Optical coherence: A convenient fiction

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We conjecture that optical coherences, i.e., quantum-mechanical coherences between states separated by Bohr frequencies in the optical regime, do not exist in optics experiments. We claim the exact vanishing of optical field amplitudes and atomic dipole expectation values, and we discuss the seemingly contradictory success of assigning finite values to such quantities in theoretical calculations. We show that our conjecture is not at variance with the observed interference between different light sources. The connection to the concept of spontaneous symmetry breaking and the identification of entangled states as pointer basis states is discussed. [S1050-2947(97)06904-7]

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I. INTRODUCTION

In the classical theory of light, electric and magnetic fields obey Maxwell's equations. Due to their high frequency we cannot observe the amplitudes of optical fields, and as the radiation is not generated by classically moving charges but is radiated from, e.g., atomic sources, a proper description of the light detection and preparation requires quantization of the field along with the radiating systems. Maxwell's equations and propagation properties from the classical theory are through mode functions central for quantized electromagnetic fields, but the field amplitudes are replaced by quantum operators, and it is the vanishing of their expectation values that is the topic of this paper.

If there are no mean fields, there is no mean polarization induced in media illuminated by the light field, and the classical theory of light is not merely a ''theory of mean values'' of the quantum theory. Large parts of optics and of atomic physics, e.g., the theory of dielectrics, the coupled Maxwell-Bloch equations, and the role of atomic coherences, should in principle be reexamined to verify that predictions based on postulated mean amplitudes can also be obtained more rigorously.

A conclusion of this paper is that it does not matter whether coherences exist or not; observable phenomena in optics and quantum optics are unchanged, and in this way optical coherences may be regarded as a convenient fiction. A discussion of the applicability of this fiction, or myth, may, however, contribute to our understanding of the quantum classical correspondence, e.g., in the process of measurement. In addition, we note the recent emergence of collective atomic effects and many-body physics concepts in optics and quantum optics. Atoms, unlike photons, are not created or annihilated in experiments, but a conclusion of the present paper is that this is not a fundamental difference between, e.g., a laser and a coherent source of bosonic atoms. In turn, the validity of many-body physics concepts such as spontaneous symmetry breaking might be examined along the same lines as the ones applied here for optics.

In Sec. II we recall the role of entanglement and correlation functions in the interaction between matter and light. The ambiguity of interpretation of density matrices is pointed out both as an origin of unjustified conclusions on the existence of optical coherence and as an element of validation of the convenient application of such coherence. In Sec. III we discuss the intensity oscillations recorded by a photodiode illuminated by two independent well-stabilized lasers. With the aid of the quantum theory of measurement it is shown that we may understand these oscillations without appeal to nonvanishing amplitudes of either field. In Sec. IV we discuss some further consequences of the present work, and we comment on the in-principle feasibility of making optical fields with finite mean amplitudes. In Sec. V we point to the similarities and differences with the spontaneous symmetry breaking concept in other fields of physics. We conclude the paper with a discussion of the rigidity of classes of quantum entangled states qualifying such states as pointer basis states, i.e., states in accord with observed classical-like behavior of the systems.

II. ENTANGLEMENT, DENSITY MATRICES, AND CORRELATION FUNCTIONS

We shall now briefly illustrate why optical coherence is not easily generated: For a medium represented by quantized, e.g., atomic, systems enumerated by the index α with bare energy levels $E_{\alpha,i}$ and states $|i_\alpha\rangle$, and quantized electromagnetic fields described by field mode operators $a_\lambda, a_\lambda^\dagger$, the Hamiltonian can be written

$$H = H_F + H_M + H_{\text{int}}, \quad H_F = \sum_\lambda \hbar\omega_\lambda (a_\lambda^\dagger a_\lambda + \tfrac{1}{2}),$$

$$H_M = \sum_{\alpha,i} E_{\alpha,i} |i_\alpha\rangle\langle i_\alpha|,$$

$$H_{\text{int}} = \hbar \sum_{\lambda,\alpha,ij} [f_{\alpha,\lambda}^{ij} a_\lambda^\dagger |i_\alpha\rangle\langle j_\alpha| + \text{H.c.}]. \tag{1}$$

The coefficients $f_{\alpha,\lambda}^{ij}$ contain the position dependence of the mode functions (evaluated at the position of the particle α) and the matrix elements corresponding to the motion of the charges in the quantized systems (e.g., atomic dipole matrix elements).

Lasers and conventional light sources rely on near-resonant interactions between an incoherently pumped medium and the radiated light field. The coupling is only effective among bare medium and field states of approximately the same energy, and the sum in $H_{\rm int}$ in Eq. (1) reduces to one where the atomic transition operators $|i_\alpha\rangle\langle j_\alpha|$ only appear in terms with a_λ ($a_\lambda^\dagger$) if $E_{\alpha,i}-E_{\alpha,j}\simeq\hbar\omega_\lambda$ ($-\hbar\omega_\lambda$). The Heisenberg equations of motion for field amplitude operators contain only atomic transition operators having essentially the same free evolution frequencies,

$$\frac{d}{dt}a_\lambda(t)=-i\omega_\lambda a_\lambda(t)-i\sum_{\alpha,ij} f^{ij}_{\alpha,\lambda}|i_\alpha\rangle\langle j_\alpha|(t), \tag{2}$$

and these in turn are coupled only to operators with the same rapid evolution. Hence, if all such quantities have vanishing mean values at some initial time, they will never develop nonvanishing expectation values. If only one frequency range is considered for the field and the atomic transitions, this can be attributed to the conservation of number of photons plus atomic excitations in the system governed by the Hamiltonian.

Entangled superposition states of the atoms and quantized fields are prepared. The dimensionality of the Hilbert space prohibits a treatment incorporating the quantum state of all systems contributing to an optics experiment: the state of an atomic target for a laser beam is entangled with the field states, as inferred from the Hamiltonian (1), but the field state is already entangled with the state of the light inside the laser cavity, which in turn is entangled with the states of the gain medium, which are entangled with the states of the incoherent pump (thermal reservoir) ... . It is not only convenient, it is absolutely necessary to break this hierarchy of entanglement as close as possible to the target system of interest, and this is what we do when we replace field operators by c numbers.

Note, however, that for light propagation problems, the wave equation for the field contains the polarization of the medium, but the spatiotemporal character of this equation is the same whether it describes the classical quantities (alledged mean values of field and dipole operators) or the quantum-mechanical operators. If we are in the linear regime, for example, the steady state atomic dipole operator is proportional to the field operator of the driving field with the same constant of proportionality as between the mean dipole and an injected mean electric field, hence the field is refracted the same way, and in an interferometric setup the same interference pattern in an intensity signal will be recorded.

A quantum system, interacting weakly with its surroundings, can be described by a reduced density matrix, obtained as the trace over the Hilbert space of the surrounding quantum system. No optical coherences exist in the (system +surroundings) state vector, and this is not changed by the trace procedure. What is changed, however, is that a pure state is replaced by a mixed state, represented by a density matrix.

In quantum descriptions of the laser a near-Poissonian photon number distribution (diagonal density matrix) is obtained when the entanglement with the states of the gain medium has been traced out [1]. Alternatively, one may suggest that any specific laser at a given instant of time is in a pure coherent state $|\alpha\rangle=\exp(-|\alpha|^2/2)\Sigma(\alpha^n/\sqrt{n!})|n\rangle$ but *a priori* we do not know the value of its diffusing phase $\arg(\alpha)$, and the average over this quantity yields the Poissonian number distribution with vanishing mean amplitude. Complementarity implies that there is no physical difference between an ensemble of number states and an ensemble of coherent states if the ensemble averaged density matrix is the same. Therefore, even if precise knowledge of the exact (field + surrounding matter) state vector precludes a mean field, we may still utilize coherent states in calculations.

The idea of associating pure states with density matrix problems has attracted much interest. For dissipative problems of a very wide class, it has been shown that one may consistently evolve an ensemble of wave functions, so that they, on the average, reproduce the reduced density matrix for the system. These approaches have been described by the names "quantum trajectories" [2], "Monte Carlo wave functions" [3], "quantum state diffusion," and "decoherent histories" [4]. The state vectors can be ascribed physical meaning as the states conditioned on certain measurements performed on the surroundings of the system.

The emergence of near-coherent field states in cavities has thus been seen when the simulation scheme involves stochastic differential equations for the state vectors, the quantum state diffusion picture. All such calculations, however, are of a kind where the wave function evolution is consistent with homodyne or heterodyne detection, which requires a coherent local oscillator field. As such fields do not exist, the simulations do not represent a rigorous analysis of the time evolution of the system. Simulations associated with feasible measurements of excitation and/or intensity (slow variables) do not produce coherences. In the next section we shall see which kinds of states are produced during homodyne or heterodyne detection.

As real-time monitoring of a signal oscillating in the optical regime is not possible with classical devices, we are bound to measure slowly varying or stationary quantities such as power spectra. Historically, the field of quantum optics has been strongly rooted in the growing awareness starting around 1960 of the consequences in photodetection experiments of the difference between means of operator products and products of operator means [5]. Considering t as a fixed time argument, we may multiply Eq. (2) from the left by, e.g., the operator $a_\lambda^\dagger(t)$, and we obtain for the two-time operator $a_\lambda^\dagger(t)a_\lambda(t+\tau)$ the equivalent equation

$$\frac{d}{d\tau}a_\lambda^\dagger(t)a_\lambda(t+\tau)=-i\omega_\lambda a_\lambda^\dagger(t)a_\lambda(t+\tau)-i\sum_{\alpha,ij} f^{ij}_{\alpha,\lambda}a_\lambda^\dagger(t)|i_\alpha\rangle\langle j_\alpha|(t+\tau). \tag{3}$$

The operator quantities in this equation do not have vanishing mean values, e.g., at $\tau=0$ the mean photon number appears on the right-hand side, and later the entanglement of the field and the atoms contributes in the last sum.

From a formal perspective, one- and two-time expectation values obey the same set of equations, merely with different initial conditions. In the context of open dissipative systems this connection is known as the quantum regression theorem

[6] (valid under the Markov assumption for the system's interaction with its surroundings). This provides another validation for applying alleged nonvanishing coherences in place of the mean values of the operators in Eq. (2): they produce the correct quantitative results, not for one-time averages and products like $\langle a^\dagger(t)\rangle\langle a(t+\tau)\rangle$, but for the experimentally relevant two-time expectation values like $\langle a^\dagger(t)a(t+\tau)\rangle$.

III. INTERFERENCE BETWEEN TWO INDEPENDENT LIGHT SOURCES

One of the cornerstones in quantum optics is the analysis of photodetection experiments, and one lesson learned from this analysis is the apparent preservation of quantum character of a signal into the classical electronic circuitry of the detector. In heterodyne spectroscopy, for example, light from a source is mixed with the field from a local oscillator. This mixture is incident on a photodetector, and the photocurrent is spectrum analyzed. The power spectrum does not determine the Fourier transform of the autocorrelation function of the mean current $\overline{i(t)}\propto\langle E^{(-)}(\vec{r},t)E^{(+)}(\vec{r},t)\rangle$, where $E^{(+)}(\vec{r},t)$ denotes the positive frequency part of the quantized electric field at the location $\vec{r}$ of the detector, but the Fourier transform of the mean autocorrelation function $\overline{i(t)i(t+\tau)}\propto\langle E^{(-)}(\vec{r},t)E^{(-)}(\vec{r},t+\tau)E^{(+)}(\vec{r},t+\tau)E^{(+)}(\vec{r},t)\rangle$.

The field operator $E^{(+)}(\vec{r},t)$ is a sum of free field parts (annihilation operators for unpopulated field modes) and a source part, which is here a coherent superposition of the fields emitted from the two sources. As only the source part is relevant for the intensity measurement, we introduce the relevant combination of source annihilation operators, e.g., for a 50/50 lossless beam splitter, $c=(a+b)/\sqrt{2}$. Even if the two modes are uncorrelated so that $\overline{i(t)}\propto\langle c^\dagger c\rangle$ receives no contribution from the cross terms involving $a^\dagger b, b^\dagger a$, the correlation function contains a term proportional to $\langle a^\dagger(t)b^\dagger(t+\tau)a(t+\tau)b(t)\rangle$, and there will be an interference signature in the spectrum at the difference in frequency between the source and the oscillator, proportional to the product of the autocorrelation functions for each mode $\langle a^\dagger(t)a(t+\tau)\rangle$ and $\langle b^\dagger(t+\tau)b(t)\rangle$. Hence, an interference peak in the power spectrum does not imply a field amplitude in either of the beams impinging on the detector.

There are experiments in which fields are derived from such well stabilized lasers and with such close optical frequencies that a photocurrent can be observed to oscillate sinusoidally on the time scale of seconds. The oscillations are in agreement with the analysis based on the photocurrent correlation function, but the question is whether optical coherences in the field modes are required in order to explain the interference observed.

We now consider the following simple model, see Fig. 1: two single mode cavities are assumed to be populated by photon number states, so that the state of the field is the product state $|n,n\rangle=|n\rangle\otimes|n\rangle$ at $t=0$. The modes have different frequencies ω_a and ω_b, and both cavities have a partially transmitting mirror causing damping of the intensity with the same decay rate Γ. The fields escaping the cavities are combined on a 50/50 lossless beam splitter, and the resulting fields measured by two detectors have positive frequency parts involving the operators $c=1/\sqrt{2}(a+b)$ and $d=1/\sqrt{2}(a-b)$. What do we predict for the outcome of such an experiment ?

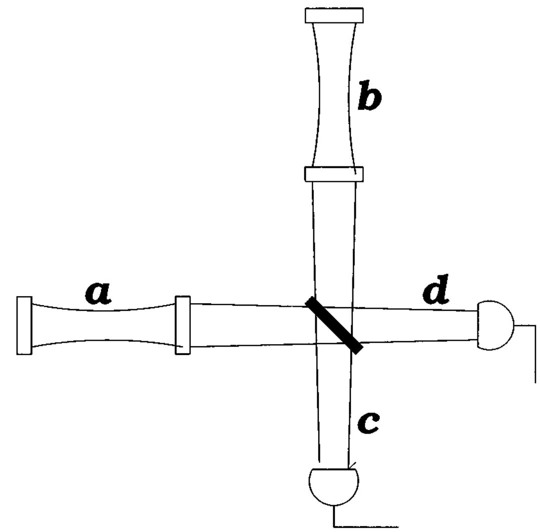

FIG. 1. Optical setup where the output beams from two cavities are mixed on a beam splitter and the intensities of the resulting beams are measured by two photodetectors.

The density matrix treatment predicts no interference. Indeed, the two cavities will independently produce binomial distributions, $p(l,t)=\binom{n}{l}\exp(-\Gamma t)^l[1-\exp(-\Gamma t)]^{n-l}$, and the expectation values of the photon fluxes $\Gamma c^+c, \Gamma d^+d$ follow exponential decay laws $\Gamma n\exp(-\Gamma t)$.

We shall now see that such a density matrix treatment does not provide an adequate description of the experiment. In a photodetection measurement the photocurrent represents the counts of photons in certain time intervals. From the combined wave function of the two modes we know the probabilities for detecting $0,1,\ldots$ photons with the two detectors, and according to the quantum theory of measurement a detection event leads to a collapse of the wave function on the subspace corresponding to the selected eigenvalue. We can now build a quantum trajectory, modifying the wave function gradually according to the simulated detection events in the two detectors. Our numerical procedure is described in detail below, but let us first indicate how entanglement of the modes is introduced. In the initial state both detectors have the same photodetection probability; assume that a photon is detected in the detector illuminated by the $c=1/\sqrt{2}(a+b)$ combination of the cavity fields. The state vector after this detection is then obtained as $c|n,n\rangle=\sqrt{n}(|n-1,n\rangle+|n,n-1\rangle)/\sqrt{2}$ (to be subsequently normalized). Now, the two cavity field modes have become entangled, $\langle a^+b\rangle\neq 0$, and the two detectors no longer have the same detection rate. If the two eigenfrequencies differ, the wave function is not stationary, and the detection probabilities will oscillate at the frequency difference $\Delta=\omega_a-\omega_b$.

We examine the details of the evolution by a quantum jump simulation appropriate to the given detection scheme [3]: a wave function is propagated with the effective Hamiltonian

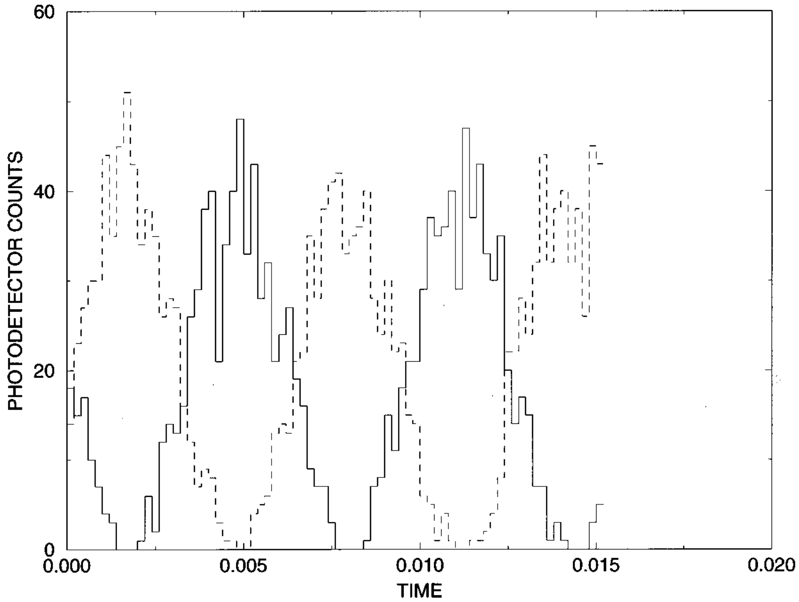

FIG. 2. Number of photons counted by the two detectors in time intervals of $\delta t = 0.0002\Gamma^{-1}$. Time is given in units of Γ^{-1}. Solid (dashed) line: mode-c (-d) detector. Repeated simulations with the same initial number states $n = 10^5$ in the two cavities show the same periodicity of the two intensity signals, but the phase of the oscillations varies from simulation to simulation.

$$H_{\text{eff}} = \hbar\omega_a(a^\dagger a + \tfrac{1}{2}) + \hbar\omega_b\left(b^\dagger b + \frac{1}{2}\right) - i\hbar\frac{\Gamma}{2}(a^\dagger a + b^\dagger b) \tag{4}$$

and the evolution is interrupted at random instants of time by quantum jumps $|\psi\rangle \to c|\psi\rangle$ or $d|\psi\rangle$, occurring with $|\psi\rangle$-dependent rates $\gamma_c = \Gamma\langle\psi(t)|c^\dagger c|\psi(t)\rangle$ and $\gamma_d = \Gamma\langle\psi(t)|d^\dagger d|\psi(t)\rangle$. The last term in H_{eff} can also be written as $-i\hbar(\Gamma/2)(c^\dagger c + d^\dagger d)$, so this is really the standard Monte Carlo wave-function procedure for a master equation on the so-called Lindblad form [3].

The action of either of the jumps is to reduce the total photon number by unity. Hence starting from a state with definite total photon number, $2n$, at a later time T the field state of the two modes will be an eigenstate of the total photon number operator $a^\dagger a + b^\dagger b$ with eigenvalue $2n - q$, where q is the total number of simulated detection events. This implies that we can write the wave function as

$$|\psi(t)\rangle = \sum_{k=0}^{q} c_k(t)|n-k, n-q+k\rangle. \tag{5}$$

As the non-Hermitian part of H_{eff} acts identically on all terms in $|\psi(t)\rangle$, it is sufficient to consider the Hermitian part in the determination of the evolution of the amplitudes between jumps, and by choosing an appropriate rotating frame we obtain the equation $\dot{c}_k = ik\Delta c_k$ with the solution

$$c_k(t+\tau) = c_k(t)\exp(ik\Delta\tau). \tag{6}$$

The total jump rate $\gamma_c + \gamma_d = \Gamma(2n-q)$ is independent of the values of the amplitudes c_k, hence the time between jumps is exponentially distributed, and we select the instant of the next jump from the time increment τ solving $\exp(-\Gamma(2n-q)\tau) = \varepsilon$, where ε is a random number between zero and unity. Which one of the jumps to take is determined from the ratio between the current values of the rates $\gamma_{c(d)} \propto (2n-q) + (-)2\ \mathrm{Re}[Q]$, where

$$Q = \langle\psi(t+\tau)|a^\dagger b|\psi(t+\tau)\rangle$$
$$= \sum_{k=0}^{q-1} \sqrt{n-k}\sqrt{n-q+k+1}\,c_k^* c_{k+1}. \tag{7}$$

The value of q is increased by unity, and an expression analogous to Eq. (5) is again valid after the action of the c or d annihilation operator

$$(a \pm b)\sum_{k=0}^{q} c_k|n-k, n-q+k\rangle$$
$$= \sum_{k=0}^{q+1} c'_k|n-k, n-(q+1)+k\rangle, \tag{8}$$

where

$$c'_k = \sqrt{n-(k-1)}\,c_{k-1} \pm \sqrt{n-q+k}\,c_k, \tag{9}$$

and where a subsequent normalization should be introduced.

This semianalytic evolution is easily implemented on a computer, and in Fig. 2 we show the outcome of a single run. In the calculation we have taken $n = 10^5$ photons initially in each cavity, and a frequency difference $\Delta = 1000\Gamma$. We plot the number of jumps of each type performed within time windows of duration $\delta t = 0.0002\Gamma^{-1}$, and the simulation proceeds until a total of $q = 3000$ photons have been detected. In repeated runs, one obtains the same picture after a short entanglement period, but the oscillations are shifted in time.

The simulated signal is identical to the one from two cavities initially in a product of coherent states $|\alpha,\beta\rangle = |\alpha\rangle\otimes|\beta\rangle$. This product state evolves as a two-mode coherent state, $|\alpha\exp(-i\omega_a t - (\Gamma/2)t), \beta\exp(-i\omega_b t - (\Gamma/2)t)\rangle$, and the rates of detection events have the expectation values $\frac{1}{2}\Gamma[|\alpha|^2 + |\beta|^2 \pm 2|\alpha^*\beta|\cos(\Delta t + \phi)]\exp(-\Gamma t)$, where ϕ is the relative phase of the two complex numbers α,β. In our simulation we have no mean fields in the cavities, neither in the initial state nor during the time evolution, but after the

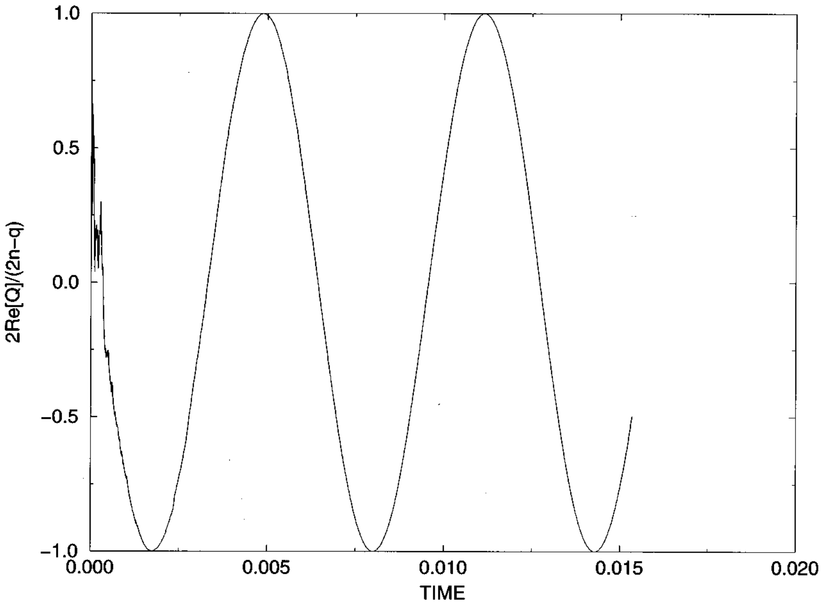

FIG. 3. Time dependence of the quantity Q, controlling the count rates in the two detected field modes; time is in units of Γ^{-1}. The value of $2\,\mathrm{Re}[Q]/(2n-q)$ is presented at 3000 different instants for the simulation yielding the intensities in Fig. 2. Q, and the rates, are smoothly varying quantities.

first few photodetections, the resulting entangled state behaves just as a product coherent state with a random relative phase ϕ.

It is amazing that the wave function (5) for some interval of time leads to a nearly uninterrupted sequence of detections in one detector, then, a quarter of a period later, to a completely random detection in the two detectors without destruction of the memory in the wave function so that another quarter of a period later only photons in the other detector are recorded. The interference is due to the evolution of the quantity Q introduced in Eq. (7). At $t=0$, Q vanishes, but already after one detection $Q\simeq\pm n/2$, and during the first $\simeq 100$ jumps $|Q|$ gradually approaches the value n. The phase of Q at this point depends on the explicit sequence of jumps. In between jumps we obtain the differential equation, $\dot{Q}=i\Delta Q$, which provides the harmonic variation in intensity at the two detectors. In Fig. 3 we show the time evolution of $\mathrm{Re}[Q]$, displaying clearly the transition between an initial randomness and a subsequent harmonic evolution. As we see, the random events have a negligible effect on the relevant physical quantities at this point. $\mathrm{Re}[Q]$ is here plotted at each jump performed in the program and it is observed to evolve continuously (the noise in Fig. 2 reflects the usual count statistics for small count rates). This resembles the effect of photon detection on a coherent state, where jumps have no effect because the state is an eigenstate of the jump operator. In our problem, both types of jump project the state vector in the $2n-q$ photon number space onto state vectors with the same character residing, however, in the $2n-(q+1)$ number space. Numerically we verify that the c_k amplitudes evolve smoothly during jumps (after the frustrated transient), and to an excellent approximation we find a binomial distribution after the detection of q photons, $|c_k|^2\simeq 2^{-q}\binom{q}{k}$.

Let us comment on the status of correlation functions in connection with real time evolution. The density matrix treatment of the interference problem yields nonoscillating photocurrents in the two detectors, but if these are connected to power spectrum analyzers the frequency difference Δ is readily identified. To calculate this result we must invoke the photocurrent correlation function, assuming that the photocurrent, although macroscopic, preserves the quantum character of the field operators. Now, this macroscopic quantum behavior, mentioned earlier as a special aspect of quantum optics, is indeed mysterious, and as pointed out by Carmichael [2] in the case of subshotnoise detection, the physics is more readily understood in terms of quantum trajectories. Here, the recorded signal is classical but it is accompanied by collapses of the quantum state of the light source and this gives the signal its nonclassical temporal correlations. The signal in such a record necessarily shows the temporal behavior present in the correlation function because the average over many records reproduces this function. Structures in two-time correlation functions are not predicted as one-time averages by the density matrix, but if looked for experimentally they will be seen as such in individual realizations, and many experiments in quantum optics are individual realizations, e.g., the joint intensity measurement of two laser beams just discussed in detail.

The analysis just presented is closely connected to recent work by Javanainen and Yoo [7], in which an interference pattern in the detection of atoms populating two spatially overlapping Bose condensates is predicted due to the backaction of measurements. For further analyses of this problem, see also Ref. [8]. We shall come back to the discussion of matter waves below; here we just point out differences between the situation described by us and the one considered by Javanainen and Yoo. Our interference occurs in time, which does not play a significant role in the spatial interference. In fact, the spatial structure is already present in their multiatom wave function prior to detection in terms of spatial correlation functions. The effect of the first detection events is to pin down the location of the periodic pattern. More remarkably, whereas the spatial interference appears during measurement of the continuous variable in which the interference occurs, we monitor two discrete variables in our simulation. It is the ratio between the number of detection events in the two detectors that is able to establish an entanglement which evolves so as to produce the results shown in Fig. 2.

Our work is also closely related to a recent theoretical study by Castin and Dalibard [9] of the measurements on atoms macroscopically populating two possible states, and being detected sequentially in a set of superposition states. Also here interference compatible with a common initial random superposition state for all the atoms is observed.

IV. OTHER VANISHING COHERENCES AND CANDIDATES FOR NONVANISHING COHERENCES

We have focused on the vanishing of field amplitudes associated with the fact that superpositions between states differing by optical Bohr frequencies do not exist. Other properties of fields are also affected by this fact. Or, rather, a number of phenomena should remain entirely unaffected, as it should be possible to identify the quantities appropriately replaced by alledged nonvanishing coherences in the description of the relevant systems.

One example is squeezed light. Amplitude squeezing is a reduction of the fluctuations of, e.g., the Hermitian field operator $a+a^\dagger$ below the vacuum level. The squeezed vacuum state involves a superpositon of all even-number photon states, and in the computation of $\langle (a+a^\dagger)^2\rangle$, it is the nonvanishing expectation values of a^2 and $(a^\dagger)^2$ that produce the reduction in fluctuations compared to the coherent state with the same mean photon number $\langle a^\dagger a\rangle$. Now, these coherences are completely negligible in optical systems, hence squeezed states of light are also part of the convenient myth spanned by postulated coherences. A closer scrutiny of the mechanism responsible for squeezed light generation (four-wave-mixing, down-conversion) reveals the absence of squeezing, provided the pump beams are not assumed to be in coherent states, and it reveals the entanglement between pump and signal beams in squeezing experiments. The analysis of different experiments may point to the pump-signal entanglement, or the backaction in measurement records like in the previous section, as the true ''squeezing'' mechanism, and definitely shows that fictitious, ''mythological'' squeezed states represent much more efficient ways to obtain the same results.

We have intentionally emphasized that coherences in the *optical* regime are not created, and we explicitly appealed to the fact that normal sources of optical radiation are not classical oscillators but quantum systems, and the vanishing of their dipole moments is crucial for the argument. For low frequencies, a moving charge distribution gives rise to a time dependent electromagnetic field, and most RF sources, radio stations, and microwave ovens are likely to produce radiation with nonvanishing amplitude. Of course, this distinction between high and low frequencies is only acceptable together with the acknowledgment of classical physics, being correct for the description of such macroscopic phenomena. Rather than entering the discussion of the transition between the quantum and the classical world at this point, let us note that if one assumes that domains of physics are correctly described by classical physics, one must accept that classically moving charged objects exist and that they emit coherent radiation.

Another aspect about low frequencies is related to the terms that effectively contribute in the standard coupling of matter and light. The smallness of the coupling constants $f^{ij}_{\alpha,\lambda}$ in Eq. (1) excludes coupling of states differing by energies in the optical regime, but low frequency coherences may be established as the energy may be sufficiently well conserved in a microwave transition between two atomic levels, both if a single or two field quanta are involved, so that an atomic excited state with no photons $|j_\alpha\rangle|n_\lambda=0\rangle$ may effectively be coupled by the Hamiltonian in Eq. (1) to both $|i_\alpha\rangle|n_\lambda=1\rangle$ and $|i_\alpha\rangle|n_\lambda=2\rangle$. The resulting superposition of all three states then exhibits a finite expectation value of the photon annihilation operator. If the coupling strengths were large enough in Eq. (1), the gap to optical coherences could be bridged, but this situation is not likely to be achieved. Deviations from the rotating wave approximation in the atomic case would not lead to optical mean fields, but to coherences between states separated by two photon energies. These, however, would be so weak that they would play no role in the evolution of the fields we study in experiments.

Accepting mean fields at low frequencies, we are also forced to accept the possibility of producing coherent optical radiation by high harmonic generation, or, maybe more efficiently, by the backaction in an experiment where the interference between the initially incoherent light and the coherent low-frequency radiation in a nonlinear crystal is monitored.

Another possible source of coherent optical radiation is charged particles oscillating at optical frequencies as, e.g., in the free-electron laser. In this system a static magnetic field with a spatial periodicity on the order of centimeters, in a so-called wiggler, is experienced by a relativistic electron beam as a time varying field. The frequency of this field may well be in the optical range. The electron beam undergoes density fluctuations due to the interaction with the field, and the rapid oscillations cause emission of radiation, which may have a nonvanishing mean value.

Recently, sonoluminescence has been suggested to be a quantum optical effect where the rapid contraction of a bubble in a dielectric causes creation of photon pairs via the change in mode structure [10]. Like the free-electron laser for one-photon coherence, this may be a real source of two-photon coherent, i.e., squeezed, light.

The point of this paper is not to demonstrate that it is not possible to create optical coherence. I am suggesting that it has, with few possible exceptions, not been done so far and, more importantly, that coherences are not needed to account for the rich variety of phenomena observed in optics. Some phenomena, e.g., the properties of squeezed light, should probably be further examined to clearly display the validity, or rather usefulness, of alleged coherences.

V. SPONTANEOUS SYMMETRY BREAKING AND MEAN FIELDS IN MANY-BODY PHYSICS

There are cases where systems choose states with nonvanishing expectation values of certain operators, although the governing Hamiltonian does not lead to these values. When, for example, the state of a system does not have the same symmetry as the Hamiltonian, one refers to ''symmetry breaking,'' and theories of symmetry breaking are wide-spread. Without entering a discussion of the qualitative differences between different systems we mention localization and orientation of macroscopic bodies, the macroscopic magnetic moment of a ferromagnet, and the magnetic ordering in an antiferromagnet, where it is meaningful to talk about *spontaneous* symmetry breaking. One way to introduce spontaneous symmetry breaking is through a weak external perturbation. The larger the system, the closer the energy eigenvalues and the weaker is the perturbation needed to make a symmetry-breaking state the preferred state of the system—observations may have the same effect due to the back action mentioned earlier; see also Ref. [11].

Our argument against the existence of optical coherences is that states possessing such mean amplitudes belong to parts of Hilbert space that are not coupled to any conceivable initial state for our systems. To introduce an oscillating amplitude in the quantum system by spontaneous symmetry breaking, the symmetry breaking perturbation would have to contain a quantity oscillating at an optical frequency. Such a perturbation cannot be thought of as a random inhomogeneity in the environment and even if it could, to be significant for the production of optical coherence, for example in a laser cavity, the perturbation needs to be strong and persistent to overcome dissipation—otherwise, the injected coherence will rapidly spread out on a large number of degrees of freedom. We cannot take the limit of a vanishing perturbation and keep a mean amplitude. This is not at variance with semiclassical laser theories in which the motion of a c-number amplitude is governed by a ''Mexican hat'' potential. In these mathematical models a weak symmetry breaking perturbation is sufficient to establish a nonvanishing ampliude, but in such theories one first postulates mean amplitudes and then factorizes operator products accordingly.

Rather than deal with spontaneous symmetry breaking in our exact quantum optics systems, we may refer to the mean field approximations as ''symmetry-breaking approximations.'' The system and the interactions are simply replaced by something different which (i) is easier to deal with formally and conceptually and which (ii) yields nearly the same results as the more cumbersome exact approach, if such an approach within the symmetry conserving framework is feasible at all. The discussion in the preceding sections shows that in optics we cannot distinguish experimentally between the physical situations described by the exact treatment and the symmetry breaking approximation.

Laser cooling and evaporative cooling have recently made it possible to achieve Bose-Einstein condensation in a dilute gas of atoms [12]. A related line of research is to produce a coherent source of matter waves, an atom laser [13,14]. This has caused an interest in applying concepts from the atomic physics and quantum optics communities to the description of physics with many atoms, and in particular to make descriptions that do not invoke those concepts of many-body physics which are directly at variance with the atomic physics starting point. For example, the fact that atoms are not created or destroyed rules out the existence of nonvanishing mean fields for atoms, but interference phenomena may exist nonetheless [7–9,15].

For atoms, the mean fields may effectively represent nonvanishing correlation functions. The off-diagonal long range order, introduced by Penrose and Onsager [16], who ascribe finite values to the expectation value of matter field products $\langle \psi^\dagger(x)\psi(y)\rangle$ for very distant positions x and y, is quite analogous to the two-time corelation function (or two space points correlation function) of the electromagnetic field. The successful application of mean field amplitudes in many body physics is here reminiscent of the equally successful application of mean electromagnetic fields in optics. In both cases the long range order is due to the existence of a mode extending over a larger spatial range (Maxwell mode of the field, Schrödinger wave function of individual particles); the mode does not need to be populated in a superposition of different number states for long range order to persist.

In many-body physics, atoms or electrons make transitions between different states, so that the dynamics of a certain part or phase of the system is entangled with the surrounding system, and also here mean fields may substantially facilitate calculations. For example, Bogoliubov transformed operators [16] and states without definite particle number, e.g., in BCS theory of superconductivity and in nuclear physics, are more convenient to work with than states with a definite particle number. It is noteworthy, however, that when Bardeen, Cooper, and Schrieffer introduce the BCS wave function in their seminal paper on superconductivity [17], they only '' ... for the moment relax the requirement that the wave function describes a system with a fixed number of particles, ...'' In the following discussion they explicitly extract the projection of this wave function on the space of N electron pairs, bringing the problem into analogy with the entangled states in our analysis of the interference between two light sources in Sec. III.

A general and very illuminating discussion of the usefulness of violating conservation laws has been given by Lipkin [18] (in addition to particle number, also linear and angular momentum are offered as examples). Stating in the Introduction the same goals as we have had with the present paper—to explain why and how we can use bad wave functions to calculate real properties of a system—Lipkin derives an approach of model Hamiltonians, effectively subtracting a part from the Hamiltonian ($\hbar\omega a^\dagger a$ in the case of an optical field mode) so that different eigenstates of the subtracted operator are degenerate and superposition states become eigenstates of the new model Hamiltonian H'. Similarities in the physics described by H and H' ensure the usefulness of the results obtained with the latter. The mechanism of introducing H' has found practical applications in the Lipkin-Nogami pairing scheme in self-consistent nuclear structure calculations [19]. It provides conservation law conserving wave functions which, in particular for low nucleon numbers, are more adequate than the BCS states with their relatively large number fluctuations in these cases.

VI. DISCUSSION

In this paper we have discussed coherences in optics. We have argued that mean optical amplitudes are not created and not detected in experiments, and we have shown that a number of properties of light and atoms, which are usually understood in terms of mean fields, can be accounted for by quantum states of the systems with no coherences in the optical regime. Apart from their representation of autocorrelation functions, mean fields, in a compact form, exhibit the essential features of entanglement, e.g., between atoms and quantized fields and between different field modes as in the detailed study of the two-mode problem in Sec. III. Some explanations of the success of optical mean field descriptions presented in this paper may justify the use of mean fields for atoms—in both cases mean fields serve as convenient mathematical tools. On the practical side there are many differences between a conventional optical laser and an atom laser, as the roles of fields and particles are essentially exchanged, but a mean field description is not, in principle, more valid in one case than the other.

Let us comment on an aspect of the more general relationship between classical behavior, mean fields, and entangled states, which is pointed out by the discussion in Sec. III, and which may be at the origin of the understanding of a wide range of optical systems and experiments. We have discussed why the coherent states reproduce very well and very efficiently the results based on the entangled states; we must now ask ourselves why these entangled states behave like coherent (classical) states. In contrast to a number of studies discussing the fragility of entangled states and the rigidity of coherent states, our identified entangled states are robust against the influence of the environment, e.g., of physical observation.

Consider the simple optics example of a product of two coherent states for two oscillators, $|\psi\rangle=|\alpha\rangle\otimes|\beta\rangle$, projected onto a number state eigenspace,

$$|\psi_N\rangle=\mathcal{N}\sum_{k=0}^{N}\frac{\alpha^k}{\sqrt{k!}}\frac{\beta^{N-k}}{\sqrt{(N-k)!}}|k\rangle\otimes|N-k\rangle, \qquad (10)$$

where $\mathcal{N}$ is a normalization constant. With a conventional view of entanglement, the state $|\psi_N\rangle$ would be described as very nonclassical. It preserves, however, its character after being acted upon by any linear combination of the annihilation operators of the two oscillators; the resulting state is simply the projection of the product coherent state $|\psi\rangle$ (eigenstate of the annihilation operators) on the $(N-1)$ photon number space. Of course $|\psi_N\rangle$ and this state $|\psi_{N-1}\rangle$ are orthogonal, but when N is large they are experimentally indistinguishable. Here, measures of identity and/or difference other than the inner product are called for.

The concept of a *pointer basis*, i.e., a basis of states naturally populated by quantum systems and preferred in the analysis of a probabilistic density matrix result, has been introduced as a means to understand the emergence of classical behavior in systems described by quantum mechanics [20]. Entangled states of the kind derived in this paper seem to have all the properties required by such pointer basis states, and in addition they exist over the whole range from the macroscopic to the microscopic, or they are brought into existence when the appropriate measurement is performed—which is not true for the coherent states underlying the mean field theories.

The states derived in Sec. III are different from $|\psi_N\rangle$. We found a binomial distribution of the photon number difference in the two modes, $|c_k^{\mathrm{coh}}|^2\simeq 2^{-q}\binom{q}{k}$, which is narrower than the distribution for the wave function $|\psi_N\rangle$ with $|\alpha|^2=|\beta|^2$ and with $N=|\alpha|^2+|\beta|^2=2n-q$. $|\psi_N\rangle$ may be obtained by N successive applications of the creation operator $\alpha a^+ +\beta b^+$ on the vacuum state $|0,0\rangle$, whereas the state (5) is derived ''from above'' by q applications of the field annihilation operators $a\pm b$ on the state $|n,n\rangle$. But, as shown by the simulations, the entanglement is sufficient to preserve the character of the state under the action of the annihilation operators $a+b$ and $a-b$ with only a small increase in the range of k values due to the increase in q in each detection. We may imagine physical situations leading to other classes of entangled states with well-defined total photon number. Closer examination of the properties of such states may add a new direction of investigation to the recently very active field of entanglement in quantum physics; see, e.g., Ref. [21] and references herein.

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