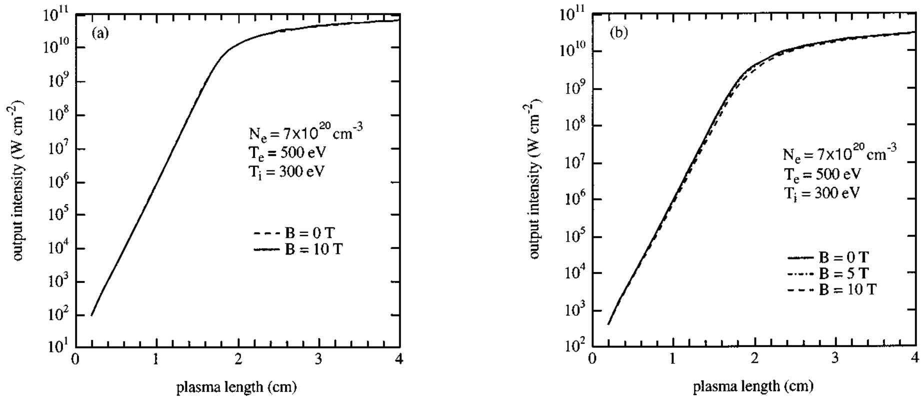



FIG. 3. Integrated intensity of the 0-1 (a) and 2-1 (b) radiations in Ne-like Ge, as a function of plasma length for three values of the spontaneously created magnetic field. Saturation is reached at 2.5–3.5 cm, depending on the radiation and on the magnetic field. The 0-1 radiation saturates before the 2-1 one.

It is worth noting that the above treatment is able to account for the possible inhomogeneities of the plasma along the propagation axis of the radiation. Large-scale inhomogeneities in the line plasma emission have actually been observed in a qualitative way by many groups [26,27]. Kieffer and co-workers [28] (see also Ref. [29]) have shown that such inhomogeneities arise from nonuniform pump laser irradiation along the line focus, and affect the gain of the x-ray laser. However, the results presented in this work do not include those effects.

The total intensity $I(z)$ is obtained by adding the partial intensities $I^{(q)}(\nu,z)$, $q=1$, and $q=-1$, and integrating with respect to frequency:

$$I(z)=\int d\nu[I^{(1)}(\nu,z)+I^{(-1)}(\nu,z)].$$

In Fig. 3 we show the evolution with $z$ of the amplified intensity of the 0-1 and 2-1 radiations in Ne-like germanium, for three values of the magnetic field. We choose an electron density of $7\times10^{20}\ \mathrm{cm}^{-3}$, which may be accessed by the use of curved targets or prepulses and we assume an electron temperature of 500 eV and an ion temperature of 300 eV. For a given plasma length and for increasing magnetic fields we observe a small attenuation of the intensity, due to the broadening of the intensity profile, generated by the increasing splitting of the various $\sigma_+$ and $\sigma_-$ profiles. While the saturation intensity is reached at larger $z$ values its magnitude does not depend on the magnetic field value, because amplification overcomes the effect of the magnetic field on line shapes. As is clearly seen for intermediate plasma lengths, the 2-1 radiation is more affected by the Zeeman interaction than the 0-1 radiation. This behavior is attributed to a larger broadening of the 2-1 radiation, which is constituted of $3\sigma_+$ and $3\sigma_-$ transitions, while the 0-1 radiation is formed of $1\sigma_+$ and $1\sigma_-$ transition only (see Fig. 4).

The plasma-column radius and length, and consequently the solid angle $\Theta$, have no effect on the gain-coefficient values. In order to calculate an effective gain [30], which is defined in the same way as in experiments and which is thus comparable to the *observed* gain, let us consider a fictitious transition $U$-$L$ giving rise to a Gaussian-shape line. Following Linford *et al.* [31] the corresponding integrated intensity is

$$I_{UL}(z)\propto\frac{[\exp(G_0z)-1]^{3/2}}{[G_0z\ \exp(G_0z)]^{1/2}},\tag{19}$$

where $G_0$ is the gain value at line peak. The effective gain is the value of $G_0$, which, treated as a parameter, allows a

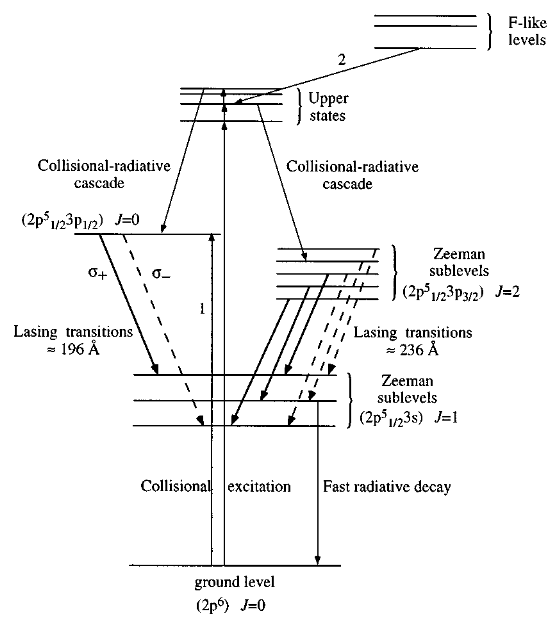
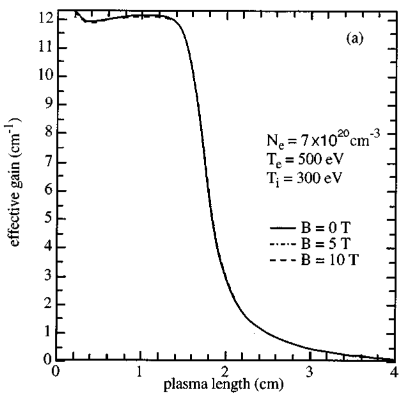



FIG. 4. Schematic representation of the $3p$-$3s$ transitions involved in the collisional-excitation scheme in the Ne-like sequence. Energies are not scaled. The collisional excitations from the ground level are followed by collisional-radiative cascades, which populate the upper lasing states $|(2p^5_{1/2}3p_{3/2})2M\rangle$ and $|(2p^5_{1/2}3p_{1/2})00\rangle$. These states are metastable, i.e., E1 radiative decay to the ground level is forbidden. The transition labeled 1 designates the monopole collisional excitation, which is the leading mechanism in the population of the $J=0$ level. The transitions 2 represent the recombination from the $F$-like stage, dominated by the dielectronic recombination. Inversions may occur because the lower lasing states $|(2p^5_{1/2}3s)1M\rangle$ have a strong radiative decay to the ground level.



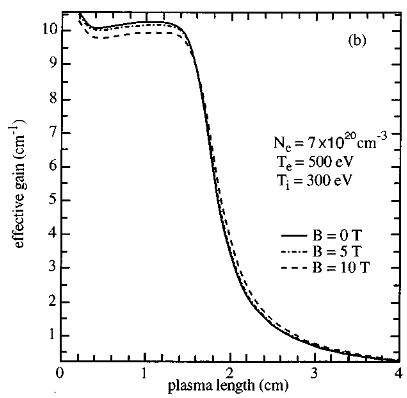




FIG. 5. Effective gain of the 0-1 (a) and 2-1 (b) radiations in Ne-like Ge, as a function of plasma length for three values of the spontaneously created magnetic field. The effective gain involves all the transitions that contribute to the output signal (two transitions for the 0-1 radiation and six transitions for the 2-1 one) and is derived from the intensities calculated for various plasma lengths. Saturation is reached at 2.5–3.5 cm, depending on the radiation and on the magnetic field.

matching of the integrated intensities $I$ and $I_{UL}$. Figure 5 illustrates, for various magnetic fields, the evolution of the effective gain for the 2-1 and 0-1 radiations, as a function of the plasma length. At the electron density of $7\times10^{20}$ cm$^{-3}$ it is obvious that the 0-1 radiation is dominant. For the same reason as above, the 2-1 effective gain is more influenced by the magnetic field than the 0-1 one.

## IV. QUANTUM-STATE POPULATIONS

Taking into account the Maxwell wave equation for radiation through the globally neutral plasma and the Bloch equation for the density operator [5] we can obtain the rate equations for the population densities $n$ in the presence of the radiation propagating along the $z$ axis, which is associated with the $JM-J'M'$ transitions. We have

$$\partial_t n_{JM}(z)=r_{JM}(z)-\Gamma_{JM}(z)n_{JM}(z) - \sum_{q,|q|=1}\langle JM|d_q|J'M-q\rangle^2[n_{JM}(z) - n_{J'M-q}(z)] \times\frac{1}{2\hbar^2\epsilon_0 c}\int d\nu\, I^{(q)}_{J,J'}(\nu,z)P_{JM,J'M-q}(\nu) \tag{20}$$

for the upper Zeeman sublevels and

$$\partial_t n_{J'M'}(z)=r_{J'M'}(z)-\Gamma_{J'M'}(z)n_{J'M'}(z) + \sum_{q,|q|=1}\langle JM'+q|d_q|J'M'\rangle^2 \times[n_{JM'+q}(z)-n_{J'M'}(z)]\frac{1}{2\hbar^2\epsilon_0 c} \times\int d\nu\, I^{(q)}_{J,J'}(\nu,z)P_{JM'+q,J'M'}(\nu) \tag{21}$$

for the lower sublevels. The above equations when coupled to the radiative-transfer equation determine the quantum-state populations at each $z$ value, and generalize the usual equation system that describes the energy level populations. The $r$ and $\Gamma$ coefficients result from all populating and depopulating processes except those of absorption and stimulated emission, which are associated with the $J$-$J'$ radiation. Using the property $\langle JM|d_1|J'M-1\rangle^2 =\langle J-M|d_{-1}|J'-M+1\rangle^2$, and dropping the $z$ coordinate for ease of notation, the population rate equations in the presence of the 0-1 and 2-1 radiations may be written as

$$\begin{aligned}
\partial_t n_{2\pm2}&=r_{2\pm2}-\Gamma_{2\pm2}n_{2\pm2}-[n_{2\pm2}-n_{1\pm1}]3\Lambda_{2\pm2,1\pm1},\\
\partial_t n_{2\pm1}&=r_{2\pm1}-\Gamma_{2\pm1}n_{2\pm1}-[n_{2\pm1}-n_{10}]\tfrac{3}{2}\Lambda_{2\pm1,10},\\
\partial_t n_{20}&=r_{20}-\Gamma_{20}n_{20}-\{[n_{20}-n_{11}]\tfrac{1}{2}\Lambda_{20,11} +[n_{20}-n_{1-1}]\tfrac{1}{2}\Lambda_{20,1-1}\},\\
\partial_t n_{1\pm1}&=r_{1\pm1}-\Gamma_{1\pm1}n_{1\pm1}+\{[n_{2\pm2}-n_{1\pm1}]3\Lambda_{2\pm2,1\pm1} +[n_{20}-n_{1\pm1}]\tfrac{1}{2}\Lambda_{20,1\pm1}+[n_{00}-n_{1\pm1}]\Lambda_{00,1\pm1}\},\\
\partial_t n_{10}&=r_{10}-\Gamma_{10}n_{10}+\{[n_{21}-n_{10}]\tfrac{3}{2}\Lambda_{21,10} +[n_{2-1}-n_{10}]\tfrac{3}{2}\Lambda_{2-1,10}\},\\
\partial_t n_{00}&=r_{00}-\Gamma_{00}n_{00}-\{[n_{00}-n_{11}]\Lambda_{00,11} +[n_{00}-n_{1-1}]\Lambda_{00,1-1}\},
\end{aligned} \tag{22}$$

with

$$\Lambda_{JM,J'M'}=\frac{B_{J,J'}}{c}\int d\nu\, I^{(q)}_{J,J'}(\nu)P_{JM,J'M'}(\nu) \quad (q=M-M'), \tag{23}$$

where $B_{J,J'}=(J\|d\|J')^2/[6\hbar^2\epsilon_0(2J+1)]$ is the Einstein coefficient for stimulated emission. Owing to the Zeeman inter-

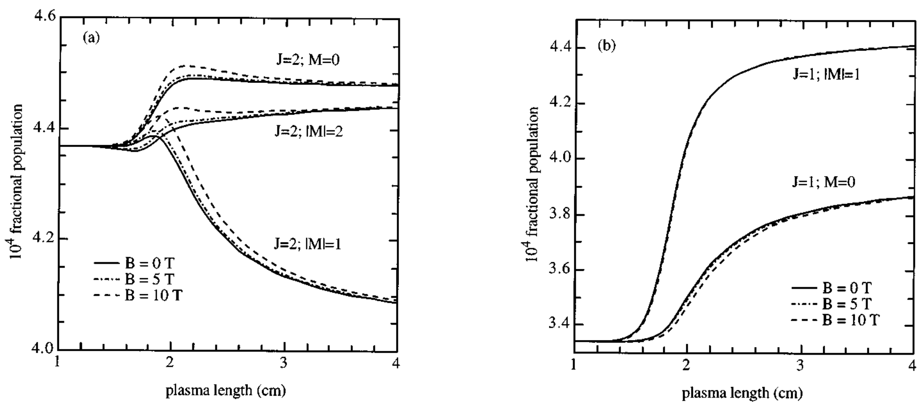



FIG. 6. Steady-state populations of the quantum states $|(2p_{1/2}^5 3p_{3/2})2M\rangle$ (a) and $|(2p_{1/2}^5 3s)1M\rangle$ (b), as a function of plasma length for three values of the spontaneously generated magnetic field, the collisional population redistribution among the Zeeman sublevels of a same multiplet being ignored. Saturation is reached at 2.5–3.5 cm, depending on the radiation and on the magnetic field. The 0-1 radiation saturates before the 2-1 one (see text).

action, the individual profiles of the various $\sigma_+$ and $\sigma_-$ components involved in a given $J$-$J'$ radiation have the same shape but are shifted with respect to each other, symmetrically with respect to the line center $\nu_0=(\omega_0+\Delta\omega_e)/(2\pi)$. Let us consider the integrals $\Lambda_{JM,J'M'}$ and $\Lambda_{J-M,J'-M'}$. Omitting the $B_{J,J'}/c$ factor, the first one is equal to $\int d\nu\, I_{J,J'}^{(q)}(\nu)P_{JM,J'M'}(\nu)$, and the second one is equal to $\int d\nu\, I_{J,J'}^{(-q)}(\nu)P_{J-M,J'-M'}(\nu)$. The first integral is obviously equal to $\int d\nu' I_{J,J'}^{(q)}(\nu_0+\nu')P_{JM,J'M'}(\nu_0+\nu')$ and the second one to $\int d\nu' I_{J,J'}^{(-q)}(\nu_0-\nu')P_{J-M,J'-M'}(\nu_0-\nu')$. Now it is clear from Fig. 2 that $P_{JM,J'M'}(\nu_0+\nu')=P_{J-M,J'-M'}(\nu_0-\nu')$. So, in the case of a nonpolarized global beam, and whenever $I_{J,J'}^{(q)}(\nu_0+\nu')=I_{J,J'}^{(-q)}(\nu_0-\nu')$, one obtains

$$\Lambda_{JM,J'M'}=\Lambda_{J-M,J'-M'}. \qquad (24)$$

This property clearly shows that the two systems of equations, for the $\{|JM\rangle,|J'M'\rangle\}_{M,M'\geqslant 0}$ states, on the one hand, and the $\{|JM\rangle,|J'M'\rangle\}_{M,M'\leqslant 0}$ states, on the other hand, are identical, yielding $n_{JM}=n_{J-M}$ and $n_{J'M'}=n_{J'-M'}$. Therefore the resolution of the population rate equations needs to consider only one sign, e.g., the upper sign, viz.,

$$\begin{aligned}
\partial_t n_{22}&=r_{22}-\Gamma_{22}n_{22}-[n_{22}-n_{11}]3\Lambda_{22,11},\\
\partial_t n_{21}&=r_{21}-\Gamma_{21}n_{21}-[n_{21}-n_{10}]\,\tfrac{3}{2}\Lambda_{21,10},\\
\partial_t n_{20}&=r_{20}-\Gamma_{20}n_{20}-[n_{20}-n_{11}]\Lambda_{20,11},\\
\partial_t n_{11}&=r_{11}-\Gamma_{11}n_{11}+\{[n_{22}-n_{11}]3\Lambda_{22,11}+[n_{20}-n_{11}]\\
&\quad\times\tfrac{1}{2}\Lambda_{20,11}+[n_{00}-n_{11}]\Lambda_{00,11}\},\\
\partial_t n_{10}&=r_{10}-\Gamma_{10}n_{10}+[n_{21}-n_{10}]3\Lambda_{21,10},\\
\partial_t n_{00}&=r_{00}-\Gamma_{00}n_{00}-[n_{00}-n_{11}]2\Lambda_{00,11}.
\end{aligned} \qquad (25)$$

This set of time-dependent equations is resolved with the help of the collisional-radiative model of Pert [32], which we have adapted for our purpose in order to account for the integrals $\Lambda$, which describe the effect of the lasing radiations on the quantum-state populations, and for the Zeeman interaction. The rates contain all significant interlevel terms and have yielded a very satisfactory description of the laser in the small-signal limit [33]. The problem is modeled in a piecewise fashion through the plasma column, assuming uniform conditions inside each interval, for the purpose of (i) finding the atomic populations and (ii) amplifying the beam through the interval of plasma thus obtained, appending the appropriate ASE term from the segment [see Eq. (17)]. We assume that an aperture of 10 mrad ($\Theta\approx 3\times 10^{-4}$ sr) of the spontaneous emission contributes to the x-ray beam. The populations are calculated in the steady state.

Figures 6 and 7 represent the fractional populations of the quantum states involved in the 2-1 radiation as a function of length, for three values of the magnetic field. In Fig. 6 the population transfers between Zeeman sublevels $JM$, fixed $J$, by electron-ion collisions are ignored, in order to exhibit clearly the different effects of the magnetic field on the various individual states. At the onset of saturation the populations of the five states—represented as three due to the $M,-M$ symmetry discussed above—of the $J=2$ multiplet separate as a consequence of the asymmetry in the stimulated-emission rates imposed by the absence of $\pi$-polarized radiation. The populations of the $|2\pm 1\rangle$ states fall most rapidly as these states interact only with $\sigma_\pm$ radiation in any case, and the increase of population for the $|1\pm 1\rangle$ states, due to the effect of the 0-1 radiation, which saturates first, reinforces the absorption processes $|1\pm 1\rangle\rightarrow|2\pm 2\rangle$ and $|1\pm 1\rangle\rightarrow|20\rangle$, yielding thus a smaller decrease for the $|2\pm 2\rangle$ and $|20\rangle$ populations. A small distance beyond the onset of 0-1 saturation, the 2-1 radiation itself begins to saturate, and stimulated emission begins to affect populations. The result is a surge in population for the $|1\pm 1\rangle$ states, accompanied by a second-order effect on the $|2\pm 2\rangle$ and $|20\rangle$, which are coupled to these states. In other words, the 0-1 radiation dominates the lasing and is responsible for the saturation behavior; the initial effect is then a rapid increase in the population of the $|1\pm 1\rangle$ states with consequent effects on the connected $|2\pm 2\rangle$ and $|20\rangle$ states.

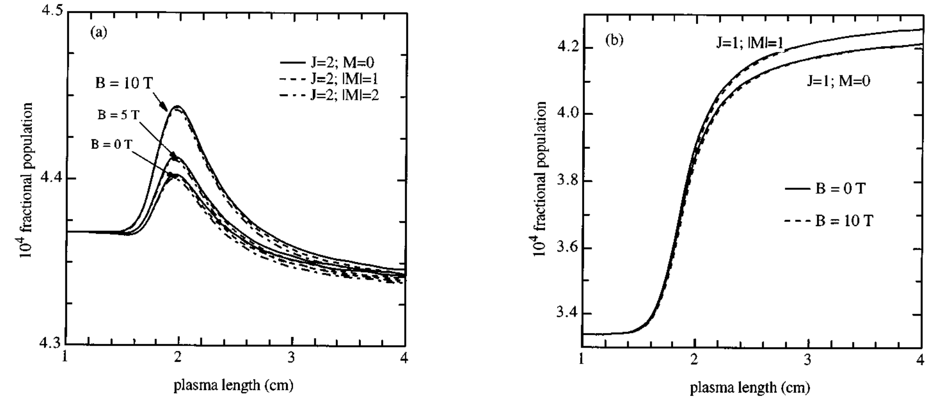



FIG. 7. Steady-state populations of the quantum-states $|(2p_{1/2}^5 3p_{3/2})2M\rangle$ (a) and $|(2p_{1/2}^5 3s)1M\rangle$ (b), as a function of plasma length for three values of the spontaneously generated magnetic field, taking into account the population redistribution among the Zeeman sublevels of a same multiplet by electron-ion collisions. Comparison with Fig. 6 shows the importance of this last mechanism.

For increasing magnetic fields the populations of the lower states decrease while those of the upper states increase. We have stressed in the preceding section that in this case the intensity decreases (see Fig. 3), yielding a decrease for the lower states populations [+ sign in Eq. (25)] and an increase for the upper states populations [− sign in Eq. (25)]. Figure 7 shows the complementary effect produced by the electron-ion collisions of the type $(\alpha JM)+e^-\rightarrow(\alpha JM')+e^-$, which tend to restore an equal population among the various Zeeman sublevels.

## V. CONCLUSION

This work presents a theoretical description of quantum-state populations and radiation intensities of collisionally pumped lasers, in the presence of spontaneously created magnetic fields. The coupled intensities and populations are calculated in the framework of the Maxwell-Bloch formalism, which is the most convenient frame for plasma samples exhibiting population inversions between two groups of Zeeman sublevels connected by electric-dipole interaction. The equations are valid with any radiation intensity, provided the coherence envelopes may be assumed to be in steady state relative to their production and decay processes. When the ASE intensities become large this approach is appropriate to describe the continuous transition to saturation of x-ray lasers in plasmas.

For intensities approaching saturation and in the presence of a strong magnetic field one needs to consider the specific quantum-state interactions with radiation. At low intensity the solutions, as they should, tend continuously towards the usual ASE regime with a constant gain value along the $z$ axis, and the gain value is then consistent with the Einstein coefficient for stimulated emission as calculated by the Fermi ‘‘golden rule.’’

Our calculations are applied to Ne-like Ge ions because the germanium slab targets have been proved to allow one of the highest efficiencies in collisionally pumped lasers with large gain-length products. We have shown that the line splitting of the circularly polarized transitions associated with an x-ray beam propagating along the $z$ axis increases with increasing magnetic fields. This expected behavior yields a small depletion of the integrated intensity, more pronounced for the 2-1 radiation than for the 0-1 one, due to the larger number of transitions involved. In accordance with a recent work in the recombination scheme (Ref. [30]), we have calculated an effective gain suitable for direct comparison with experiment. This definition is consistent with the experimental determination of the gain, and takes into account the whole line shape as well as the overlapping, which may arise when several lasing components contribute to the output signal. For the lines considered it has been checked that the effective gain is comparable to the peak gain, which is not surprising due to the smallness of the inhomogeneous Stark broadening. For a given target length we observe that increasing the magnetic field lessens the saturation (the components are more split and the total lasing efficiency is less). In this case the population of the lower quantum states decreases whereas that of the upper states increases by a larger amount.

We have investigated the 100-ps regime where the self-generated magnetic field is essentially due to the existence of noncollinear gradients of the electron density and temperature. In the last three years a new kind of plasma, namely, the transient sources (see, e.g., Refs. [34] and [35]), has been investigated. These sources are obtained by a succession of two laser pulses, the first, of ns duration, creates a plasma with nearly uniform electron density and temperature distributions, and the second one of ps duration heats the plasma when the proportion of Ne-like ions is large enough. The second pulse produces population inversions via electron collisions. The large intensity associated with the second pulse gives rise to other magnetic field sources, which compete with the analytical term given by relation (3). A better knowledge of the magnetic field would require a complete simulation involving at least the ponderomotive force in combination with resonance absorption. Furthermore, the presence of high-velocity electrons also contributes to the magnetic field generation. A study of this process under the assumption of a two-temperature electron fluid has been presented by Mason [36].

Further investigations can be undertaken, such as the study of the mutual interactions between an incident polarized radiation and a plasma. The incident beam may polarize the lasing ions of the plasma which in turn react back on the polarization state of the output signal. The MB formalism used in the present work is suitable to treat problems of this type.

## ACKNOWLEDGMENTS

We are indebted to G. J. Pert (York University) for providing the collisional-radiative code that we have adapted for this purpose. We are also grateful to P. B. Holden for helpful discussion during this work.


[1] D. L. Matthews, P. L. Hagelstein, M. D. Rosen, M. J. Eckart, N. M. Ceglio, A. Hazi, H. Medecki, B. J. McGowan, J. E. Trebes, B. J. Whitten, E. M. Campbell, C. W. Hatcher, A. M. Hawryluk, R. L. Kauffman, L. D. Plaseance, G. Rambach, G. H. Scofield, G. Stone, and T. Weaver, Phys. Rev. Lett. **54**, 110 (1985).

[2] A. Carillon, H. Z. Chen, P. Dhez, L. Dwivedi, J. Jacoby, P. Jaeglé, G. Jamelot, Jie Zhang, M. H. Key, A. Kidd, A. Klisnick, R. Kodama, J. Krishnan, C. L. S. Lewis, D. Neely, P. Norreys, D. O'Neill, G. J. Pert, S. A. Ramsden, J. P. Raucourt, G. J. Tallents, and J. Uhomoibhi, Phys. Rev. Lett. **68**, 2917 (1992).

[3] P. Jaeglé, A. Carillon, P. Dhez, P. Goedtkindt, G. Jamelot, A. Klisnick, B. Rus, Ph. Zeitoun, S. Jacquemot, D. Mazataud, A. Mens, and J. P. Chauvineau, in *X-ray Lasers 1994, Proceedings of the Fourth International Colloquium on X-ray Lasers*, edited by D. C. Eder and D. L. Matthews, AIP Conf. Proc. No. 332 (AIP, New York, 1994).

[4] T. N. Lee, E. A. McLean, and R. C. Elton, Phys. Rev. Lett. **59**, 1185 (1987).

[5] A. Sureau and P. B. Holden, Phys. Rev. A **52**, 3110 (1995).

[6] A. E. Siegman, *Lasers* (University Science Books, Mill Valley, CA, 1986).

[7] J. A. Koch, B. J. McGowan, L. B. DaSilva, D. L. Matthews, J. H. Underwood, B. J. Batson, and S. Mrowka, Phys. Rev. Lett. **68**, 3291 (1992).

[8] G. J. Pert, J. Opt. Soc. Am. B **11**, 1425 (1994).

[9] A. Dulieu and S. Jacquemot, in *X-ray Lasers 1992, Proceedings of the 3rd International Colloquium on X-ray Lasers*, edited by E. E. Fill, IOP Conf. Series No. 125 (Institute of Physics, Bristol, 1992).

[10] M. D. Feit and J. A. Fleck, J. Opt. Soc. Am. B **7**, 2048 (1990).

[11] R. A. London, M. D. Rosen, M. S. Maxon, D. C. Eder, and P. L. Hagelstein, J. Phys. B **22**, 3363 (1989).

[12] R. P. Rathowsky, R. S. Craxton, M. D. Feit, R. A. London, R. W. Wallings, G. M. Shimkaveg, A. L. Osterheld, and M. R. Carter, in *X-ray Lasers 1992, Proceedings of the 3rd International Colloquium on X-ray Lasers* (Ref. [9]).

[13] M. Strauss, Phys. Fluids B **1**, 907 (1989).

[14] G. Hazak and A. Bar-Shalom, Phys. Rev. A **38**, 1300 (1988).

[15] R. A. London, M. Strauss, and M. D. Rosen, Phys. Rev. Lett. **65**, 563 (1990).

[16] P. Amendt, R. A. London, and M. Strauss, Phys. Rev. A **44**, 7478 (1991).

[17] E. A. Watson, H. M. Gibbs, F. P. Mattar, M. Cormier, Y. Claude, S. L. McCall, and M. S. Feld, Phys. Rev. A **27**, 1427 (1983).

[18] A. Crubellier, S. Liberman, and P. Pillet, J. Phys. B **19**, 2959 (1986).

[19] W. L. Kruer, *The Physics of Laser Plasma Interactions* (Addison-Wesley, Redwood City, CA, 1988).

[20] B. Rus, C. L. S. Lewis, G. F. Cairns, P. Dhez, P. Jaeglé, M. H. Key, D. Neely, A. G. McPhee, S. A. Ramsden, C. G. Smith, and A. Sureau, Phys. Rev. A **51**, 2316 (1995).

[21] *Handbook of Mathematical Functions*, edited by M. Abramowitz and I. A. Stegun (Dover, New York, 1970).

[22] C. L. S. Lewis, D. Neely, J. Uhomoibhi, D. M. O'Neill, S. A. Ramsden, G. J. Tallents, Y. A. Hadithi, M. H. Key, S. J. Rose, and G. J. Pert, in *X-ray Lasers 1990, Proceedings of the 2nd International Colloquium on X-Ray Lasers*, edited by G. J. Tallents, IOP Conf. Series No. 116 (Institute of Physics, Bristol, 1990); see also *X-ray Lasers 1992, Proceedings of the 3rd International Colloquium on X-ray Lasers* (Ref. [9]).

[23] M. H. Key, J. Mod. Opt. **35**, 575 (1988).

[24] B. Rus, A. Carillon, B. Gauthé, P. Goedtkindt, P. Jaeglé, G. Jamelot, A. Klisnick, A. Sureau, and Ph. Zeitoun, J. Opt. Soc. Am. B **11**, 564 (1994).

[25] I. I. Sobelman, *Atomic Spectra and Radiative Transitions* (Springer-Verlag, Berlin, 1992).

[26] T. N. Lee, W. A. Molander, J. L. Fow, and R. C. Elton, Rev. Sci. Instrum. **57**, 2052 (1986).

[27] Z. Xu, S. Chen, L. Lin, Z. Jiang, W. Zhang, and A. Qian, Phys. Rev. A **39**, 808 (1989).

[28] J. C. Kieffer, M. Chaker, H. Pépin, M. Nantel, H. A. Baldis, J. Dunn, G. D. Enright, and D. M. Villeneuve, Opt. Commun. **84**, 208 (1991).

[29] M. Nantel, J. C. Kieffer, B. La Fontaine, H. Pépin, G. D. Enright, D. M. Villeneuve, J. Dunn, H. A. Baldis, and O. Peyrusse, Phys. Fluids B **5**, 4465 (1993).

[30] D. Benredjem, A. Calisti, A. Sureau, and B. Talin, J. Quant. Spectrosc. Radiat. Transfer **55**, 439 (1996).

[31] G. J. Linford, E. R. Peressini, W. R. Sooy, and M. L. Spaeth, Appl. Opt. **13**, 379 (1974).

[32] G. J. Pert, J. Comput. Phys. **39**, 251 (1980).

[33] P. B. Holden, S. B. Healy, M. T. M. Lightbody, G. J. Pert, J. A. Plowes, A. E. Kingston, E. Robertson, C. L. S. Lewis, and D. Neely, J. Phys. B **27**, 341 (1994).

[34] V. N. Shlyaptsev, P. V. Nickles, Th. Schlegel, M. P. Kalashnikov, and A. L. Osterheld, SPIE Proc. Int. Soc. Opt. Eng. **2012**, 212 (1993).

[35] P. V. Nickles, M. Schnürer, M. P. Kalashnikov, I. Will, W. Sandner, and V. N. Shlyaptsev, SPIE Proc. Int. Soc. Opt. Eng. **2520**, 373 (1995).

[36] R. J. Mason, Phys. Rev. Lett. **42**, 429 (1979).