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Experimental dynamical variables of a chaotic CO_2 laser with saturable absorber

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A weak continuous beam of a CO_2 laser was used to probe the time changing gain and absorption inside the cavity of a passive Q-switching CO_2 laser with SF_6 gas in an intracavity saturable absorber cell. The probe variations detected simultaneously with the laser intensity pulses gave experimental projection planes of the chaotic system attractor. [S1050-2947(97)01002-0]

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The single-mode CO_2 laser with an intracavity saturable absorber (LSA) has been extensively studied in terms of its dynamical instabilities [1–7]. Various models have been introduced to describe the LSA, all within the rate-equation approximation for homogeneously broadened gain and absorbing media. The common point in these models is the adiabatic elimination of the polarization of the amplifying and absorbing media [2]. The light intensity is therefore used as a dynamical variable along with the population differences of the material media inside the cavity. The single-mode operation of the laser permits the treatment with space-independent dynamics. Accordingly, the mean intensity $I(t)$, the mean gain medium population inversion $U(t)$, the mean absorber population difference $\overline{U}(t)$, and another variable $W(t)$, which represents an effective population describing the three-level population conservation of the gain medium, are the independent variables modeling this laser. These dynamical variables undergo time changes and their measurements allow the direct determination of the attractors of the laser dynamics. Varying a control parameter of a LSA, such as the gain medium current or the cavity tuning [7], produces cascades of period doubling and alternating periodic and chaotic passive Q-switching pulsations as the system approaches Shil'nikov homoclinic orbits to a saddle cycle [4,5].

Experimentally, to our knowledge, this type of laser has only been studied by the detection of its output intensity. However, it is known that in these lasers, for a wide range of dynamical operation, the pulses have nearly zero value for long-time intervals. Meanwhile, the other variables may have significant time variation. Here we report the observation of other dynamical variables of a single-mode CO_2 laser with intracavity saturable absorber (LSA) using a weak probe beam generated by another cw operated laser. The amount of absorption or amplification of this probe will follow the corresponding time variation of the associated variable in the main laser. The results were applied to verify features of the dynamics such as the adiabatic elimination of the absorber variable, in the high-pressure regime, and the correlation dimension of attractors reconstructed from different variables.

The LSA consisted of a 75-cm-long gain tube containing a flowing gas mixture of CO_2, N_2, and He in the proportion 1:1:3, respectively, with a total average pressure of 7 Torr and a 5-cm-long cell containing SF_6, mixed to a buffer gas (CO_2 at room temperature), as saturable absorber. The Fabry-Pérot optical cavity had 150 cm and was formed by a 150-lines/mm grating with 2% output coupling and a 80% reflector 5-m-radius coated germanium mirror. This mirror was mounted on a piezoelectric transducer (PZT) to enable the tuning of the laser frequency, i.e., the variation of the gain and so the changing of the dynamical behavior. An overall round-trip cavity loss estimated as 40% represented a 6.5-MHz cavity relaxation rate. The cw laser, without absorber, gave typically 1-W peak power output through the germanium mirror and tuned 60 MHz over each selected line [8]. Assuming a plane-wave approximation of a 0.5-cm-diameter beam, the inside cavity intensity at line center was 25 $\mathrm{W/cm}^2$.

The probe beam was generated by another CO_2 laser. Without an intracavity cell, this laser gave monomode tunable cw output stable to within 100 kHz. Following the standard scheme, its PZT tuning could cover near 60 MHz over each of the many lines that could be selected by the grating of the optical cavity. The probe beam was attenuated to less than 10 mW to ensure that no effect on the dynamics of main laser was produced by its presence [9–11]. Two $\mathrm{Hg}_{1-x}\mathrm{Cd}_x\mathrm{Te}$ photodiodes, with 3-MHz bandwidth amplifiers, detected simultaneously the intensity of the main laser and the probe transmitted through its gain medium, measuring the amplifier population inversion, or through the absorber cell, measuring the absorber population difference. Data series were then collected with a two-channel eight-bit analog-to-digital oscilloscope. The LSA operated on the $10P(18)$ line, which is resonant with the $A_2^1P(33)$ transition of the SF_6 molecular gas [12]. This line has 30-MHz Doppler width, 0.5-$\mathrm{cm}^{-1}\,\mathrm{Torr}^{-1}$ linear absorption coefficient, a 17-MHz Torr^{-1} homogeneous broadening linewidth, and an 11-$\mathrm{W\,cm}^{-2}\,\mathrm{Torr}^2$ effective saturation intensity at low pressure, in the inhomogeneous broadening regime [13].

The gain tube was long compared to its diameter (≈ 1

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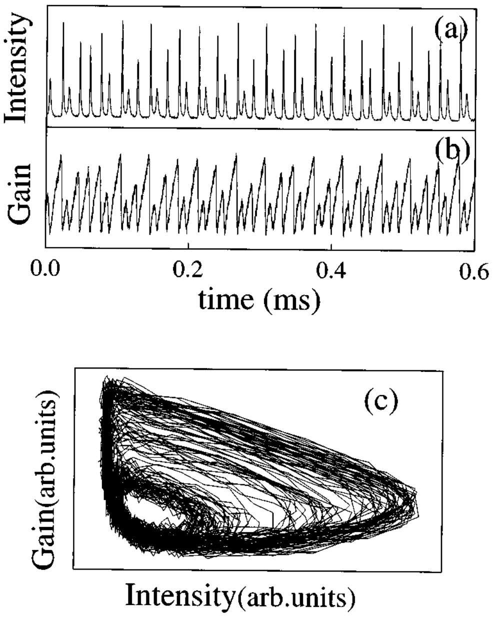

FIG. 1. Experimental (a) intensity and (b) gain of the LSA in a chaotic $C^{(1)}$ regime and (c) the projection of the attractor on the (intensity, gain) subspace.

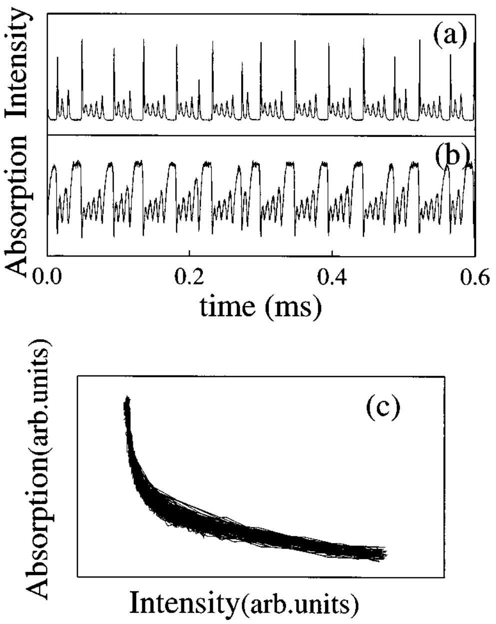

FIG. 2. Experimental (a) intensity and (b) absorption of the LSA in a chaotic $C^{(3)}$ regime. In (c) the projection of the attractor on the (intensity, absorption) subspace shows the absorption following adiabatically the intensity of the LSA.

cm), so the probe beam could not go through it without crossing the absorbing cell. The fast rotational relaxation rate ($\tau < 0.1$ μsec) of the CO_2 gain mixture allows the measurement of the gain of the $10P(18)$ line by probing any of the 10-μm lines. To avoid SF_6 absorption the probe laser line was $10P(28)$. This absorber probe beam entered the cavity through the germanium mirror and was detected after the grating output, thus being spatially separated from the LSA beam. The line-center small signal gain of our LSA amplifier, for the probe line, was about 70%. With the LSA operating in a cw regime, this gain reduced to about 65%. Such a reduction in $10P(28)$ line gain is an indication of the rotational coupling among the CO_2 lines. The rotational partition of population in CO_2 lasers has a ratio of approximately 1.3 between the $P(18)$ and $P(28)$ lines [14] and so we estimate a 90% gain for the $P(18)$ line (180% round-trip). The cw operation gain became equal to the 40% cavity loss for a 30-MHz detuning corresponding to a Lorentzian homogeneously broadened gain with 16 MHz of half-width at half maximum linewidth. The pressure broadening of typical mixtures [14] at 7 Torr would give 26 MHz. All these measurements will be used below to compare with the parameters of the three-level–two-level model.

During the pulsed operation the degree of amplification varied from below the estimated 40% round-trip cavity loss, with the laser below oscillation threshold, to values near the small signal gain, just before a big pulse spike fired. The absence of coherent effects involving the media polarizations prevented changes of sign of the gain and the absorber. The time evolution of the LSA intensity and the gain population inversion when the LSA was in a chaotic $C^{(1)}$ (following the notation of [4]) regime are shown in Figs. 1(a) and 1(b), respectively. The projection of the attractor in the subspace of these two dynamical variables is given in Fig. 1(c). Notice that the hole in the projected trajectory is well preserved despite the noise in the signal. Such a condition is essential for the use of the data to study the topology of the attractor [6].

Probing the absorber population difference was done with the $P(18)$ line itself. The small cell permitted a probe beam misalignment (10 mrad) to avoid both crossing the amplifier and resonating in the optical cavity. For this probe of the LSA we observed that during the emission spike of the light pulse the absorber is in general highly saturated and thus becomes almost completely transparent. The pulse shapes for the intensity and the transmission through the absorber are in Figs. 2(a) and 2(b), respectively, when there was 100 mTorr of SF_6 plus 500 mTorr of CO_2 acting as a buffer in the absorber cell. Figure 2(c) shows the projection of the attractor in the subspace of these two dynamical variables. A nonlinear functional relation, corresponding to the quasi-steady-state saturation of the absorber, is observed between the two variables. The absorber variable can be written as a function of the intensity, with the elimination of the Eq. (4), given below. This adiabatic elimination was discussed by Lefranc *et al.* [4], Dangoisse *et al.* [15], and Zambon [5] and is clearly demonstrated here. For lower absorber pressures the projections of the attractor did show projections with holes as in Fig. 2(c), demonstrating that the absorber population difference was an independent variable.

To describe the LSA the two-level–three-level rate equations with the variables explained above are [4]

$$\dot{I}=I(U-\bar{U}-1), \tag{1}$$

$$\dot{U}=\epsilon[W-U(1+I)], \tag{2}$$

$$\dot{W}=\epsilon(A+bU-W), \tag{3}$$

$$\dot{\bar{U}}=\bar{\epsilon}[\bar{A}-\bar{U}(1+aI)]. \tag{4}$$

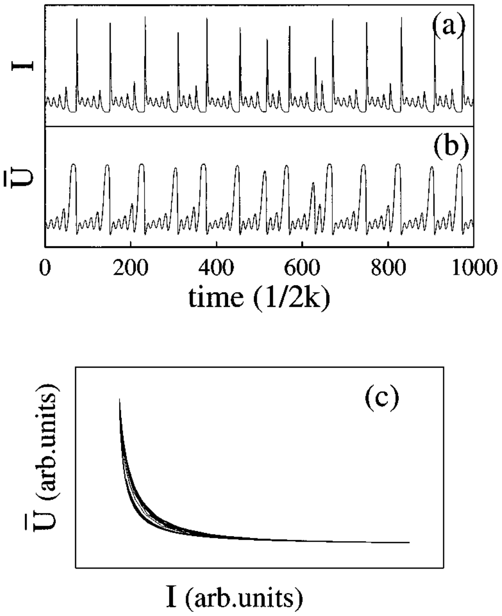

FIG. 3. Numerical results of the (a) intensity and (b) absorption of the LSA in a chaotic regime. In (c) the projection of the attractor on the (intensity, absorption) subspace, to be compared with Fig. 2, shows the absorption following adiabatically the intensity of the LSA. The time scale is in units of cavity damping rate [4].

The parameters in the equations are ϵ and $\bar{\epsilon}$, the relaxation rates of the amplifier and absorber, respectively, normalized to the cavity relaxation rate (which is the inverse time unit). A and $\bar{A}$ are the pumping rates in the amplifier and absorber population differences, respectively, normalized to the cavity loss. The coefficient b is the difference between the population relaxation rates of the lower and the upper level of the gain transition, normalized to the sum of these relaxation rates. Thus $0<b<1$ and $b=0$ for the gain medium in the two-level limit. The saturability coefficient for the absorber, normalized to the two-level limit saturability of the gain, is a. A more detailed explanation of these dimensionless quantities and the physical behavior contained in Eqs. (1)–(4) are given in Ref. [4]. The steady-state solution of these equations shows that $A/(1-b)$ is the small signal gain, normalized to the cavity loss, and $a(1-b)$ is the relative saturability between the absorber and gain media.

The choice of parameters to numerically solve the equations was done by inspection of the resulting integration and comparison with the experimental behavior of the laser. For instance, taking the relaxation rate of the absorber greater than the inverse of the cavity damping rate, we can reach the regime where the population of the absorber follows adiabatically the intensity. This result is shown in Fig. 3 and describes well the experimental results of Fig. 2. The numerical parameters used were $A=3.155$, $\bar{A}=3.315$, $\epsilon=0.151$, $\bar{\epsilon}=1.800$, $a=2.719$, and $b=0.847$.

The reported measurements made for the laser can give approximate values for some of the equation parameters. The round-trip small signal gain at resonance and the cavity loss having a ratio 190/40 will correspond to $A/(1-b)=4.7$ at line center. This value should drop with frequency detuning following a Lorentzian with the homogeneous width of the gain medium. The numerical calculation above had a much bigger value, 19.3. The experimental small signal gain together with the inside cavity intensity also gives the amplifier saturation intensity I_{SG}. Substituting this into the threshold condition $G/(1+I/I_{\mathrm{SG}})=1$, where G is the gain, results in $I_{\mathrm{SG}}=6.8\ \mathrm{W/cm}^2$. The saturation intensity in other reported work (for lasers with higher gas mixture pressure) was 40 $\mathrm{W/cm}^2$ [5] and 18 $\mathrm{W/cm}^2$ [14]. Assuming the same collision rate for CO_2-SF_6 as for SF_6 self-collision and considering the homogeneous broadening regime, the saturation intensity for the absorber at 600 mTorr is $I_{\mathrm{SA}}=6\ \mathrm{W\ cm}^{-2}$. So the ratio $I_{\mathrm{SA}}/I_{\mathrm{SG}}=a(1-b)=1.3$. For comparison, the numerical value used was 0.42. The other parameters of the equations were the relaxation rates, which can be obtained from the gain and absorber pressure once we normalize to the 6.5 MHz of the cavity rate. For the gain the width of 16 MHz gives $\epsilon=2.5$. The absorber, assuming again the same rate for buffer and self-broadening on the SF_6, had a 10-MHz rate, giving $\bar{\epsilon}=1.5$. The respective numerical values were 0.151 and 1.8. Here one sees that our measured values suffer the same discrepancies verified in previous work [3,7]. The most serious is the need of a much higher numerical gain as compared to the experimental ones. All comparisons above are highly qualitative for the detuning from line center was not taken in account in estimating experimental gain or saturation intensities.

The correlation dimension [16] of an attractor should be the same whatever the time series from one single variable of the system. However, different variables may produce different convergence rates for the calculation when one of them has a larger number of points in the series having almost the same value. Such question was discussed by Lefranc *et al.*[17] in experiments with a CO_2 laser with modulated loss. Using a logarithmic amplifier in detecting the intensity signals, they increase the separation between data points that were close to zero, thus improving their correlation calculations. To investigate the applicability of our experimental results in such convergence rate, we calculated the correlation dimension, using the Grassberger-Procaccia algorithm [16], for the attractor of our LSA. Reconstruction of the attractors was done by using the method of delays on equal length time series collected simultaneously for different variables.

In our measurements, opposite to the intensity, the gain variable changes almost all the time. Reconstructing the attractor and calculating metric properties from this variable should give better convergence. We used series of 8000 experimental data points like the ones partially shown in Fig. 1 to calculate correlation integrals for the intensity and gain variables of the LSA. Logarithmic plots of the correlation integrals and their derivatives were established for embedding dimensions from 2 to 14. An equivalent calculation was done for simultaneous series of the intensity and absorption variables. The range where correlation dimension could be determined was affected by the noise of the measured signals. In spite of this, the value obtained from the experimental time series for both the absorption and gain variables was 1.6 ± 0.3. This value is in agreement with 1.60 ± 0.05 obtained from the numerically generated data resulting from Eqs. (1)–(4) and verified for the four variables. The numerical solution giving such agreement had the parameters chosen to ensure qualitative agreement with the experimental time series, like in Figs. 3 and 2. All the comparison above could only be made in a narrow range of the radius of the m-dimensional sphere in the reconstructed space. Unfortunately, completely new improvements in the experimental apparatus would be necessary to get a wider range of convergence. To deal with the noise contribution to the experimental result one needs to use longer time series. Furthermore, higher than eight-bit analog-to-digital conversion for the data series would be necessary to improve the signal-to-noise ratio in the variables detection.

In conclusion, we have observed the dynamical variables associated with absorption and gain during chaotic operation of a laser with a saturable absorber. Noise in the weak probe detection technique limited the accuracy of the measured gain and absorption variables, but distinct projections of the chaotic attractors could be obtained. With a high-pressure absorber we verified explicitly that the population difference of the absorber follows adiabatically the intensity. Experimental data series of the different variables were shown to give a correlation dimension of 1.6 ± 0.3, in agreement with the numerical value of 1.60 ± 0.05 calculated from the three-level–two-level rate equations. Investigation on how far the power of such a probe can be increased without affecting the dynamics of the system is under way in the context of dynamics of coupled laser systems [11]. Further properties of this [6] and other laser systems should be better elucidated by simultaneous measurement of dynamical variables using the probing technique.

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[1] M. Tachikawa, F. L. Hong, K. Tanii, and T. Shimizu, Phys. Rev. Lett. **60**, 2266 (1988).
[2] D. Dangoisse, A. Bekkali, F. Papoff, and P. Glorieux, Europhys. Lett. **6**, 335 (1988).
[3] B. Zambon, F. De Tomasi, D. Hennequin, and E. Arimondo, Phys. Rev. A **40**, 3782 (1989).
[4] M. Lefranc, D. Hennequin, and D. Dangoisse, J. Opt. Soc. Am. B **8**, 239 (1991).
[5] B. Zambon, Phys. Rev. A **44**, 688 (1991).
[6] F. Papoff, A. Fioretti, E. Arimondo, G. B. Mindlin, H. Solari, and R. Gilmore, Phys. Rev. Lett. **68**, 1128 (1992).
[7] P. Alcantara, Jr., L. Guidoni, A. Barsella, A. Fioretti, and E. Arimondo, J. Opt. Soc. Am. B **12**, 1326 (1995), and references cited therein.
[8] Y. Liu and J. R. Rios Leite, Opt. Commun. **88**, 391 (1992).
[9] Y. Liu and J. R. Rios Leite, Phys. Lett. A **191**, 134 (1994).
[10] T. Sugawara, M. Tachikawa, T. Tsukamoto and T. Shimizu, Phys. Rev. Lett. **72**, 3502 (1994).
[11] Y. Liu, P. C. de Oliveira, M. B. Danailov, and J. R. Rios Leite, Phys. Rev. A **50**, 3464 (1994).
[12] B. Bobin, Ch. J. Borde, J. Borde, and Ch. Breant, J. Mol. Spectrosc. **121**, 91 (1987).
[13] O. Di Lorenzo-Filho, P. C. de Oliveira, and J. R. Rios Leite, Opt. Lett. **16**, 1768 (1991).
[14] See, for instance, W. J. Witteman, *The* CO_2 *Laser* (Springer-Verlag, New York, 1987).
[15] D. Dangoisse, P. Glorieux, and D. Hennequin, Phys. Rev. A **42**, 1551 (1990).
[16] P. Grassberger and I. Procaccia, Phys. Rev. Lett. **50**, 346 (1983).
[17] M. Lefranc, D. Hennequin, and P. Glorieux, Phys. Lett. A **163**, 269 (1992).