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Nuclear reaction rates in four-body muon molecules

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The basic properties of the ground states in the four-body muon molecules $pp\mu\mu$, $pd\mu\mu$, $pt\mu\mu$, $dd\mu\mu$, $dt\mu\mu$, and $tt\mu\mu$ are calculated. It is found that the nuclear reaction rates (R_f) in such four-body molecules are significantly larger than for the corresponding three-body ions with the same nuclei: $pp\mu$, $pd\mu$, $pt\mu$, $dd\mu$, $dt\mu$, and $tt\mu$. In particular, for the $dt\mu\mu$ system $R_f(dt)\approx 3\times10^{13}$–$6\times10^{13}$ sec^{-1}, which is ≈ 40 times greater than the respective $R_{dt}(dt\mu)$ value. [S1050-2947(97)00403-4]

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In the present Brief Report we give the results of extensive variational calculations for the bound ground states in the four-body muon molecules $pp\mu\mu$, $pd\mu\mu$, $pt\mu\mu$, $dd\mu\mu$, $dt\mu\mu$, and $tt\mu\mu$. The boundedness of the ground $S(L=0)$ states in such systems was established long ago [1]. Actually, for the symmetrical systems such as $X^+X^+Y^-Y^-$ this follows from the stability of the ground states in the Ps_2 and H_2 molecules. Recently we have studied [2] the symmetrical systems $pp\mu\mu$, $dd\mu\mu$, and $tt\mu\mu$. Now, we extend our approach to the case of nonsymmetrical systems.

Our initial goal was to compute the so-called basic properties, i.e., energetic and geometric parameters for the four-body muonic molecules: $pp\mu\mu$, $pd\mu\mu$, $pt\mu\mu$, $dd\mu\mu$, $dt\mu\mu$, and $tt\mu\mu$. Below we shall designate these neutral systems as muon molecules, in contrast with the related three-body systems $pp\mu$, $pd\mu$, $pt\mu$, $dd\mu$, $dt\mu$, and $tt\mu$, where the description ''muon-molecular ions'' or even ''ions'' would be appropriate, since in each case one muon has been removed from the neutral (four-body) molecule. Already after the first few calculations we found a remarkable difference in properties between the three-body ions and the four-body molecules. Namely, the expectation values for the $(++)$ δ functions (i.e., $\langle\delta_{++}\rangle$) in the four-body case are significantly larger (by 10–50 times) than those values for the three-body systems with the same nuclei. Further analysis showed that such a large difference produces a proportional deviation in the nuclear reaction rates for the appropriate three- and four-body systems. For instance, the nuclear dt-fusion reaction in the $dt\mu\mu$ molecule is ≈ 50 times faster than in the $dt\mu$ ion.

Let us compare the nuclear reaction rates (or fusion rates R_f, for short), e.g., for the $dt\mu\mu$ molecule and $dt\mu$ ion. The appropriate expressions are (see, e.g., [3–5])

$$R_f(dt\mu)=K_{s,3}\langle\delta_{++}(dt\mu)\rangle=\left(\lim_{v\to 0}\frac{\sigma_f(v)v}{f(v)}\right)\langle\delta_{++}(dt\mu)\rangle \tag{1}$$

for the $dt\mu$ ion, and

$$R_f(dt\mu\mu)=K_{s,4}\langle\delta_{++}(dt\mu\mu)\rangle=\left(\lim_{v\to 0}\frac{\sigma_f(v)v}{f(v)}\right)\times\langle\delta_{++}(dt\mu\mu)\rangle \tag{2}$$

for the $dt\mu\mu$ molecule. In these equations $\langle\delta_{++}\rangle\equiv\langle\delta_{dt}\rangle$, v is the relative velocity of the d and t nuclei, $\sigma_f(v)$ is the appropriate dt-fusion cross section, and the universal function $f(v)$ takes the form

$$f(v)=\frac{2\pi}{v}\left[\exp\left(\frac{2\pi}{v}\right)-1\right]^{-1}. \tag{3}$$

Now $K_{s,4}=K_{s,3}$, since the expressions for both of these values are identical, and they depend only on certain nuclear parameters for the d and t nuclei, which are the same for both $dt\mu\mu$ and $dt\mu$ systems. Therefore from the first two equations we find

$$\frac{R_f(dt\mu\mu)}{R_f(dt\mu)}=\frac{\langle\delta_{++}(dt\mu\mu)\rangle}{\langle\delta_{++}(dt\mu)\rangle}, \tag{4}$$

i.e., the ratio of fusion rates for the appropriate four- and three-body systems equals the ratio of their pair $(++)$ δ functions. Analogous formulas can be written for all of the other four- and three-body muonic systems mentioned above. This means that in order to compare the nuclear reaction rates we need to compute the respective $(++)$ δ function expectation values.

In general, the accurate values of $(++)$ δ -function expectation values for three- and four-body systems can be found only from the results of numerical calculations. *A priori* for the pair of systems $ab\mu\mu$ and $ab\mu$, we would expect that the positively charged particles a and b are closer together in $ab\mu\mu$ than in $ab\mu$, and hence the δ-function ex-

TABLE I. The total energies E,$\langle r_{++}\rangle$, and $\langle \delta_{++}\rangle$ expectation values (in muon atomic units) for a number of four-body ($ab\mu\mu$) and three-body ($ab\mu$) systems.

$ab\mu\mu$	E	$\langle r_{++}\rangle$	$\langle \delta_{++}\rangle$	$\langle \delta_{++}\rangle$	$\langle r_{++}\rangle$	E	$ab\mu$
$pp\mu\mu$	-0.9654742	2.34202	3.801×10^{-4}	3.9372×10^{-5}	3.299486184	-0.494386820249	$pp\mu$
$pd\mu\mu$	-0.9995455	2.17506	2.283×10^{-4}	1.4617×10^{-5}	3.100710404	-0.512711796501	$pd\mu$
$pt\mu\mu$	-1.0123990	2.11527	1.506×10^{-4}	8.9750×10^{-6}	3.036524321	-0.519880089782	$pt\mu$
$dd\mu\mu$	-1.0365953	2.00237	6.610×10^{-5}	2.4395×10^{-6}	2.834451766	-0.531111135402	$dd\mu$
$dt\mu\mu$	-1.0509269	1.93798	3.183×10^{-5}	8.871×10^{-7}	2.747914133	-0.538594975058	$dt\mu$
$tt\mu\mu$	-1.0661515	1.87102	1.015×10^{-5}	2.187×10^{-7}	2.652824760	-0.546374225598	$tt\mu$

pectation value for the $ab\mu\mu$ system should be larger than for $ab\mu$ [6]. For instance, in the $dt\mu\mu$ and $dt\mu$ systems $\langle \delta_{++}(dt\mu\mu)\rangle$ should be larger than $\langle \delta_{++}(dt\mu)\rangle$. This means that for fusion rates calculated by means of Eq. (4) we would expect that $R_f(dt\mu\mu)>R_f(dt\mu)$.

The correctness of this prediction can be tested only by extensive numerical calculations. To compute the bound states in the three-body systems we used the so-called exponential variational expansion (for more details see, e.g., [7]). For the four-body systems the so-called six-dimensional Gaussoid variational expansion (proposed in [8]) has been applied. This expansion has the form

$$\Psi_{L=0}=\sum_{i=1}^{N} C_i\exp(-\alpha_{12}^i r_{12}^2-\alpha_{13}^i r_{13}^2-\alpha_{23}^i r_{23}^2-\alpha_{14}^i r_{14}^2 -\alpha_{24}^i r_{24}^2-\alpha_{34}^i r_{34}^2), \quad (5)$$

where N is the total number of basis functions, C_i are the linear (variational) parameters, while the α_{kl}^i are the $6N$ nonlinear parameters.

The results of our calculations are given in Table I. In Table I the so-called muon-atomic units are used. In these units $m_\mu=1$, $\hbar=1$, and $|e|=1$. To evaluate the corresponding binding energies in electron volts (eV) the conversion factor $m_\mu\times 27.211\,396\,1$ must be used. The lowest decay channel for these four-body systems is the dissociation into two neutral muonic atoms, e.g., $dt\mu\mu\rightarrow d\mu+t\mu$. For the three-body systems the lowest channel is dissociation into the heavier muonic atom and the lightest nuclear ion, e.g., $dt\mu\rightarrow t\mu+d^+$ (instead of $dt\mu\rightarrow d\mu+t^+$). In our present calculations we used the following values of the masses: $m_\mu=206.768\,262m_e$, $m_p=1836.152\,701m_e$, $m_d=3670.483\,014m_e$, and $m_t=5496.521\,58m_e$ [9].

In Table I the total energies, $\langle r_{++}\rangle$, and $\langle \delta_{++}\rangle$ expectation values are given for the ground bound states in the four-body muon molecules $pp\mu\mu$, $pd\mu\mu$, $pt\mu\mu$, $dd\mu\mu$, $dt\mu\mu$, and $tt\mu\mu$, and in the three-body muon ions $pp\mu$, $pd\mu$, $pt\mu$, $dd\mu$, $dt\mu$, and $tt\mu$. It follows from Table I that the nuclear reaction rates in these four-body systems would be expected to be $\approx$10–50 times larger than those values for the corresponding three-body systems. The most interesting case is the $dt\mu\mu$ system, where the time for a nuclear dt reaction is $\approx$40 times shorter than in the $dt\mu$ ion. In terms of recent experimental results for $R_{dt}(dt\mu)$ [10] we can estimate the numerical value of $R_{dt}(dt\mu\mu)$ as $\approx 3\times10^{13}$–6×10^{13} sec^{-1}.

Such a large value of $R_{dt}(dt\mu\mu)$ and similar fusion rates for other four-body systems suggest that these four-body systems should be of interest for applications related with low-temperature plasma ignition. Indeed, an intense muon beam can be used together with laser or electron beams to form the thermonuclear burn wave in the initially low-temperature ($T_{\rm in}\leqslant 50$ eV) DT target. This will require further investigation, however, at the present time it is clear only that the presence of the muon beams would make easier low-temperature ignition in a DT plasma. Actually, this opens a new avenue for initiating micro and supermicro DT explosions: the total mass of fuel can be as low as $\approx 4\times10^{-15}$–1×10^{-7} g, and the total energy release $\approx 2.5\times10^{-4}$–6.5×10^{4} J. Such small explosions cannot be produced in terms of the traditional technique (see, e.g., [11]).

It should be noted that the realization of nuclear fusion in the four-body, bi-muon systems differs significantly from the traditional muon-catalyzed fusion (see, e.g., [5] or [12]). In particular, such bi-muonic systems cannot be produced repeatedly inside of the small DT target. At the same time the helium-muonic sticking coefficient probably has a quite large value (as well as for the traditional muon-catalyzed fusion [13]). However, the nuclear fusion in the four-body, bi-muon systems has a great advantage, since the nuclear reaction proceeds significantly faster. This could be very important in practice.

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[6] Within the series $ab\mu\mu$ or the series ($ab\mu$), the heavier the nuclei a and b are, the closer together they are, i.e., $\langle r_{++}\rangle$ decreases. The expectation values $\langle \delta_{++}\rangle$ decrease as well. However, this is not contrary to intuition, since the particle masses are different, and $\langle \delta_{++}\rangle$ depends not only on $\langle r_{++}\rangle$,

but also (and very significantly, see reference below) on the values of the particle masses m_a and m_b. For the very heavy (infinite) particles, e.g., in the DT^+ ion the value $\langle r_{++} \rangle$ becomes minimal ≈ 2.00 a.u. and $\langle \delta_{++} \rangle \approx 1 \times 10^{-60}$ [the minimal value equals 0 (i.e., no fusion) for the ${}^{\infty}H_2{}^+$ ion]. For the DT molecule the value $\langle r_{++} \rangle$ also approaches its minimal value ≈ 1.40 a.u., while $\langle \delta_{++} \rangle \approx 1 \times 10^{-58}$ (the minimal value equals 0 for the ${}^{\infty}H_2$ molecule). For more details see, e.g., A.M. Frolov, J. Phys. B **26**, L845 (1993).

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