Macroscopic averages in QED in material media

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This article addresses the problem of whether it is possible to describe the effect of the atoms of material media on the field only in terms of a dielectric constant, in the regime where the field has to be treated quantum mechanically. Using a simple model of a linear lossless material medium, we start from first principles and determine the validity of the approximations required to obtain such a quantum analogue of classical macroscopic electrodynamics. This theory is derived here from the fundamental microscopic QED description of a medium, in terms of its constituent atoms in the vacuum, by taking macroscopic averages of the dynamical variables. The condition of the validity of the macroscopic approximation is obtained as the proviso for neglecting the contribution of the atoms of the medium to the quantum noise of the field. We show that macroscopic averaging is compatible with a quantum theory and does not imply any smoothening of the intrinsic quantum fluctuations of the field. Although this theory is based on a simple one-dimensional model of a single-mode cavity, it is able to describe the frequency dependence of the dielectric constant. [S1050-2947(97)00805-6]

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I. INTRODUCTION

Many physical systems where the quantum nature of electromagnetic radiation is important involve the presence of material media. There have been several examples of these systems in the literature recently, including research on quantum optics in semiconductors [1], semiconductor microlasers [2–4], cavity QED in microspheres [5–11], photonic band materials [12–19], the generation of squeezed light in nonlinear material media [20–25], and the discovery of nonlocal dispersion cancellation in single-photon propagation through material media [26–28]. Although it is possible to use ordinary quantum electrodynamics to describe the physics of these systems [28–38], it would be convenient if there were a quantum version of macroscopic classical electrodynamics. Then instead of accounting for each atom of the medium in a fundamental way, their effect would be described by a dielectric constant only. There have been many attempts to develop such a theory [39–59]. This article is concerned with the validity of these macroscopic approaches.

The starting point of macroscopic theories of quantum electrodynamics in material media is usually the classical macroscopic Maxwell equations, which are then quantized. In this article, we adopt the point of view that there is no need to quantize the macroscopic Maxwell equations because ordinary QED already provides a quantum description of electrodynamics in material media at least in principle. If a macroscopic description is possible, it should appear as an approximation, under certain conditions, to the fundamental microscopic theory. Here, we obtain such a macroscopic approximation and derive its condition of validity.

We discuss in particular three main questions about the validity of macroscopic descriptions of QED in material media. The first one concerns dielectric constants that depend on the frequency [37,49]. This is the case of dispersive media. The problem is that, if the fields vary faster than the response time of the medium, the displacement field at a given time will depend on the value of the electric field at all previous times [37,49,60]. Then the effective Lagrangian is nonlocal in time and cannot be used in a quantization scheme [49]. Huttner *et al.* [36] argue that the energy of the medium [61] has to be introduced in any rigorous Lagrangian or Hamiltonian formalism. Nevertheless, some authors have devised ways of avoiding the problem in certain special cases [46,56,58]. We show in Sec. V that, when the condition for a macroscopic description to be a good approximation to the fundamental microscopic theory is fulfilled, we recover Milonni's results [58].

The second question is about whether a macroscopic approach can be used at all in the quantum regime. The atoms that form a medium exhibit quantum fluctuations that can affect the fields. If a material medium is to be described only by a dielectric constant, we must be able to neglect the effect of these quantum fluctuations. Usually, it is assumed that this can be done when the frequencies involved are far from the resonance frequency of the medium [47]. Rosewarne [35], however, has calculated the variance of the electric field in a scalar version of the Hopfield model [62] and has shown that, even far away from the resonance of the medium in the nondispersive region, there is an atomic contribution to the variance larger than the medium correction to the dielectric constant of the vacuum. In Sec. III, we calculate the variance of the field in our microscopic model and find only a negligible contribution from the atoms of the medium in this regime. The difference between our result and Rosewarne's is due to him having assumed a continuous polarization for the medium, implying a macroscopic average. That leads us to the third question addressed in this article.

The third question is about the legitimacy of taking macroscopic averages in a quantum theory. The idea of a macroscopic average is implicit in any macroscopic theory of electrodynamics in material media. It is based on the following assumption. Electromagnetic fields are rapidly varying functions of position and time on the scale of the atomic constituents of a material medium. In many cases, however,

the particular features of the fields in this fine scale are irrelevant for the physics of the phenomena at hand; for example, macroscopic measuring apparatuses are often insensitive to detail in the atomic scale. Then it is possible to average over the fine detail, performing some sort of coarse graining. In the realm of classical electrodynamics, such averaging yields the macroscopic Maxwell equations with smoothly varying fields and continuous charge and current distributions. A key issue in the development of a macroscopic theory of QED in material media is whether averaging the fields in this way will not also average their quantum fluctuations. Is a macroscopic version of QED a contradiction of terms? This is a very important question connected to the exact nature of macroscopic averages that macroscopic theories of QED in a material media [39–59] have often avoided discussing, but which we address in this article in Sec. IV. Our results suggest that there is no such contradiction for most practical purposes, as in fact some experiments seem to demonstrate [20,22–25]. The most important consequence of macroscopically averaging the fields is the well-known problem of the local field [63–66] which differs from the macroscopic field. Nevertheless, because the atoms in a medium are sources of quantum noise as well, we find that macroscopic fields can exhibit different values for their variances when compared to microscopic fields. Our calculations reproduce Rosewarne's result for the variance of the electric field in a material medium [35]. We show that Rosewarne's result does not rule out the possibility of a macroscopic theory of QED in a material medium because, under certain conditions, a guest atom immersed in the medium becomes insensitive to the large atomic medium contribution to the variance of the field. We use this property in Sec. V to obtain a macroscopic theory from our microscopic model.

This article is organized as follows. In Sec. II we introduce the microscopic model of a material medium we adopt. Then, in Sec. III, we discuss the problem of the extra quantum noise that is introduced by the atoms of the medium. In Sec. IV, we address the problem of macroscopic averages. In Sec. V, we show that under certain conditions a macroscopic description, incorporating the frequency dependence of the dielectric constant, provides a good approximation to the physics of the system. In this domain, we recover Milonni's results [58]. Finally, we summarize the main points of this article in Sec. VI.

II. MODEL

The simplest and most fundamental case of interaction between atoms and radiation is where a single mode of the field is coupled to a single atomic transition [67,68]. We think that the essential features of material media can be described in terms of this basic case. So we adopt what we believe to be the simplest one-dimensional microscopic model of lossless material media: N two-level atoms having the same resonance frequency ω_0 in a single-mode cavity of resonance frequency ω. Although there is no wave packet propagation in a single-mode cavity where dispersion can act, we are able to describe the frequency dependence of the dielectric constant with this model. We deal only with linear media where, as Fano has shown [69], the two-level atoms can be described approximately by harmonic oscillators. We also consider, however, a guest two-level atom of resonance frequency ω_a immersed in the material medium and strongly coupled to the field so that it will not be approximated by a harmonic oscillator (see Fig. 1). A similar model was adopted by Knoester and Mukamel [34] to study impurity molecules in a dielectric host crystal, but their model does not involve a single mode cavity.

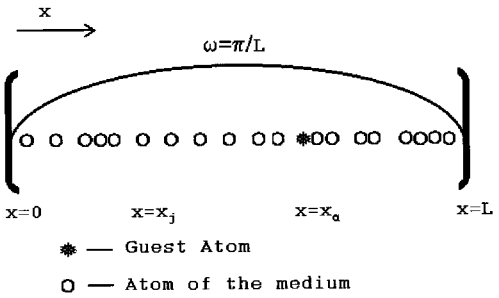

FIG. 1. This is a schematic representation of our one-dimensional model of a single-mode cavity filled with a material medium with an immersed guest atom. The medium is composed of N atoms located at the positions $x_1, \ldots, x_j, \ldots, x_N$ and the guest atom sits at $x = x_a$. As can be seen from this scheme, the frequency ω of the single mode is related to the length of the cavity L by $\omega = \pi/L$.

The displacement field in the cavity is given by [70,71]

$$D(x) = \varepsilon_0 \sqrt{\frac{\hbar\omega}{\varepsilon_0 L}}(\hat{a} + \hat{a}^\dagger)\sin\left(\frac{\omega}{c}x\right) \tag{1}$$

and the polarization of the medium by

$$P(x) = \sqrt{\frac{\hbar}{2\omega_0}}\sum_{j=1}^{N}(\hat{b}_j + \hat{b}_j^\dagger)q_j\delta(x - x_j), \tag{2}$$

where ε_0 is the dielectric constant of the vacuum (we are adopting SI units), L is the length of the cavity, and $\sqrt{\hbar/2\omega_0}(\hat{b}_j + \hat{b}_j^\dagger)$ is the position operator of an oscillator of effective charge q_j; the product $\sqrt{\hbar/2\omega_0}q_j(\hat{b}_j + \hat{b}_j^\dagger)$ is the electric dipole moment operator of the atom of the medium that is located at x_j which we are approximating by a harmonic oscillator. We notice that ω and L for the single-mode cavity are related by

$$\omega = \frac{\pi c}{L} \tag{3}$$

and that the operators $\hat{a}$, $\hat{a}^\dagger$, $\hat{b}_j$, and $\hat{b}_j^\dagger$ satisfy the commutation relations

$$[\hat{a}, \hat{a}^\dagger] = 1, \tag{4}$$

$$[\hat{b}_j, \hat{b}_{j'}^\dagger] = \delta_{jj'}, \tag{5}$$

$$[\hat{b}_j, \hat{b}_{j'}] = 0, \tag{6}$$

and $\hat{a}$, $\hat{a}^\dagger$commute with $\hat{b}_j$, $\hat{b}_j^\dagger$.

The Hamiltonian is given by

$$\hat{H}=\hbar\omega\hat{a}^{\dagger}\hat{a}+\hbar\omega_0\sum_{j=1}^{N}\hat{b}_j^{\dagger}\hat{b}_j-\int dx\frac{D(x)}{\varepsilon_0}P(x)+\frac{\hbar\omega_a}{2}\hat{\sigma}_z$$
$$+\hbar\Omega(\hat{a}+\hat{a}^{\dagger})(\hat{\sigma}+\hat{\sigma}^{\dagger}), \tag{7}$$

where

$$\Omega=-d\sqrt{\frac{\omega}{\varepsilon_0 L\hbar}}\sin\left(\frac{\omega}{c}x_a\right) \tag{8}$$

is the Rabi frequency of the guest atom located at x_a whose electric dipole moment strength is d. We notice that we are neglecting in Eq. (7) the terms describing the self-energies of the dipoles [71], as is usually done in treatments based on the dipole interaction Hamiltonian [67,68].

Writing the solution of the integral in Eq. (7) explicitly, we obtain

$$\hat{H}=\hbar\omega\hat{a}^{\dagger}\hat{a}+\hbar\omega_0\sum_{j=1}^{N}\hat{b}_j^{\dagger}\hat{b}_j+\hbar\sum_{j=1}^{N}g_j(\hat{a}+\hat{a}^{\dagger})(\hat{b}_j+\hat{b}_j^{\dagger})$$
$$+\frac{\hbar\omega_a}{2}\hat{\sigma}_z+\hbar\Omega(\hat{a}+\hat{a}^{\dagger})(\hat{\sigma}+\hat{\sigma}^{\dagger}), \tag{9}$$

where g_j is given by

$$g_j=-q_j\sqrt{\frac{\omega}{2\varepsilon_0 L\omega_0}}\sin\left(\frac{\omega}{c}x_j\right). \tag{10}$$

We see that the interaction of the atoms of the material medium with the field depends only on the weighted sum of all the atomic creation and annihilation operators, $\Sigma_{j=1}^{N}g_j(\hat{b}_j+\hat{b}_j^{\dagger})$. In fact, if we define the new operator

$$\hat{B}_1=\frac{1}{\sqrt{\sum_{j'=1}^{N}g_{j'}^2}}\sum_{j=1}^{N}g_j\hat{b}_j, \tag{11}$$

which is also an annihilation operator satisfying the standard commutation relation

$$[\hat{B}_1,\hat{B}_1^{\dagger}]=1, \tag{12}$$

as can be verified from Eqs. (5), (6), and (11), we find that, because all the atoms of the medium have the same resonance frequency ω_0, the Hamiltonian in the interaction picture will depend only on $\hat{B}_1$ and $\hat{B}_1^{\dagger}$. This suggests a potential simplification in the Hamiltonian (9). Such a simplification is possible if we express the Hamiltonian (9) in terms of the transformed atomic medium operators

$$\hat{B}_1=\frac{1}{G_N}\sum_{j=1}^{N}g_j\hat{b}_j \tag{13}$$

and

$$\hat{B}_k=\frac{1}{G_{k-1}G_k}\sum_{j=1}^{k-1}(g_j^2\hat{b}_k-g_k g_j\hat{b}_j), \tag{14}$$

where $k=2,\ldots,N$ and

$$G_n=\sqrt{\sum_{j'=1}^{n}g_{j'}^2}. \tag{15}$$

As the operators $\hat{b}_j$ and $\hat{b}_j^{\dagger}$ are annihilation and creation operators obeying the commutation relations (5) and (6), it follows from the definitions (13) and (14) that the same is true of the $\hat{B}_j$ and $\hat{B}_j^{\dagger}$ operators, i.e.,

$$[\hat{B}_j,\hat{B}_{j'}^{\dagger}]=\delta_{jj'} \tag{16}$$

and

$$[\hat{B}_j,\hat{B}_{j'}]=0. \tag{17}$$

The simplification in the Hamiltonian (9), which we are about to achieve, is possible only because

$$\sum_{j=1}^{N}\hat{b}_j^{\dagger}\hat{b}_j=\sum_{k=1}^{N}\hat{B}_k^{\dagger}\hat{B}_k. \tag{18}$$

For the case of a ''medium'' made up of only two atoms, Eqs. (13) and (14) are reduced to

$$\hat{B}_1=\frac{g_1\hat{b}_1+g_2\hat{b}_2}{\sqrt{g_1^2+g_2^2}}, \tag{19}$$

$$\hat{B}_2=\frac{g_1\hat{b}_2-g_2\hat{b}_1}{\sqrt{g_1^2+g_2^2}}, \tag{20}$$

and a straightforward calculation shows that Eq. (18) is verified. In order to demonstrate that Eq. (18) remains true for a medium made up of any arbitrary number N of atoms, we will show that if Eq. (18) is valid for n atoms, then it is also valid for $n+1$.

Because $\hat{B}_1$ depends on the number of atoms of the medium, in our demonstration, we will attach a superscript to it to denote this number; i.e., $\hat{B}_1^n$ is defined by

$$\hat{B}_1^n=\frac{1}{G_n}\sum_{j=1}^{n}g_j\hat{b}_j. \tag{21}$$

Now, we assume that Eq. (18) holds for n atoms, i.e.,

$$\hat{B}_1^{n\dagger}\hat{B}_1^n+\sum_{k=2}^{n}\hat{B}_k^{\dagger}\hat{B}_k=\sum_{j=1}^{n}\hat{b}_j^{\dagger}\hat{b}_j. \tag{22}$$

So for $n+1$ atoms, we can write, using Eq. (22),

$$\hat{B}_1^{n+1\dagger}\hat{B}_1^{n+1}+\sum_{k=2}^{n+1}\hat{B}_k^{\dagger}\hat{B}_k=\hat{B}_1^{n+1\dagger}\hat{B}_1^{n+1}+\hat{B}_{n+1}^{\dagger}\hat{B}_{n+1}-\hat{B}_1^{n\dagger}\hat{B}_1^n$$
$$+\sum_{j=1}^{n}\hat{b}_j^{\dagger}\hat{b}_j. \tag{23}$$

From Eqs. (14) and (21), we obtain

$$\hat{B}_1^{n+1}=\frac{G_n\hat{B}_1^n+g_{n+1}\hat{b}_{n+1}}{\sqrt{G_n^2+g_{n+1}^2}} \tag{24}$$

and

$$\hat{B}_{n+1}=\frac{G_n\hat{b}_{n+1}-g_{n+1}\hat{B}_1^n}{\sqrt{G_n^2+g_{n+1}^2}}. \tag{25}$$

A straightforward calculation yields

$$\hat{B}_1^{n+1\dagger}\hat{B}_1^{n+1}+\hat{B}_{n+1}^{\dagger}\hat{B}_{n+1}=\hat{B}_1^{n\dagger}\hat{B}_1^{n}+\hat{b}_{n+1}^{\dagger}\hat{b}_{n+1}. \tag{26}$$

Substituting Eq. (26) into Eq. (23), we find

$$\hat{B}_1^{n+1\dagger}\hat{B}_1^{n+1}+\sum_{k=2}^{n+1}\hat{B}_k^{\dagger}\hat{B}_k=\sum_{j=1}^{n+1}\hat{b}_j^{\dagger}\hat{b}_j. \tag{27}$$

As we have already shown that Eq. (18) is valid for two atoms, we have now demonstrated by induction that Eq. (18) is valid for an arbitrary number N of atoms in the medium.

Written in terms of the new set of atomic operators, the Hamiltonian (9) takes a much simpler form. It can be expressed as the sum of two uncoupled Hamiltonians

$$\hat{H}=\hat{H}_b+\hat{H}_m, \tag{28}$$

where

$$\hat{H}_m=\hbar\omega\hat{a}^{\dagger}\hat{a}+\hbar\omega_0\hat{B}_1^{\dagger}\hat{B}_1+\hbar G_N(\hat{a}+\hat{a}^{\dagger})(\hat{B}_1+\hat{B}_1^{\dagger})+\frac{\hbar\omega_a}{2}\hat{\sigma}_z$$
$$+\hbar\Omega(\hat{a}+\hat{a}^{\dagger})(\hat{\sigma}+\hat{\sigma}^{\dagger}) \tag{29}$$

and

$$\hat{H}_b=\hbar\omega_0\sum_{k=2}^{N}\hat{B}_k^{\dagger}\hat{B}_k. \tag{30}$$

The atomic operators $\hat{B}_k$, $\hat{B}_k^{\dagger}$ with $k=2,\ldots,N$ have their free time evolution, given by

$$\hat{B}_k(t)=\hat{B}_k(0)\exp(-i\omega_0 t), \tag{31}$$

undisturbed by the rest of the system described by $\hat{H}_m$.

The new set of atomic operators $\hat{B}_j$ $(j=1,\ldots,N)$ is composed of collective bosonic operators that involve all the atomic oscillators. The $N-1$ operators $\hat{B}_k$ $(k=2,\ldots,N)$ represent collective excitations that cannot be excited by the single-field mode $\hat{a}$. However, the physical meaning of $\hat{B}_1$ is more subtle. This is the only collective excitation that can be excited by the field mode with a strength that is dependent on the effective coupling constant G_N, Eq. (29). As will become clear in Secs. IV and V, $\hat{B}_1$ is related to the macroscopic polarization density of the medium and G_N to the effective oscillator strength of the medium.

Because $\hat{B}_k$ $(k=2,\ldots,N)$ are left undisturbed by the rest of the system, we will discuss only $\hat{H}_m$ from now on. In order not to make the notation unnecessarily complicated, we will drop the subscripts 1 from $\hat{B}_1$, N from G_N, and m from $\hat{H}_m$. So we will be concerned with the Hamiltonian

$$\hat{H}=\hat{H}_0+\hat{H}_A, \tag{32}$$

where

$$\hat{H}_0=\hbar\omega\hat{a}^{\dagger}\hat{a}+\hbar\omega_0\hat{B}^{\dagger}\hat{B}+\hbar G(\hat{a}+\hat{a}^{\dagger})(\hat{B}+\hat{B}^{\dagger}), \tag{33}$$

$$\hat{H}_A=\frac{\hbar\omega_a}{2}\hat{\sigma}_z+\hbar\Omega(\hat{a}+\hat{a}^{\dagger})(\hat{\sigma}+\hat{\sigma}^{\dagger}), \tag{34}$$

$$\hat{B}=\frac{1}{G}\sum_{j=1}^{N}g_j\hat{b}_j, \tag{35}$$

and

$$G=\sqrt{\sum_{j'=1}^{N}g_{j'}^2}. \tag{36}$$

We can now deal with the polariton problem diagonalizing $\hat{H}_0$. Following Hopfield's procedure [62,72], we define two pairs of dressed annihilation and creation operators given by

$$\hat{c}_k=x_1^k\hat{a}+y_1^k\hat{a}^{\dagger}+x_2^k\hat{B}+y_2^k\hat{B}^{\dagger}, \tag{37}$$

with $k=1,2$, satisfying the usual commutation relations

$$[\hat{c}_k,\hat{c}_{k'}^{\dagger}]=\delta_{kk'}, \tag{38}$$

$$[\hat{c}_k,\hat{c}_{k'}]=0, \tag{39}$$

and diagonalizing $\hat{H}_0$,

$$[\hat{c}_k,\hat{H}_0]=\hbar\Omega_k\hat{c}_k. \tag{40}$$

Equations (4), (16), (37), (38), and (40) yield

$$x_j^k=\tfrac{1}{2}(v_j^k+u_j^k), \tag{41}$$

$$y_j^k=\tfrac{1}{2}(v_j^k-u_j^k), \tag{42}$$

where

$$u_1^k=\frac{\omega}{\Omega_k}v_1^k, \tag{43}$$

$$u_2^k=\frac{\Omega_k^2-\omega^2}{2G\Omega_k}v_1^k, \tag{44}$$

$$v_2^k=\frac{\Omega_k^2-\omega^2}{2G\omega_0}v_1^k, \tag{45}$$

$$v_1^k=\sqrt{\frac{4G^2\Omega_k\omega_0}{(\Omega_k^2-\omega^2)^2+4\omega_0\omega G^2}}, \tag{46}$$

and

$$\Omega_k^2=\frac{1}{2}\{\omega_0^2+\omega^2+(-1)^k\Lambda\}, \tag{47}$$

$$\Lambda=(\omega_0^2-\omega^2)\sqrt{1+\frac{16\omega_0\omega G^2}{(\omega_0^2-\omega^2)^2}}. \tag{48}$$

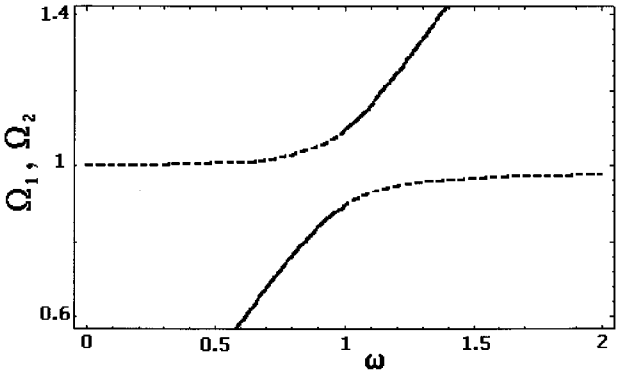

FIG. 2. This is a plot of the frequencies of the polariton modes in units of ω_0 as a function of ω/ω_0 for $G'=0.1\omega_0$. The thick line shows Ω_1 and the dotted line shows Ω_2. Our definition of Ω_1 and Ω_2 makes them discontinuous at $\omega=\omega_0$. In the usual definition, Ω_1 is the entire upper curve and Ω_2 the entire lower curve.

Equation (47) shows the usual total absence of polariton modes in the frequency interval between $\sqrt{\omega_0^2-4G'^2}$ and ω_0, where

$$G'=\sqrt{\frac{\omega_0}{\omega}}G \tag{49}$$

is the effective oscillator strength (in units of ω_0), which is held constant. Unlike Hopfield [62], however, we have chosen the sign of Λ in Eq. (47) and (48) so that when $G'\to 0$, Ω_1 approaches the field frequency ω and Ω_2 approaches the atomic frequency ω_0. This way of defining Ω_1 and Ω_2 turns both of them into discontinuous functions of the wave number ω/c at $\omega=\omega_0$. Usually, Ω_1 and Ω_2 are defined as two continuous functions of ω/c corresponding to the lower and the upper polariton branches. Figure 2 illustrates this point. The reason for defining Ω_1 and Ω_2 in this nonstandard way is that we want to have the electric permittivity given by $\varepsilon_r=(\omega/\Omega_1)^2$ but ε_r must approach 1 when $G'\to 0$. As we can see in Fig. 3, the plot of $\varepsilon_r=(\omega/\Omega_1)^2$ as a function of the cavity frequency ω shows the usual features of ε_r for lossless media. Another peculiarity of our model is

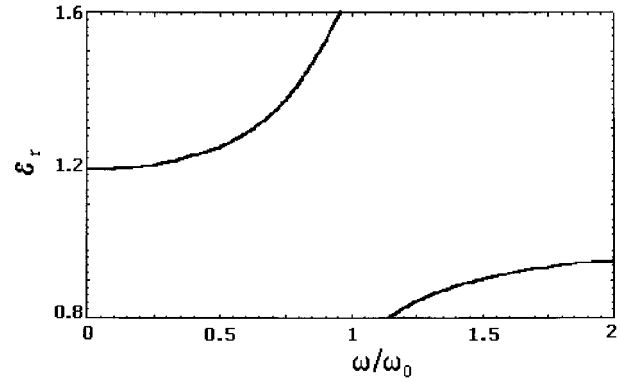

FIG. 3. This is a plot of the relative permittivity $\epsilon_r=\omega/\Omega_1$ as a function of ω/ω_0 with $G'=0.2\omega_0$. For $\omega\ll\omega_0$, the relative permittivity is independent of the frequency ω and takes its static value. At $\omega=\omega_0$, there is a singularity because the medium is lossless. For $\omega\gg\omega_0$, the relative permittivity approaches one (as in the vacuum).

that in order for Ω_1 not to assume imaginary values, we must satisfy the condition $4G'^2\leqslant\omega_0^2$. This does not happen in the Hopfield model where the polariton frequencies are always real for all values of the parameters of the system. The apparently extra condition $4G'^2\leqslant\omega_0^2$ we have here is a consequence of neglecting the self-energies of the dipoles in our model [71]. The effect of these self-energy terms in the Hamiltonian is to shift the oscillator frequencies ω_0. Such a shift can be neglected if it is much smaller than ω_0 which is when $4G'^2\leqslant\omega_0^2$ is satisfied making our treatment consistent. We also notice that although we have chosen v_1^k to be real, this does not imply any loss of generality.

From the commutation relations satisfied by $\hat{a}$, $\hat{a}^\dagger$, $\hat{B}$, $\hat{B}^\dagger$, $\hat{c}_k$, and $\hat{c}_k^\dagger$ we obtain the following expressions for $\hat{a}$ and $\hat{B}$ in terms of $\hat{c}_k$ and $\hat{c}_k^\dagger$ [72]:

$$\hat{a}=x_1^{1*}\hat{c}_1-y_1^1\hat{c}_1^\dagger+x_1^{2*}\hat{c}_2-y_1^2\hat{c}_2^\dagger, \tag{50}$$

$$\hat{B}=x_2^{1*}\hat{c}_1-y_2^1\hat{c}_1^\dagger+x_2^{2*}\hat{c}_2-y_2^2\hat{c}_2^\dagger. \tag{51}$$

Using Eq. (50) in Eq. (34), we can rewrite Eq. (32) in terms of the dressed operators $\hat{c}_k$ and $\hat{c}_k^\dagger$:

$$\hat{H}=\hbar\Omega_1\hat{c}_1^\dagger\hat{c}_1+\hbar\Omega_2\hat{c}_2^\dagger\hat{c}_2+\frac{\hbar\omega_a}{2}\hat{\sigma}_z+\hbar\Omega u_1^1(\hat{c}_1+\hat{c}_1^\dagger)(\hat{\sigma}+\hat{\sigma}^\dagger)$$
$$+\hbar\Omega u_1^2(\hat{c}_2+\hat{c}_2^\dagger)(\hat{\sigma}+\hat{\sigma}^\dagger). \tag{52}$$

The original problem is reduced now to the case of a single atom coupled to two polariton modes. In the next section, we use the dressed operator formalism presented here to study the influence of the atoms of the medium on the quantum noise of the field.

III. QUANTUM NOISE DUE TO THE ATOMS OF THE MEDIUM

In the absence of a guest atom and when the total system is in its ground state defined by $\hat{c}_1|g\rangle=0$ and $\hat{c}_2|g\rangle=0$, we find the following expression for the variance of $\hat{a}+\hat{a}^\dagger$:

$$\langle[\Delta(\hat{a}+\hat{a}^\dagger)]^2\rangle\equiv\langle(\hat{a}+\hat{a}^\dagger)^2\rangle-\langle\hat{a}+\hat{a}^\dagger\rangle^2$$
$$=\sum_{k=1}^{2}\frac{\omega}{\Omega_k}\frac{\Omega_k^2-\omega_0^2}{2\Omega_k^2-\omega_0^2-\omega^2}. \tag{53}$$

The simplest macroscopic theories of electrodynamics in material media assume no dispersion. Then the dielectric constant is independent of the frequency and takes its static value. In our model, this regime corresponds to $\omega\ll\omega_0$. If we also consider the case where the atoms of the medium are only weakly coupled to the field, i.e., $G',\omega\ll\omega_0$, with G' given by Eq. (49), we find from Eqs. (47), (48), and (53) that

$$\langle[\Delta(\hat{a}+\hat{a}^\dagger)]^2\rangle\approx 1+2\frac{G'^2}{\omega_0^2}-2\frac{G'^2}{\omega_0^2}\left(\frac{\omega}{\omega_0}\right)^2, \tag{54}$$

up to second order in ω/ω_0 and G'/ω_0. So if we neglect the higher order term that is second order in both ω/ω_0 and G'/ω_0, we obtain

$$\langle[\Delta(\hat{a}+\hat{a}^{\dagger})]^2\rangle \approx \sqrt{\varepsilon_r}, \tag{55}$$

where ε_r is the relative permittivity defined in terms of the change in the cavity resonance frequency by

$$\Omega_1 = \frac{\omega}{\sqrt{\varepsilon_r}} \tag{56}$$

and, in this approximation, found to be

$$\varepsilon_r \approx 1 + 4\frac{G'^2}{\omega_0^2}, \tag{57}$$

in agreement with the expression for ε_r found in Ref. [35].

Equation (55) reproduces the ratio between the variance of the displacement field in the material medium and in the vacuum calculated by Glauber and Lewenstein [57]. Rosewarne [35], however, computed the variance of the electric field in a scalar version of the Hopfield model and found that, in this regime ($G', \omega \ll \omega_0$), it had a contribution of the order of $G'^2/(\omega_0\omega)$. Such a contribution is larger than the $4G'^2/\omega_0^2$ medium correction to the relative permittivity of the vacuum and is not accounted for when the medium is described only in terms of a dielectric constant.

The electric field is given in terms of the displacement field and the polarization by

$$E = \frac{D}{\varepsilon_0} - \frac{P}{\varepsilon_0}. \tag{58}$$

As we have just shown that the variance of D has no contribution of order $G'^2/(\omega_0\omega)$, this contribution can only come from the polarization. Nevertheless, because the microscopic polarization, given by Eq. (2), vanishes whenever we are not at one of the positions occupied by atoms of the medium, except for these positions the variance of E will be of the same order in G'/ω_0 as that of D when $G', \omega \ll \omega_0$. This apparently contradicts the result of Ref. [35].

The reason why Rosewarne finds a different value for the variance of the electric field is that he adopts a continuous distribution of atoms in the medium instead of a more realistic discrete one. This implies that the dynamical variables he uses are macroscopic averages already. In particular, the electric field at a given position in his calculation [35] is not the field an atom situated at that position would experience, but a macroscopically averaged electric field whose variance is rather different from that of the microscopic field. In the next section, we show this point explicitly taking the macroscopic average of the microscopic electric field in our model and calculating the variance to demonstrate that we can recover the result of Ref. [35].

IV. MACROSCOPIC AVERAGES

Although macroscopic theories of QED in a material media [39–59] have often avoided discussing the macroscopic averaging procedure, the exact nature of these averages is one of the most important issues in any such theory. The answer to the question posed in the Introduction about the possibility of the macroscopic averaging procedure washing out quantum fluctuations in the field will clearly depend on what is meant by macroscopic average. Schram [73] advocates a quantum mechanical averaging. If this is the case, then quantum fluctuations will certainly be absent from the macroscopic theory.

The problem of defining exactly what should be the macroscopic averaging procedure has been the source of much confusion that has plagued, for many years, attempts to derive the classical Maxwell equations for a material medium from the more fundamental microscopic equations for the fields and charged particles in the vacuum. It was Lorentz [74] who, in the beginning of this century, first tried such a derivation using an averaging procedure. He introduced a macroscopic average over ''physically infinitesimal'' volume elements. These elements had to be small enough to be treated as infinitesimals in the macroscopic sense but large enough to contain many atoms so that the spatial variations in the microscopic densities and fields would vanish after averaging. There are two main objections to the Lorentz procedure [75,76]. First, the division of a macroscopic system into ''physically infinitesimal'' volume elements will only lead to uniquely defined macroscopic polarization and magnetization densities if these elements are strictly neutral and no currents leave the elements [75–78]. So the contributions of the bound charges to the charge and current densities have to vanish always, which is absurd. Second, in many problems there are no such ''physically infinitesimal'' volumes that are both macroscopically small and have a large number of atoms, and yet the macroscopic equations seem to hold true. This is the case of optics, for example, where the volume element cannot exceed 10^9 Å^3; otherwise, every oscillation at optical frequencies would be averaged to zero. The addition of a single extra electron to such a volume causes a change in the charge density of about 160 C m^{-3} which is extremely high to be regarded as an infinitesimal increment.

For many years, not much progress was made in improving Lorentz's original ideas (a historical account of the subject can be found in De Groot's book [79] or in the review article by van Kranendonk and Sipe [80]). The first authors to introduce a different macroscopic averaging procedure were Mazur and Nijboer [81]. They have shown that under certain circumstances the macroscopic Maxwell equations can be obtained from a statistical ensemble average of the microscopic equations. Mazur and Nijboer's statistical ensemble average does not suffer from the two problems mentioned above about Lorentz's average over ''physically infinitesimal'' volume elements, and it has been adopted by other authors [79,82]. The problem with ensemble averaging is that it can only be used to define quantities independent of details of the microscopic structure of the system, if the ensemble distribution function varies slowly over distances of the order of the interatomic spacing. This is true for fluids but not for crystals [75–77] or certain other physical systems such as some vacuum electronic devices to which, nevertheless, the macroscopic Maxwell equations can be applied [75,76].

Robinson [75,76] has proposed a different kind of macroscopic average that keeps the essence of Lorentz's original ideas but which is free from the problems mentioned before and does not involve ensemble averaging. He regards a macroscopic description as a description where spatial Fourier components of the field variables above some limiting spatial

frequency k_0 are irrelevant. The value of k_0 is to be determined only from the sort of problem and calculation involved, not by the physical specification of the system. While the use of an ensemble presupposes that our knowledge of the microscopic state of the system is incomplete, the use of Robinson's truncation procedure implies that we do not wish to make use of this knowledge even though it might be available to us [75,76]. Because in our view Robinson's definition of macroscopic average is the most sensible definition so far, we adopt it in this article.

As there can be no atoms of the medium at the positions occupied by the cavity walls, i.e., at $x=0$ and at $x=L$, we can express the Dirac δ functions in Eq. (2) as a Fourier sine series inside the cavity:

$$\delta(x-x_j)=\frac{2}{L}\sum_{n=1}^{\infty}\sin\left(\frac{n\pi}{L}x_j\right)\sin\left(\frac{n\pi}{L}x\right). \tag{59}$$

In our model, involving only a single mode of the field, Fourier components with spatial frequencies above ω/c are irrelevant in a macroscopic description. So following Robinson [75,76], we take as the macroscopically averaged variables those where such Fourier components have been discarded. Then from Eqs. (2), (3), and (59), we can write the following expression for the macroscopic polarization:

$$\overline{P(x)}=\frac{1}{L}\sqrt{\frac{2\hbar}{\omega_0}}\sin\left(\frac{\omega}{c}x\right)\sum_{j=1}^{N}(\hat{b}_j+\hat{b}_j^{\dagger})q_j\sin\left(\frac{\omega}{c}x_j\right). \tag{60}$$

Using the definitions of g_j, Eq. (10), $\hat{B}$, Eq. (35), G, Eq. (36), and G', Eq. (49), we obtain

$$\overline{P(x)}=-\frac{2G'}{\sqrt{\omega_0\omega}}\varepsilon_0\sqrt{\frac{\hbar\omega}{\varepsilon_0 L}}(\hat{B}^{\dagger}+\hat{B})\sin\left(\frac{\omega}{c}x\right), \tag{61}$$

and the role played by the collective operator $\hat{B}_1$ of Sec. II now becomes explicit. Then, the macroscopic electric field operator (58) is given by

$$\overline{E(x)}=\sqrt{\frac{\hbar\omega}{\varepsilon_0 L}}\left\{\hat{a}^{\dagger}+\hat{a}+\frac{2G'}{\sqrt{\omega_0\omega}}(\hat{B}^{\dagger}+\hat{B})\right\}\sin\left(\frac{\omega}{c}x\right). \tag{62}$$

Now we can calculate the variance in the ground state of the term enclosed with curly brackets in Eq. (62):

$$\left\langle\left\{\Delta\left[\hat{a}^{\dagger}+\hat{a}+\frac{2G'}{\sqrt{\omega_0\omega}}(\hat{B}^{\dagger}+\hat{B})\right]\right\}^2\right\rangle$$
$$=\langle(\hat{a}^{\dagger}+\hat{a})^2\rangle+\frac{2G'}{\sqrt{\omega_0\omega}}\langle(\hat{B}^{\dagger}+\hat{B})(\hat{a}^{\dagger}+\hat{a})$$
$$+(\hat{a}^{\dagger}+\hat{a})(\hat{B}^{\dagger}+\hat{B})\rangle+\frac{4G'^2}{\omega_0\omega}\langle[\Delta(\hat{B}^{\dagger}+\hat{B})]^2\rangle. \tag{63}$$

If we write $\hat{a}$, $\hat{a}^{\dagger}$, $\hat{B}$, and $\hat{B}^{\dagger}$ in terms of $\hat{c}_1$, $\hat{c}_1^{\dagger}$, $\hat{c}_2$, and $\hat{c}_2^{\dagger}$ using Eqs. (50) and (51), we find that

$$\langle(\hat{B}^{\dagger}+\hat{B})(\hat{a}^{\dagger}+\hat{a})+(\hat{a}^{\dagger}+\hat{a})(\hat{B}^{\dagger}+\hat{B})\rangle$$
$$=\sqrt{\frac{\omega_0}{\omega}}\sum_{k=1}^{2}\frac{4\omega^2G'^2(\Omega_k^2-\omega^2)}{[(\Omega_k^2-\omega^2)^2+4\omega^2G'^2]\Omega_k} \tag{64}$$

and

$$\langle[\Delta(\hat{B}^{\dagger}+\hat{B})]^2\rangle=\sum_{k=1}^{2}\frac{\omega_0}{\Omega_k}\frac{(\Omega_k^2-\omega^2)^2}{(\Omega_k^2-\omega^2)^2+4\omega^2G'^2}. \tag{65}$$

Then up to second order in G'/ω_0, we obtain

$$\left\langle\left\{\Delta\left[\hat{a}^{\dagger}+\hat{a}+\frac{2G'}{\sqrt{\omega_0\omega}}(\hat{B}^{\dagger}+\hat{B})\right]\right\}^2\right\rangle\approx 1-6\frac{G'^2}{\omega_0^2}+4\frac{G'^2}{\omega_0\omega}. \tag{66}$$

The first two terms on the right-hand side of Eq. (66) yield the $\varepsilon_r^{-3/2}$ variance obtained in Glauber and Lewenstein's macroscopic theory [57]. The last term on the right-hand side of Eq. (66), however, represents some extra noise that comes from the atoms of the medium. This noise is larger than the medium correction $4G'^2/\omega_0$ to the relative permittivity and agrees with Rosewarne's result [35].

Thus we have shown that the macroscopic and the microscopic fields exhibit different variances. The variance of the macroscopic electric field, as Rosewarne demonstrated [35], derives a large contribution from the atoms of the medium. This seems to suggest that any macroscopic theory will give wrong results for the quantum fluctuations of its variables. In the next section, however, we show that we can still construct a macroscopic theory of QED in a material medium that is free from the problems discussed here.

V. RECOVERING MACROSCOPIC DESCRIPTIONS

We have been concerned so far only with the atoms of the medium and the cavity field. This is the material medium analogue of the field in the vacuum in the absence of sources. Most situations of interest, however, involve guest atoms ''immersed'' in the medium, the main difference between an ''immersed'' atom and an atom of the medium being that the transition frequency of the former is often closer to the field frequency than that of the latter. The ''immersed'' atoms are usually also coupled more strongly to the field than the atoms of the medium. In such situations, the only relevant fluctuations are those that the ''immersed'' atoms can experience. So we should extend our criterion of what is irrelevant to eliminate, in the process of taking the macroscopic average, the part of the microscopic field that has a negligible effect on the ''immersed'' guest atom. In this section we show that when the extended criterion is adopted, the macroscopic description obtained will be a good approximation to the microscopic theory if the transition frequency of the guest atom is far from the resonance of the medium. Then we derive expressions for the macroscopic fields that agree with those found by Milonni [58].

The large atomic contribution to the variance of the macroscopic electric field that we have calculated in the previous section comes from the variance of its $\hat{c}_2$ and $\hat{c}_2^{\dagger}$ components. If, however, the resonance frequency of the guest atom ω_a is very far from Ω_2, the guest atom will not be affected

much by this polariton mode. How far from Ω_2 the resonance frequency ω_a has to be, for the effect of this polariton mode on the guest atom to be negligible, depends on the coupling between them. An analysis of the probability of this mode inducing transitions of the guest atom shows that such probability is negligible when

$$|u_2^2\Omega| \ll |\Omega_2 - \omega_a|. \tag{67}$$

This is analogous to the condition for neglecting the counter-rotating terms in the Jaynes-Cummings model (JCM) [67,68]. As in the JCM case the nonresonant terms can induce a frequency shift in the guest atom similar to the Bloch-Siegert shift [83], but far from Ω_2 such a shift is negligible. We also notice that condition (67) assumes a simpler form when the coupling between the atoms of the medium and the field is weak, i.e., $G' \ll \omega_0, \omega$. Then if ω_a is close to the field frequency ω, condition (67) becomes $|\omega_0 - \omega| \gg |\Omega|$; i.e., the detuning between the frequency of the field and the resonance of the medium must be much larger than the Rabi frequency of the guest atom.

In the regime described by Eq. (67), we extend our definition of macroscopic average to both a truncation in the mode expansion of the fields, leaving out the polariton mode described by $\hat{c}_2$ and $\hat{c}_2^\dagger$ and the truncation of the Fourier series at the spatial frequency ω/c. Then from the microscopic Hamiltonian (52), we obtain the macroscopic Hamiltonian

$$\hat{H}_{\rm mac} = \hbar\Omega_1 \hat{c}_1^\dagger \hat{c}_1 + \frac{\hbar\omega_a}{2}\hat{\sigma}_z + \hbar\Omega u_1^1 (\hat{c}_1 + \hat{c}_1^\dagger)(\hat{\sigma} + \hat{\sigma}^\dagger). \tag{68}$$

From Eqs. (8), (43), and (46), we find that we can rewrite the macroscopic Hamiltonian (68) as

$$\hat{H}_{\rm mac} = \hbar\Omega_1 \hat{c}_1^\dagger \hat{c}_1 + \frac{\hbar\omega_a}{2}\hat{\sigma}_z - \frac{d}{\varepsilon_0} D_{\rm mac}(x_a)(\hat{\sigma} + \hat{\sigma}^\dagger), \tag{69}$$

where x_a is the position of the guest atom, d is its electric dipole strength, and

$$D_{\rm mac}(x) = \left[\frac{\hbar\Omega_1 \varepsilon_0 \varepsilon_r \sqrt{\varepsilon_r}}{L\gamma}\right]^{1/2} (\hat{c}_1 + \hat{c}_1^\dagger) \sin\left(\sqrt{\varepsilon_r}\frac{\Omega_1}{c}x\right) \tag{70}$$

is the macroscopic displacement field. The relative permittivity ε_r appearing in Eq. (70) is derived from Eq. (47) and found to be

$$\varepsilon_r = 1 + \frac{4G'^2}{\omega_0'^2 - \Omega_1^2}, \tag{71}$$

with G'^2 clearly appearing as the effective oscillator strength of the medium, justifying the physical meaning anticipated for G_N in Sec. II, and

$$\omega_0'^2 = \omega_0^2 - 4G'^2 \tag{72}$$

being the correction on the resonance frequency of the medium due to the interaction with the field [84]. The parameter γ in Eq. (70) is given by

$$\gamma = \left(\frac{1}{v_1^1}\right)^2. \tag{73}$$

Using Eqs. (46) and (71), we can express γ in terms of Ω_1 and ε_r as

$$\gamma = \frac{d}{d\Omega_1}(\Omega_1\sqrt{\varepsilon_r}), \tag{74}$$

which can be identified as the ratio between the speed of light in vacuum and the group velocity in the medium.

The macroscopic electric field is obtained from Eq. (58) with $D_{\rm mac}$ substituted for D and with $\overline{P}$, Eq. (61), without its $\hat{c}_2$, $\hat{c}_2^\dagger$ polariton component, substituted for P. From Eqs. (44) and (71), we find

$$\frac{2G'}{\sqrt{\omega_0\omega}}u_2^1 = \frac{1-\varepsilon_r}{\sqrt{\varepsilon_r}}v_1^1. \tag{75}$$

Then from Eqs. (58), (61), (70), (73), and (75), we obtain

$$\begin{aligned} E_{\rm mac}(x) &= \left(\sqrt{\varepsilon_r} + \frac{1-\varepsilon_r}{\sqrt{\varepsilon_r}}\right)\left(\frac{\hbar\Omega_1\varepsilon_r\sqrt{\varepsilon_r}}{\varepsilon_0 L\gamma}\right)^{1/2} \\ &\quad \times (\hat{c}_1 + \hat{c}_1^\dagger)\sin\left(\sqrt{\varepsilon_r}\frac{\Omega_1}{c}x\right) \\ &= \left(\frac{\hbar\Omega_1}{\varepsilon_0\sqrt{\varepsilon_r}L\gamma}\right)^{1/2}(\hat{c}_1 + \hat{c}_1^\dagger)\sin\left(\sqrt{\varepsilon_r}\frac{\Omega_1}{c}x\right). \end{aligned} \tag{76}$$

We notice that our expressions for the macroscopic fields coincide with those derived by Milonni [58] if his results are particularized for the case of a single mode. This is interesting because Milonni adopted a completely different approach in his derivations of these expressions for the macroscopic fields. He started from the macroscopic Maxwell equations and showed that given a narrow range of frequencies where absorption is negligible, it is possible to define a Hamiltonian that is local in time, enabling him to quantize the macroscopic fields in the usual way (as for the vacuum) within this frequency range.

VI. CONCLUSION

We have adopted a simple microscopic model for the interaction between an atom and radiation in a linear lossless material medium: a guest two-level atom inside a single-mode cavity filled with a host medium composed of other two-level atoms that are approximated by harmonic oscillators. The guest atom works as a probe for the field, giving us a definite criterium for deciding what is field and what is matter inside the material medium; namely, field is what affects the guest atom. Although the presence of the guest atom proved to be a very convenient way of introducing a probe for the field, in principle any other probe, such as a photodetector, would do equally well.

We have shown that when the transition frequency of the guest atom is far from the frequency of one of the polariton modes, we can obtain an approximate macroscopic description for this model where the medium appears only through a

dielectric constant. Hopfield had already mentioned in 1958 that the medium could be treated as a classical dielectric in this regime [62] but, to the best of our knowledge, it is the first time that such a result is explicitly derived from a microscopic theory by taking macroscopic averages. We notice that the frequency discrimination carried out by the probe (guest atom) is crucial here. If the probe could sense the frequency of the other polariton mode, it would be affected by the excess fluctuations arising from the medium and the macroscopic approach would break down. However, in most of the situations of physical interest, the system that interacts with the field is sensitive only to a narrow range of frequencies as the probe (guest atom) in our model, allowing then the use of a macroscopic theory.

Our method has helped to clarify some questions about the validity of macroscopic approaches. In particular we have shown that, contrary to Rosewarne's suggestion [35], quantum fluctuations are properly described in the regime mentioned above. Moreover, although a single-mode cavity cannot allow for dispersive propagation of wave packets, we have been able to preserve the frequency dependence of the relative permittivity in our macroscopic approximation, recovering Milonni's results [58]. This seems to suggest that a macroscopic description of QED in material media can include dispersion as long as the frequencies involved are far from those of the other polariton branch. We are currently working on an extension of this method to free space where wave packet propagation under the influence of a frequency-dependent dielectric permittivity would be possible.

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