

Transverse excess noise factor in geometrically stable laser resonators

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The excess noise factor due to the nonorthogonality of the transverse modes of a geometrically stable cavity subject to large diffraction losses is calculated. This calculation is based on an exact determination of the transverse field distribution of the cavity fundamental eigenmode. It is shown that when the modes become essentially determined by diffraction, the transverse modes are far from being orthogonal, leading to the appearance of a large excess noise factor that must multiply the standard Schawlow-Townes linewidth. Moreover, in the presence of two diffracting apertures, the excess noise factor is shown to exhibit a resonant behavior reminiscent of the one observed in unstable cavities. We estimate that, in the case of circular apertures, excess noise factors as large as 100 can be experimentally measured using a high-gain gas micro-laser. [S1050-2947(97)09706-0]

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I. INTRODUCTION

The linewidth of a monomode laser is fundamentally limited by spontaneous emission into the laser mode, leading to the well-known Schawlow-Townes linewidth [1]. This theory predicts a Lorentzian line shape for the laser power spectral density, which has been found in agreement with experiments [2–4]. However, more recently, it has been shown that the nonorthogonality of the different laser modes can give rise to an enhancement of this fundamental laser linewidth. This so-called excess noise factor was first discussed [5,6] and observed [7,8] in the case of gain-guided semiconductor lasers and amplifiers. Independently, Siegman [9–11] has shown that the non-Hermitian nature of the Huygens-Fresnel operator for one round-trip in unstable resonators leads to peculiar nonorthogonality properties of the transverse modes of such resonators, and hence to large excess noise factors. In such geometrically unstable cavities, excess noise factors as large as a few 10^3 have been predicted [11–17] and observed [18–21]. Physically, this effect has been attributed to the adjoint coupling of the vacuum fluctuations into the laser resonator [22,23], exhibiting resonances for the values of the equivalent Fresnel number of the cavity that lead to the higher losses [16,21]. Concerning geometrically stable laser cavities, it has recently been shown that large output coupling could lead to a nonorthogonality of the longitudinal modes of the cavity, and thus to an enhancement of the Schawlow-Townes linewidth of the laser [24–26]. However, it is well known that even in the case of a geometrically stable cavity, the Huygens-Fresnel kernel is in general non-Hermitian [10]. Consequently, the transverse modes of a geometrically stable resonator must also be non-orthogonal. This fact has already been experimentally confirmed through the peculiar evolution of the diffraction losses in a multiapertured stable resonator [27]. Indeed, in a stable cavity, once a first aperture has been introduced to select the fundamental mode, the diffraction losses exhibit an oscillating behavior versus the diameter of a second aperture, a behavior exactly equivalent to the one observed in unstable cavities. Nevertheless, the possibility for the existence of a

non-negligible excess noise factor due to the nonorthogonality of the transverse modes in stable resonators has, to our best knowledge, been overlooked. The aim of this paper is consequently to compute the excess noise factor in a non-Hermitian stable resonator and to discuss the possibility to observe it experimentally. Section II is hence devoted to the exact calculation of the fundamental mode of a stable cavity containing a single aperture and to the determination of the corresponding excess noise factor. In Sec. III, we wonder whether the resonant behavior of the excess noise factor in unstable cavities can equivalently occur in a geometrically stable cavity, when a second aperture is introduced. Section IV is then devoted to a discussion of the possible observation of these phenomena.

II. SINGLE APERTURE STABLE CAVITY

A. Exact calculation of the fundamental mode of a stable cavity

Let us consider the cavity schematized in Fig. 1(a), which consists of a spherical mirror M_1 (radius of curvature R) and a plane mirror M_2 . If the cavity length L is smaller than R , the cavity is geometrically stable. Then, if the transverse dimensions of the cavity are large enough and if the cavity obeys a cylindrical symmetry, the transverse modes can be obtained from the resolution of the Huygens-Fresnel eigenequation given by

$$\gamma u(r) = \int_0^\infty dr' r' K(r, r') u(r'), \quad (1)$$

with the kernel

$$K(r, r') = \frac{2\pi}{B\lambda} j^{l+1} \exp\left[-j \frac{\pi}{B\lambda} (Ar'^2 + Dr^2)\right] J_l\left(\frac{2\pi}{B\lambda} rr'\right), \quad (2)$$

where the wave front at the considered plane inside the cavity is $u(r)\exp[jl\varphi]$ in the (r, φ) cylindrical coordinate system, γ is the associated eigenvalue, λ is the vacuum wave-

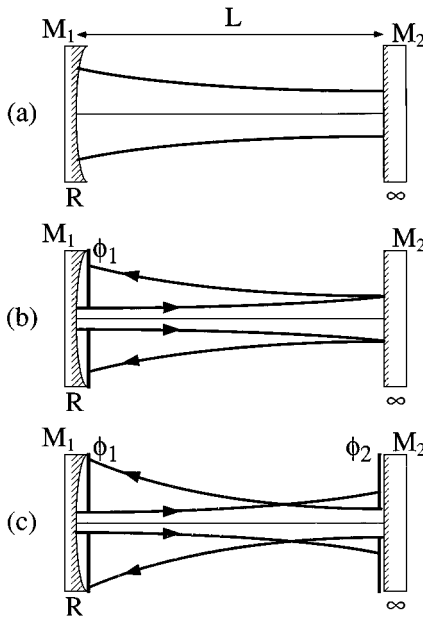


FIG. 1. (a) Geometrically stable cavity of length L built with a spherical mirror M_1 (radius of curvature $R > L$) and a plane mirror M_2 . The fundamental TEM_{00} eigenmode has the same shape in both directions of propagation. (b) The cavity now contains a circular diffracting aperture of diameter ϕ_1 , which alters the mode profile. In particular, the two directions of propagation are no longer equivalent. (c) A second aperture of diameter ϕ_2 is introduced near the plane mirror.

length of light, J_l is the Bessel function of order l , and l is an integer. The coefficients A , B , and D are those of the $ABCD$ matrix for one round trip inside the cavity starting from the chosen reference plane. For reasons of symmetry, we choose this reference plane at the middle of the lens equivalent to the reflection on the spherical mirror M_1 . Then the $ABCD$ matrix is given by

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 - 2L/R & 2L \\ -2/R(1 - 2L/R) & 1 - 2L/R \end{pmatrix}. \quad (3)$$

As is well known, in this case, the Huygens-Fresnel transformation of Eq. (1) is Hermitian. This means that the two opposite directions of propagation inside the cavity are equivalent. In particular, the fundamental eigenmode of the cavity is the usual TEM_{00} Gaussian mode, which has the same shape in both directions [10], as seen in Fig. 1(a).

However, in general, to select this fundamental mode, one has to introduce a diffracting aperture inside the cavity. If we locate this circular aperture on the spherical mirror M_1 , Eq. (1) then becomes

$$\begin{aligned} \gamma u_1(r) &= \frac{2\pi}{B\lambda} j^{l+1} \int_0^{\phi_1/2} dr' r' u_1(r') \\ &\times \exp\left[-j \frac{\pi}{B\lambda} (Ar'^2 + Dr^2)\right] J_l\left(\frac{2\pi}{B\lambda} rr'\right), \end{aligned} \quad (4)$$

where ϕ_1 is the diameter of the aperture and where we call $u_1(r)$ the field distribution on mirror M_1 . Then the Huygens-Fresnel operator of Eq. (4) is no longer Hermitian.

This leads to modifications of the mode profile with respect to the usual Gaussian mode [28–32]. In particular, the mode profile is in general different for the two directions of propagation [see Fig. 1(b)].

In order to illustrate this point, we have numerically computed the fundamental mode profile of the cavity of Fig. 1(b) with $R = 0.6$ m, $L = 0.59$ m, $\lambda = 3.39$ μm , and for different values of ϕ_1 . To compute this mode, one can solve Eq. (4) with $l = 0$ iteratively using a quasifast Hankel transform algorithm [33,34]. In particular, such a Fox-Li-type calculation [35,36] provides the modulus $|\gamma|$ of the eigenvalue that is related to the round-trip intensity diffraction losses Γ of the fundamental mode in the following manner:

$$\Gamma = 1 - |\gamma|^2. \quad (5)$$

The mode profile $u_2(r)$ on mirror M_2 can be obtained by propagating $u_1(r)$ through the cavity using the following integral:

$$\begin{aligned} u_2(r) &= \frac{2\pi}{B_{12}\lambda} \int_0^{\phi_1/2} dr' r' u_1(r') \\ &\times \exp\left[-j \frac{\pi}{B_{12}\lambda} (A_{12}r'^2 + D_{12}r^2)\right] J_0\left(\frac{2\pi}{B_{12}\lambda} rr'\right), \end{aligned} \quad (6)$$

where the $ABCD$ matrix corresponding to propagation from mirror M_1 to mirror M_2 is given by

$$\begin{pmatrix} A_{12} & B_{12} \\ C_{12} & D_{12} \end{pmatrix} = \begin{pmatrix} 1 - L/R & L \\ -1/R & 1 \end{pmatrix}. \quad (7)$$

The resulting mode intensity profiles on mirrors M_1 and M_2 for different values of ϕ_1 are shown in Figs. 2(a) and 2(b), respectively. These figures also display the Gaussian profiles of the unaltered TEM_{00} mode (obtained for $\phi_1 = \infty$) whose sizes are $w_1 = 2.23$ mm and $w_2 = 288$ μm on mirrors M_1 and M_2 , respectively. These figures illustrate the fact that the diffracting aperture does not only introduce losses for the fundamental cavity mode, it also strongly modifies its shape [27–32]. In particular, one can observe in Fig. 2(a) that the mode profile on the spherical mirror oscillates around the TEM_{00} mode shape when the diffracting aperture diameter ϕ_1 is increased. On the contrary, on the plane mirror M_2 [see Fig. 2(b)], when ϕ_1 is increased, the mode diameter decreases until it reaches the unaltered TEM_{00} Gaussian profile. In particular, these profiles confirm the fact that in the presence of strong diffraction, the mode becomes different for the two directions of propagation, as schematized in Fig. 1(b). We can consequently expect the appearance of a non-negligible excess noise factor [15].

B. Calculation of the excess noise factor

In the general case of non-Hermitian cavities [10,11], the excess noise factor is obtained from the biorthogonality properties of the modes of the cavity. If $\{u^{(n)}(r, \varphi)\}$ is the set of the eigenmodes of the cavity, associated with the eigenvalues $\{\gamma^{(n)}\}$ [see Eq. (4)], these eigenmodes are in general not orthogonal:

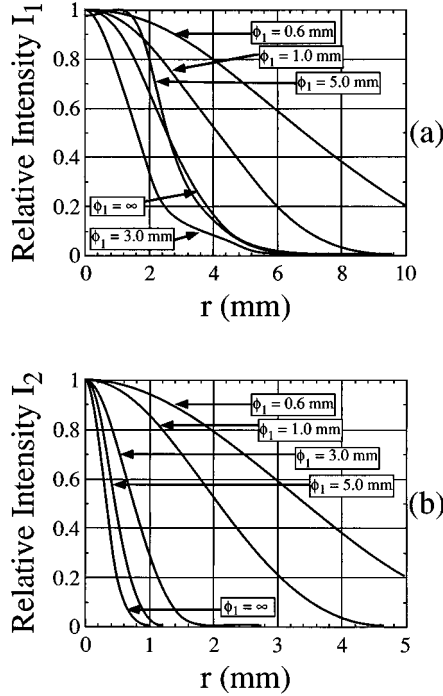


FIG. 2. Intensity profiles of the fundamental mode of the cavity of Fig. 1(b) on the (a) spherical and (b) plane mirrors computed for different values of ϕ_1 . The values of the parameters are $\lambda = 3.39 \mu\text{m}$, $L = 0.59 \text{ m}$, and $R = 0.6 \text{ m}$. The profiles corresponding to the usual TEM_{00} mode are labeled $\phi_1 = \infty$.

$$\langle u^{(m)}, u^{(n)} \rangle = \int_0^{2\pi} d\varphi \int_0^\infty dr r [u^{(m)}(r, \varphi)]^* u^{(n)}(r, \varphi) \neq \delta_{mn}. \quad (8)$$

However, it is always possible to define a set of fields $\{\phi^{(m)}(r, \varphi)\}$ that obey the following biorthogonality relations:

$$\langle \phi^{(m)}, u^{(n)} \rangle = \int_0^{2\pi} d\varphi \int_0^\infty dr r \phi^{(m)}(r, \varphi) u^{(n)}(r, \varphi) = \delta_{mn}. \quad (9)$$

Then these fields $\{\phi^{(m)}(r, \varphi)\}$ can be shown to be the eigenmodes of the cavity propagating in the opposite direction, associated with the same eigenvalues $\gamma^{(m)}$ [14]. Then, if we normalize the modes of the cavity according to

$$\langle u^{(n)}, u^{(n)} \rangle = \int_0^{2\pi} d\varphi \int_0^\infty dr r |u^{(n)}(r, \varphi)|^2 = 1, \quad (10)$$

the excess noise factor associated to mode n is given by [11]

$$K_n = \langle \phi^{(n)}, \phi^{(n)} \rangle = \int_0^{2\pi} d\varphi \int_0^\infty dr r |\phi^{(n)}(r, \varphi)|^2 > 1. \quad (11)$$

In the cavity of Fig. 1(b) that we consider here, the problem is simplified since the plane mirror M_2 is a plane of symmetry of the cavity [13,14]. Then, on mirror M_2 , the adjoint modes $\{\phi^{(n)}(r, \varphi)\}$ have the same profiles as the eigenmodes $\{u^{(n)}(r, \varphi)\}$. Then, if $u_2(r)$ is the transverse dependence of

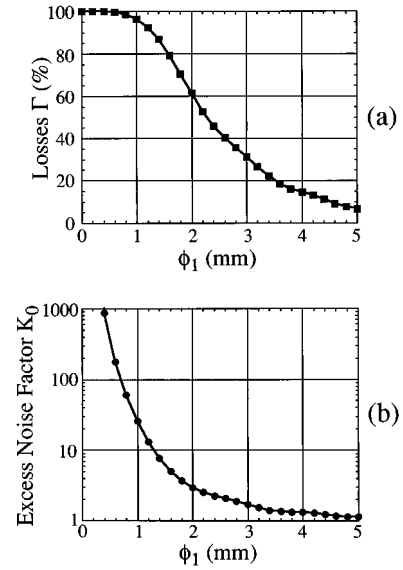


FIG. 3. Computed evolutions of (a) the intensity losses per round-trip Γ and (b) the excess noise factor K_0 of the fundamental mode of the cavity of Fig. 1(b) vs ϕ_1 , with the parameters used in Fig. 2.

the fundamental eigenmode (which has no dependence in φ) on the plane mirror, the excess noise factor for this fundamental mode is given by

$$K_0 = \frac{\left[\int_0^\infty dr r |u_2(r)|^2 \right]^2}{\left| \int_0^\infty dr r [u_2(r)]^2 \right|^2}. \quad (12)$$

Figures 3(a) and 3(b) display the evolutions of the losses Γ and the excess noise factor K_0 computed versus ϕ_1 from the mode profiles of Fig. 2(b). One can notice that the usual decrease of the losses versus ϕ_1 [37] is accompanied by a decrease of the excess noise factor. One must also notice that this excess noise factor can reach large values, as large as 10^3 , when ϕ_1 gets very small compared to the unaltered mode diameter $2w_1$. Of course, such high values of the excess noise factor occur for high losses, as will be discussed in Sec. IV.

III. MULTIAPERTURED STABLE CAVITY

As recalled in the Introduction, once a first aperture has been introduced inside a stable cavity, as in Sec. II, this cavity becomes non-Hermitian. It then behaves in a manner similar to an unstable cavity. In particular, if one introduces a second aperture inside the cavity, the losses exhibit oscillations versus the diameter of this second aperture [27]. Moreover, the diffraction losses Γ for one round-trip inside the cavity can be reduced by the introduction of the second aperture. In unstable cavities, the oscillation of the losses is accompanied by resonances of the excess noise factor. To explore the existence of such resonances in a stable cavity, we consider the cavity of Fig. 1(c), which now contains two apertures of diameters ϕ_1 and ϕ_2 , located on mirrors M_1 and M_2 , respectively. To compute the fundamental eigenmode of such a cavity, one must solve the two following coupled Huygens-Fresnel equations:

$$\gamma_{12}u_2(r_2) = \frac{2\pi}{B_{12}\lambda} \int_0^{\phi_1/2} dr_1 r_1 u_1(r_1) \exp\left[-j \frac{\pi}{B_{12}\lambda} (A_{12}r_1^2 + D_{12}r_2^2)\right] J_0\left(\frac{2\pi}{B_{12}\lambda} r_1 r_2\right), \quad (13a)$$

$$\gamma_{21}u_1(r_1) = \frac{2\pi}{B_{21}\lambda} \int_0^{\phi_2/2} dr_2 r_2 u_2(r_2) \exp\left[-j \frac{\pi}{B_{21}\lambda} (A_{21}r_2^2 + D_{21}r_1^2)\right] J_0\left(\frac{2\pi}{B_{21}\lambda} r_1 r_2\right), \quad (13b)$$

where $u_1(r_1)$ and $u_2(r_2)$ are the wave fronts on mirrors M_1 and M_2 , respectively, and γ_{12} and γ_{21} give the fundamental mode eigenvalue γ :

$$\gamma = \gamma_{12}\gamma_{21}. \quad (14)$$

The coefficients A_{12} , B_{12} , D_{12} , A_{21} , B_{21} , and D_{21} of Eqs. (13) are deduced from the $ABCD$ matrices for the two directions of propagation through the cavity, given, respectively, by Eq. (7) and by

$$\begin{pmatrix} A_{21} & B_{21} \\ C_{21} & D_{21} \end{pmatrix} = \begin{pmatrix} A_{12} & B_{12} \\ C_{12} & D_{12} \end{pmatrix}^T = \begin{pmatrix} 1 & L \\ -1/R & 1-L/R \end{pmatrix}. \quad (15)$$

Here again, the problem can be solved iteratively. Now, to compute the excess noise factor, one must notice that mirror M_2 no longer constitutes a plane of symmetry of the resonator. Equation (12) is hence no longer valid and, using Eq. (11), the excess noise factor for the fundamental eigenmode is given by

$$K_0 = \frac{\left[\int_0^\infty dr r |u_2(r)|^2 \right] \left[\int_0^{\phi_2/2} dr r |u_2(r)|^2 \right]}{\left| \int_0^{\phi_2/2} dr r [u_2(r)]^2 \right|^2}. \quad (16)$$

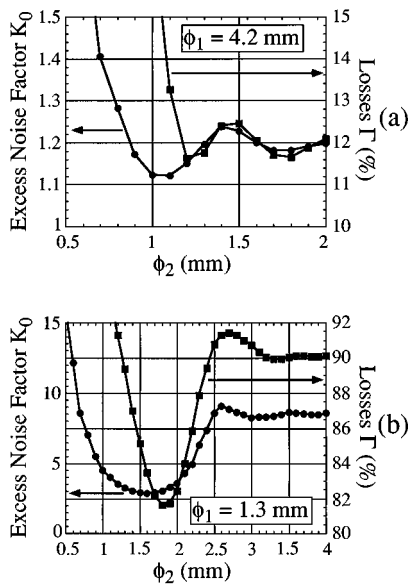


FIG. 4. Computed evolutions of the intensity losses per round-trip Γ (■) and the excess noise factor K_0 (●) of the fundamental mode of the cavity of Fig. 1(c) vs ϕ_2 , with the parameters used in Fig. 2 and with (a) $\phi_1 = 4.2$ mm and (b) $\phi_1 = 1.3$ mm.

The result of such a calculation is shown in Fig. 4. Figure 4(a) displays the evolution of the losses and the excess noise factor in our cavity versus diameter ϕ_2 of the second aperture, for a fixed value $\phi_1 = 4.2$ mm of the diameter of the first aperture. Such an aperture alone leads to a reasonable amount of diffraction losses ($\Gamma = 13\%$), usually sufficient to select the fundamental mode in high-gain lasers. As already seen in Ref. [27], when ϕ_2 is increased, the losses Γ of the mode oscillate near the value ($\Gamma = 13\%$) obtained in the presence of the first aperture only. In particular, for $\phi_2 = 1.2$ mm, these losses can be reduced because of the introduction of the second aperture. Moreover, one can see that the excess noise factor K_0 oscillates with ϕ_2 , its maxima (minima) corresponding to the maxima (minima) of Γ . In particular, for $\phi_2 = 1.0$ mm, its value can decrease from $K_0 = 1.27$ to $K_0 = 1.1$.

This oscillating behavior is still more visible in the results of Fig. 4(b), for which the first aperture ($\phi_1 = 1.3$ mm) alone introduces much more diffraction losses ($\Gamma = 89.9\%$). In this case, in the presence of the second aperture, the excess noise factor oscillates between $K_0 = 2.8$ (for $\phi_2 = 1.7$ mm) and $K_0 = 9$ (for $\phi_2 = 2.6$ mm). Here again, the oscillations of K_0 match the oscillations of the losses Γ , as in unstable cavities, but the resonances are far less sharp than in unstable cavities. These results show that in such a stable cavity, the excess noise factor can increase with the aperture diameter, and not only decrease monotonically.

IV. DISCUSSION AND CONCLUSION

The results of Fig. 3(b) show that contrary to what is usually stated, geometrically stable cavities can exhibit excess noise factors far larger than 1. Of course, this occurs just below the cavity stability limit ($L \leq R$) and for large values of diffraction losses, i.e., when the eigenmode of the cavity is essentially determined by diffraction, as in the case of geometrically unstable cavities. However, it seems possible to observe such large values of the excess noise factor in a stable cavity. Let us recall that to observe the fundamental linewidth of a laser, one must heterodyne the output beam. This has been performed either by heterodyning the laser output with another laser [3,4], by self-heterodyning the output beam of the laser thanks to a Michelson interferometer including a long delay line [20,38–40], or by monitoring the $\sigma^+ - \sigma^-$ beat note inside a quasi-isotropic high-gain gas laser submitted to a longitudinal magnetic field [21,41,42]. In this latter case, using a He-Xe laser, unsaturated gain coefficients as large as 250 dB/m can be reached [42]. Consequently, using a small 5-cm-long laser, one can expect a 12-dB single-pass gain, and hence reach threshold even with 99.6% losses per round-trip inside the cavity. If one refers to the results of Fig. 3, this means that we could probably observe excess noise factors due to the nonorthogonality of the transverse eigenmodes of a stable cavity as large as 100 in such a

high-gain gas microlaser. Such a large value confirms the fact that once diffraction losses become important, a geometrically stable cavity behaves as an unstable one [27]. This fact has also been confirmed by the fact that in a stable cavity containing two apertures, the excess noise factor behaves resonantly with the diameter of one of the apertures. These resonances are of course far less sharp in stable cavities than in unstable cavities. However, the results of Fig. 4(b) show that such variations of the excess noise factor are large enough to be experimentally observable. One can thus expect

deviations from standard predictions for the fundamental laser linewidth, due to nonorthogonality of the transverse eigenmodes in stable cavities.

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