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Multiple Landau-Zener crossings and quantum interference in atoms driven by phase modulated fields

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The excitation amplitude after multiple crossings is not a mere product of Landau-Zener transition probabilities at each crossing, due to coherent evolution of the system in between crossings. In the three-level ladder system, the trapping of population by frequency modulated fields ensures coherent evolution, and inclusion of phase effects for population redistribution after multiple crossings becomes necessary. The relative phase accumulated by various adiabatic states as they evolve along different paths is tailored to show the existence of quantum interference effects. We present a method of inverting the population in a three-level system, without affecting the population in the intermediate state. We also present an *all optical* implementation of the three-level ladder system, where these effects can be realized. [S1050-2947(97)01503-5]

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I. INTRODUCTION

The Landau-Zener (LZ) [1] formula for the calculation of the transition probability at a crossing of two potential energy curves of atomic systems has been widely used. Particularly, in problems of slow atomic and ionic collisions, where the motion of the heavy nuclei is treated classically and the various inelastic processes are investigated by solving a nonstationary Schrödinger equation, LZ theory is used extensively to calculate various transition probabilities. The LZ formula deals with nonadiabatic transitions between two adiabatic states as the system traverses a crossing of the energy levels. The states involved should depend weakly on the parameter, which when varied causes the crossing, leading to transition between them. This means that the LZ formula is valid only if the departure from adiabatic behavior is not too large [2]. Bates [3] pointed out that the validity of the LZ formula is more restrictive than is commonly supposed. The major objections raised were the failure of the LZ formula to take into account the transitions that occur away from the crossings, the atomic orbitals being spherically unsymmetric, and the variation of interaction (coupling) energy with nuclear separation. Extension of the LZ formula for multilevel (even infinite) crossing, when they are *well separated*, has been shown to decompose into elementary LZ factors at each crossing [4], each of which mixes only a pair of states. On the other hand, multilevel crossing in the context of scattering of atoms and ions has been dealt with in detail by Nakamura [5], where the dependence of the probability amplitude on the Stokes phase is noticed. Its inclusion leads to a better matching of the LZ theory with the exact quantum calculation.

In this paper, we choose the parameters in the domain such that LZ-like behavior is valid at every crossing and examine the dynamics when there are multiple crossings with the system evolving coherently in between these crossings. We had earlier shown the existence of population trapping in a two-level system driven by a frequency modulated field [6], where there is a redistribution of population at the crossing of energy levels. We present here a detailed study of these crossings in multilevel systems, typically for the three-level cascade system. We show interference effects which arise due to the phase accumulated as the system evolves along various paths in between multiple crossings of energy levels. By an appropriate choice of detuning, this path and thus the phase accumulated along it is tailored. We also demonstrate a mechanism for achieving nearly complete population inversion across multilevel system, without stepwise transfer of population through intermediate states. This inversion can be completely undone at the next crossing. We discuss the advantages of this method over the usual rapid adiabatic passage process (RAP), which uses unconventional sequence of pulses [7] to achieve such inversion. Moreover, we also propose an all optical implementation of the three-level ladder system in *optical atoms*, where these various effects can be realized.

The outline of the paper is as follows. In Sec. II, we set up the equations that would govern the dynamics of three-level system in presence of frequency modulated fields. In Sec. III, we demonstrate the effects of phase and quantum interference in this system, and discuss various features. We also describe a mechanism of inverting population in a three-level system. In Sec. IV, we present a scheme to realize a three-level system in optical atoms. We present our conclusions in Sec. V.

II. MODEL

As we had earlier demonstrated [6], a two-level system in the presence of a frequency modulated electromagnetic field shows trapping of population, and its redistribution at periodic intervals of time. These times correspond to the times at which the adiabatic levels cross each other. The applied field being coherent ensures coherent evolution between such crossings. As a generalization, we consider here a three-level ladder system in presence of frequency modulated electro-

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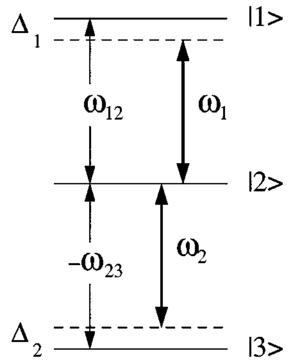

FIG. 1. Three-level ladder system with the energies being measured from the middle level $|2\rangle$. Transitions $|1\rangle \leftrightarrow |2\rangle$ and $|2\rangle \leftrightarrow |3\rangle$ are coupled by frequency modulated fields centered at ω_1 and ω_2, respectively. Δ_1, Δ_2 and $2G_1$, $2G_2$ are the detunings and Rabi frequencies of the corresponding transitions, respectively.

magnetic field. The frequency modulated field at the atom is given as

$$\mathbf{E}=\mathbf{E}_1 e^{-i[\omega_1 t+\phi_1(t)]}+\mathbf{E}_2 e^{-i[\omega_2 t+\phi_2(t)]}+\text{c.c.};$$
$$\phi_i(t)=M_i \sin(\Omega_i t), \quad i=1,2, \tag{1}$$

where M_i and Ω_i are the index of modulation and the frequency of modulation, respectively. This field interacts with a three-level cascade system Fig. 1. The field centered at ω_1 couples the transitions $|1\rangle \leftrightarrow |2\rangle$ and ω_2 couples $|2\rangle \leftrightarrow |3\rangle$. The total Hamiltonian of the system is

$$H=\hbar \omega_{12}|1\rangle\langle 1|-\hbar \omega_{23}|3\rangle\langle 3|-\mathbf{d}\cdot\mathbf{E}, \tag{2}$$

where $\mathbf{d}=\mathbf{d}_{12}|1\rangle\langle 2|+\mathbf{d}_{23}|2\rangle\langle 3|+\text{c.c.}$ The first two terms in H correspond to the unperturbed system, the energies being measured from the middle level $|2\rangle$, and the last term is the interaction term in dipole approximation. We describe the dynamics of the atom plus field system by the Schrödinger equation

$$i\hbar \frac{\partial |\psi\rangle}{\partial t}=H|\psi\rangle, \tag{3}$$

where, $|\psi\rangle=C_1|1\rangle+C_2|2\rangle+C_3|3\rangle$. We now discuss the various conditions for localization of population in a multilevel system in presence of a frequency modulated field.

The generating function for the Bessel functions [8] gives the various spectral components of the frequency modulated field

$$e^{iM\sin(\Omega t)}=\sum_{k=-\infty}^{\infty} J_k(M) e^{ik\Omega t}. \tag{4}$$

We observe, that for large Ω the major contribution on the time scales slower than the periodic exponentials in Eq. (4) would be from the $J_0(M)$ term, moreover, magnitudes of $J_n(M)$ decay with increase in $|n|$ for a fixed M. Hence, for large Ω

$$H_{int}\approx -\mathbf{d}\cdot[\mathbf{E}_1 J_0(M_1)+\mathbf{E}_2 J_0(M_2)]. \tag{5}$$

By choosing M_i to be a zero of the zeroth-order Bessel function, i.e., $J_0(M_i)=0$ for $i=1$ and 2, the dominant term in the interaction Hamiltonian goes to zero, which effectively leads to trapping of the population on the appropriate time scales. At times $t\sim \pi/2\Omega, 3\pi/2\Omega, 5\pi/2\Omega, \cdots$, (for the resonant case) the other exponentials would dominate the interaction term, causing transitions between different levels.

We now transform Eq. (3), where H denotes the *complete Hamiltonian* given by Eq. (2), into a frame rotating with the instantaneous frequency of the field by defining the following transformation for the complex amplitudes C_i's:

$$\tilde{C}_1=C_1 e^{i[\omega_1 t+\phi_1(t)]}, \quad \tilde{C}_2=C_2,$$
$$\tilde{C}_3=C_3 e^{-i[\omega_2 t+\phi_2(t)]}. \tag{6}$$

We neglect the rapidly rotating terms at twice the optical frequency, like $e^{\pm 2i[\omega_i t+\phi_i(t)]}$, (for $i=1,2$), then the equations of evolution for the slowly varying $\tilde{C}_i$'s are

$$\begin{bmatrix} \dot{\tilde{C}}_1 \\ \dot{\tilde{C}}_2 \\ \dot{\tilde{C}}_3 \end{bmatrix}=\begin{bmatrix} -i[\Delta_1-M_1\Omega_1\cos(\Omega_1 t+\theta)] & iG_1 & 0 \\ iG_1^* & 0 & iG_2 \\ 0 & iG_2^* & i[\Delta_2-M_2\Omega_2\cos(\Omega_2 t)] \end{bmatrix}\begin{bmatrix} \tilde{C}_1 \\ \tilde{C}_2 \\ \tilde{C}_3 \end{bmatrix}, \tag{7}$$

where, $\Delta_1=\omega_{12}-\omega_1$ and $\Delta_2=\omega_{23}-\omega_2$, the parameters $2G_1=(2\mathbf{d}_{12}\cdot\mathbf{E}_1)/\hbar$ and $2G_2=(2\mathbf{d}_{23}\cdot\mathbf{E}_2)/\hbar$ are the Rabi frequencies of the corresponding transitions, θ gives the initial phase mismatch between the fields $\mathbf{E}_1$ and $\mathbf{E}_2$, and the overdot denotes the first-order time derivative [9]. Here, its the G_i's that provide the nonadiabatic coupling between various adiabatic states $|i\rangle$'s and cause transitions between them whenever their energies cross each other. Thus, the conditions for observing trapping of population and its redistribution due to jumps (at the crossings), are firstly, the choice of the modulation index M_i such that $J_0(M_i)=0$, for both $i=1,2$ and secondly, the choice of modulation frequency Ω_i, which would set the times at which jumps (crossings) would occur. Radmore, Tarzi, and Tang [10] studied the steady-state ionization spectra induced by a phase-varying field, which exhibited redistribution of population. On the other hand, by modulating away the resonant component of the applied field (which is similar to our condition for trapping), Lam and Savage [11] obtained complete population inversion in two-level system due to correlated sidebands.

The trapping states we demonstrate are limited by the decay processes in the atom, as was shown in Ref. [6]. We require $\Omega \gg \gamma$, where 2γ is the spontaneous emission rate of the excited state, to observe these jumps. Here, its not a mere generalization of the earlier spin 1/2 system [6] to a spin 1 system, but to a more general three-level ladder system, apart from consideration of phase accumulation between various kinds of multiple crossings. Kenkre *et. al.* [12] have demonstrated dynamic localization in solid-state systems, in the motion of a charged particle in an infinite lattice driven by a harmonic time-dependent electric field, which essentially is an equispaced infinite multilevel system. The condition for dynamic localization in Ref. [12] is closely related to what is obtained here, a brief comparison has been made by a treatment for finite lattice sites [12].

III. CALCULATIONS AND DISCUSSION

In this section, we demonstrate the phenomenon of localization and jumps, and the need to consider phase effects due to coherent evolution of the population along different interfering pathways. To observe trapping, we choose the index of modulation such that its a zero of the Bessel J_0 function. The jumps occur at times when the bare energy levels cross. Figure 2 shows the trapping of population and its transfer at the times when the energy levels cross. The energy levels and the various crossings depicted in the figures correspond to the zero-order Hamiltonian in the rotating frame. As is well known, the consideration of the nonadiabatic coupling G_i's between the adiabatic states would transform these crossings into avoided crossings. We initially put in the population in state $|1\rangle$ and show the importance of the choice of M for trapping. In Fig. 2(a) we do not have any trapping, as M is not a zero of the Bessel J_0 function (for simplicity we take $M_1 = M_2$ and $\Omega_1 = \Omega_2$, general case is discussed later). For $M_i = 30.6346$ (*tenth zero of the J_0 Bessel function*) we have trapping of population Fig. 2(c). The zero-order energies (i.e., without the nonadiabatic coupling term) of the adiabatic levels $|1\rangle$ and $|3\rangle$ as measured from $|2\rangle$, in the frame corotating with the instantaneous field frequency are

$$E_1(t) = \Delta_1 - M_1\Omega_1\cos(\Omega_1 t + \theta), \quad E_2(t) = 0,$$

$$E_3(t) = -[\Delta_2 - M_2\Omega_2\cos(\Omega_2 t)]. \tag{8}$$

Hence, whenever $E_i = E_j$ (for $i \neq j$; $i,j = 1,2,3$), the levels $|i\rangle$ and $|j\rangle$ cross at those times causing a population transfer Fig. 2(b). The first nonadiabatic crossing takes place when $E_1 = E_2$ at $\Omega t \sim 1.23$; the second crossing is at $\Omega t \sim 1.57$ when $E_1 = E_3$; the next crossing takes place when $E_2 = E_3$ at $\Omega t \sim 1.90$ and so on. The population transfer probability at the crossing of *two levels* can be approximated by the Landau-Zener transition probability [1], by taking only the linear terms in the expansion of $\cos(\Omega t)$ in Eq. (8) about the crossing time. The probability turns out to be $P = 1 - e^{-2\pi\kappa}$, $\kappa = [G^2/|d/dt(E_1 - E_2)|]$ at $\Omega t \sim 1.23$, $P = 0.8$, which is comparable to the observed probability af-

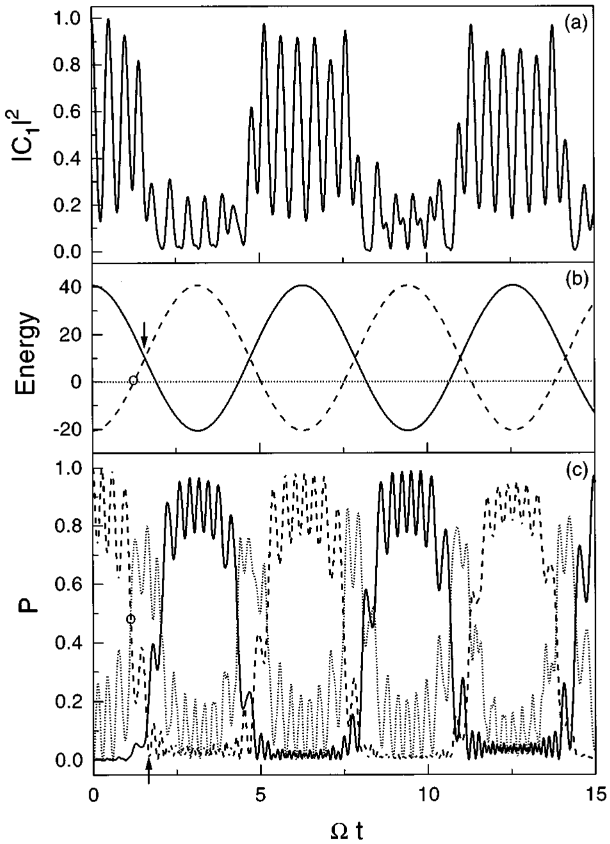

FIG. 2. (a) There is no trapping of population if the M_i's are chosen such that $J_0(M_i) \neq 0$. For $M_1 = M_2 = 7$, $\Omega_1 = \Omega_2 = 1$, $\Delta_1 = 10$, $\Delta_2 = -10$, $G_1 = G_2 = 6$, with the initial condition $|C_1|^2 = 1$, $|C_2|^2 = |C_3|^2 = 0$. (b) The evolution of the energy levels, given by Eq. (8) and the crossings of these levels for a choice of M such that $J_0(M) = 0$, $M = 30.6346$ and the other parameters being the same as in (a). The dashed, dotted, and solid lines are for energies of levels $|1\rangle$, $|2\rangle$, and $|3\rangle$, respectively. (c) Dynamic evolution of the trapping of population in various states. At every crossing of the energy levels (the first two crossings are denoted by a circle and arrow) there is redistribution of population, for $M = 30.6346$ and the other parameters being the same as in (a). Similar to (b) the dashed, dotted, and solid lines denote $|C_1|^2$, $|C_2|^2$, and $|C_3|^2$, respectively. Δ_i and G_i are in units of Ω.

ter the first crossing. To illustrate the need to consider the phase accumulated between two crossings along the various paths of evolution, we choose the values of $\Delta_{1,2}$ such that level $|1\rangle$ does not cross the other levels, Fig. 3(a). There is a periodic exchange of population predominantly between levels $|2\rangle$ and $|3\rangle$, whereas the population in $|1\rangle$ remains practically unaffected. We begin initially with the population in $|3\rangle$. The probability of it being transferred to $|2\rangle$, due to the nonadiabatic coupling term G_2, at the first crossing is $P = 0.9$, using the Landau-Zener theory. If one considers each crossing independently, the probability at every crossing is the same because the absolute value of the slope of the energy difference $E_2 - E_3$ is the same at all crossings, Fig. 3(a). To determine the population after multiple crossings, one cannot merely take the individual Landau-Zener probabilities [4] at each crossing, as it would imply *a steady decrease in the probability of population transfer with in-*

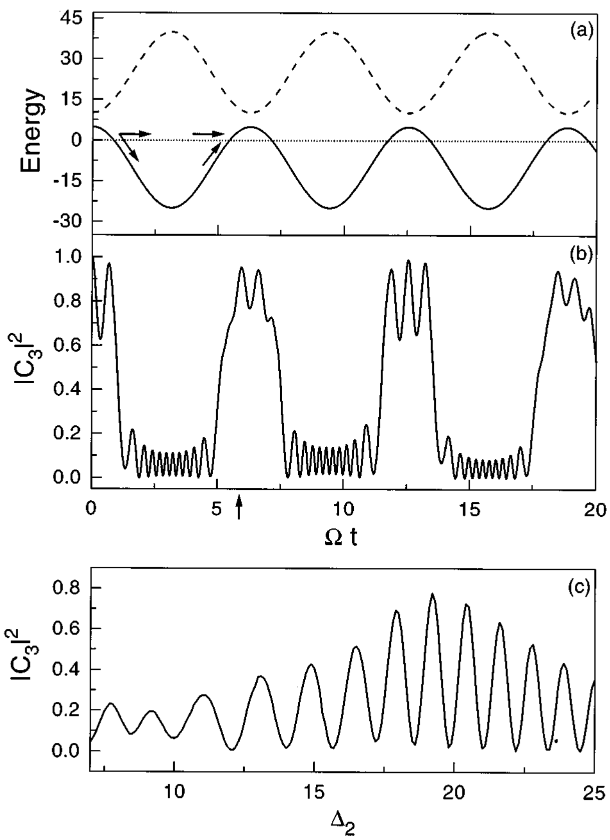

FIG. 3. (a) The energy level $|1\rangle$ (dashed) does not cross with $|2\rangle$ (dotted) and $|3\rangle$ (solid) for $M=14.9309$, $\Delta_1=25$, $\Delta_2=10$, $G_1=7$, $G_2=3$, with the initial condition $|C_3|^2=1$, $|C_1|^2=|C_2|^2=0$. The arrows denote the various paths along which the system evolves in between crossings. (b) The population in $|3\rangle$, where after multiple crossings the probability is not a mere product of Landau-Zener probabilities at each crossing. (c) Quantum interference effect observed in the net population in level $|3\rangle$ at $\Omega t=6.0$ [shown by arrow in (b)], by changing the path (which depends on Δ_2) along which the system evolves and thus, accumulates a different phase. Δ_i and G_i are in the units of Ω.

creasing number of crossings. While the observed probability is more or less independent of the number of crossings the system has undergone. The population in $|3\rangle$ revives completely after even number of crossings, Fig. 3(b). Thus, the probability after multiple crossings requires consideration of the *phase accumulated* by the system between such crossings along with the Landau-Zener probabilities at each crossing.

It was shown by Berry [13] that when a quantum system is forced round a cycle by an adiabatic change, it will return with an extra phase which is purely geometric in nature. Bouwmeester *et al.* [14] have implemented the twisted Landau-Zener model in optical atoms and showed the effect of phase, due to curvature of the path followed in the parameter space. As a result, even for open paths, the geometric phase strongly influences transition probability, as is demonstrated in our system. The effect of phase is also considered by Vitanov and Knight [15], where they studied multiple pulse excitation of two-level system by a train of pulses. They accounted for the phase accumulation, during and in between these pulses. In an experiment by Gatzke, Watkins, and Gallagher [16], they have observed quantum interference in multiphoton transitions. When a pulse is applied to the system, it traverses resonance (akin to crossing in our case) twice, once, on the rising edge of the pulse, and another, on the falling edge. The superposition of the two states involved, evolves coherently in between these resonances. The Landau-Zener probabilities are considered at each traversal of the resonance and also, the phase accumulated in between these crossings, which leads to interference in the transition probability when the relative phase accumulated by each state during the pulse is varied.

As is shown in Fig. 3(a), there are two distinct pathways (indicated by the arrows) at each crossing along which the system evolves, thereafter, the levels cross again and so on. In between these crossings, the superposition of the corresponding states evolves *coherently* in time. The net transition probability between the two states depends on the relative phase accumulated along each path. The path traversed by the system can be varied by varying the detunings. We examine, the probability of population being in level $|2\rangle$ and its dependence on the path traversed between the first two crossings. We observe the evolution of the system from $\Omega t=0$ to 6.0, by which time level $|2\rangle$ has crossed a second time with level $|3\rangle$. By varying Δ_2 the phase accumulated by the system, as it evolves coherently along the two paths between the two crossings, is varied and we see quantum interference effect, Fig. 3(c). The probability of population transfer varies from 0.2 to 0.8 by varying the relative paths of evolution of the states between crossings.

When *all three levels* cross (the LZ formula cannot be used due to the two-state approximation) for $|\Delta_1|=|\Delta_2|$ at a time $\Omega t=\cos^{-1}(\Delta/M\Omega)$, there is a periodic exchange of population between the top and the bottom level at every crossing, whereas the population in the middle level remains intact. If the population is initially put in level $|1\rangle(|3\rangle)$, after the first crossing the population is transferred to $|3\rangle$ $(|1\rangle)$, with the intermediate level experiencing transient population, Fig. 4(a). Note further, that the population transfer to the intermediate state is not complete, only $\sim$10–20 % goes into it. The inversion achieved at the first crossing is completely undone at the next crossing. This is quite different from the usual population transfer in three-level systems by RAP [7]. In RAP one requires a sequence of unconventional pulses, so that first the empty set of levels is pumped and only thereafter—with some temporal overlap—the initially populated levels are addressed. In this way the intermediate level remains practically unpopulated. Another RAP proposal, is to apply temporarily coincident pulses with frequency sweep in an anti-intuitive manner, so that again the unpopulated levels experience the resonance and there after the populated level. All these proposals are sensitive to the precise times at which the pulses are applied or are frequency swept. In contrast, we deal with cw fields and as the application of the FM field leads to simultaneous crossing of all the levels (unlike in RAP, where the unpopulated levels are dressed first and thereafter the populated level crosses with the appropriate state), and, hence, is experimentally more attractive. Another advantage is that the presence of finite population in the intermediate level renders the RAP process very inefficient. Whereas, in our scheme, population in the intermediate state remains untouched. Only the population in $|1\rangle$ and $|3\rangle$ gets

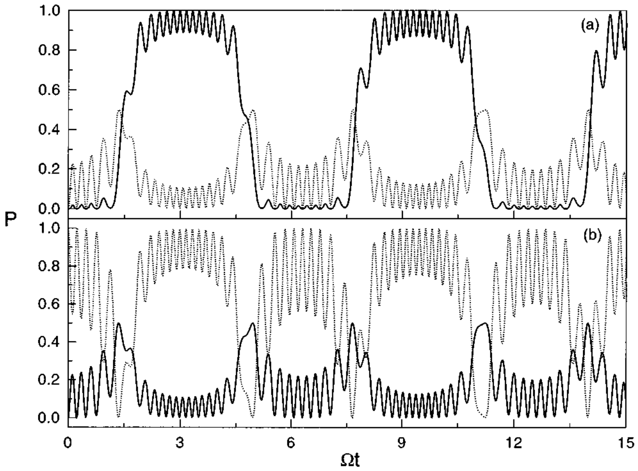

FIG. 4. (a) When all three-levels cross at every crossing, there is a periodic exchange of population between $|1\rangle$ and $|3\rangle$ (solid), whereas, the level $|2\rangle$ (dotted) experiences partial population transfer. For $M=30.6346$, $\Delta_1=\Delta_2=5$, $G_1=G_2=7$, with the initial condition $|C_1|^2=1$, $|C_2|^2=|C_3|^2=0$. (b) Same as (a) with initial condition $|C_2|^2=1$, $|C_1|^2=|C_3|^2=0$, the population in the intermediate state (dotted) remains unaffected even after multiple crossings with other levels. Population in $|1\rangle$ and $|3\rangle$ are denoted by the solid line. Δ_i and G_i are scaled in terms of Ω.

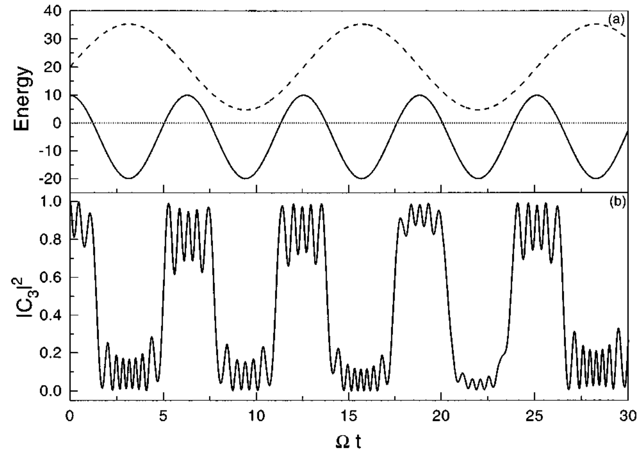

FIG. 5. (a) Level $|1\rangle$ (dashed) does not cross any other level for a choice of $\theta=\pi/2$ and $M_1=30.6346$, $M_2=14.9309$, $\Omega_1=1/2$, $\Omega_2=1$, $\Delta_1=20$, $\Delta_2=5$, $G_1=8$, $G_2=3$ and initially the population is in level $|3\rangle$. The energies of $|2\rangle$ and $|3\rangle$ are denoted by dotted and solid lines, respectively. (b) Trapping is clearly seen even when $M_1\neq M_2$, $\Omega_1\neq\Omega_2$ and a finite θ, all parameters being the same as in (a). Δ_i and G_i are in units of Ω_2.

exchanged at every crossing. Figure 4(b) shows this feature, when all the population is initially in $|2\rangle$ it remains unchanged even after multiple crossings with levels $|1\rangle$ and $|3\rangle$.

The crossing of energy levels is essential for population transfer, otherwise it remains trapped in various levels depending on the initial condition. In general, trapping is also observed when $M_1\neq M_2$, however, they have to be chosen as zeros of the J_0 Bessel function. Choice of unequal Ω_1 and Ω_2 does not affect these trapping states. Trapping is also unaffected by initial phase mismatch between the fields $\mathbf{E}_1$ and $\mathbf{E}_2$. If the phase mismatch θ, or choice of Ω_i's, does not result in extra crossings of the energy levels, the characteristic trapping dynamics of the system does not suffer. We show in Fig. 5, for a general choice of parameters, i.e., when $M_1\neq M_2$, $\Omega_1\neq\Omega_2$ and a finite phase difference θ, trapping persists.

IV. OPTICAL ATOMS REALIZATION

The basic unit one deals with in the physics of atomic-optical resonance is a two-level atom interacting with an electromagnetic radiation of a frequency that matches the energy level separation in the atom. Woerdman and coworkers [17] have shown in detail that a system formed using two distinct coupled classical optical modes show analogous characteristics, this system has been termed as an *optical atom*. They have also shown that multilevel systems with even number of levels like four, six, etc., can be simulated in optical atoms by an appropriate choice of various longitudinal modes in the cavity.

We propose, an optical implementation of a *three-level* atom. The highlight of studies with optical atoms is that, they being macroscopic in nature one has precise control on all the parameters over ranges that are sometimes not accessible in experiments with real atoms. In the trapping phenomenon we describe using frequency modulated field, the localization is more effective if the interaction term of the Hamiltonian is minimal, which can be ensured by choosing a high index of modulation. The experimental difficulty with real atoms lies in achieving high index of modulation at optical frequencies, and, hence, *optical atoms* would be best suited in this regime.

In contrast to a single ring cavity used in Ref. [17] for the two-level atom, here, we require two identical coupled ring cavities Fig. 6. One possibility could be two *identical* fiber-optic ring resonators coupled using a 2×2 fiber-optic coupler. We distinguish the various modes by their *direction of propagation*. The eigenmodes, for a particular polarization state of a single longitudinal mode in the coupled cavity,

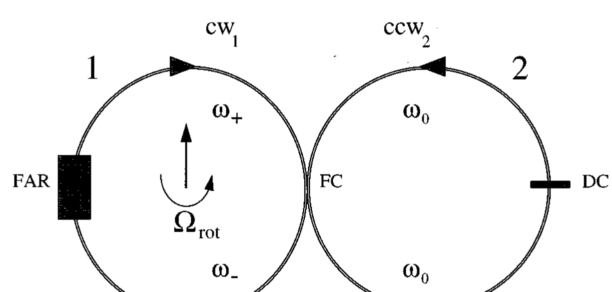

FIG. 6. Schematic of the optical implementation of the three-level ladder system. Ω_{rot} depicts the effect of the Faraday rotator (FAR). The *conservative* coupling of both cavities is gotten by a lossless 2×2 fiber coupler (FC). The *dissipative* coupler (DC) is a thin localized absorber. The three nondegenerate *propagation* modes simulating the three-level ladder system are cw_1, ccw_1, and cw_2-ccw_2.

would be two degenerate set of modes traveling clockwise (cw) and counterclockwise (ccw) in each cavity. Thus, there would be four degenerate modes cw_1, ccw_1, cw_2, and ccw_2 (where 1 and 2 label the two cavities). Now, the basic idea is to lift the degeneracies of two modes in one of the cavities and couple all these modes appropriately.

The Faraday rotator in the ring resonator 1 would simulate mechanical rotation of only the ring cavity 1 (we assume weak coupling between the two cavities), this would result in a round-trip phase difference between the cw_1 and ccw_1 modes due to the *Sagnac effect*, thus lifting the degeneracy between these counter propagating modes.

We require that the coupling between modes cw_1 and ccw_2 (and ccw_1 and cw_2) should be *conservative* [18], which would result in frequency splitting, in the passive mode structure of the coupled cavity. Whereas, the coupling (by backscattering) between cw_2 and ccw_2 should be of the *dissipative* kind, which would cause frequency locking of these two modes. These various couplings can be realized in the following way. To realize a conservative coupling, a *lossless* 2×2 fiber-optic coupler can be used, which would merely redistribute the intensity between the cw_1 and ccw_2 (and correspondingly between ccw_1 and cw_2) modes, causing a frequency splitting between them and would result in an anticrossing in the passive coupled cavity mode structure. The width of this frequency splitting would be proportional to the coupling ratio of the 2×2 coupler. On the other hand, the dissipative coupler (DC) could be a localized absorber, i.e., a thin (as compared to the wavelength of the input field) absorbing layer placed perpendicular to the mode axis, that would cause frequency locking of the modes in cavity 2. This phenomenon of frequency locking is same as the *locking problem* of the counterpropagating modes due to injection signal (caused by scattering, etc.) in laser gyroscope at low rotation rates [19]. As there is no rotation of cavity 2 the cw_2 and ccw_2 modes get frequency locked and become degenerate. Without this dissipative coupler one would have two two-level systems shifted in frequencies, such that cw_2 and ccw_2 would be uncoupled and near degenerate. Now there would be three nondegenerate modes in the system cw_1, ccw_1, and cw_2-ccw_2 (as the modes in the cavity 2 are coupled and frequency locked) [20].

On similar lines as in Ref. [17], we introduce two parameters S and W, where the Faraday isolator strength is proportional to S. W is proportional to intensity coupling of the modes $cw_1 \leftrightarrow ccw_2$ ($\propto W_+$) and $ccw_1 \leftrightarrow cw_2$ ($\propto W_-$) via the 2×2 coupler Fig. 6. We define the eigenvectors of the Hamiltonian H_S for $W=0$ as the S basis and those for $S=0$ as the W basis. In the S basis we have the following Hamiltonian:

$$H_S=\begin{bmatrix} S & W_+ & 0 \\ W_+ & 0 & W_- \\ 0 & W_- & -S \end{bmatrix}. \tag{9}$$

For simplicity we assume a symmetrical 2×2 coupler, i.e., $W=W_+=W_-$. On diagonalizing Eq. (9) we get the eigenfrequencies, which are $\omega_o=0$ and $\omega_\pm=\pm\sqrt{2S^2+W^2}$. On varying the parameter S one obtains an avoided crossing due to the coupling W.

Now let us consider the three-level optical atom in presence of a harmonically time-dependent field at the frequency ω_l i.e., $S=S_0 e^{-i\omega_l t}+\text{c.c.}$ We transform the Hamiltonian to the W basis, which after the rotating-wave approximation is

$$H_W=\begin{bmatrix} \Delta_+ & S_0 & 0 \\ S_0 & 0 & S_0 \\ 0 & S_0 & -\Delta_- \end{bmatrix}. \tag{10}$$

The detunings are defined as $\Delta_+=\omega_l-W_+$ and $\Delta_-=\omega_l-W_-$. The generalized Rabi frequency of the optical atom for $W_+=W_-$, between the various transitions is proportional to $\sqrt{\Delta^2+2S_0^2}$. A comparison of Eqs. (9) and (10) shows that the analogy between the three-level atomic system and its optical implementation is complete if the following connections are made:

$$\omega_{12} \leftrightarrow \omega_+$$

$$\omega_{23} \leftrightarrow \omega_-$$

$$\Delta_{12} \leftrightarrow \Delta_\pm$$

$$\mathbf{d}_{12},\mathbf{d}_{23} \leftrightarrow (\text{magneto-optic coefficient of FAR})\times(\text{its length}).$$

To observe the phenomenon of trapping states discussed earlier, one would have to generate an appropriate field S at the Faraday rotator, so that $\partial B/\partial t$ simulates frequency modulated field, with a modulation index M so that $J_0(M)=0$, and the frequency of modulation Ω, the choice of which would determine the times at which various levels cross each other.

V. CONCLUSIONS

(a) We have shown parameter regimes where various kinds of multilevel crossings occur, causing population redistribution. (b) The phase accumulated by various states in between these crossings plays a vital role in determining the transition probabilities after multiple crossings, leading to quantum interference effect. (c) We have demonstrated a method for near 100% population inversion in three-level ladder system, without affecting the population in the intermediate level. (d) Trapping of population persists, even if there is an initial phase mismatch between the two applied fields, or unequal choice of the modulation index and modulation frequency. (e) We have proposed an optical implementation of the three-level ladder system in which such a phenomenon can be observed.

Furthermore, a three-level Λ configuration is also possible with an appropriate choice of a set of longitudinal modes in the cavity and the corresponding coupling of these modes. Unlike the fiber-optic possibility, one could even go in for bulk optic cavities where, a variable coupling between the cavities can be obtained using various methods of evanescent coupling [21]. This optical atom implementation would open avenues to observe richer dynamics in three-level systems in hitherto unexplored parameter regime.

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$$\omega_{\pm} = \omega_o^i \pm \tfrac{1}{2}[(L^2/n^2\pi^2c^2)(\Omega_{rot} - \Omega_{rot}^j)^2 + (\Delta_{\pm}/\omega_o^i)]^{1/2},$$

where L is the length of each cavity, Ω_{rot} is the uniform angular frequency with which cavity 1 rotates, n is the refractive index and $\Delta_{\pm} = (\sqrt{\gamma_{\pm}}/\pi)\Delta\omega_{fsr_1}$. Here, $\Delta\omega_{fsr_1}$ is the free spectral range of the ring resonator 1, and $\gamma_{\pm}$ is the intensity coupling coefficient per round trip between the $\omega_{\pm}$ and ω_o modes, respectively (which depend on the coupling ratio of the 2×2 fiber coupler).

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