Two-level atom in an electromagnetic wave of circular polarization

Gao-Jian Zeng,[1,2] Shi-Lun Zhou,[2] Sheng-Mei Ao,[2] and Zhao-Yang Zeng[2]
[1]*CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China*
[2]*Department of Physics, Hunan Normal University, Hunan 410006, China*
(Received 16 November 1995; revised manuscript received 25 September 1996)

In this paper, we study the behavior of a two-level atom in an electromagnetic wave of circular polarization. A general solution to the Schrödinger equation is obtained, and the action of the electromagnetic wave on the atom is analyzed in detail. It is shown that the levels of the atom would be split, with the energy difference between the split levels the same for the two states of the atom. The atom would acquire an average momentum, with its direction the same as the propagating direction of the electromagnetic wave, showing the expelling action of the electromagnetic wave on the atom; or with its direction opposite to the propagating direction of the electromagnetic wave, showing the trapping action of the electromagnetic wave on the atom. The properties of the action are dependent on the initial state of the atom. The average expelling and trapping forces exerted on the atom by the electromagnetic wave are determined. Two cases of ''resonance'' and ''strict detuning'' are discussed particularly. Under the resonance case, the transition probability reaches its maximum for a given time. In the strict detuning case, the atom is not influenced by the electromagnetic wave. [S1050-2947(97)07903-1]

PACS number(s): 42.50.Vk, 32.80.−t, 03.65.Ge

I. INTRODUCTION

The subject of atomic motion in an electromagnetic wave has attracted much attention because of its important applications, such as the deflection of atomic beams, the selection of atomic species in an atomic beam (isotope separation) [1–3], the cooling and trapping of atoms, etc. [4–9].

The plane electromagnetic wave is of the simplest electromagnetic waves. Studying the motion of an atom in a plane electromagnetic wave is a fundamental and important subject. However, it seems that the study of such a subject has so far produced no good solution.

In this paper, we study the motion of an atom in an electromagnetic wave of circular polarization. This subject is also fundamental and important, since a circularly polarized electromagnetic wave may be formed from two plane electromagnetic waves; conversely, a plane electromagnetic wave may be also formed from two circularly polarized electromagnetic waves.

We assume that the atom has only two levels and assume that there is a dipole interaction between the atom and electromagnetic wave. In studying many problems on the interaction of atoms with electromagnetic waves, these assumptions have been used [1–9]. Our procedure is first to solve the Schrödinger equation, in which the motion of the center of mass of the atom is treated quantum mechanically, and then discuss in detail the physical significance of the solution, in particular calculate the momentum the atom acquires from the electromagnetic wave and the force exerted on the atom by the electromagnetic wave.

We analyze particularly the two cases of ''resonance'' and ''strict detuning,'' both of which are important. In the resonance case, the transition probability reaches its maximum (for a given time), and momentum transfer between the atom and field would be effectively completed. In the strict detuning case, the transition probability tends to 0, and the state of the atom is not influenced by the electromagnetic wave. From these results, conditions for which the two-level approximation is appropriate may be found.

The two initial conditions that the atom initially is in its ground or excited state are separately considered. The results show that under the different initial conditions, the distributions of the momenta and energies of the atom are the same, but the actions of the electromagnetic wave on the atom are different. It displays either expelling or trapping properties. That the motion of the center of mass of the atom is treated quantum mechanically is a basic reason leading to this important result.

The contents in the following sections are organized as follows. In Sec. II, the Schrödinger equation is solved and a general form of the wave function of the atom is obtained (in momentum space). In Secs. III and IV, assuming the atom is in its ground and excited states at $t=0$, respectively, the concrete forms of the wave functions of the atom are given (in momentum and coordinate spaces) and the average momentum the atom acquires from the electromagnetic wave and the average force exerted on the atom by the electromagnetic wave are determined. In Sec. V, the tuning problem and variations of the wave functions and observables with time are discussed further, the physical contents of the two solutions obtained in Secs. III and IV, respectively, are compared, and the quantum feature of the motion of the center of mass is shown. In Sec. VI, the contents of this paper are summed up and its significance is described.

II. GENERAL FORM OF THE SOLUTION

We consider a two-level atom of mass m, transition angular frequency ω, and dipole moment $\mathbf{D}$. The atom starts out moving in the z direction with momentum p_0, and then is irradiated by a circularly polarized electromagnetic wave of wave vector k and angular frequency ω_L. The electromagnetic wave propagates along the positive z direction and its electronic field $\mathbf{E}$ is assumed to be the form $\mathbf{E}=(E_1,E_2)$,

$E_1 = A\cos(\omega_L t - kz)$, $E_2 = -A\sin(\omega_L t - kz)$, where A is the amplitude of **E**.

Because the electromagnetic wave includes only the space variable z, the case is a question of one dimension. In this case, the Hamiltonian of the two-level atom interacting with the electromagnetic wave is given by

$$H = \frac{P^2}{2m} + \frac{1}{2}\hbar\omega\sigma_3 + V, \tag{1}$$

where $P^2/2m$ is the kinetic energy associated with the center-of-mass momentum along the z direction, $\frac{1}{2}\hbar\omega\sigma_3$ is the Hamiltonian associated with the internal motion of the atom, and $V = -\mathbf{D}\cdot\mathbf{E}$ is the interaction energy between the atom and electromagnetic wave. We here have used dipole approximation.

We denote the state vectors of the two-level atom by $|\psi_1\rangle$ and $|\psi_2\rangle$. According to the feature of the dipole transition [10], the actions of V on these state vectors are $V|\psi_1\rangle = v_{21}|\psi_2\rangle$ and $V|\psi_2\rangle = v_{12}|\psi_1\rangle$, where $v_{12} = \langle\psi_1|V|\psi_2\rangle$ and $v_{21} = \langle\psi_2|V|\psi_1\rangle$ are the nonzero matrix elements of V.

Setting $D^{\pm} = D_1 \pm iD_2$ and $E^{\pm} = E_1 \pm iE_2$, V may be written as

$$V = -\tfrac{1}{2}(D^+E^- + D^-E^+). \tag{2}$$

Noting $D^-_{12} = D^+_{21} = \langle\psi_1|D^-|\psi_2\rangle = 0$ and denoting $D^+_{12} = D^-_{21} = \langle\psi_1|D^+|\psi_2\rangle = 2\langle\psi_1|D_1|\psi_2\rangle$ by $2D$ (which is assumed to be real), one shows easily that $v_{12} = -DE^- = -D(E_1 - iE_2)$ and $v_{21} = -DE^+ = -D(E_1 + iE_2)$. Substituting the expressions of E_1 and E_2 into v_{12} and v_{21}, then it gets

$$v_{12} = -\tfrac{1}{2}\hbar\Omega e^{i(\omega_L t - kz)}, \quad v_{21} = -\tfrac{1}{2}\hbar\Omega e^{-i(\omega_L t - kz)}, \tag{3}$$

where we have set $DA = \hbar\Omega/2$; Ω is called induced rate, which describes the interaction intensity.

In order to study the motion of the atom, we solve the Schrödinger equation

$$i\hbar\frac{d}{dt}|\psi\rangle = H|\psi\rangle \tag{4}$$

for an arbitrary state $|\psi\rangle$. Setting $|\psi\rangle = (|\psi_1\rangle, |\psi_2\rangle)^T$, projecting Eq. (4) onto $|\psi_1\rangle$ and $|\psi_2\rangle$, and using Eqs. (1)–(3), Eq. (4) is reduced to a pair of the coupled Schrödinger equations:

$$i\hbar\frac{d}{dt}|\psi_1\rangle = \left(\frac{P^2}{2m} + \frac{1}{2}\hbar\omega\right)|\psi_1\rangle - \frac{1}{2}\hbar\Omega e^{-i(\omega_L t - kz)}|\psi_2\rangle, \tag{5}$$

$$i\hbar\frac{d}{dt}|\psi_2\rangle = \left(\frac{P^2}{2m} - \frac{1}{2}\hbar\omega\right)|\psi_2\rangle - \frac{1}{2}\hbar\Omega e^{i(\omega_L t - kz)}|\psi_1\rangle. \tag{6}$$

Both Eqs. (5) and (6) include the momentum operator P; therefore, $|\psi_1\rangle$ and $|\psi_2\rangle$ may be expanded in terms of the eigenvector $|p\rangle$ of the momentum operator P:

$$|\psi_1\rangle = \int \phi_1(p)dp|p\rangle, \quad |\psi_2\rangle = \int \phi_2(p)dp|p\rangle. \tag{7}$$

Substituting these expansions into Eqs. (5) and (6) and using the orthogonality of the eigenvectors $|p\rangle$, Eqs. (5) and (6) are reduced to

$$i\hbar\frac{d\phi_1(p)}{dt} = \left(\frac{p^2}{2m} + \frac{1}{2}\hbar\omega\right)\phi_1(p) - \frac{1}{2}\hbar\Omega e^{-i\omega_L t}\phi_2(p - \hbar k), \tag{8}$$

$$i\hbar\frac{d\phi_2(p)}{dt} = \left(\frac{p^2}{2m} - \frac{1}{2}\hbar\omega\right)\phi_2(p) - \frac{1}{2}\hbar\Omega e^{i\omega_L t}\phi_1(p + \hbar k). \tag{9}$$

Equations (8) and (9) can be decoupled and solved exactly. For example, eliminating $\phi_1(p)$ from Eqs. (8) and (9), we obtain the equation for $\phi_2(p)$:

$$\frac{d^2\phi_2(p)}{dt^2} + ia_1\frac{d\phi_2(p)}{dt} + a_2\phi_2(p) = 0, \tag{10}$$

where

$$a_1 = \frac{1}{\hbar}\left[\frac{p^2}{2m} + \frac{(p+\hbar k)^2}{2m} - \hbar\omega_L\right], \tag{11}$$

$$a_2 = \frac{1}{\hbar^2}\left\{\left(\frac{p^2}{2m} - \frac{1}{2}\hbar\omega\right)\left[\hbar\omega_L - \frac{1}{2}\hbar\omega - \frac{(p+\hbar k)^2}{2m}\right] + \frac{\hbar^2\Omega^2}{4}\right\}. \tag{12}$$

The general solution to Eq. (10) is

$$\phi_2(p,t) = c_2(p)e^{-i\omega_2(p)t} + c_2'(p)e^{-i\omega_2'(p)t}, \tag{13}$$

where

$$\omega_2(p) = \alpha_2(p) + \beta(p), \quad \omega_2'(p) = \alpha_2(p) - \beta(p), \tag{14}$$

while

$$\alpha_2(p) = \frac{a_1}{2} = \frac{1}{2\hbar}\left[\frac{p^2}{2m} + \frac{(p+\hbar k)^2}{2m} - \hbar\omega_L\right], \tag{15}$$

$$\beta(p) = \sqrt{\left(\frac{a_1}{2}\right)^2 + a_2} = \frac{1}{2\hbar}[(\Delta e_p + \hbar\Delta\omega)^2 + \hbar^2\Omega^2]^{1/2}, \tag{16}$$

with $\Delta\omega = \omega - \omega_L$ and $\Delta e_p = [(p+\hbar k)^2/2m] - p^2/2m$; $c_2(p)$ and $c_2'(p)$ are coefficients to be determined.

One easily sees that both $\alpha_2(p)$ and $\beta(p)$ are real and $\beta(p) \neq 0$. Thus there are two oscillators with frequencies $\omega_2(p)$ and $\omega_2'(p)$, respectively, in $\phi_2(p,t)$.

We have derived a solution for $\phi_2(p,t)$. Substituting the expression of $\phi_2(p,t)$ into Eq. (9), $\phi_1(p,t)$ can be obtained:

$$\phi_1(p+\hbar k,t) = c_1(p)e^{-i\omega_1(p)t} + c_1'(p)e^{-i\omega_1'(p)t}, \tag{17}$$

where

$$\omega_1(p) = \alpha_1(p) + \beta(p), \quad \omega_1'(p) = \alpha_1(p) - \beta(p), \tag{18}$$

while

$$\alpha_1(p)=\alpha_2(p)+\omega_L=\frac{1}{2\hbar}\left[\frac{p^2}{2m}+\frac{(p+\hbar k)^2}{2m}+\hbar\omega_L\right]. \tag{19}$$

The coefficients $c_1(p)$ and $c_1'(p)$ are associated with $c_2(p)$ and $c_2'(p)$:

$$c_1(p)=\frac{2[\varepsilon(p)-\beta(p)]}{\Omega}\,c_2(p), \tag{20}$$

$$c_1'(p)=\frac{2[\varepsilon(p)+\beta(p)]}{\Omega}\,c_2'(p), \tag{21}$$

where

$$\varepsilon(p)=-\frac{1}{2\hbar}\,(\Delta e_p+\hbar\Delta\omega). \tag{22}$$

Equations (13) and (17) have given a general solution for Eqs. (8) and (9), and the coefficients $c_2(p)$, $c_2'(p)$, and $c_1(p)$, $c_1'(p)$ in it may be determined from initial conditions.

$\phi_1(p,t)$ and $\phi_2(p,t)$ are in fact the wave functions of the atom in momentum space. With the help of the transformation

$$\psi(z,t)=\frac{1}{(2\pi\hbar)^{1/2}}\int_{-\infty}^{\infty}e^{ipz/\hbar}\phi(p,t)dp, \tag{23}$$

they would be reduced to the wave functions in coordinate space.

In the following sections, we will present two different initial conditions to determine the concrete forms of the solutions and discuss in detail their physical contents. The calculating processes in the two cases are the same, but their physical consequences are not totally the same. In the discussions, one needs to use two important relations, i.e.,

$$[\varepsilon(p)-\beta(p)][\varepsilon(p)+\beta(p)]=-\tfrac{1}{4}\Omega^2, \tag{24}$$

$$[\varepsilon(p)-\beta(p)]^2+[\varepsilon(p)+\beta(p)]^2=4\beta^2(p)-\tfrac{1}{2}\Omega^2, \tag{25}$$

which may be derived from the properties of the roots of an algebraic equation of degree 2.

III. SOLUTION WHEN THE ATOM IS IN ITS GROUND STATE AT $t=0$

We assume that at $t=0$ the atom is in its ground state; i.e., one has $|\psi_1\rangle_{t=0}=0$, $|\psi_2\rangle_{t=0}\neq 0$. According to this assumption, one may set $\phi_1(p,0)=0$ and $\phi_2(p,0)=\delta(p-p_0)$. The form of $\phi_2(p,0)$ means that $\phi_2(p,0)$ is the eigenstate of the momentum operator and the corresponding eigenvalue is p_0. Using this initial condition, a simple calculation yields

$$c_2(p)=\frac{\varepsilon(p)+\beta(p)}{2\beta(p)}\,\delta(p-p_0), \tag{26}$$

$$c_2'(p)=-\frac{\varepsilon(p)-\beta(p)}{2\beta(p)}\,\delta(p-p_0), \tag{27}$$

$$c_1(p)=-c_1'(p)=\frac{[\varepsilon(p)-\beta(p)][\varepsilon(p)+\beta(p)]}{\Omega\beta(p)}\,\delta(p-p_0). \tag{28}$$

Equations (13), (17), and (26)–(28) have given the solution under the above initial conditions. Using the relation (24), Eq. (28) can be simplified to

$$c_1(p)=-c_1'(p)=-\frac{\Omega}{4\beta(p)}\,\delta(p-p_0). \tag{29}$$

For convenience, we substitute Eqs. (26), (27), and (29) into Eqs. (13) and (17): then, $\phi_1(p,t)$ and $\phi_2(p,t)$ may be rewritten as

$$\phi_1(p+\hbar k,t)=\frac{\Omega}{4\beta(p)}\,[-e^{-i\omega_1(p)t}+e^{-i\omega_1'(p)t}]\delta(p-p_0), \tag{30}$$

$$\phi_2(p,t)=\frac{1}{2\beta(p)}\,\{[\varepsilon(p)+\beta(p)]e^{-i\omega_2(p)t} -[\varepsilon(p)-\beta(p)]e^{-i\omega_2'(p)t}\}\delta(p-p_0). \tag{31}$$

These are the wave functions of the atom in momentum space. If they are transformed into coordinate space, then one has

$$\psi_1(z,t)=\frac{\Omega}{(2\pi\hbar)^{1/2}4\beta(p_0)}\,\{-e^{i[(p_0+\hbar k)z-E_1(p_0)t]/\hbar} +e^{i[(p_0+\hbar k)z-E_1'(p_0)t]/\hbar}\}, \tag{32}$$

$$\psi_2(z,t)=\frac{1}{(2\pi\hbar)^{1/2}2\beta(p_0)}\,\{[\varepsilon(p_0)+\beta(p_0)]e^{i[p_0z-E_2(p_0)t]/\hbar} -[\varepsilon(p_0)-\beta(p_0)]e^{i[p_0z-E_2'(p_0)t]/\hbar}\}. \tag{33}$$

These are the wave functions of the atom in coordinate space, where $E_1(p_0)=\hbar\omega_1(p_0)$, $E_1'(p_0)=\hbar\omega_1'(p_0)$, $E_2(p_0)=\hbar\omega_2(p_0)$, and $E_2'(p_0)=\hbar\omega_2'(p_0)$. Using the expressions of $\omega_1(p_0)$, etc., then $E_1(p_0)$, etc. may be written explicitly as

$$E_1(p_0)=\frac{(p_0+\hbar k)^2}{2m}+\frac{1}{2}\,\hbar\omega_L-\frac{1}{2}\,\Delta e_0 +\frac{1}{2}\,[(\Delta e_0+\hbar\Delta\omega)^2+\hbar^2\Omega^2]^{1/2}, \tag{34}$$

$$E_1'(p_0)=\frac{(p_0+\hbar k)^2}{2m}+\frac{1}{2}\,\hbar\omega_L-\frac{1}{2}\,\Delta e_0 -\frac{1}{2}\,[(\Delta e_0+\hbar\Delta\omega)^2+\hbar^2\Omega^2]^{1/2}, \tag{35}$$

$$E_2(p_0)=\frac{p_0^2}{2m}-\frac{1}{2}\,\hbar\omega_L+\frac{1}{2}\,\Delta e_0 +\frac{1}{2}\,[(\Delta e_0+\hbar\Delta\omega)^2+\hbar^2\Omega^2]^{1/2}, \tag{36}$$

$$E_2'(p_0)=\frac{p_0^2}{2m}-\frac{1}{2}\hbar\omega_L+\frac{1}{2}\Delta e_0 - \frac{1}{2}[(\Delta e_0+\hbar\Delta\omega)^2+\hbar^2\Omega^2]^{1/2}, \quad (37)$$

where

$$\Delta e_0=\frac{(p_0+\hbar k)^2}{2m}-\frac{p_0^2}{2m}.$$

We now examine the physical content of the solution. One sees from Eqs. (32) and (33) that both $\psi_1(z,t)$ and $\psi_2(z,t)$ include two plane waves, in which the momenta of the atom are the same, but the energies of the atom are different. Explicitly, when the atom is in its ground state, its momentum is always p_0 and its energy is $E_2(p_0)$ or $E_2'(p_0)$, while when the atom is in its excited state, its momentum is always $p_0+\hbar k$ and its energy is $E_1(p_0)$ or $E_1'(p_0)$. Thus the two levels of the atom are split, the energy difference ΔE between the split levels being the same for the two states of the atom: $\Delta E=E_1(p_0)-E_1'(p_0)=E_2(p_0)-E_2'(p_0)$, and

$$\Delta E=[(\Delta e_0+\hbar\Delta\omega)^2+\hbar^2\Omega^2]^{1/2}. \quad (38)$$

(It is noted that Δe_0 is in fact a difference of the kinetic energies of the atom in its two states.)

Although the two states of the atom have definite momenta, $p_0+\hbar k$ and p_0, respectively, but the probabilities of these momenta are generally different and dependent on time. We denote the probabilities of the momenta $p_0+\hbar k$ and p_0 by $P_1=P_1(p_0+\hbar k)$ and $P_2=P_2(p_0)$, respectively. They are obtained easily from Eqs. (30) and (31) or Eqs. (32) and (33):

$$P_1=\left|\frac{\Omega}{4\beta(p_0)}[-e^{-i\omega_1(p_0)t}+e^{-i\omega_1'(p_0)t}]\right|^2=\frac{\Omega^2}{4\beta_0^2}\sin^2\beta_0 t, \quad (39)$$

$$P_2=\left|\frac{1}{2\beta(p_0)}\{[\varepsilon(p_0)+\beta(p_0)]e^{-i\omega_2(p_0)t} -[\varepsilon(p_0)-\beta(p_0)]e^{-i\omega_2'(p_0)t}\}\right|^2 =1-\frac{\Omega^2}{4\beta_0^2}\sin^2\beta_0 t, \quad (40)$$

where $\beta_0=\beta(p_0)$. It is clear that $P_1+P_2=1$, which shows that the overall probability is conserved.

Using P_1 and P_2, the average values of the momenta and kinetic energies of the atom are calculated to be

$$\overline{P}=(p_0+\hbar k)P_1+p_0P_2=p_0+\frac{\Omega^2}{4\beta_0^2}\hbar k\,\sin^2\beta_0 t, \quad (41)$$

$$\overline{E_k}=\frac{(p_0+\hbar k)^2}{2m}P_1+\frac{p_0^2}{2m}P_2=\frac{p_0^2}{2m}+\frac{\Omega^2}{4\beta_0^2}\Delta e_0\,\sin^2\beta_0 t. \quad (42)$$

$\overline{P}\neq p_0$ and $\overline{E_k}\neq p_0^2/2m$, generally, except for $t=n\pi/\beta_0$ ($n=0,1,2,\ldots$). At these times, the atom is always in its ground state, and thus $\overline{P}=p_0$ and $\overline{E_k}=p_0^2/2m$.

Setting $\overline{\Delta P}=\overline{P}-p_0$ and $\overline{\Delta E_k}=\overline{E_k}-p_0^2/2m$, it gets

$$\overline{\Delta P}=\frac{\Omega^2}{4\beta_0^2}\hbar k\,\sin^2\beta_0 t, \quad (43)$$

$$\overline{\Delta E_k}=\frac{\Omega^2}{4\beta_0^2}\Delta e_0\,\sin^2\beta_0 t. \quad (44)$$

$\overline{\Delta P}$ and $\overline{\Delta E_k}$ are the average momentum and the average kinetic energy the atom acquires from the electromagnetic wave, respectively.

It is clear that $\overline{\Delta P}\geqslant 0$. Therefore, if $p_0>0$, then $\overline{P}>p_0$, and the momentum and, thus, the kinetic energy of the atom increase. If $p_0<0$, then $|\overline{P}|<|p_0|$, and the momentum and, thus, the kinetic energy of the atom decrease.

The change of the momentum of the atom is due to the action of the electromagnetic wave on the atom. Using Ehrenfest's theorem, the average value of the force exerted on the atom by the electromagnetic wave is obtained as

$$\overline{F}=\frac{d}{dt}\overline{P}=\frac{d}{dt}\overline{\Delta P}=\frac{\Omega^2\hbar k}{4\beta_0}\sin 2\beta_0 t. \quad (45)$$

$\overline{F}$ can take positive or negative values. The positive values of F show that the average momentum of the atom increases with t, and the negative values of F show that the average momentum of the atom decreases with t.

We have shown that when $p_0>0$, $\overline{P}>p_0$; when $p_0<0$, $|\overline{P}|<|p_0|$. This shows that the direction of the average momentum the atom acquires from the electromagnetic wave is the same as the propagating direction of the electromagnetic wave, the action of the electromagnetic wave on the atom showing the expelling property.

Now we examine further the significance of the probabilities P_1 and P_2. They have been defined as the probabilities of the momenta $p_0+\hbar k$ and p_0 at time t, respectively. Because the momenta of the atom at its excited state and ground state are $p_0+\hbar k$ and p_0, respectively, therefore P_1 and P_2 are also the probabilities that the atom is in its excited state and ground state at time t, respectively. In particular, because the atom is in its ground state at $t=0$, P_1 is also the probability that the atom transits to its excited state from its ground state at time t. It is clear that the transition probability P_1 is dependent on the frequency of the electromagnetic wave. Noting expression (16) of $\beta(p)$, one sees that if the frequency of the electromagnetic wave is modulated such that

$$\Delta e_0+\hbar\Delta\omega=0, \quad (46)$$

then the transition probability P_1 would reach its maximum (for a given t); this case is called ''resonance.'' If the frequency of the electromagnetic wave is modulated such that

$$|\Delta e_0+\hbar\Delta\omega|\gg\hbar\Omega, \quad (47)$$

then the transition probability $P_1 \to 0$; this case is called ''strict detuning.'' In the following, we discuss these two cases, respectively.

A. Resonance

Under the resonance condition (46), $\beta(p_0)=\Omega/2$, $\varepsilon(p_0)-\beta(p_0)=-\Omega/2$, and $\varepsilon(p_0)+\beta(p_0)=\Omega/2$; thus, the wave functions of the atom have a simpler form

$$\psi_1(z,t)=\frac{1}{2(2\pi\hbar)^{1/2}}\{-e^{i[(p_0+\hbar k)z-E_1(p_0)t]/\hbar} + e^{i[(p_0+\hbar k)z-E_1'(p_0)t]/\hbar}\}, \tag{48}$$

$$\psi_2(z,t)=\frac{1}{2(2\pi\hbar)^{1/2}}\{e^{i[p_0 z-E_2(p_0)t]/\hbar}+e^{i[p_0 z-E_2'(p_0)t]/\hbar}\}, \tag{49}$$

where

$$E_1(p_0)=\frac{(p_0+\hbar k)^2}{2m}+\frac{1}{2}\hbar\omega+\frac{1}{2}\hbar\Omega, \tag{50}$$

$$E_1'(p_0)=\frac{(p_0+\hbar k)^2}{2m}+\frac{1}{2}\hbar\omega-\frac{1}{2}\hbar\Omega, \tag{51}$$

$$E_2(p_0)=\frac{p_0^2}{2m}-\frac{1}{2}\hbar\omega+\frac{1}{2}\hbar\Omega, \tag{52}$$

$$E_2'(p_0)=\frac{p_0^2}{2m}-\frac{1}{2}\hbar\omega-\frac{1}{2}\hbar\Omega. \tag{53}$$

They are the energies of the split levels. The energy difference between the split levels is

$$\Delta E=\hbar\Omega. \tag{54}$$

Under the resonance condition, the probabilities of the momenta $p_0+\hbar k$ and p_0 at time t are

$$P_1=\sin^2\frac{\Omega}{2}t, \quad P_2=\cos^2\frac{\Omega}{2}t, \tag{55}$$

and the average values of the momenta and kinetic energies of the atom are

$$\overline{P}=p_0+\hbar k\,\sin^2\frac{\Omega}{2}t, \quad \overline{E_k}=\frac{p_0^2}{2m}+\Delta e_0\,\sin^2\frac{\Omega}{2}t. \tag{56}$$

The average momentum and the average kinetic energy the atom acquires from the electromagnetic wave are

$$\overline{\Delta P}=\hbar k\,\sin^2\frac{\Omega}{2}t \tag{57}$$

and

$$\overline{\Delta E_k}=\Delta e_0\,\sin^2\frac{\Omega}{2}t, \tag{58}$$

respectively, and the average value of the force exerted on the atom by the electromagnetic wave is

$$\overline{F}=\frac{\Omega\hbar k}{2}\sin\Omega t. \tag{59}$$

Under the resonance condition, the energy and momentum transport between the atom and electromagnetic wave would be effectively completed, because when $t=[(2n+1)\pi]/\Omega$ $(n=0,1,2,\ldots)$, the atom must transit to its excited state from its ground state, and the average momentum the atom acquires from electromagnetic wave reaches its maximum $\hbar k$.

B. Strict detuning

In the strict detuning case (47), it is easy to show that $\Omega/\beta(p_0)\simeq 0$, $\varepsilon(p_0)-\beta(p_0)\simeq -2\beta(p_0)$, and $\varepsilon(p_0)+\beta(p_0)\simeq 0$; thus, the wave functions of the atom would be simplified in form

$$\psi_1(z,t)\simeq 0, \quad \psi_2(z,t)\simeq\frac{1}{(2\pi\hbar)^{1/2}}e^{i[p_0 z-E_2''(p_0)t]/\hbar}, \tag{60}$$

where

$$E_2''(p_0)=(p_0^2/2m)-\tfrac{1}{2}\hbar\omega,$$

which is just the total energy of the atom before it is irradiated by the electromagnetic wave. Equations (60) are just the wave functions of the atom before it is irradiated by the electromagnetic wave, and so we conclude that in the strict detuning case, the atom is not influenced by the electromagnetic wave.

IV. SOLUTION WHEN THE ATOM IS IN ITS EXCITED STATE AT $t=0$

We now change the initial condition, assuming that at $t=0$ the atom is in its excited state and its momentum is $p_0+\hbar k$ (it is the same as the momentum of the atom in its excited state in Sec. III), i.e., one has $\phi_1(p,0)=\delta(p-p_0-\hbar k)$ and $\phi_2(p,0)=0$. Using the same procedure as that used in Sec. III, one may obtain the following results. The solutions for $\phi_1(p,t)$ and $\phi_2(p,t)$ are

$$\phi_1(p+\hbar k,t)=\frac{1}{2\beta(p)}\{-[\varepsilon(p)-\beta(p)]e^{-i\omega_1(p)t} + [\varepsilon(p)+\beta(p)]e^{-i\omega_1'(p)t}\}\delta(p-p_0), \tag{61}$$

$$\phi_2(p,t)=\frac{\Omega}{4\beta(p)}[-e^{-i\omega_2(p)t}+e^{-i\omega_2'(p)t}]\delta(p-p_0). \tag{62}$$

Their transformations in coordinate space are

$$\psi_1(z,t)=\frac{1}{(2\pi\hbar)^{1/2}}\frac{1}{2\beta(p_0)} \times\{-[\varepsilon(p_0)-\beta(p_0)]e^{i[(p_0+\hbar k)z-E_1(p_0)t]/\hbar} + [\varepsilon(p_0)+\beta(p_0)]e^{i[(p_0+\hbar k)z-E_1'(p_0)t]/\hbar}\}, \tag{63}$$

$$\psi_2(z,t)=\frac{1}{(2\pi\hbar)^{1/2}}\frac{\Omega}{4\beta(p_0)}\{-e^{i[p_0z-E_2(p_0)t]/\hbar}+e^{i[p_0z-E_2'(p_0)t]/\hbar}\},\tag{64}$$

where $E_1(p_0)$, etc., are the same as those expressed in Eqs. (34)–(37).

According to Eqs. (61)–(64), when the atom is in its excited state, its momentum always is $p_0+\hbar k$ and its energy is $E_1(p_0)$ or $E_1'(p_0)$; when the atom is in its ground state, its momentum always is p_0 and its energy is $E_2(p_0)$ or $E_2'(p_0)$. These relations between the momenta and energies are the same as those in Sec. III. The probabilities of the momenta $p_0+\hbar k$ and p_0 are

$$P_1=P_1(p_0+\hbar k)=1-\frac{\Omega^2}{4\beta_0^2}\sin^2\beta_0 t\tag{65}$$

and

$$P_2=P_2(p_0)=\frac{\Omega^2}{4\beta_0^2}\sin^2\beta_0 t,\tag{66}$$

respectively. The average values of the momenta and kinetic energies of the atom are

$$\overline{P}=p_0+\hbar k-\frac{\Omega^2}{4\beta_0^2}\hbar k\,\sin^2\beta_0 t\tag{67}$$

and

$$\overline{E_k}=\frac{(p_0+\hbar k)^2}{2m}-\frac{\Omega^2}{4\beta_0^2}\Delta e_0\,\sin^2\beta_0 t,\tag{68}$$

respectively. The average momentum and average kinetic energy the atom acquires from the electromagnetic wave are

$$\overline{\Delta P}=-\frac{\Omega^2}{4\beta_0^2}\hbar k\,\sin^2\beta_0 t\tag{69}$$

and

$$\overline{\Delta E_k}=-\frac{\Omega^2}{4\beta_0^2}\Delta e_0\,\sin^2\beta_0 t,\tag{70}$$

respectively. When $p_0>0$, the momentum and, thus, the kinetic energy of the atom decrease. Conversely, if $p_0<0$, the momentum and, thus, the kinetic energy of the atom increase. The average value of the force exerted on the atom by the electromagnetic wave is

$$\overline{F}=-\frac{\Omega^2\hbar k}{4\beta_0}\sin 2\beta_0 t.\tag{71}$$

Now the direction of the average momentum the atom acquires from the electromagnetic wave is converse to the propagating direction of the electromagnetic wave, and therefore the action of the electromagnetic wave on the atom shows the trapping property.

A. Resonance

In the present case, the transition probability that the atom transits to its ground state from its excited state at time t is P_2. If the frequency of the electromagnetic wave satisfies the condition (46), i.e., $\Delta e_0+\hbar\Delta\omega=0$, then P_2 reaches its maximum (for a given time t). In this case, $\beta_0=\Omega/2$, and the wave functions of the atom have the forms

$$\psi_1(z,t)=\frac{1}{2(2\pi\hbar)^{1/2}}\{e^{i[(p_0+\hbar k)z-E_1(p_0)t]/\hbar}+e^{i[(p_0+\hbar k)z-E_1'(p_0)t]/\hbar}\},\tag{72}$$

$$\psi_2(z,t)=\frac{1}{2(2\pi\hbar)^{1/2}}\{-e^{i[p_0z-E_2(p_0)t]/\hbar}+e^{i[p_0z-E_2'(p_0)t]/\hbar}\},\tag{73}$$

where $E_1(p_0)$, etc., are the same as those expressed in Eqs. (50)–(53). The probabilities of the momenta $p_0+\hbar k$ and p_0 are

$$P_1=\cos^2\frac{\Omega}{2}t,\quad P_2=\sin^2\frac{\Omega}{2}t.\tag{74}$$

The average values of the momenta and kinetic energies of the atom are

$$\overline{P}=p_0+\hbar k-\hbar k\,\sin^2\frac{\Omega}{2}t,$$

$$\overline{E_k}=\frac{(p_0+\hbar k)^2}{2m}-\Delta e_0\,\sin^2\frac{\Omega}{2}t,\tag{75}$$

and the average momentum and average kinetic energy the atom acquires from the electromagnetic wave are

$$\overline{\Delta P}=-\hbar k\,\sin^2\frac{\Omega}{2}t\tag{76}$$

and

$$\overline{\Delta E_k}=-\Delta e_0\,\sin^2\frac{\Omega}{2}t,\tag{77}$$

respectively. The average value of the force exerted on the atom by the electromagnetic wave is

$$\overline{F}=-\frac{\Omega\hbar k}{2}\sin\Omega t.\tag{78}$$

Under the resonance condition, the energy and momentum transport between the atom and the electromagnetic wave would be effectively completed as well.

B. Strict detuning

If the frequency of the electromagnetic wave satisfies Eq. (47), i.e., $|\Delta e_0+\hbar\Delta\omega|\gg\hbar\Omega$, the transition probability $P_2\to 0$. In this case, the wave functions of the atom have the forms

$$\psi_1(z,t)\simeq\frac{1}{(2\pi\hbar)^{1/2}}\,e^{i[(p_0+\hbar k)z-E_1''(p_0)t]/\hbar},\quad \psi_2(z,t)\simeq 0, \tag{79}$$

where

$$E_1''(p_0)=\frac{(p_0+\hbar k)^2}{2m}+\frac{1}{2}\,\hbar\omega,$$

which is just the total energy of the atom in its excited state before it is irradiated by the electromagnetic wave. Equations (79) are the wave functions of the atom before it is irradiated by the electromagnetic wave; it shows also that in the strict detuning case the atom is not influenced by the electromagnetic wave.

V. DISCUSSION

We have used two different initial conditions to obtain two different solutions and discussed their physical contents in detail. In this section, we show still the following points.

A. Variation of transition probability with frequency ω_L

We have shown that when the ground state of the atom is taken as the initial state, the transition probability is $P_1(p_0+\hbar k)$, but if the excited state of the atom is taken as the initial state, the transition probability is $P_2(p_0)$. They have the same form. Now we denote unitedly the transition probability by P_t, i.e., set

$$P_t=\frac{\Omega^2}{4\beta^2(p_0)}\sin^2\beta(p_0)t=f\,\sin^2\beta(p_0)t \tag{80}$$

and make further analysis for P_t, where $f=\Omega^2/[4\beta^2(p_0)]$, which represents the maximum P_t can reach when it varies with t.

Substituting Eq. (16) into f, one has

$$f=\frac{\Omega^2}{[(\Delta e_0/\hbar)+\Delta\omega]^2+\Omega^2}. \tag{81}$$

f varies with ω_L. In general, $f<1$. When condition (46), i.e., $\Delta e_0+\hbar\Delta\omega=0$, is satisfied, f reaches its maximum 1. When condition (47), i.e., $|\Delta e_0+\hbar\Delta\omega|\gg\hbar\Omega$, is satisfied, f tends to 0.

Substituting further the expression for Δe_0 into Eq. (81) and noting $mc^2\gg\hbar\omega_L$ for an arbitrary ω_L, then f may be reduced to

$$f=\frac{\Omega^2}{(\omega-s\omega_L)^2+\Omega^2}, \tag{82}$$

where $s=1-(p_0/mc)$. According to Eq. (82), the condition that f reaches its maximum also may be written as

$$\omega-s\omega_L=0. \tag{83}$$

In general, $mc\gg p_0$, $s\simeq 1$, and thus $\omega_L\simeq\omega$. If p_0 is greater, then there would be some difference between ω_L and ω. For example, if $s=0.9$, then $\omega_L=\omega/0.9$. On the other hand, if

$$(\omega-s\omega_L)^2\gg\Omega^2, \tag{84}$$

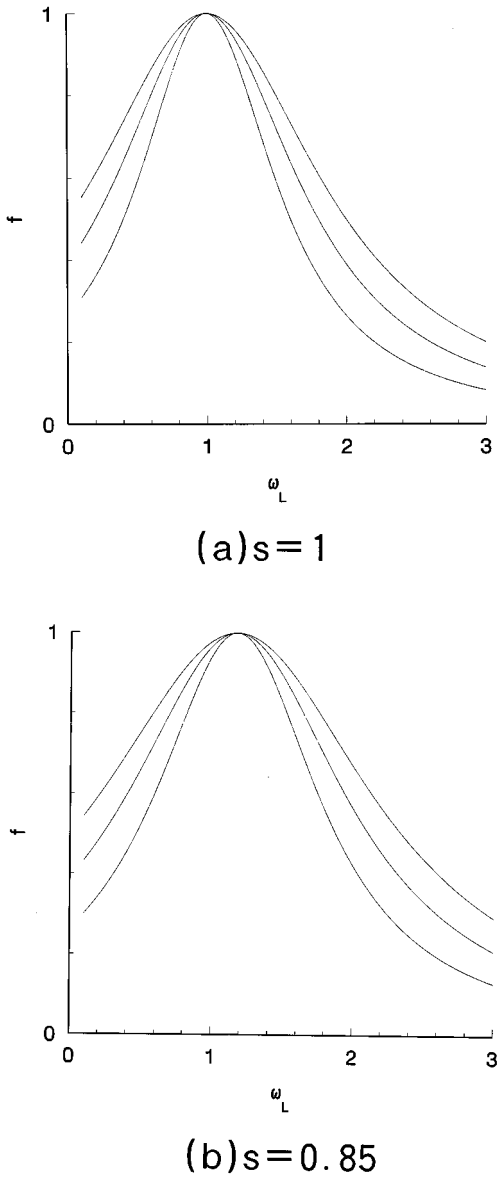

FIG. 1. This figure plots the variation of f with ω_L, in which $\omega=1,\Omega=1,0.8,0.6$ (from up to down). The units of ω_L,ω and Ω are 10^{16} $\sec^{-1}$.

then f tends to 0.

For the transition probability under the resonance condition, which is very important, we now give a sum [noting $\beta(p_0)=\Omega/2$ in this case].

Theorem 1. The transition probability P_t of the two-level atom interacting with the electromagnetic wave of circle polarization reaches its maximum (for a given time t) under the resonance condition $\omega-s\omega_L=0$ and

$$P_t=\sin^2\frac{\Omega}{2}\,t. \tag{85}$$

The transition probability in the general case is given in Eqs. (80) and (82). In Fig. 1, we plot the variation of f with ω_L. The curves are dependent on the parameter s and the interaction intensity Ω. The width $\Delta\omega_L$ of a curve may be

defined as the distance between the two points ω_L' and ω_L'' at which $f=\frac{1}{2}$, i.e., $\Delta\omega_L=\omega_L''-\omega_L'$. From Eq. (82), one easily obtains

$$\Delta\omega_L=\frac{2\Omega}{s}. \tag{86}$$

It is clear that the greater s, the smaller Ω, and then the narrower the curve.

B. Variations of wave functions and observables with time t

The results in Secs. III and IV show that the probabilities P_1 and P_2, and the measurement values for an observable, for example, $\overline{\Delta P}$, $\overline{\Delta E_k}$, and $\overline{F}$, etc., vary with time. The variations appear in an oscillator form, and the oscillating frequency depends on only β_0. Under the resonance condition, the oscillating frequency depends on only Ω, the interaction intensity, since $\beta_0=\Omega/2$ in this case. (Because $\hbar\Omega=2DA$, therefore, the oscillating frequency in turn depends on the dipole moment of the atom and the intensity of the electromagnetic wave.) If the interaction intensity is weaker, the oscillator is slower.

This feature of observables comes from the feature of the wave functions. From Eqs. (48), (49), (72), and (73), one sees that under the resonance condition, the wave functions may be reduced to the forms

$$\psi_1(z,t)=\frac{i}{(2\pi\hbar)^{1/2}}\sin\frac{\Omega}{2}\,te^{i[(p_0+\hbar k)z-E_1''(p_0)t]/\hbar}, \tag{87}$$

$$\psi_2(z,t)=\frac{1}{(2\pi\hbar)^{1/2}}\cos\frac{\Omega}{2}\,te^{i[p_0z-E_2''(p_0)t]/\hbar}, \tag{88}$$

when the ground state is taken as the initial state, and

$$\psi_1(z,t)=\frac{1}{(2\pi\hbar)^{1/2}}\cos\frac{\Omega}{2}\,te^{i[(p_0+\hbar k)z-E_1''(p_0)t]/\hbar}, \tag{89}$$

$$\psi_2(z,t)=\frac{i}{(2\pi\hbar)^{1/2}}\sin\frac{\Omega}{2}\,te^{i[p_0z-E_2''(p_0)t]/\hbar}, \tag{90}$$

when the excited state is taken as the initial state, where

$$E_1''(p_0)=\frac{(p_0+\hbar k)^2}{2m}+\frac{1}{2}\hbar\omega$$

and

$$E_2''(p_0)=\frac{p_0^2}{2m}-\frac{1}{2}\hbar\omega.$$

They are all the sum of the center-of-mass kinetic energy and internal energy. Equations (87)–(90) show that the center-of-mass kinetic energy and internal energy influence the phases of the wave functions, but do not influence the absolute values of the wave functions. The latter are dependent on only the interaction intensity Ω, and thus the probability densities $\rho_1=|\psi_1|^2$ and $\rho_2=|\psi_2|^2$ and the measurement values of observables are dependent also on only the interaction intensity Ω.

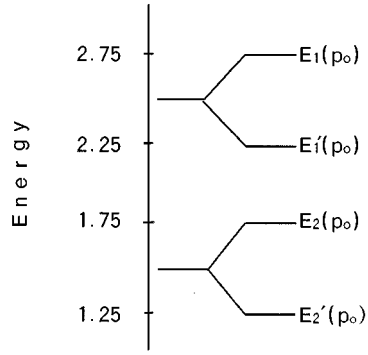

FIG. 2. The energies of the split levels, calculated in units 10^{-18} J.

C. Physical consequences and initial conditions

Now we compare the solutions obtained under two different initial conditions. This is important and interesting.

In the two solutions, the distributions of the momenta of the atom in its two states are the same: If the momentum of the atom in its ground state is p_0, then the momentum of the atom in its excited state must be $p_0+\hbar k$. This result does not depend on the initial conditions. In fact, it does not depend on tuning as well.

In the two solutions, the distributions of the energies of the atom in its two states are also the same: Each level is split into two levels; the level in the excited state is split into $E_1(p_0)$ and $E_1'(p_0)$, and the level in the ground state is split into $E_2(p_0)$ and $E_2'(p_0)$. The energy differences between the two split levels are the same for the two states of the atom. In particular, under the resonance condition, the energies of the split levels in the excited state are

$$E_1(p_0)=\frac{(p_0+\hbar k)^2}{2m}+\frac{1}{2}\hbar\omega+\frac{1}{2}\hbar\Omega, \tag{91}$$

$$E_1'(p_0)=\frac{(p_0+\hbar k)^2}{2m}+\frac{1}{2}\hbar\omega-\frac{1}{2}\hbar\Omega, \tag{92}$$

and the energies of the split levels in the ground state are

$$E_2(p_0)=\frac{p_0^2}{2m}-\frac{1}{2}\hbar\omega+\frac{1}{2}\hbar\Omega, \tag{93}$$

$$E_2'(p_0)=\frac{p_0^2}{2m}-\frac{1}{2}\hbar\omega-\frac{1}{2}\hbar\Omega. \tag{94}$$

These energies are all the algebraic sum of the center-of-mass kinetic energy, internal energy, and interaction energy; the energy difference between the two split levels in each state is

$$\Delta E=\hbar\Omega. \tag{95}$$

Since Ω is dependent on the amplitude of the electric field, the split of the levels in fact represents a kind of ac Stark effect.

Figure 2 gives a concrete example, in which a moderately massive atom of mass $m\simeq10^{-23}$ G, transition angular frequency $\omega=10^{16}$ sec^{-1}, original velocity $v_0\simeq20$ m sec^{-1}, and

dipole moment $D \simeq 1.6\times 10^{-29}$ C m, and a circularly polarized electromagnetic wave of amplitude $A \simeq 1.5\times 10^{10}$ V m^{-1} and angular frequency $\omega_L \simeq \omega = 10^{16}$ sec^{-1} are considered. When the atom is irradiated by the electromagnetic wave, the system would reach its resonance state, since $p_0 \simeq 2\times 10^{-19}$ N sec, $mc \simeq 3\times 10^{-12}$ N sec, $mc \gg p_0$; the resonance condition is $\omega_L \simeq \omega$, which is satisfied. In this case, the energies of the split levels are $E_1(p_0) \simeq 2.75\times 10^{-18}$ J, $E_1'(p_0) \simeq 2.25\times 10^{-18}$ J, $E_2(p_0) \simeq 1.75 \times 10^{-18}$ J, and $E_2'(p_0) \simeq 1.25\times 10^{-18}$ J. The energy difference between the two split levels is $\Delta E = 0.50\times 10^{-18}$ J for each state of the atom.

However, in the two solutions, the wave functions of the atom are not the same, and the momentum and energy transfers between the atom and the electromagnetic wave are different as well. In particular, under the resonance condition, the average momentum the atom acquires from the electromagnetic wave and the average force exerted on the atom by the electromagnetic wave are

$$\overline{\Delta P} = \tfrac{1}{2}\hbar k(1-\cos\Omega t), \quad \overline{F} = \tfrac{1}{2}\hbar k\Omega \sin\Omega t, \qquad (96)$$

if the ground state is taken as the initial state, and

$$\overline{\Delta P} = -\tfrac{1}{2}\hbar k(1-\cos\Omega t), \quad \overline{F} = -\tfrac{1}{2}\hbar k\Omega \sin\Omega t, \quad (97)$$

if the excited state is taken as the initial state. There is a difference of a sign between the two results, which means that when the initial state of the atom is its ground state, the direction of the average momentum the atom acquires from the electromagnetic wave is the same as the propagating direction of the electromagnetic wave; this shows the expelling action of the electromagnetic wave on the atom. When the initial state of the atom is its excited state, the direction of the average momentum the atom acquires from the electromagnetic wave is opposite to the propagating direction of the electromagnetic wave; this shows the trapping action of the electromagnetic wave on the atom. The property of the action of the electromagnetic wave on the atom is dependent on the initial state of the atom.

The variations of $\overline{\Delta P}$ and $\overline{F}$ with time t depend on Ω. In the above example, the angular frequencies corresponding to $E_1(p_0)$, etc., are $\omega_1(p_0) \simeq 2.75\times 10^{16}$ sec^{-1}, $\omega_1'(p_0) \simeq 2.25 \times 10^{16}$ sec^{-1}, $\omega_2(p_0) \simeq 1.75\times 10^{16}$ sec^{-1}, and $\omega_2'(p_0) \simeq 1.25 \times 10^{16}$ sec^{-1}, respectively, while $\Omega \simeq 0.5\times 10^{16}$ sec^{-1}. Comparing Ω with $\omega_1(p_0)$, etc., one sees that the variations of $\overline{\Delta P}$ and $\overline{F}$ with time are slower than that of the wave functions.

D. Quantum feature of the motion of the center of mass

We have treated the motion of the center of mass quantum mechanically. Now we show the physical consequence of this treatment. As in the above description, the momenta of the atom in its two states are not the same, and there is a difference of $\hbar k$ between them. For this difference, the average momentum of the atom varies with time, and the average force exerted on the atom by the electromagnetic wave does not equal generally 0. The force is proportional to $\hbar k$ and appears in different signs under different initial conditions, which show different properties of the action under different initial conditions. These results mirror sufficiently the quantum features of the motion of the center of mass. They come from the quantum-mechanical treatment for the motion of the center of mass. If one sets $\hbar \to 0$ (which means the transition to classical case), then the above quantum features would vanish.

The above result—that the force exerted on a two-level atom by an electromagnetic wave of circle polarization does not equal 0 generally and is dependent on the initial state of the atom—is very important and may be generalized to a system interacting with any electromagnetic running wave; therefore, it is fundamental for studying more complicated cases.

It is useful to compare our work with Ref. [11], which concluded that if spontaneous emission is not considered, then the force exerted on a two-level atom by a plane electromagnetic wave equals 0. We have not considered spontaneous emission, but our result clearly is not the same as this conclusion. In considering this difference, we recall that Ref. [11] used the wave function derived for the case that the kinetic energy term in the Hamiltonian was dropped; in other words, there the motion of the center of mass of the atom was not treated quantum mechanically.

The quantum feature of the motion of the center of mass is observable. One has already found experimentally that when an atomic beam is irradiated by an electromagnetic standing wave, it would be split and deflected [1–3]. This is a kind of momentum diffusion phenomenon. When the motion of the center of mass is treated quantum mechanically, this phenomenon can be explained theoretically. In the problem studied in this paper, the average force exerted on the atom by the electromagnetic wave is very small, and so the average momentum the atom acquires from the electromagnetic wave is also very small. However, if we consider a system (gas, liquid, or solid) which consists of many atoms, the action of the electromagnetic wave on the system would generate an observable (macroscopic) effect, provided the two-level atoms in the system are initially prepared in a definite state—their ground states or their excited states.

In our work, although the electromagnetic wave is treated classically, the momentum exchange between the atom and the field occurs in a definite quantity $\hbar k$. This result, which comes from the quantum-mechanical treatment for the motion of the center of mass, is very interesting.

VI. CONCLUSION

We have studied the motion of a two-level atom in an electromagnetic wave of circle polarization, given a general solution, and discussed in detail its physical significance.

In studying the force exerted on an atom by an electromagnetic wave, the deflection of an atomic beam in an electromagnetic standing wave, and other problems, one usually separates first high-frequency oscillating factor from the wave function, then omits the high-frequency oscillating terms in the result. This approximation method (which could possibly lose some useful information) has not been used in this paper, because it is not needed. For this reason, one can think that our solution is exact.

At the beginning of this paper, we have assumed that the

atom has only two levels. From the previous sections one may find the condition in which the two-level approximation may be used for a many-level atom. For example, we consider a three-level atom with the internal levels E_1, E_2, and E_3, and assume that there is dipole transition between its levels 1 and 2, and 1 and 3, the corresponding transition angular frequencies are $\omega_{21}=(E_2-E_1)/\hbar$ and $\omega_{31}=(E_3-E_1)/\hbar$, respectively. If ω_{21}, ω_{31}, and the laser angular frequency ω_L satisfy $\omega_{21}-\omega_L\simeq 0$ and $(\omega_{31}-\omega_L)^2 \gg \Omega_{31}$, where Ω_{31} is the induced rate between levels 3 and 1 (for simplicity s here is assumed $\simeq 1$), then, the atom, which initially is in level 1, could transit to level 2, but cannot transit to level 3 from level 1. In this case, level 3 may be ignored, and thus the three-level atom may be treated as a two-level atom. These arguments may be generalized clearly to any many-level atom.

A plane wave may be formed from two circularly polarized waves, while an arbitrary wave may be expanded in terms of plane waves. Therefore, in studying generally the interaction of atom with electromagnetic waves, one may find that our work would be useful.

[1] M. H. Mittleman, K. Rubin, R. H. Callender, and J. I. Gersten, Phys. Rev. A **16**, 583 (1977).
[2] R. J. Cook and A. F. Bernhardt, Phys. Rev. A **18**, 2533 (1978).
[3] E. Arimondo, A. Bambini, and S. Stenholm, Phys. Rev. A **24**, 898 (1981).
[4] D. J. Wineland and Wayne M. Itano, Phys. Rev. A **20**, 1521 (1979).
[5] S. Stenholm, Rev. Mod. Phys. **58**, 699 (1986).
[6] J. Dalibard and C. Cohen-Tannoudji, J. Opt. Soc. Am. B **6**, 2023 (1989).
[7] J. I. Cirac, L. J. Garay, R. Blatt, A. S. Parkins, and P. Zoller, Phys. Rev. A **49**, 421 (1994).
[8] P. Marte, R. Dum, R. Taïeb, P. Zoller, M. S. Shahriar, and M. Prentiss, Phys. Rev. A **49**, 4826 (1994).
[9] R. Taïeb, R. Dum, J. I. Cirac, P. Marte, and P. Zoller, Phys. Rev. A **49**, 4876 (1994).
[10] Amnon Yariv, *Quantum Electronics* (Wiley, New York, 1989).
[11] R. J. Cook, Phys. Rev. A **20**, 224 (1979).