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Light amplification mechanisms in a coherently coupled atomic system

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We analyze transient properties of light amplification in a coherently coupled three-level Λ-type system and study the system evolution from the transient regime into the steady state. From the time evolution of the atomic coherence and population distribution, we discuss the transient light amplification mechanism with and without population inversion. We also solve the density-matrix equations in the steady state and derive the conditions for the existence of specific light amplification mechanisms, such as light amplification without inversion in any standard state basis, light amplification by stimulated Raman scattering, and light amplification from population inversion in the dressed states. [S1050-2947(97)10206-2]

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I. INTRODUCTION

There have been considerable interests recently in the study of light amplification and lasing in coherently coupled multilevel atomic systems without population inversion [1]. In many aspects, the inversionless laser gain studied in multilevel atomic systems can be viewed as a generalization of earlier results observed in studies of coherently driven two-level systems [2–8]. However, the multilevel systems are important in that they may be used to create laser gain at frequencies far removed from that of the coherent coupling fields, thus provide possibilities for the generation of coherent radiation in the short-wave length regime. Many coherent coupling schemes for laser gain without inversion have been proposed and dependence of the gain on various system parameters has been examined [9–14]. Experimental observations of inversionless gain and lasing have been reported by several groups [15–21].

Mechanisms of phase-insensitive optical amplification can be divided into several categories: light amplification from population inversion between the upper and lower transition states; light amplification from stimulated Raman transition; light amplification from population inversion in the dressed states; and light amplification without population inversion in any standard atomic states: the bare atomic states and the dressed states. Imamoglu, Field, and Harris showed that the steady-state light amplification without inversion in any standard atomic states can be found in a resonantly driven Λ-type three-level atomic system [9]. Here we present a detailed analysis of the Λ-type three-level system, including laser gain in the time-dependent transient regime and the steady state. In particular, we distinguish different gain mechanisms in the Λ system and derive conditions in the steady state under which the different gain mechanism may take place. In Sec. II, we present an analysis of the time evolution of the atomic coherence and population distribution in the closed Λ-type atomic system. We consider the situation in which there is an applied incoherent pump field as well as the situation in which there is no incoherent pump field. The steady-state responses of the system under the two situations are qualitatively different: with an incoherent pump field, the Λ system may exhibit steady-state laser gain without inversion in any standard atomic states; without the incoherent field, the Λ system does not exhibit steady-state laser gain. This is expected from the requirement of energy conservation. However, transient laser gain can exist in the Λ system with or without the incoherent pump. From these analyses, we identify the physical mechanisms to which the transient gain may be attributed. In Sec. III, we show that in the steady state, when the frequency of the coupling laser is near the atomic resonance frequency, there is laser gain without population inversion in any standard atomic states. For intermediate coupling-laser detunings, there is a Raman inversion in atomic states, the existing laser gain in the Λ system can be attributed to stimulated Raman scattering. Finally, for sufficiently large coupling-laser detunings, both the Raman inversion and population inversion in the dressed states occur, the laser gain is due to both the Raman gain and the gain from the dressed state inversion.

II. TRANSIENT ANALYSIS

We consider a closed Λ-type, three-level system with the ground states $|1\rangle$ and $|2\rangle$, and excited state $|3\rangle$, as illustrated in Fig. 1. The transition $|2\rangle \leftrightarrow |3\rangle$ of frequency ω_{23} is driven by a strong-coupling laser of frequency ω_1 with Rabi frequency 2Ω. The transition $|1\rangle \leftrightarrow |3\rangle$ of frequency ω_{31} is pumped with a rate Λ by an incoherent field (broadband excitation). γ_{31} (γ_{32}) is the spontaneous decay rate from state $|3\rangle$ to state $|1\rangle$ ($|2\rangle$). There is no dipole allowed coupling between states $|2\rangle$ and $|1\rangle$. A weak probe laser of frequency

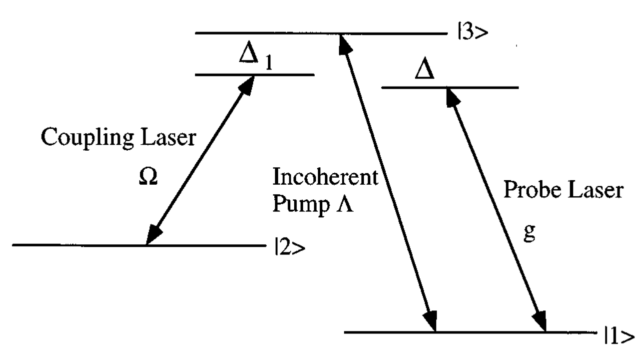

FIG. 1. Coherently and incoherently coupled Λ-type three-level system.

ω_p with Rabi frequency $2g$ is applied to the transition $|1\rangle \leftrightarrow |3\rangle$. Ω and g are chosen to be real. The semiclassical density-matrix equations of motion under the electric-dipole and the rotating-wave approximations can be written as

$$\frac{d\rho_{11}}{dt} = -\Lambda\rho_{11} + (\Lambda+\gamma_{31})\rho_{33} + ig(\rho_{31}-\rho_{13}),$$

$$\frac{d\rho_{22}}{dt} = \gamma_{32}\rho_{33} + i\Omega(\rho_{32}-\rho_{23}),$$

$$\frac{d\rho_{33}}{dt} = \Lambda\rho_{11} - (\Lambda+\gamma_{31}+\gamma_{32})\rho_{33} + ig(\rho_{13}-\rho_{31}) + i\Omega(\rho_{23}-\rho_{32}),$$

$$\frac{d\rho_{12}}{dt} = -\left(\frac{\Lambda}{2} + i(\Delta-\Delta_1)\right)\rho_{12} - i\Omega\rho_{13} + ig\rho_{32},$$

$$\frac{d\rho_{13}}{dt} = -\left(\frac{2\Lambda+\gamma_{31}+\gamma_{32}}{2} + i\Delta\right)\rho_{13} - i\Omega\rho_{12} + ig(\rho_{33}-\rho_{11}),$$

$$\frac{d\rho_{23}}{dt} = -\left(\frac{\Lambda+\gamma_{31}+\gamma_{32}}{2} + i\Delta_1\right)\rho_{23} - ig\rho_{21} + i\Omega(\rho_{33}-\rho_{22}), \tag{1}$$

where $\Delta_1 = \omega_{32} - \omega_1$, and $\Delta = \omega_{31} - \omega_p$ are the coupling-laser and probe-laser detunings, respectively. The closure of the system requires $\rho_{11}+\rho_{22}+\rho_{33}=1$. The gain-absorption coefficient for the probe laser (the coupling laser) coupled to the transition $|1\rangle \leftrightarrow |3\rangle$ ($|2\rangle \leftrightarrow |3\rangle$) is proportional to $\mathrm{Im}(\rho_{13})$ [$\mathrm{Im}(\rho_{23})$]. If $\mathrm{Im}(\rho_{13})>0$, the probe laser will be amplified. Similarly if $\mathrm{Im}(\rho_{23})>0$, the coupling laser will be amplified. Taking $\Delta_1=\Delta=0$, we begin by examining time-dependent numerical solutions of Eq. (1) with and without the incoherent pump. The parameters for the numerical solutions are chosen such that the conditions for laser gain in the steady state are satisfied [9]. Explicitly, the normalized parameters are $\Omega=20\gamma_{31}$, $\gamma_{32}=2\gamma_{31}$, $g=0.1\gamma_{31}$, $\Lambda=3\gamma_{31}$, or 0.

With a resonant coupling laser and a resonant probe laser ($\Delta_1=\Delta=0$), we found that $\rho_{13}(t)=i\,\mathrm{Im}[\rho_{13}(t)]$ and $\rho_{23}(t)=i\,\mathrm{Im}[\rho_{12}(t)]$, and $\rho_{12}(t)=\mathrm{Re}[\rho_{12}(t)]$. The dispersive response for the probe laser and the coupling laser vanishes, and the two-photon coherence ρ_{12} is real. The time evolution of the atomic response is plotted in Fig. 2 with the initial condition $\rho_{11}(0)=\rho_{22}(0)=0.5$ and the other $\rho_{ij}=0$ $(i,j=1-3)$. Figure 2(a) shows the time evolution of the population distribution in the Λ system without the incoherent pump ($\Lambda=0$). As expected, ρ_{22} and ρ_{33} show oscillatory behavior and reach the steady-state values $\rho_{22}=\rho_{33}\sim 0$. Since $g \ll \Omega$ and γ_{ij} $(i,j=1-3)$, the probability for the atoms being excited to state $|3\rangle$ is very small, and the optical pumping from the coupling laser results in $\rho_{11}\sim 1$ in the steady state. The atomic population evolution with the incoherent pump ($\Lambda=3\gamma_{31}$) is plotted in Fig. 2(b). It is seen that the ρ_{22} and ρ_{33} oscillation is similar to that in Fig. 2(a), but now ρ_{33} and ρ_{22} eventually reach a nonzero steady-state value. Note that the additional damping due to Λ causes a faster decay of the atomic response, and the atomic system reaches the steady-state faster with $\Lambda=3\gamma_{31}$ than with $\Lambda=0$. The time evolution of $\mathrm{Im}(\rho_{13})$ is plotted in Fig. 2(c). It shows similar oscillatory behavior versus time, i.e., the probe laser exhibits periodic amplification and absorption. The time evolution of $\mathrm{Im}(\rho_{23})$ is plotted in Fig. 2(d). With and

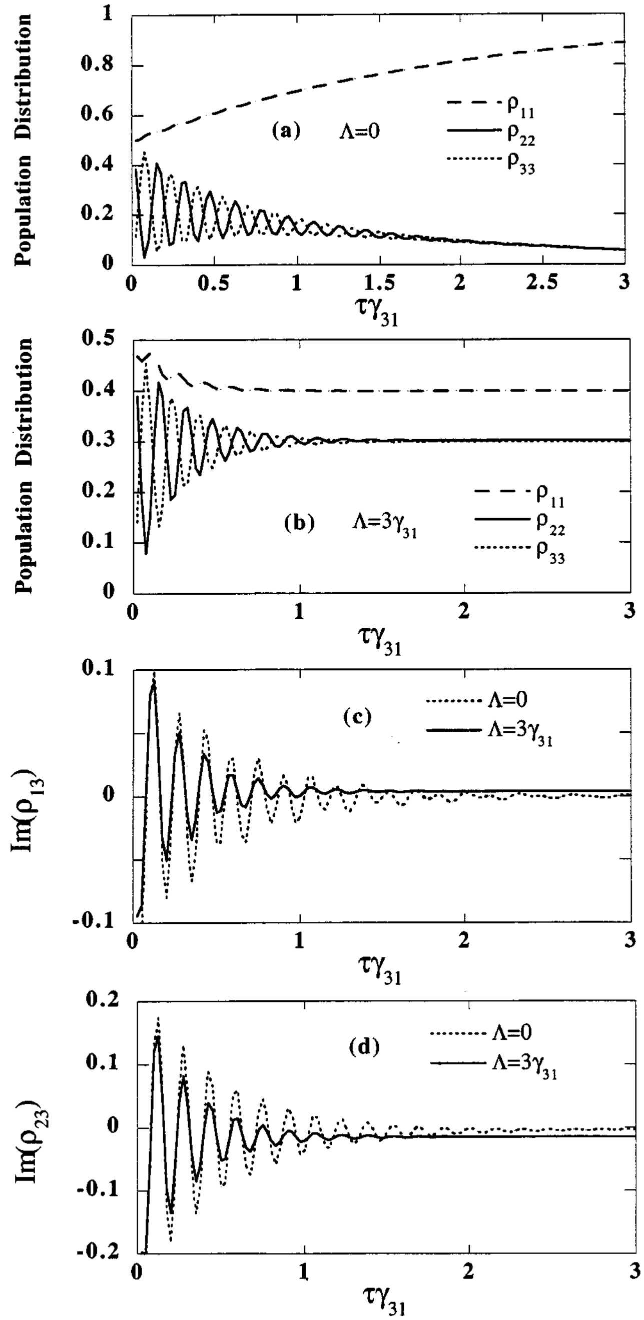

FIG. 2. Calculated time evolution of the atomic response in the three-level Λ system. (a) Without the incoherent pump field ($\Lambda=0$), the population distribution ρ_{ii} $(i=1-3)$ vs the normalized time $\tau\gamma_{31}$. (b) With the incoherent pump field ($\Lambda=3\gamma_{31}$), the population distribution ρ_{ii} $(i=1-3)$ vs the normalized time $\tau\gamma_{31}$. (c) $\mathrm{Im}(\rho_{13})/g$ (proportional to the probe gain-absorption coefficient) vs the normalized time $\tau\gamma_{31}$. (d) $\mathrm{Im}(\rho_{23})/\Omega$ (proportional to the coupling-laser gain-absorption coefficient) vs the normalized time $\tau\gamma_{31}$. The chosen parameters are $\gamma_{32}=2\gamma_{31}$, $\Omega=20\gamma_{31}$, $g=0.1\gamma_{31}$, $\Delta_1=0$, and $\Delta=0$. The initial conditions are $\rho_{11}(0)=\rho_{22}(0)=0.5$, and the other $\rho_{ij}(0)=0$ $(i,j=1-3)$.

without the incoherent pump, the transient behavior of $\mathrm{Im}(\rho_{13})$ and $\mathrm{Im}(\rho_{23})$ is qualitatively the same. They all oscillate with the same frequency. Comparing Figs. 2(c) and 2(d), it is seen that $\mathrm{Im}(\rho_{13})$ and $\mathrm{Im}(\rho_{23})$ are oscillating nearly in phase with each other. The two lasers experience gain or absorption at the same time. There is a phase difference $\sim\pi/2$ between $\mathrm{Im}(\rho_{13})$ $[\mathrm{Im}(\rho_{23})]$ and ρ_{33}. The transient amplification of the probe laser and the coupling laser occurs after ρ_{33} reaches the maximum values. The amplification of the probe laser and the coupling laser occurs in the time interval in which $d\rho_{33}/dt<0$ and the peak amplification coincides with the time at which the change in the slope of ρ_{33} is steepest. The transient gain-absorption of the coupling laser is similar to the Rabi oscillation of a strongly driven two-level system. However, the origin of the probe-laser amplification is quite different. Note from Figs. 2(a) and 2(b), one always has $\rho_{33}<\rho_{11}$ and $\rho_{22}<\rho_{11}$. Therefore, there is no population inversion for the probe transition $|1\rangle\leftrightarrow|3\rangle$. As will be shown later, this is the necessary and sufficient condition of population noninversion in either the bare atomic states or the dressed states, and the transient probe amplification is induced by the oscillatory atomic coherence ρ_{12} only. One may wonder if this transient light amplification is from the stimulated Raman scattering $|2\rangle\rightarrow|3\rangle\rightarrow|1\rangle$ between states $|2\rangle$ and $|1\rangle$. Examination of Figs. 2(c) and 2(d) rules out such a possibility. If the probe laser is amplified by stimulated Raman scattering $|2\rangle\rightarrow|3\rangle\rightarrow|1\rangle$, the coupling laser has to be attenuated in the Raman scattering process ($|2\rangle\rightarrow|3\rangle$ corresponds to the absorption of a coupling-laser photon, $|3\rangle\rightarrow|1\rangle$ corresponds to the emission of a probe-laser photon), and vice versa: i.e., $\mathrm{Im}(\rho_{13})$ and $\mathrm{Im}(\rho_{23})$ have to have a π phase difference between them in order for the stimulated Raman gain to occur. This is in contradiction with the in-phase solutions presented in Figs. 2(c) and 2(d). Furthermore, for the Raman gain to occur for the probe laser, a Raman inversion condition, $\rho_{22}>\rho_{11}$, must be satisfied. This is impossible for the resonantly coupled Λ system as shown by Figs. 2(a) and 2(b) with or without the incoherent pump, ρ_{22} is always less than ρ_{11}, there is no Raman-type population inversion. Raman gain cannot be attributed to the transient probe amplification. The time evolution of the two-photon coherence ρ_{12} is plotted in Fig. 3. In the bare state picture, ρ_{12} is responsible for laser gain in the inversionless system [14]. Note that the magnitude of $\mathrm{Re}(\rho_{12})$ is reduced with the addition of the incoherent pump field, yet the steady-state laser gain occurs in the Λ system only with a sufficiently strong incoherent field. To understand this behavior, we write the steady state ρ_{13} as

$$\rho_{13}=2i\,\frac{g(\rho_{33}-\rho_{11})-\Omega\rho_{12}}{2\Lambda+\gamma_{31}+\gamma_{32}}. \tag{2}$$

The probe gain $[\propto\mathrm{Im}(\rho_{13})]$ is contributed to by two terms: the population difference $\rho_{33}-\rho_{11}(<0)$ and the two-photon coherence $\rho_{12}(<0)$. Without the incoherent pump ($\Lambda=0$), the negative contribution from the first term is greater than the positive contribution from the ρ_{12} term, and the probe laser can only be attenuated in the steady state. With an incoherent pump field, even though the two-photon coherence ρ_{12} is reduced, the much greater increase of the population difference $\rho_{33}-\rho_{11}$ results in a positive value for $\mathrm{Im}(\rho_{13})$. Therefore, the steady-state laser gain without population inversion occurs in the Λ system as a combined effect of the increased population probability ρ_{33} (decreased ρ_{11}) and the residual two-photon coherence ρ_{12}.

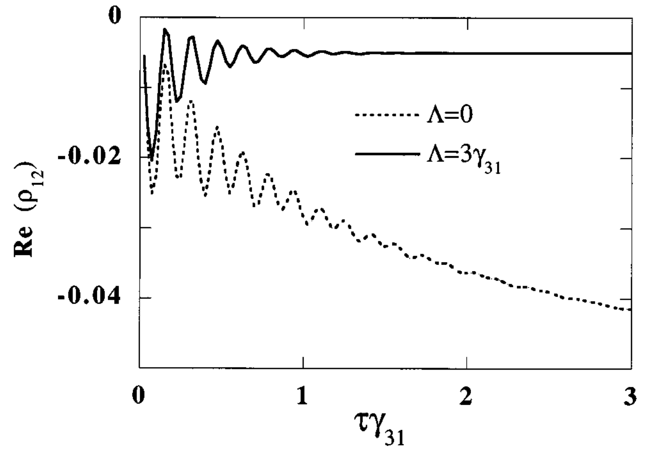

FIG. 3. The two-photon atomic coherence $\mathrm{Re}(\rho_{12})$ vs the normalized time $\tau\gamma_{31}$. The parameters are the same as that in Fig. 2. Note that with the incoherent pump field (the curve with $\Lambda=3\gamma_{31}$), the magnitude of the two-photon coherence is reduced in comparison with $\mathrm{Re}(\rho_{12})$ at $\Lambda=0$. The initial conditions are the same as that in Fig. 2.

The transient evolution of the atomic response in laser fields depends on the initial condition. Figure 4 shows the time evolution of the population distribution and the atomic coherence in the Λ system when the atoms are initially in the state $|1\rangle$ $[\rho_{11}(0)=1$ and other $\rho_{ij}(0)=0]$ and the incoherent pumping rate is $\Lambda=3\gamma_{31}$. It is seen in Fig. 4(a) that ρ_{11} monotonically decreases to its steady-state value while ρ_{22} and ρ_{33} reach the steady-state value oscillatorily. In the steady state, $\rho_{11}>\rho_{22}>\rho_{33}$. It is interesting to note in Fig. 4(b) that the oscillatory frequency of $\mathrm{Im}(\rho_{13})$ is about half the oscillatory frequency of $\mathrm{Im}(\rho_{23})$. Similar behavior can also be observed when the atom is initially in state $|2\rangle$ as shown in Fig. 5. Figure 5(a) shows the time evolution of the population distribution. Here ρ_{11}, ρ_{22}, and ρ_{33} oscillate with the same frequency, and reach the same steady-state values as that in Fig. 4(a). Under these two initial conditions, the time evolution of the atomic coherence is such that for some periods of time, both the coupling laser and probe laser are amplified or absorbed simultaneously; for other times, the coupling laser is amplified (absorbed) while the probe laser is absorbed (amplified). The oscillation of $\mathrm{Im}(\rho_{13})$ is noticeably anharmonic. A Fourier analysis indicates that it contains two frequency components: the main component is the oscillation with half the Rabi frequency Ω while the other contribution comes from the oscillation with the Rabi frequency 2Ω. There is a subtle yet important difference for the atomic response under these two initial conditions. For $\rho_{11}(0)=1$, the time evolution is such that $\rho_{11}>\rho_{22}$ is always valid, no transient probe amplification from the stimulated Raman scattering $|2\rangle\rightarrow|1\rangle$. However, for $\rho_{22}(0)=1$, there are initial periods of times in which $\rho_{22}>\rho_{11}$, i.e., the transient Raman inversion exists and the transient probe amplification can occur from the stimulated Raman scattering $|2\rangle\rightarrow|1\rangle$. In

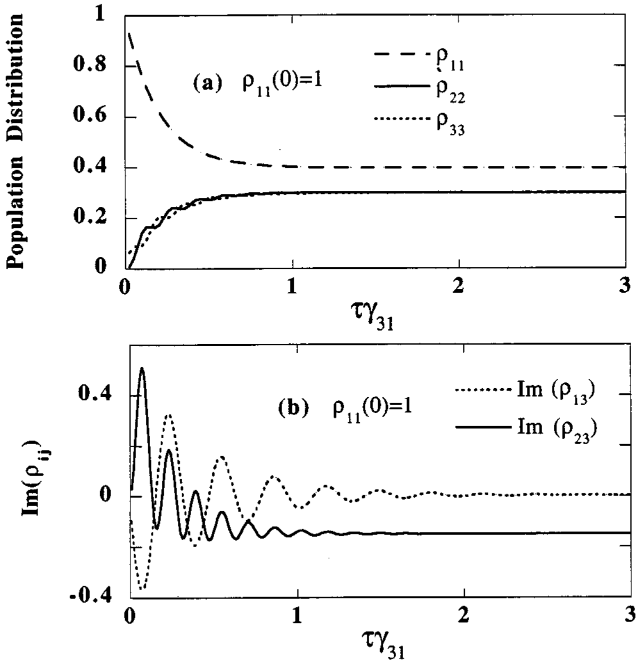

FIG. 4. Calculated time evolution of the atomic response in the three-level Λ system under the initial conditions: $\rho_{11}(0)=1$ and the other $\rho_{ij}(0)=0$ $(i,j=1-3)$. The incoherent pump $\Lambda=3\gamma_{31}$ and the other parameters are the same as that in Fig. 2. (a) The population distribution ρ_{ii} $(i=1-3)$ vs the normalized time $\tau\gamma_{31}$. (b) $\mathrm{Im}(\rho_{13})/g$ and $\mathrm{Im}(\rho_{23})/\Omega$ vs the normalized time $\tau\gamma_{31}$. Note that the oscillation frequency of $\mathrm{Im}(\rho_{23})$ is about twice the oscillation frequency of $\mathrm{Im}(\rho_{13})$.

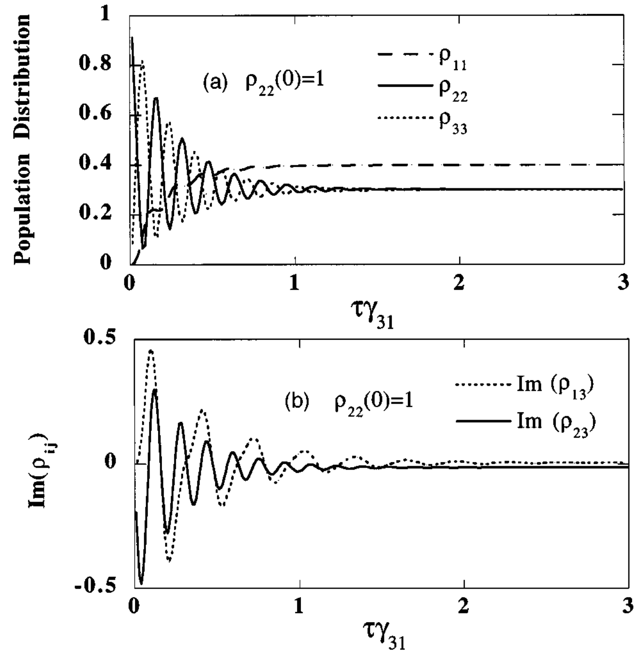

FIG. 5. Calculated time evolution of the atomic response in the three-level Λ system under the initial conditions: $\rho_{22}(0)=1$ and the other $\rho_{ij}(0)=0$ $(i,j=1-3)$. The incoherent pump $\Lambda=3\gamma_{31}$ and the other parameters are the same as that in Fig. 2. (a) The population distribution ρ_{ii} $(i=1-3)$ vs the normalized time $\tau\gamma_{31}$. (b) $\mathrm{Im}(\rho_{13})/g$ and $\mathrm{Im}(\rho_{23})/\Omega$ vs the normalized time $\tau\gamma_{31}$. Note that the oscillation frequency of $\mathrm{Im}(\rho_{23})$ is about twice the oscillation frequency of $\mathrm{Im}(\rho_{13})$ as shown in Fig. 4(b).

spite of this difference, the time evolution of $\mathrm{Im}(\rho_{13})$ is similar under the two different initial conditions. The two-photon coherence $\mathrm{Re}(\rho_{12})$ corresponding to the initial conditions $\rho_{11}(0)=1$ and $\rho_{22}(0)=1$ is plotted in Fig. 6. The anharmonic oscillation of $\mathrm{Re}(\rho_{12})$ with the initial condition $\rho_{22}(0)=1$ is clearly seen. There is a phase difference $\sim\pi$ for $\mathrm{Re}(\rho_{12})$ between the two initial conditions.

Our calculations show the rich transient dynamics of the three-level Λ system coupled by multiple laser fields. With a strong-coupling laser ($\Omega\gg\gamma_{ij}$, Λ, and g) and a weak probe laser ($g\ll\gamma_{ij}$ and Λ), the atomic coherence ρ_{23} oscillates dominantly with the Rabi frequency 2Ω, irrespective of the initial conditions. The oscillation of the atomic coherence ρ_{13} and ρ_{12} contains two frequencies: the Rabi frequency 2Ω and half of the Rabi frequency Ω. Depending on the initial conditions, one frequency component may dominate the other [see Figs. 2(c), 3, and 4(b)] or both frequency components have to be accounted for [see Figs. 5(b) and 6]. The characteristics of the transient oscillation is similar with and without the incoherent pump filed. As expected, we found that with a strong-coupling laser, the oscillation frequencies are independent of the ratio of the spontaneous decay rates γ_{32} and γ_{31}.

The density-matrix equation provides a statistical description of atomic systems and its solutions provide a probability analysis of the atomic responses, i.e., the statistical behavior of many identical atoms. The two-photon coherence ρ_{12} that has been attributed to the laser gain in the Λ system is induced by several nonlinear optical processes and different initial conditions may favor or suppress a certain process. As shown in Fig. 2, $\mathrm{Im}(\rho_{13})$ and $\mathrm{Im}(\rho_{23})$ are in phase, and the probe laser and the coupling laser can be amplified simultaneously when the atom is in state $|3\rangle$. When $\rho_{11}(0)=1$, near $\tau\sim0$, we have $\mathrm{Im}(\rho_{23})>0$ and $\mathrm{Im}(\rho_{13})<0$ [see Fig. 4(b)],

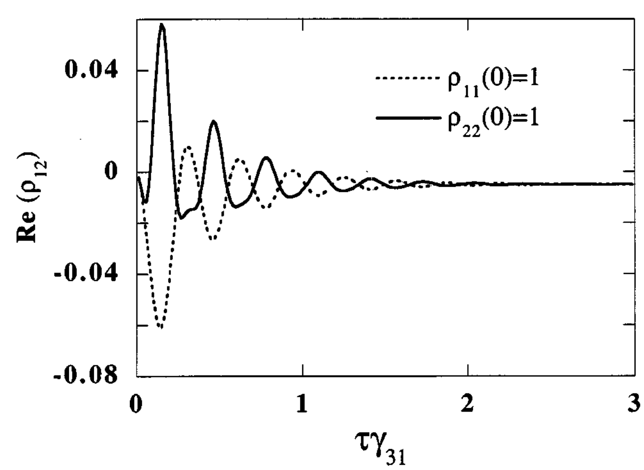

FIG. 6. Two-photon atomic coherence $\mathrm{Re}(\rho_{12})$ vs the normalized time $\tau\gamma_{31}$ under the two different initial conditions shown in Figs. 4 and 5, respectively. The parameters are the same as that in Fig. 2. Note that the oscillation frequency of $\mathrm{Re}(\rho_{12})$ is the same as that of $\mathrm{Im}(\rho_{13})$ in Figs. 4 and 5.

indicating the stimulated two-photon Raman process $|1\rangle\rightarrow|3\rangle\rightarrow|2\rangle$; when $\rho_{22}(0)=1$, near $\tau\sim 0$, we have $\mathrm{Im}(\rho_{13})>0$ and $\mathrm{Im}(\rho_{23})<0$ [see Fig. 5(b)], indicating the reverse stimulated two-photon Raman process $|2\rangle\rightarrow|3\rangle\rightarrow|1\rangle$. At later times, $\mathrm{Im}(\rho_{23})$ and $\mathrm{Im}(\rho_{13})$ may have the same sign or opposite sign with different population probabilities. The statistical analysis derived from solving the density-matrix equation does not separate out different nonlinear optical processes and it is not clear to us that unambiguous determination of a given nonlinear process that results dominantly in the inversionless probe gain can be resolved from the numerical calculations presented here. Better understanding of the atomic response can certainly be obtained if each individual, nonlinear photon scattering process can be separated and accounted for. Recently Sellin *et al.* discussed inversionless gain and lasing in a ladder-type three-level system [22]. They suggested that an irreversible nonlinear process involving spontaneous emission may contribute to the inversionless gain. In the three-level Λ-type system, the existence of steady-state inversionless gain requires $\gamma_{32}>\gamma_{31}$. This suggests that an irreversible multiphoton scattering process similar to that proposed by Sellin *et al.* for a ladder-type system may be responsible for the inversionless probe amplification in the Λ-type system, in which the spontaneous emission occurs from $|3\rangle\rightarrow|2\rangle$, followed by the stimulated absorption of a coupling-laser photon from $|2\rangle\rightarrow|3\rangle$, and finally the stimulated emission of a probe-laser photon from $|3\rangle\rightarrow|1\rangle$ leads to the inversionless gain. Because $\gamma_{32}>\gamma_{31}$, the reverse process, in which the spontaneous emission first occurs from $|3\rangle\rightarrow|1\rangle$, followed by the absorption of a probe-laser photon $|1\rangle\rightarrow|3\rangle$, and finally the emission of a coupling-laser photon $|3\rangle\rightarrow|2\rangle$, is not as strong. Since a spontaneous-emission triggered process does not require population inversion, the net effect of these two nonlinear processes may result in the inversionless gain for the probe transition $|3\rangle\rightarrow|1\rangle$. It is interesting to note that in a three-level V-type system, the existence of inversionless gain also requires that the spontaneous-emission rate for the coupling transition is greater than that of the probe transition [14], which is consistent with the physical picture of inversionless gain associated with an irreversible, spontaneous-emission triggered nonlinear process. Further studies along these lines should be worthwhile.

III. STEADY-STATE ANALYSIS

It is easy to derive the analytical steady-state solutions to Eq. (1) with $\Delta_1=\Delta=0$. With a strong-coupling laser, $\Omega \gg \gamma_{ij}$, Λ, and g is satisfied. Then, the first-order steady-state solutions to Eq. (1) become very simple. The steady-state atomic polarizations ρ_{13} and ρ_{12}, and the two-photon coherence ρ_{23} are found to be

$$\rho_{13}=i\,\frac{g\Lambda(\gamma_{32}-\gamma_{31})}{\Omega^2(6\Lambda+2\gamma_{31})}, \tag{3}$$

$$\rho_{23}=-i\,\frac{\Lambda\gamma_{32}}{\Omega(6\Lambda+2\gamma_{31})}, \tag{4}$$

and

$$\rho_{12}=-\frac{2g\gamma_{31}}{\Omega(6\Lambda+2\gamma_{31})}. \tag{5}$$

The steady-state population probabilities are given by

$$\rho_{33}=\rho_{22}=\frac{\Lambda}{3\Lambda+\gamma_{31}}, \tag{6}$$

and

$$\rho_{11}=\frac{\Lambda+\gamma_{31}}{3\Lambda+\gamma_{31}}. \tag{7}$$

These solutions are consistent with the numerical results presented in Figs. 2–6. If there is no incoherent pump. No steady-state light amplification can be observed in the Λ system. With an incoherent pump field, $\mathrm{Im}(\rho_{13})>0$ (assuming that $\gamma_{32}>\gamma_{31}$ is always satisfied), i.e., the probe laser is amplified while the coupling laser is attenuated.

Next, we address the question of different light amplification mechanisms in the Λ system. With an incoherent pump field $(\Lambda\neq 0)$, the probe amplification occurs both in the transient regime and in the steady state. In the steady state $\rho_{11}>\rho_{22}\geqslant\rho_{33}$, there is no population inversion in the bare state basis consisting of states $|1\rangle$, $|2\rangle$, and $|3\rangle$. The only other meaningful state basis is the dressed state basis consisting of states $|1\rangle$, $|+\rangle$, and $|-\rangle$. For a resonant coupling laser, the semiclassical dressed states $|+\rangle$ and $|-\rangle$ are simply given by [23]: $|+\rangle=1/\sqrt{2}[|2\rangle+|3\rangle]$ and $|-\rangle=1/\sqrt{2}[|2\rangle-|3\rangle]$. Then the population distribution in the dressed states is given by

$$\rho_{++}=\rho_{--}=\frac{\rho_{22}+\rho_{33}}{2}=\rho_{22}=\rho_{33}. \tag{8}$$

Thus, in the dressed state basis, one has $\rho_{++}<\rho_{11}$ and $\rho_{--}<\rho_{11}$: no population inversion in the dressed states either. Furthermore, since $\rho_{22}<\rho_{11}$, there is no Raman inversion, and the probe amplification cannot be attributed to the stimulated Raman gain $|2\rangle\rightarrow|3\rangle\rightarrow|1\rangle$. The light amplification is solely due to the coherence induced interference. The coupling laser generates a pair of dressed states $|+\rangle$ and $|-\rangle$ separated by the Rabi frequency 2Ω. In the dressed state picture, the atoms will not absorb the probe laser at $\Delta=0$ since the probability amplitudes for transitions of $|1\rangle\rightarrow|+\rangle$ and $|1\rangle\rightarrow|-\rangle$ interfere destructively. However, for the stimulated emission $|+\rangle\rightarrow|1\rangle$ and $|-\rangle\rightarrow|1\rangle$, the probability amplitudes add at $\Delta=0$ and result in the probe amplification [24,25]. In the bare-state picture, the two-photon coherence ρ_{12} is induced between the two ground states $|1\rangle$ and $|2\rangle$ by the coupling laser and the probe laser, which results in light amplification without population inversion [14].

From the above analysis, it is clear that the criterion of light amplification without population inversion in any standard state basis is $\rho_{11}\geqslant\rho_{22}\geqslant\rho_{33}$, which is satisfied at $\Delta_1=0$. It is obvious that the population distribution in the Λ system depends on the coupling laser detuning Δ_1. When $\Delta_1\neq 0$, other light amplification mechanisms may take place. With a weak probe laser ($g\ll\Omega$, γ_{ij}, and Λ), the steady-state population probabilities with an arbitrary detuning Δ_1 in the bare states are given by

$$\rho_{11}=\frac{4\Omega^2(\Lambda+\gamma_{31}+\gamma_{32})(\Lambda+\gamma_{31})}{4\Omega^2(\Lambda+\gamma_{31}+\gamma_{32})(3\Lambda+\gamma_{31})+\Lambda\gamma_{32}[4\Delta_1^2+(\Lambda+\gamma_{31}+\gamma_{32})^2]}, \tag{9}$$

$$\rho_{22}=\frac{4\Omega^2\Lambda(\Lambda+\gamma_{31}+\gamma_{32})+\Lambda\gamma_{32}[4\Delta_1^2+(\Lambda+\gamma_{31}+\gamma_{32})^2]}{4\Omega^2(\Lambda+\gamma_{31}+\gamma_{32})(3\Lambda+\gamma_{31})+\Lambda\gamma_{32}[4\Delta_1^2+(\Lambda+\gamma_{31}+\gamma_{32})^2]}, \tag{10}$$

and

$$\rho_{33}=\frac{4\Omega^2\Lambda(\Lambda+\gamma_{31}+\gamma_{32})}{4\Omega^2(\Lambda+\gamma_{31}+\gamma_{32})(3\Lambda+\gamma_{31})+\Lambda\gamma_{32}[4\Delta_1^2+(\Lambda+\gamma_{31}+\gamma_{32})^2]}. \tag{11}$$

Setting $\rho_{22}-\rho_{11}=0$, we obtain a critical value of $|\Delta_1|$

$$\Delta_{c1}=\left(\frac{\Omega^2\gamma_{31}(\Lambda+\gamma_{31}+\gamma_{32})}{\Lambda\gamma_{31}}-\frac{(\Lambda+\gamma_{31}+\gamma_{32})^2}{4}\right)^{1/2}$$
$$\approx\Omega\left(\frac{\gamma_{31}(\Lambda+\gamma_{31}+\gamma_{32})}{\Lambda\gamma_{31}}\right)^{1/2}. \tag{12}$$

When $|\Delta_1|<\Delta_{c1}$, the population distribution satisfies $\rho_{11}>\rho_{22}\geqslant\rho_{33}$. The Λ system exhibits light amplification by coherence only, as discussed before. When $|\Delta_1|>\Delta_{c1}$, the population distribution satisfies $\rho_{22}>\rho_{11}>\rho_{33}$, and a population inversion for the Raman transition $|2\rangle\rightarrow|1\rangle$ occurs. The probe amplification may be viewed as due to the stimulated Raman scattering $|2\rangle\rightarrow|3\rangle\rightarrow|1\rangle$ in which the atoms absorb a coupling-laser photon and then emit a probe-laser photon. In the dressed state picture, the semiclassical dressed states $|+\rangle$ and $|-\rangle$ are given by [23]

$$|+\rangle=\sin\,\theta|2\rangle+\cos\,\theta|3\rangle, \tag{13}$$

$$|-\rangle=\cos\,\theta|2\rangle-\sin\,\theta|3\rangle. \tag{14}$$

Where $\tan(2\theta)=\Omega/\Delta_1$ $(0\leqslant2\theta\leqslant\pi)$. The corresponding population probabilities are given by

$$\rho_{++}=(\sin\,\theta)^2\rho_{22}+(\cos\,\theta)^2\rho_{33}, \tag{15}$$

and

$$\rho_{--}=(\cos\,\theta)^2\rho_{22}+(\sin\,\theta)^2\rho_{33}. \tag{16}$$

Let $\rho_{++}-\rho_{11}=0$ (or $\rho_{--}-\rho_{11}=0$), one obtains the second critical value Δ_{c2} $(>\Delta_{c1})$ for $|\Delta_1|$. When $|\Delta_1|<\Delta_{c2}$, one has $\rho_{++}<\rho_{11}$ and $\rho_{--}<\rho_{11}$: there is no population inversion in the dressed states. However, when $|\Delta_1|>\Delta_{c2}$, the population distribution satisfies $\rho_{--}>\rho_{11}$ (for $\Delta_1>0$) or $\rho_{++}>\rho_{11}$ (for $\Delta_1<0$), and a population inversion in the dressed states occurs. The probe amplification may proceed through the stimulated emission between the inverted dressed state transition. As shown by Eqs. (9)–(11), ρ_{22} increases with increasing $|\Delta_1|$, and ρ_{11} and ρ_{33} decrease with increasing $|\Delta_1|$. In summary, when $|\Delta_1|<\Delta_{c1}$, the population distribution satisfies $\rho_{11}>\rho_{22}\geqslant\rho_{33}$, the light amplification results from the laser induced atomic coherence (no population inversion in any standard atomic states). When $|\Delta_1|$ becomes greater than Δ_{c1}, the population distribution satisfies $\rho_{22}>\rho_{11}>\rho_{33}$. Raman inversion takes place and light amplification occurs from the stimulated Raman transition $|2\rangle\rightarrow|1\rangle$. Further increases of $|\Delta_1|$ above the second critical value Δ_{c2} will bring the population distribution to $\rho_{--}>\rho_{11}>\rho_{++}$, $(\Delta_1>0)$ or $\rho_{++}>\rho_{11}>\rho_{--}$, $(\Delta_1<0)$, i.e., besides the Raman inversion, population inversion also occurs in the dressed states. The light amplification can occur from the stimulated Raman transition and the dressed state inversion. To show graphically the three regions of population distribution and the associated light amplification mechanisms, we have calculated numerically the steady-state atomic response versus Δ_1 and the results are plotted in Fig. 7. The relevant parameters are the same as those chosen in Fig. 2. Figure 7(a) shows the atomic population distribution versus Δ_1 in the bare-state basis and Fig. 7(b) shows the atomic population distribution versus Δ_1 in the dressed state basis. Three regions of population distribution are separated by the vertical dashed lines. In region I ($|\Delta_1|\leqslant\Delta_{c1}$), population distribution satisfies $\rho_{11}>\rho_{22}\geqslant\rho_{33}$ in the bare states and $\rho_{11}>\rho_{--}\geqslant\rho_{++}$ in the dressed states. Light amplification occurs from the atomic coherence. In region II ($\Delta_{c1}<|\Delta_1|\leqslant\Delta_{c2}$), the population distribution satisfies $\rho_{22}>\rho_{11}>\rho_{33}$ in the bare states and $\rho_{11}>\rho_{--}>\rho_{++}$ in the dressed states. Although no population inversion exists for the probe transition in either the bare states or the dressed states, the Raman inversion $\rho_{22}>\rho_{11}$ occurs. The light amplification may be viewed as due to the stimulated Raman scattering process $|2\rangle\rightarrow|3\rangle\rightarrow|1\rangle$. In region III ($|\Delta_1|>\Delta_{c2}$), the population distribution becomes $\rho_{22}>\rho_{11}>\rho_{33}$ in the bare states and $\rho_{--}>\rho_{11}>\rho_{++}$ in the dressed states. The light amplification can be attributed to both the population inversion in the dressed states and the stimulated Raman scattering. The frequencies at which the probe laser experiences amplification depend on Δ_1 and are shifted away from the resonant frequency of the atomic transition $|3\rangle\leftrightarrow|1\rangle$ as Δ_1 increases. This behavior is shown in Fig. 7(c). When Δ_1 is much smaller than Ω, the probe gain occurs near $\Delta=0$, the resonance frequency of the bare-state transition $|3\rangle\leftrightarrow|1\rangle$. As Δ_1 increases, the frequency of the probe gain is gradually shifted from $\Delta=0$ to $\Delta=(\Omega^2+\Delta_1^2)^{1/2}$, which corresponds to one of the Autler-Townes' doublet transitions. The maximum gain without population inversion in any standard atomic states is much smaller than the maximum gain with the Raman inversion and the dressed state inversion, which indicates the inefficiency of light amplification by coherence (no population inversion in any standard atomic states).

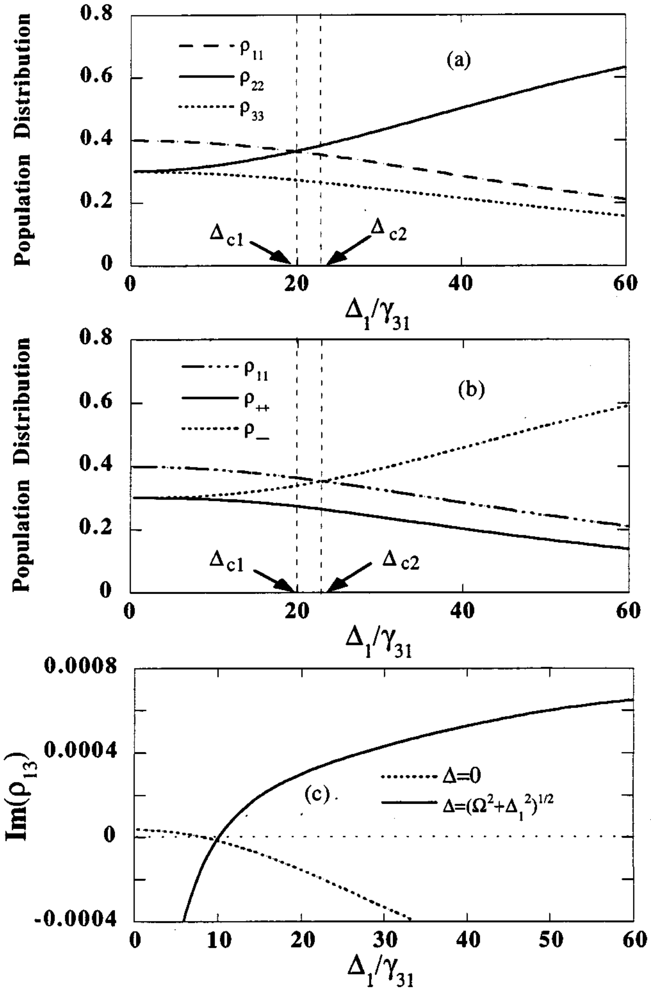

FIG. 7. (a) Steady-state atomic population distribution in the bare atomic states vs the coupling-laser detuning Δ_1/γ_{31}. (b) The steady-state atomic population distribution in the dressed states vs the coupling-laser detuning Δ_1/γ_{31}. Three regions of population distribution in (a) and (b) are separated by the vertical dashed lines. (c) The steady-state Im(ρ_{13}) at $\Delta=0$ (dashed line) and $\Delta=(\Omega^2+\Delta_1^2)^{1/2}$ (solid line) vs Δ_1/γ_{31}, respectively. The relevant parameters are $\gamma_{21}=2\gamma_{31}$, $\Omega=20\gamma_{31}$, $g=0.1\gamma_{31}$, i.e., the same as that in Fig. 2.

IV. CONCLUSION

In summary, we have presented an analysis of the time evolution of atomic response in a coherently coupled Λ-type three-level system and analyzed the steady-state conditions for the onset of different light amplification mechanisms. We have shown that transient light amplification by coherence (without population inversion in any standard atomic states) can be observed in the Λ system with or without the incoherent pump field, and the steady-state light amplification by coherence can occur only with an incoherent pump field. We have identified three regions of population distribution determined by the coupling-laser detuning Δ_1 and discussed the corresponding light amplification mechanisms. Specifically, when $|\Delta_1|\leq\Delta_{c1}$, the light amplification by coherence can be observed in the Λ system; when $\Delta_{c1}<|\Delta_1|\leq\Delta_{c2}$, the light amplification can be attributed to the stimulated Raman scattering in the bare states; when $|\Delta_1|>\Delta_{c2}$, the light amplification may be viewed as due to population inversion in the dressed states and the stimulated Raman scattering in the bare states. The three light amplification mechanisms existing in the Λ system also occur in a coherently coupled V-type three-level system and the dependence of the light amplification mechanism on the coupling detuning Δ_1 is very similar between the two systems [26]. The analysis presented here further demonstrates the existence of three phase-insensitive light amplification mechanisms in coherently coupled atomic systems and reveals a common feature in light amplification without inversion in any standard atomic states in various atomic systems, i.e., the coupling laser has to be tuned near the resonant frequency of the chosen atomic transition. In both V and Λ systems, the existence of the steady-state inversionless gain requires that the spontaneous-emission rate for the coupling transition is greater than that of the probe transition. This suggests that an irreversible, spontaneous-emission triggered multiphoton process may be important for the inversionless light amplification. It is also interesting to note that although the Λ and V systems exhibit similar light amplification mechanisms under comparable conditions, the photon statistics of the laser oscillators with these two systems as the gain media with the same light amplification mechanism (light amplification by coherence) is quite different: the quantum fluctuations in an inversionless laser with the Λ system as the gain medium can be well below the standard quantum limit (amplitude squeezing) under normal operating conditions [27], while the quantum fluctuation in an inversionless laser with the V system as the gain medium is always above the standard quantum limit [28].

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