Low-frequency external force acting on an atom in a resonant field

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The probe-wave absorption spectrum calculation is presented for the case when a two-level atom or ion in the resonant field of laser radiation is subjected to low-frequency external action in such a way that parametric resonance takes place and the fine structure appears on the probe-wave absorption coefficient plot. [S1050-2947(97)07204-1]

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I. INTRODUCTION

The two-level atom (TLA) or ion is a very convenient and widely used model to investigate the behavior of a real atom or ion in different situations, a problem which is of essential interest and has a lot of applications [1–10].

Here we concentrate upon a TLA in a strong monochromatic resonant laser field. In this case an additional frequency, characterizing the ''TLA-field'' system, appears, Rabi frequency, and this means that we can think of a parametric resonance (PR) possibility.

According to [11], the dynamic system reveals PRs when it suffers small perturbative action at a frequency equal to the difference between eigenfrequencies of the system. Physically it means that we can pump energy in through one channel and observe the result through the other. Mathematically [11], the PRs can occur when the external action reveals itself through time-dependant coefficients of the dynamics equation. Spectroscopically, we speak of the PR [12] in case the quasienergies of the atom, placed into the laser field, overlap. In [13,14] it was shown that the meanings of these definitions are one and the same and a small perturbation can lead to the essential deformation of absorption spectrum, thus providing the possibility to use the PR effects in sub-Doppler spectroscopy.

There are some additional reasons (clarifying the physical background of the effect) to investigate the TLA-field system in search for the parametric resonances. In the recent work [15] the expression for the mechanical force exerted on a two-level atom by a bichromatic laser field was found. One of the field modes Ω_1 (resonant to the atomic transition) was considered to be strong, while the other mode with the frequency Ω_2 sufficing $|\Omega_1-\Omega_2|=2\Omega_{\text{Rabi}}$ was considered to be weak. In this situation the PR took place. It appeared that besides the well known [16] constant component of the light pressure force F_c equal to

$$F_c=\hbar k\gamma\zeta(v,\Omega_1,A_1),$$

where $\hbar$ is Planck's constant, k is the wave vector, γ is the excited-state decay rate, $\zeta=O(1)$ is the combination of parameters (v is the atom's velocity, A_1 is the Rabi parameter), there was also the oscillating component

$$F_{\text{osc}}=\hbar kA_1\{H\cos[(\Omega_2-\Omega_1)t+\Psi]+O(\varepsilon)\},$$

$$\varepsilon=\gamma/(2^{1/2}A_1)\ll 1,$$

where Ψ is the phase, depending on the velocity, and $H=O(1)$ is the combination of problem parameters. The value of F_c appears to be only of the next (smaller) order in the asymptotic expansion of force in the powers of ε. Thus, if the conditions of PR are fulfilled, the TLA has not only an ''average'' acceleration in the wave-vector direction, but suffers an intensive ''shaking'' (oscillations) along this vector with the doubled Rabi frequency.

At the same time, in case of such a PR in the bichromatic field [13,14], there is a fine structure on the probe-wave absorption spectrum. By comparing the results of [15] and [13,14], we can suggest that the sidebands of the fine structure may be due to the Doppler effect, caused by the oscillations.

Turning back again to the monochromatic field, we can suggest that if we make the TLA oscillate with the help of an external force of any nature, and put it into the monochromatic laser field that is resonant to the atomic transition, the PR is possible, and it will lead to the appearance of the fine structure on the probe-wave absorption plot.

This gives us a version of a well-known Brillouin scattering, which is a powerful instrument in the investigation of the atom's structure and dynamics. It is important to notice that since Rabi frequency depends on the intensity of the laser beam, we have a possibility to vary in the experiment the position of the PR sidebands and place them into convenient frequency regions.

When a two-level ion is subjected simultaneously to the action of the resonant laser field and the low-frequency electric field [17], the effect of resonant light pressure is the heating or the cooling of the localized particles ensemble. The mechanical action of running and standing resonant waves on the moving atom, reviewed in [17,18], presents a situation that is in some sense the reverse of that which is going to be discussed here. The details of such a cooling process can be analyzed with the help of the proposed method of optic-mechanical parametric resonance.

In [19] the dynamic Stark effect was discussed. It was discovered that it resulted in the shifting of atomic energy levels and this led to new resonances in the nonlinear optical susceptibility describing probe-wave absorption. The comparison of these results with the experiment is presented in [20]. The influence of ion scattering on the light-induced electrostatic fluctuations upon the ion velocities was discussed in [21]. It was found that the noise spectrum contains the information about the nonlinear optical absorption. The resonance fluorescence due to the interference of quasienergy states being a result of dynamic Stark effect was discussed in [22]. All these processes can have profound implications to the operation of devices that rely on the resonant nonlinear optical response of atomic systems. But the possibility of the PR which appears while regarding an ion instead of an atom and correlating the frequency of the external action with Rabi frequency was not analyzed. The above-mentioned possibility to perform measurements in the convenient frequency region provides the increase of the instrumental sensibility.

Another experimental field for the proposed method is a modification of the Brillouin scattering. In [23,24] the resonances in the interaction of a saturated two-level system with stimulated Brillouin scattering were discussed. These resonances result from the coupling between the two-level system and the resonant acoustic waves that are excited in the host substance by the counterpropagating laser beams. The proposed method suggests the use of the external acoustic field, thus providing two parameters to be controlled externally: acoustic frequency and laser intensity, which is a new possibility to investigate the TLAs, admixed to the buffer gas.

One of the most effective experimental methods to investigate the features of atom's structure is the use of the counterpropagating probe wave [25,26], and commonly used setups for it are described in [25–28]. In this paper the calculation of the probe-wave absorption spectrum for the case of the PR is presented.

The material is organized in the following way. In Sec. II the initial Bloch equations are formulated with respect to the atom's low-frequency oscillations. Then the rotating-wave approximation is used, and after variable changing we get Eq. (4), which can be solved through the use of a perturbative technique. Using the conditions of parametric resonance, we get the result in the form of expressions (7) and (8) and discuss the origin and relative importance of its different terms. In Sec. III we discuss the meaning of the assumptions made and the possibilities to suffice the corresponding conditions in the experiment. In Sec. IV the conclusions are made and the acknowledgments are presented. In the Appendix the mathematical details of the solution of Eq. (4) are given.

II. THE PROBLEM AND ITS SOLUTION

Let us describe the laser field classically, and use the density matrix $\rho(v,z)$ for the atom's dynamics description. Then the Bloch equations will be

$$\frac{d}{dt}(\rho_{22}-\rho_{11}) = -\gamma(\rho_{22}-\rho_{11}) + 4i[A_1\cos(\Omega_1 t - k_1 z) + A_p\cos(\Omega_p t + k_p z)](\rho_{21}-\rho_{12}) + \Lambda, \tag{1}$$

$$\frac{d}{dt}\rho_{12} = -(\gamma_{12}+i\omega)\rho_{12} - 2i[A_1\cos(\Omega_1 t - k_1 z) + A_p\cos(\Omega_p t + k_p z)](\rho_{22}-\rho_{11}),$$

Here, k_1 and k_p stand for the wave vectors of the counterpropagating resonant and probe waves; A_1 and A_p are the amplitudes of these waves in frequency units; ρ_{22} and ρ_{11} are the levels populations; γ_{12} is the transversal decay rate; Λ is the saturation parameter. We will regard the situation when due to some external periodical action the atom's velocity v oscillates with low frequency D and amplitude V_1. Then the full time derivative can be written as

$$\frac{d}{dt} = \frac{\partial}{\partial \tau} + V\frac{\partial}{\partial z}, \quad V = v + V_1 \cos Dt.$$

Let the spatial scale, characterizing the atom-field interaction, be of the order of 10^{-2} m. Then we can assume $k_1 = k_p = k$. In case the amplitude V_1 of velocity alteration is small in comparison with v $(V_1 \ll v)$, the quasiclassical approach to the description of the atom's motion remains valid. Since we are interested in space-averaged values, it is natural to use the rotating-wave approximation [25]. Then, making the substitutions

$$R = 2^{-1/2}(\rho_{22}-\rho_{11}),$$

$$\rho_{12} = R_{12}^{+}\exp[-i(\Omega_1 t - kz)] + R_{12}^{-}\exp[-i(\Omega_p t + kz)],$$

$$\rho_{21} = R_{21}^{+}\exp[i(\Omega_1 t - kz)] + R_{21}^{-}\exp[i(\Omega_p t + kz)],$$

and changing variables to

$$2^{1/2}A_1 t = \tau, \quad \gamma/A_1 = 2^{1/2}\varepsilon,$$

$$k/A_1 = 2^{1/2}\kappa, \quad (\Omega_p-\omega)/A_1 = 2^{1/2}\Delta_p,$$

$$V_1 = v_1\varepsilon[v_1 = 0(1)], \quad Dt = \delta\tau(2^{1/2}\delta = D/A_1),$$

$$A_p/A_1 = a_p, \quad \gamma_{12}/A_1 = 2^{1/2}\Gamma\varepsilon,$$

$$(\Omega_1-\omega)/A_1 = 2^{1/2}\Delta, \quad \Lambda/A_1 = 2^{1/2}\lambda,$$

we come to the following system of equations:

$$\frac{\partial}{\partial t}R = -\varepsilon R + i[R_{21}^{+} - R_{12}^{+} - a_p(R_{21}^{-}+R_{12}^{-})] + \lambda,$$

$$\frac{\partial}{\partial t}R_{12}^{+} = -iR + [-\Gamma\varepsilon + i(\Delta-\kappa V)]R_{12}^{+},$$

$$\frac{\partial}{\partial t}R_{21}^{+} = iR + [-\Gamma\varepsilon - i(\Delta-\kappa V)]R_{21}^{+}, \tag{2}$$

$$\frac{\partial}{\partial t}R_{12}^{-} = -ia_pR + [-\Gamma\varepsilon + i(\Delta_p+\kappa V)]R_{12}^{-},$$

$$R_{21}^{-} = \overline{R_{12}^{-}}.$$

To find the probe-wave absorption coefficient

$$\chi(\Omega_p)=\int \exp(-V^2/u_T{}^2)\mathrm{Im}R_{21}^-(V)dV, \tag{3}$$

where u_T is the atom's heat velocity, we have to solve the last equation of system (2). Let's consider $\varepsilon \ll 1$, $\Gamma=0(1)$, and let the probe-wave amplitude a_p be so small that it does not affect the atom's dynamics. Then the last differential equation in system (2) can be regarded separately, and the expression for function R, which is present in this equation, can be taken as a result of solving the system, consisting of the first three equations in Eq. (2). So, at first, we have to solve the system

$$\frac{\partial}{\partial\tau}\mathbf{W}=(\mathbf{Q}_0+\varepsilon\mathbf{Q}_1(\tau))\mathbf{W}+\mathbf{C}, \tag{4}$$

where

$$\mathbf{W}=\begin{pmatrix} R \\ R_{12}^+ \\ R_{12}^- \end{pmatrix},\quad \mathbf{Q}_0=i\begin{pmatrix} 0, & -1, & 1 \\ -1, & \sigma, & 0 \\ 1, & 0, & -\sigma \end{pmatrix},\quad \mathbf{C}=\begin{pmatrix} \lambda \\ 0 \\ 0 \end{pmatrix},$$

$$\mathbf{Q}_1=\begin{pmatrix} -1 & 0 & 0 \\ 0 & -\Gamma-i\kappa v_1\cos\delta\tau & 0 \\ 0 & 0 & -\Gamma+i\kappa v_1\cos\delta\tau \end{pmatrix},$$

where we have used $\sigma=\Delta-\kappa v$. Our goal is to find the leading term of the asymptotic expansion in the powers of ε of the stable solution of system (4). The mathematical details of the solution are presented in the Appendix. Here we will only emphasize that the PR condition

$$J=\delta+\varepsilon\nu,\quad \nu=0(1), \tag{5}$$

where $J=(\sigma^2+2)^{1/2}$, is essential. Equation (5) characterizes the velocity of groups of atoms which suffice the PR conditions.

The asymptotical solution of system (4) for the first component of vector $\mathbf{W}$ is

$$R=T(v)+S(v), \tag{6}$$

$$T(v)=\lambda\sigma^2(B^2+\nu^2)J^5/\{\varepsilon[(\sigma^2+2\Gamma)(B^2+\nu^2)+2\kappa^2v_1^2B]\},$$

$$S(v)=-2\lambda\sigma\kappa v_1 J^5(\cos\delta\tau+B\sin\delta\tau)/\{\varepsilon[(\sigma^2+2\Gamma)\times(B^2+\nu^2)+2\kappa^2v_1^2B]\},$$

$$B=1+\Gamma(1+\sigma^2).$$

We can see that if there is no low-frequency action ($v_1=0$), the leading term in function R expansion (6) does not contain an oscillating component, and its form coincides (with regard to notation) with the known expression [25].

To calculate the probe-wave absorption coefficient, substitute Eq. (6) into the fourth equation of system (2). The expression for $\mathrm{Im}R_{21}^-$ is

$$\begin{aligned}\mathrm{Im}R_{21}^-=-\frac{a_p\lambda}{\mathbb{K}J^6}\Bigg\{&\frac{\Gamma\sigma^2(B^2+\nu^2)}{\Gamma^2\varepsilon^2+\sigma_p^2}+\frac{\kappa v_1\sigma}{2\varepsilon}\\&\times\Bigg[\left(\frac{-\Gamma\varepsilon\nu+B(\sigma_p+\delta)}{\Gamma^2\varepsilon^2+(\sigma_p+\delta)^2}+\frac{-\Gamma\varepsilon\nu-B(\sigma_p-\delta)}{\Gamma^2\varepsilon^2+(\sigma_p-\delta)^2}\right)\\&\times\cos\delta\tau+\left(\frac{-\Gamma\varepsilon B-\nu(\sigma_p+\delta)}{\Gamma^2\varepsilon^2+(\sigma_p+\delta)^2}\right.\\&\left.+\frac{-\Gamma\varepsilon B+\nu(\sigma_p-\delta)}{\Gamma^2\varepsilon^2+(\sigma_p-\delta)^2}\right)\sin\delta\tau\Bigg]\Bigg\},\end{aligned} \tag{7}$$

where

$$\sigma_p=\Delta_p+\kappa v,$$

$$\mathbb{K}=\det(K)=(-J^{-2})^3[(\sigma^2+2\Gamma)(B^2+\nu^2)+2\kappa^2v_1^2B].$$

Let us discuss the origin and importance of the terms in Eq. (7). The first term in large curly brackets corresponds to the Lorentz contour in the vicinity of the atomic transition frequency. Its expression coincides with the known one and provides the known form for the regular case of scanning the atom in resonant field by the probe wave.

The second term in large curly brackets is an oscillating one. We can see that its magnitude is of the next (higher) order, due to ε in the denominator. This term appears only in case the conditions of the parametric resonance are fulfilled. Thus, we can conclude that the existence of the PR (resulting in the appearance of the oscillating terms in the expression for R) really leads to the appearance of the sharp fine structure on the $\chi(\Omega_p)$ plot, which can be calculated with the help of Eqs. (3) and (7). The fine-structure sidebands have the amplitudes essentially larger than the peak corresponding to the resonance frequency ω, and are located symmetrically about ω at the distances equal to the external force frequency D.

Substituting Eq. (7) into Eq. (3), we finally get the following expression for the probe-wave absorption coefficient dependence on frequency:

$$\chi(\Omega_p)=-\int \exp(-V^2/u_T{}^2)\,\frac{a_p\lambda}{\mathbb{K}J^6}\left[\frac{\Gamma\sigma^2(B^2+\nu^2)}{\Gamma^2\varepsilon^2+\sigma_p^2}+\frac{A\sigma\kappa v_1}{2\varepsilon}\cos(\delta\tau-\phi)\right]dV,$$

$$A=\frac{2(B^2+\nu^2)^{1/2}[\sigma_p^4\delta^2+\sigma_p^2(2\Gamma^4\varepsilon^4+\Gamma^2\varepsilon^2-2\delta^4)+(\Gamma^2\varepsilon^2+\delta^2)^3]^{1/2}}{\sigma_p^4+2\sigma_p^2(\Gamma^2\varepsilon^2-\delta^2)+(\Gamma^2\varepsilon^2+\delta^2)^2} \tag{8}$$

$$\phi = \text{arctang}\ \frac{\sigma_p^2(\Gamma\varepsilon B - \nu\delta) + (\Gamma^2\varepsilon^2 + \delta^2)(\Gamma\varepsilon B + \nu\delta)}{\sigma_p^2(\Gamma\varepsilon\nu + B\delta) + (\Gamma^2\varepsilon^2 + \delta^2)(\Gamma\varepsilon\nu - B\delta)}.$$

III. DISCUSSION

To compare the results of the calculation with the experiment, we have to formulate the conditions for the physical system and the experimental setup. While solving the problem we used the following assumptions.

(i) $A_p \ll A_1$. This condition can be easily fulfilled in experiment

(ii) $\gamma/(2^{1/2}A_1) = \varepsilon \ll 1$. This is a limitation on the laser intensity. The excited-state decay rate has to be essentially less than the Rabi parameter A_1 ($Ed = \hbar A_1$, where d stands for the dipole moment and E is the amplitude of laser radiation).

(iii) $V_1 \ll v$. This condition means that the atom's velocity change during a quarter of period of the low-frequency force action is small in comparison with atoms heat velocity ($v \sim 10^3$ m/s for room temperatures). So, the amplitude of the low-frequency external force, acting on an atom or an ion, should be small. If we speak of an ion placed into the low-frequency electric field, then this field should be low intensive. If the TLAs are the admixture in the buffer gas, the acoustic field should be low intensive. In both cases condition (iii) is easily controlled. It should be noted that we considered V_1 and ε to be of the same order. If $V_1 \ll \varepsilon$, then the $\mathbf{Q}_1$ matrix in Eq. (4) degenerates and the oscillating component in the absorption coefficient vanishes. The effect of the external action in this case might have been a certain increase in the absorption coefficient value due to the larger number of atoms, taking part in the formation of the Doppler contour, because of their velocity oscillations. But as will be seen from item (iv), the scattering of velocities of atoms, taking part in the PR, is of the order of $\varepsilon^{1/2}$, i.e., the contribution of oscillating motions with amplitudes $V_1 \ll \varepsilon$ will be negligible.

(iv) $J = \delta + \varepsilon\nu$ [$\nu = O(1)$]. This is the condition of the PR, i.e., this combination of parameters provides the nontrivial form of the probe-wave absorption spectrum. Since $J = (\sigma^2 + 2)^{1/2}$, we obtain

$$\Delta - \kappa v = \pm[(\delta + \varepsilon\nu)^2 - 2]^{1/2}.$$

Substituting the initial variables, we see that the PR is possible only if $D \geqslant 2A_1(1 - 2^{-1/2}\varepsilon\nu)$. Then, if $2^{1/2}A_1 - |D| = \varepsilon\eta$, $\eta = 0(1)$ (η can be negative as well), the atoms taking part in the PR have the velocities

$$v = \frac{\Omega_1 - \omega}{k} \pm 2^{3/4}\varepsilon^{1/2}(\nu - \eta)^{1/2}\kappa^{-1}.$$

The velocity scattering is of the order of $\varepsilon^{1/2}$ in κ^{-1} units, $\kappa^{-1} = (A_1 c 2^{1/2})/\omega$, and c is the light velocity. The condition under discussion means that we have to control the relation between the frequency of the external action and Rabi frequency. This is true both for ions in the electric field and for TLAs admixed to buffer gas in the acoustic field.

IV. CONCLUSION

The result of this work is the calculation of the probe-wave absorption spectrum for the case when the two-level atom or ion in the resonant field suffers the low-frequency action of an external force. It is shown that when the parametric resonance conditions are fulfilled, the absorption coefficient obtains the oscillating component with the amplitude, which is essentially larger than the constant component. This entails the appearance of the fine structure in the probe-wave absorption spectrum, its location being dependent on the external action frequency. To reach the best sensitivity we can regulate the location of the sidebands by the simultaneous varying of the laser beam intensity and external action frequency.

In the experiment this result can be observed in different ways. One of them is the low-frequency electric action on the two-level ion. The other is a particular type of Brillouin scattering when the TLAs are admixed to the buffer gas and the acoustic action is produced. The above analysis of the demands for the experimental situation shows the possibility of using the proposed method of optic-mechanical parametric resonance for the investigation of the atom's dynamics in various applications.

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APPENDIX

We will start solving Eq. (4) by solving the corresponding homogeneous equation

$$\frac{\partial}{\partial\tau}\mathbf{N}(\tau) = (\mathbf{Q}_0 + \varepsilon\mathbf{Q}_1(\tau))\mathbf{N}(\tau), \quad \mathbf{N}(0) = \mathbf{I}, \tag{A1}$$

where $\mathbf{I}$ is the unity matrix. Let us construct the matrices $\mathbf{E} = \text{diag}\{0, iJ, -iJ\}$ and $\mathbf{U}$,

$$\mathbf{U} = \frac{1}{J}\begin{pmatrix} \sigma & 1 & 1 \\ 1 & 1/(\sigma - J) & 1/(J + \sigma) \\ 1 & 1/(J + \sigma) & 1/(J - \sigma) \end{pmatrix},$$

with the help of matrix $\mathbf{Q}_0$ eigenvalues, which are equal to 0 and $\pm J = \pm(\sigma^2 + 2)^{1/2}$. Introducing the matrix $\mathbf{L}(\tau) = \exp\{-\mathbf{E}\tau\}\mathbf{U}^{-1}\mathbf{N}(\tau)$ we transform Eq. (4) into

$$\frac{\partial}{\partial\tau}\mathbf{L}(\tau) = \varepsilon\mathbf{G}(\tau)\mathbf{L}(\tau),$$
$$\mathbf{G}(\tau) = \exp(-\mathbf{E}\tau)\mathbf{U}^{-1}\mathbf{Q}_1(\tau)\mathbf{U}\ \exp(\mathbf{E}\tau). \tag{A2}$$

To solve system (A2) we will use the many-scales method [11]. We introduce the slow time $\tau_1 = \varepsilon\tau$, transform Eq. (A2) into

$$\left[\frac{\partial}{\partial\tau}+\varepsilon\frac{\partial}{\partial\tau_1}\right]\mathbf{L}(\tau,\tau_1)=\varepsilon\mathbf{G}(\tau)\mathbf{L}(\tau,\tau_1), \tag{A3}$$

and search for its solution in the form

$$\mathbf{L}(\tau,\tau_1)=\mathbf{L}_0(\tau,\tau_1)+\varepsilon\mathbf{L}_1(\tau,\tau_1)+\cdots. \tag{A4}$$

The direct substitution gives $\mathbf{L}_0(\tau,\tau_1)=\mathbf{L}_0(\tau_1)$, and we obtain the equation for $\mathbf{L}_0(\tau_1)$, demanding that there are no secular terms on the right-hand side of Eq. (A8). Here it means that we should expel all the $\mathbf{G}(\tau)$ harmonics with frequencies of $O(1)$ out of the right-hand side of Eq. (A3). Then the equation for $\mathbf{L}_0(\tau_1)$ takes the form

$$\frac{\partial}{\partial\tau_1}\mathbf{L}_0(\tau_1)=\langle\mathbf{G}(\tau)\rangle\mathbf{L}_0(\tau_1), \tag{A5}$$

where $\langle\,\rangle$ means that there are no above-mentioned harmonics in the $\mathbf{G}(\tau)$ matrix. We regard the parametric resonance which takes place when the following condition is fulfilled:

$$J=\delta+\varepsilon\nu,\quad \nu=0(1). \tag{A6}$$

Substituting Eq. (A6) into Eq. (A5), we get

$$\frac{\partial}{\partial\tau_1}\mathbf{L}_0(\tau_1)=\langle\mathbf{G}(\tau_1)\rangle\mathbf{L}_0(\tau_1),$$
$$\langle\mathbf{G}(\tau_1)\rangle=-J^{-2}\begin{pmatrix}\sigma^2+2\Gamma & -i\kappa v_1\exp(i\nu\tau_1) & i\kappa v_1\exp(-i\nu\tau_1)\\ -i\kappa v_1\exp(-i\nu\tau_1) & 1+\Gamma(1+\sigma^2) & 0\\ i\kappa v_1\exp(i\nu\tau_1) & 0 & 1+\Gamma(1+\sigma^2)\end{pmatrix}. \tag{A7}$$

The system (A7) can be transformed into the system with constant coefficients. We take matrix $\mathbf{Z}(\tau_1)=\mathrm{diag}\{1,\exp(-i\nu\tau_1),\exp(i\nu\tau_1)\}$ and designate $\mathbf{L}_0(\tau_1)=\mathbf{Z}(\tau_1)\mathbf{S}(\tau_1)$. Then system (A7) is as follows:

$$\frac{\partial}{\partial\tau_1}\mathbf{S}(\tau_1)=\mathbf{K}\mathbf{S}(\tau_1),\quad \mathbf{K}=-J^{-2}\begin{pmatrix}\sigma^2+2\Gamma & -i\kappa v_1 & i\kappa v_1\\ -i\kappa v_1 & 1+\Gamma(1+\sigma^2)-i\nu & 0\\ i\kappa v_1 & 0 & 1+\Gamma(1+\sigma^2)+i\nu\end{pmatrix}. \tag{A8}$$

Solving Eq. (A8) we obtain

$$\mathbf{L}_0(\tau_1)=\mathbf{Z}(\tau_1)\exp(\mathbf{K}\tau_1),$$

and thus the homogeneous equation (A1) has the solution

$$\mathbf{N}_0(\tau)=\mathbf{U}\exp(\mathbf{E}\tau)\tilde{\mathbf{Z}}(\tau)\exp(\varepsilon\mathbf{K}\tau)\mathbf{U}^{-1},$$

where $\tilde{\mathbf{Z}}(\tau)=\mathrm{diag}\{1,\exp(-i\varepsilon\nu\tau),\exp(i\varepsilon\nu\tau)\}$. We can see that $\exp(\mathbf{E}\tau)\tilde{\mathbf{Z}}(\tau)=\exp(i\mathbf{d}\tau)$, where $\mathbf{d}$ is the diagonal matrix $\mathbf{d}=\mathrm{diag}\{0,\delta,-\delta\}$. Thus, the leading term of the asymptotic expansion in the powers of ε of the solution of the homogeneous equation (A1) is

$$\mathbf{N}_0(\tau)=\mathbf{U}\exp(i\mathbf{d}\tau)\exp(\varepsilon\mathbf{K}\tau)\mathbf{U}^{-1}. \tag{A9}$$

Equation (A9) can be used to obtain the solution of the nonhomogeneous problem (4),

$$\mathbf{W}(\tau)=\mathbf{N}(\tau)\mathbf{W}(0)+\mathbf{N}(\tau)\int_0^\tau[\mathbf{N}(x)]^{-1}\mathbf{C}dx.$$

Since the real parts of the matrix $\mathbf{K}$ eigenvalues are strictly negative and we search for the stable solution, we can omit the first term on the right-hand side of this equation and regard only the upper limit when integrating. Then

$$\mathbf{W}_c(\tau)=\mathbf{U}\exp(i\mathbf{d}\tau)\exp(\varepsilon\mathbf{K}\tau)\int^\tau\exp(-\varepsilon\mathbf{K}x)\times\exp(-i\mathbf{d}x)\mathbf{U}^{-1}\mathbf{C}dx. \tag{A10}$$

Fulfilling the integration and neglecting terms of the order of ε, we obtain the leading term of the asymptotic for R, the first component of vector $\mathbf{W}$, and come to Eq. (6).

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