Reflection of the Jaynes-Cummings dynamics in the spectrum of a regularly pumped micromaser

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We derive approximate analytical expressions for the spectrum of a regularly pumped micromaser. Our procedure is based on ensemble averaging rather than time averaging, and the results are obtained from the solution of a non-Markovian evolution problem. For certain values of the atom-field interaction time the spectrum is split into several equidistant peaks. We show that this line splitting reflects the phase reversal of the entire cavity field caused by the transit of a single atom. This phenomenon is due to the occurrence of quantum Rabi oscillations and it is closely related to the Jaynes-Cummings revival. [S1050-2947(97)08601-0]

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I. INTRODUCTION

Recently, renewed attention has been paid to the experimental test of field quantization in a single-mode cavity which reveals itself in the occurrence of quantum Rabi oscillations at discrete frequencies when the cavity is probed by two-level atoms. For Rydberg atoms interacting with the vacuum or with a small coherent field in a microwave cavity, the Rabi nutation in the atomic population has been observed for a wide range of interaction times, and the signal has indeed been found to exhibit discrete Fourier components at frequencies proportional to the square root of successive integers [1]. Much earlier the oscillation of the atomic population in a conventional micromaser had been detected for a limited range of interaction times [2]. The observed behavior was reminiscent of the collapse and revival of the atomic population which are obtained for a coherent field with the help of the Jaynes Cummings-model [3] describing the interaction of a single two-level atom with a single quantized mode of the radiation field. Whereas a collapse of the Rabi nutation could also be due to classical intensity fluctuations giving rise to a spread of the Rabi frequencies, the revival is a pure quantum effect which has its origin in the discreteness of the possible values of the field energy.

Another effect that arises from the granular structure of the field energy and that cannot be explained classically is the approximate reversal of the phase of the cavity field strength that occurs for certain values of the interaction time due to the transit of a single atom even at very large field amplitudes. For a coherent field this effect has been calculated numerically with the help of the Jaynes-Cummings model [3]. It has been found that almost complete phase reversal occurs when the interaction time t_{int} is an odd multiple of the revival time [3]. More generally, the Q function of the coherent field turned out to be shifted by the phase angle π in the complex plane at these values of t_{int} [4].

A phenomenon closely related to the phase reversal of the field strength consists in the phase difference of the complex field-field correlation functions relating the field strength before and after the transit of a single atom to the original field strength. With a and $a^\dagger$ being the single-mode photon annihilation and creation operators, this can be expressed by the relation $\langle a^\dagger(t+t_{\mathrm{int}})a(t)\rangle \approx -\langle a^\dagger(t)a(t)\rangle$, which also cannot be explained in classical terms. The field-field correlation functions have to be considered when the phase of the field is uncertain, i.e., $\langle a\rangle = 0$, as it is the case in lasers and micromasers which are pumped by atoms being in a definite energy state. To avoid confusion we note the following. When speaking of collapse and revivals in the Jaynes-Cummings-dynamics one is thinking of atoms that probe one and the same field with increasing interaction times. However, when the interaction time t_{int} is changed, the field which is built up by the atoms changes, too. Thus one cannot trace the dynamics of the Jaynes-Cummings model in the original sense by increasing the interaction time. Rather, for each value of t_{int} one obtains one single point of the collapse-revival curve for one specific field. For properly chosen values of t_{int} the above-mentioned nonclassical phase reversal will occur.

It is the aim of the present contribution to show by an analytical treatment that in the spectrum of a micromaser with regular pumping [5] clear evidence can be found for the nonclassical phase reversal of the field-field correlation function caused by the quantum Rabi oscillations. The phase reversal reveals itself through a splitting of the power spectrum into several equidistant peaks, alternatively, through an oscillatory decay of the correlation function, $\langle a^\dagger(\tau)a(0)\rangle$ with growing τ, for certain values of the interaction time t_{int}. The measurement of the spectrum of a regularly pumped micromaser at these values of t_{int} therefore could provide an independent experimental means for the observation of one of the most pronounced quantum features of the Jaynes-Cummings model.

We mention that spectral splitting due to regular pumping has been already found previously by numerical calculations [6] or by an approximate analytical treatment [7] using a stroboscopic approach with subsequent time averaging. In contrast to this, we make use of the recently proposed [8] unified treatment of discrete (regular) and continuous [9,10] non-Poissonian pumping which rests on ensemble averaging. The strength of this method consists in the fact that it yields an evolution equation for the field density matrix that can be easily interpreted in physical terms and that allows exact solutions [8] under trapping-state conditions [11]. As we shall show, for large mean photon numbers an approximate analytical treatment is possible.

II. BASIC EQUATIONS

We consider the conventional one-atom micromaser [12] pumped by a beam of resonant Rydberg atoms which are excited to the upper level of the maser transition. Throughout the paper we suppose that only one atom at a time is present in the cavity and that the damping of the cavity field can be neglected over the transit time $t_{\rm int}$ of a single atom. The change of the reduced density operator ρ of the field caused by the interaction with a single atom then can be formally described by the equation $\rho(t+t_{\rm int})=M(t_{\rm int})\rho(t)$, where, in the photon-number representation, the Jaynes-Cummings-superoperator M is given by

$$[M\rho]_{n,n+k}=\cos(\sqrt{n+k+1}gt_{\rm int})\cos(\sqrt{n+1}gt_{\rm int})\rho_{n,n+k}+\sin(\sqrt{n+k}gt_{\rm int})\sin(\sqrt{n}gt_{\rm int})\rho_{n-1,n+k-1} \tag{1}$$

with $n,k=0,1\ldots$ and g being the atom-field coupling constant. Due to cavity damping over a time interval t the field density operator is changed according to $\rho(t)=\exp(Lt)\rho(0)$, where

$$[e^{Lt}\rho]_{n,n+k}=e^{-\gamma t(n+k/2)}\sum_{j=0}^{\infty}(1-e^{-\gamma t})^j\times\sqrt{\binom{n+k+j}{j}\binom{n+j}{j}}\rho_{n+j,n+k+j}. \tag{2}$$

Here γ denotes the cavity damping constant and, for simplicity, thermal photons have been neglected.

The evolution of the density operator ρ in the interaction representation, resulting from the combined action of cavity damping and atom-field interaction, obeys the general equation $\rho(\tau)=V(\tau,0)\rho(0)$. For all kinds of non-Poissonian micromaser pumping the resulting dynamics, characterized by the evolution operator $V(\tau,0)$, can be shown to be non-Markovian [8]. In the following we specialize to a micromaser where the Rydberg atoms in the incoming atomic beam are regularly spaced with the time interval T between successive atoms. We assume that the atoms are brought into resonance with the field with probability p before the interaction with the cavity field. The evolution operator $V(\tau,0)$ then explicitly reads [8]

$$\begin{aligned}V(\tau,0)={}&\frac{1}{T}\int_0^{xT}dt'\,e^{L(xT-t')}[1+p(M-1)]\\&\times\{e^{Lt}[1+p(M-1)]\}^{[\tau/T]}e^{Lt'}\\&+\frac{1}{T}\int_{xT}^{T}dt'\,e^{L(xT-t')}\{e^{LT}[1+p(M-1)]\}^{[\tau/T]}e^{Lt'}\end{aligned} \tag{3}$$

where $x=\tau/T-[\tau/T]$ with $[\tau/T]$ denoting the largest integer that does not exceed τ/T. The two parts of the sum in the above equation arise from the fact that either $[\tau/T]+1$ or $[\tau/T]$ atoms may be present in an interval of length τ that is arbitrarily located with respect to the arrival times of the atoms. When the micromaser has reached a stationary state which is independent of the initial conditions and described by the steady-state density operator $\overline{\rho}=\lim_{\tau\to\infty}\rho(\tau)$, all two-time expectation values of field quantities can be obtained with the help of the evolution operator V [8]. In particular, we get [8]

$$\langle a^{\dagger k}(\tau)a^k(0)\rangle_{ss}={\rm Tr}[V(\tau,0)a^k\rho a^{\dagger k}]. \tag{4}$$

As a straightforward generalization of the usual power spectrum we introduce the k-photon spectrum $S_k(\omega)$ that could be determined by Fourier transforming the output current of a hypothetical photodetector which is based on k-photon-absorption, i.e., we define, in analogy to the recently introduced two-photon-spectrum [13],

$$S_k(\omega)=\frac{1}{\pi}{\rm Re}\int_0^\infty d\tau\frac{\langle a^{\dagger k}(\tau)a^k(0)\rangle_{ss}}{\langle a^{\dagger k}a^k\rangle_{ss}}e^{-i(\omega-k\nu)\tau}, \tag{5}$$

where ν is the frequency of the cavity mode. In Ref. [13] it has been shown that the two-photon spectrum can be determined by investigating the two-photon absorption in a weak beam of two-photon-resonant probe atoms. It yields additional information about the dynamics of the density matrix of the cavity field, which cannot obtained from the one-photon spectrum.

III. ANALYTICAL RESULTS

First let us consider the Jaynes-Cummings dynamics separately. Describing the field at $t=0$ by the density operator ρ, we calculate the field expectation value $\langle a^{\dagger k}(t_{\rm int})\rangle$ after the interaction with a single atom, and the field correlation function $\langle a^{\dagger k}(t_{\rm int})a^k(0)\rangle$. Making use of Eq. (1) and of the properties of the operators a and $a^\dagger$ we find

$$\langle a^{\dagger k}(t_{\rm int})\rangle={\rm Tr}[M\rho a^{\dagger k}]=\sum_{n=0}^{\infty}\sqrt{\frac{(n+k)!}{n!}}\rho_{n,n+k}f_n^{(k)}(t_{\rm int}), \tag{6}$$

where we introduced the abbreviation

$$\begin{aligned}f_n^{(k)}(t_{\rm int})={}&\cos(\sqrt{n+k+1}gt_{\rm int})\cos(\sqrt{n+1}gt_{\rm int})\\&+\sqrt{1+\frac{k}{n+1}}\sin(\sqrt{n+k+1}gt_{\rm int})\\&\times\sin(\sqrt{n+1}gt_{\rm int}).\end{aligned} \tag{7}$$

On the other hand, in analogy to Eq. (4) we arrive at

$$\langle a^{\dagger k}(t_{\rm int})a^k(0)\rangle={\rm Tr}[Ma^k\rho a^{\dagger k}]=\sum_{n=0}^{\infty}\frac{(n+k)!}{n!}\rho_{n+k,n+k}f_n^{(k)}(t_{\rm int}). \tag{8}$$

Obviously, for a field that is initially in a coherent state $|\alpha\rangle$, the interaction-time dependence is identical in the expressions (6) and (8) since $\sqrt{(n+k)!/n!}\rho_{n+k,n+k}=\alpha^k\rho_{n,n+k}$. The same holds true when the photon-number distribution is truncated at the maximum photon number $n_{\max}=k$ since then only the terms with $n=0$ survive in the sums. Now we assume that the photon-number distribution of the field is strongly peaked at the mean photon number $\bar{n}\gg k$. By expanding the square root in front of the sine term of Eq. (7) we get the approximation

$$f_n^{(k)}(t_{\rm int})=\cos[(\sqrt{n+1+k}-\sqrt{n+1})gt_{\rm int})]+\left(\frac{k}{2\bar{n}}-\frac{k^2}{8\bar{n}^2}\right)\sin^2(\sqrt{\bar{n}}gt_{\rm int}) \qquad (9)$$

that can be further simplified to yield $f_n^{(k)}\approx b_k$ with

$$b_k(t_{\rm int})=\cos\left(\frac{kgt_{\rm int}}{2\sqrt{\bar{n}}}\right)+\frac{k}{2\bar{n}}\left(1-\frac{k}{4\bar{n}}\right)\sin^2(\sqrt{\bar{n}}gt_{\rm int}), \qquad (10)$$

which is independent of n. Instead of Eqs. (6) and (8) we therefore may write

$$\frac{\langle a^{\dagger k}(t_{\rm int})\rangle}{\langle a^{\dagger k}(0)\rangle}=\frac{\langle a^{\dagger k}(t_{\rm int})a^k(0)\rangle}{\langle a^{\dagger k}(0)a^k(0)\rangle}=b_k(t_{\rm int})\geq -1. \qquad (11)$$

From Eq. (9) it becomes obvious that the cosine term on the right-hand side of Eq. (10) stems from the discreteness of the Rabi-oscillation frequencies due to field quantization. When $kgt_{\rm int}=2\pi\sqrt{\bar{n}}(2l+1)$ $(l=0,1,\ldots)$, i.e., when b_k is approximately equal to -1, the phase of the complex field quantity $\langle a^{\dagger k}\rangle$ is reversed by the interaction with a single atom whereas the amplitude is kept constant. For $k=1$ this corresponds to the reversal of the electric field strength due to the Jaynes-Cummings interaction which has been calculated for a coherent field to occur at odd multiples of the revival time $t_R=g^{-1}2\pi\sqrt{\bar{n}}$ [3].

When describing mere cavity damping over a time interval of length t_d for a field with a photon-number distribution which is strongly peaked at the mean value $\bar{n}\gg 1$, we can use the approximation

$$\frac{\langle a^{\dagger k}(t_d)\rangle}{\langle a^{\dagger k}(0)\rangle}=\frac{\langle a^{\dagger k}(t_d)a^k(0)\rangle}{\langle a^{\dagger k}(0)a^k(0)\rangle}=\frac{{\rm Tr}[e^{Lt_d}a^k\rho a^{\dagger k}]}{{\rm Tr}[a^k\rho a^{\dagger k}]}=e^{-(k/2)\gamma t_d}. \qquad (12)$$

We remark that for a field being initially in a coherent state, Eq. (12) holds true for arbitrary values of $\bar{n}$ since in this case for normally ordered expectation values the dynamics is identical to that of the corresponding classical quantities. In the frame of our approximation, which is valid to the lowest order of the small quantity $\bar{n}^{-1}$, the deviation of the strongly peaked photon-number distribution from the Poissonian one belonging to a coherent state is negligible.

In order to calculate the k-photon spectrum with the help of Eqs. (3)–(5), we make use of the fact that on the assumption of a strongly peaked photon-number distribution with $\bar{n}\gg 1$ the superoperators L and M approximately commute [14]. After performing the integration with respect to t' we then obtain from Eq. (3)

$$V(\tau,0)=e^{L\tau}[1+p(M-1)]^{[\tau/T]}\left\{1+p(M-1)\left(\frac{\tau}{T}-\left[\frac{\tau}{T}\right]\right)\right\}. \qquad (13)$$

Taking into account Eqs. (11), (12), and (4) it seems justified to make the approximation

$$\langle a^{\dagger k}(\tau)a^k(0)\rangle_{ss}=v_k(\tau,0)\langle a^{\dagger k}a^k\rangle_{ss}, \qquad (14)$$

where

$$v_k(\tau,0)=e^{-(k/2)\gamma\tau}[1+p(b_k-1)]^{[\tau/T]}\times\left\{1+p(b_k-1)\left(\frac{\tau}{T}-\left[\frac{\tau}{T}\right]\right)\right\}, \qquad (15)$$

with $b_k=b_k(t_{\rm int})$ being given by Eq. (10). It is interesting to note that Eqs. (14) and (15) provide an exact solution of the non-Markovian evolution problem when the photon-number distribution is truncated at $n_{\max}=k$ [8]. This can be achieved by operating the micromaser under the k-photon trapping condition $gt_{\rm int}=q\pi/\sqrt{k+1}$ $(q=1,2,\ldots)$ [11]. Instead of Eq. (10) one then has to use the quantity [8]

$$b_k=b_k(q)=\cos(q\pi)\cos\left(\frac{q\pi}{\sqrt{k+1}}\right), \qquad (16)$$

which follows from Eq. (7) with $n=0$. From Eqs. (5) and (14) we obtain the k-photon spectrum

$$S_k(\omega)=\frac{1}{\pi}{\rm Re}\left[\frac{1}{\frac{k}{2}\gamma+i(\omega-k\nu)}\left\{1+\frac{p(b_k-1)}{\left[\frac{k}{2}\gamma+i(\omega-k\nu)\right]T}\times\frac{1-e^{[(k/2)\gamma+i(\omega-k\nu)]T}}{1-[1+p(b_k-1)]e^{-[(k/2)\gamma+i(\omega-k\nu)]T}}\right\}\right]. \qquad (17)$$

When $b_k\leq 0$, the quantity $1+p(b_k-1)$ may take on negative values. In this case the function $v_k(\tau,0)$ exhibits oscillatory decay in a sawtooth like manner, cf. [6–8]. This leads to a splitting of the spectrum into several peaks separated by the distance $\Delta\omega=2\pi T^{-1}$; see Figs. 1 and 2 for the usual one-photon spectrum. Since b_k cannot be smaller than -1 [cf. Eqs. (10) and (16)], spectral splitting can only occur when $p>\frac{1}{2}$ in accordance with the results obtained numerically [6] for $k=1$. The physical origin of this spectral splitting is the reversal of the sign of the field strength (when taken to be real initially), due to the transit of a single atom, or, more precisely, the reversal of the sign of the corresponding field-field correlation function, which is expressed by a negative value of b_k. To estimate the average effect of one injected atom on the field in our model, we have to consider both the possibilities that the atom does not interact with the field since it is out of resonance, which occurs with probability $(1-p)$, and that an interaction takes place (probability p), which causes a change due to the quantum features of the Jaynes-Cummings interaction [see Eq. (11)]. Thus the factor $1-p+pb_k$ in Eq. (15) becomes plausible. With decreasing atom distance T, i.e., with increasing injection rate or in-

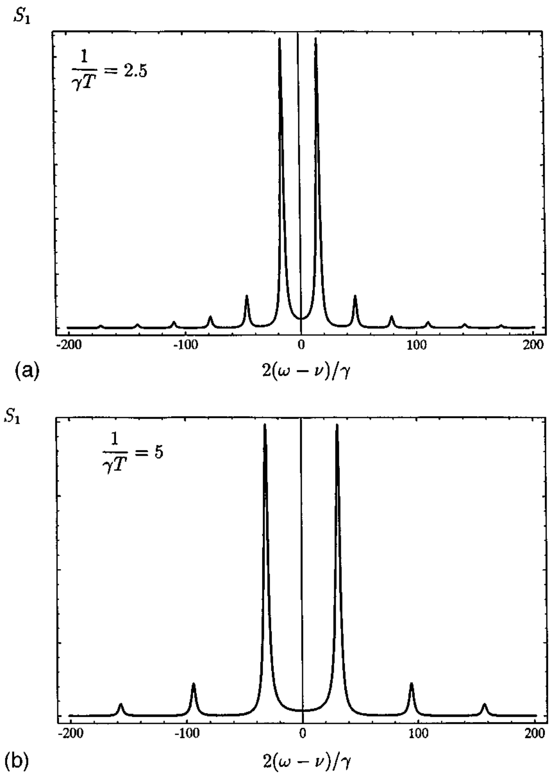

FIG. 1. One-photon spectrum of a regularly pumped micromaser for different values of $(\gamma T)^{-1}$. The quantity $1+p(b_1-1)$ has the value -0.9, where b_1 is given by Eq. (10) with $k=1$.

creasing mean photon number of the field, respectively, the distance between the spectral peaks is enlarged and the individual peaks are broadened (see Fig. 1).

Phase reversal of the field due to the Jaynes-Cummings interaction with a single resonant atom, of course, also occurs for Poissonian pumping. However, this effect cannot be observed in the spectrum since the spectral splitting is smeared out due to the statistical distribution of the arrival times of the atoms, whereas the overall linewidth remains approximately constant (see Fig. 2). To see this more explicitly, we investigate Eq. (17) in the double limit $p\rightarrow 0, T\rightarrow 0$ but $r=p/T$ finite, which corresponds to Poissonian pumping with pump rate r [5]. By expanding the exponential functions for small T we find after minor algebra

$$S_k(\omega)=\frac{1}{\pi}\frac{\frac{k}{2}\gamma+r(1-b_k)}{\left[\frac{k}{2}\gamma+r(1-b_k)\right]^2+(\omega-k\nu)^2} \tag{18}$$

describing a Lorentzian spectrum with linewidth (full width at half maximum)

$$\Delta\omega^{(k)}=k\gamma+2r(1-b_k). \tag{19}$$

The Lorentzian spectrum corresponds to an exponential decay of the correlation functions $\langle a^{\dagger k}(\tau)a^k(0)\rangle$ in accordance with the Markovian character of the field dynamics for Poissonian pumping. On the assumption of a strongly peaked photon-number distribution with large mean photon number $\overline{n}$ the average rate of photon production in the cavity due to atomic deexcitation is $r\sin^2(\sqrt{\overline{n}}gt_{\mathrm{int}})$, and in the stationary regime of the micromaser this production rate equals the photon loss rate $\gamma\overline{n}$. When we insert the expression (10) into Eq. (19), replacing $\sin^2(\sqrt{\overline{n}}gt_{\mathrm{int}})$ by $\gamma\overline{n}/r$, we obtain the linewidth of the k-photon spectrum with Poissonian pumping

$$\begin{aligned}\Delta\omega^{(k)}&=2r\left[1-\cos\left(\frac{kgt_{\mathrm{int}}}{2\sqrt{\overline{n}}}\right)\right]+k^2\frac{\gamma}{4\overline{n}}\\&=4r\sin^2\left(\frac{kgt_{\mathrm{int}}}{4\sqrt{\overline{n}}}\right)+k^2\frac{\gamma}{4\overline{n}}.\end{aligned} \tag{20}$$

For the one-photon spectrum this result is identical with the linewidth calculated previously [15] for a Poissonian pumped micromaser having a strongly peaked photon-number distribution with a sufficiently large mean photon number $\overline{n}$. The latter can be only achieved when the pump rate r is considerably larger than the cavity decay constant γ. Therefore it becomes obvious from Eq. (20) with $k=1$ that for most values of t_{int} the micromaser linewidth is much larger than the Schawlow-Townes linewidth $\gamma/2\overline{n}$ of a conventional laser operated well above threshold. An exception only occurs when (for $k=1$) $\cos(gt_{\mathrm{int}}/2\sqrt{\overline{n}})$ is close to unity, i.e., when the interaction time is an even multiple of the revival time. In these cases the phase change of the field due to the transition of a single atom is caused by spontaneous emission only and not by the superposition of Rabi oscillations with different frequencies.

In Fig. 2 the dependence of the one-photon spectrum on the value of b_1 is illustrated for Poissonian pumping and for regular pumping ($p=1$) with the same rate $r=T^{-1}$. The broadening of the overall spectral width with decreasing value of b_1 is clearly visible. On the other hand, when splitting occurs for regular pumping, the width of the individual peaks is diminished with decreasing b_1, the splitting thus becoming more and more pronounced. For complete splitting ($b_1=-1$) the overall linewidth takes its maximum value $4r=4/T$. Because the peak distance is $2\pi/T$, only two very strong peaks will be practically resolvable.

For a micromaser operated under the k-photon-trapping condition, the k-photon spectrum for Poissonian pumping is exactly of Lorentzian shape with linewidth

$$\Delta\omega^{(k)}=k\gamma+\cos(q\pi)\cos\left(\frac{q\pi}{\sqrt{k+1}}\right), \tag{21}$$

$q=(1,2,\ldots)$, following from Eqs. (16) and (18) and being in accordance with the result derived previously for $k=1$ [16].

IV. CONCLUSIONS

Finally, we stress once again that our analytical results for $\overline{n}\gg 1$ hold true only for those values of t_{int} where the photon-number distribution indeed is single-peaked. In all other cases one has to resort to numerical calculations. We mention that in these other cases also for Poissonian pumping spectral splitting can occur, however, not into regularly spaced peaks [17]. The reason then is that the approach to equilibrium is dominated by several exponentially decaying contributions. Some of them enter with negative weight, and since they decay at different rates, in various time intervals various terms may dominate the dynamics, thus giving rise to an oscillatory approach to equilibrium. In contrast to this

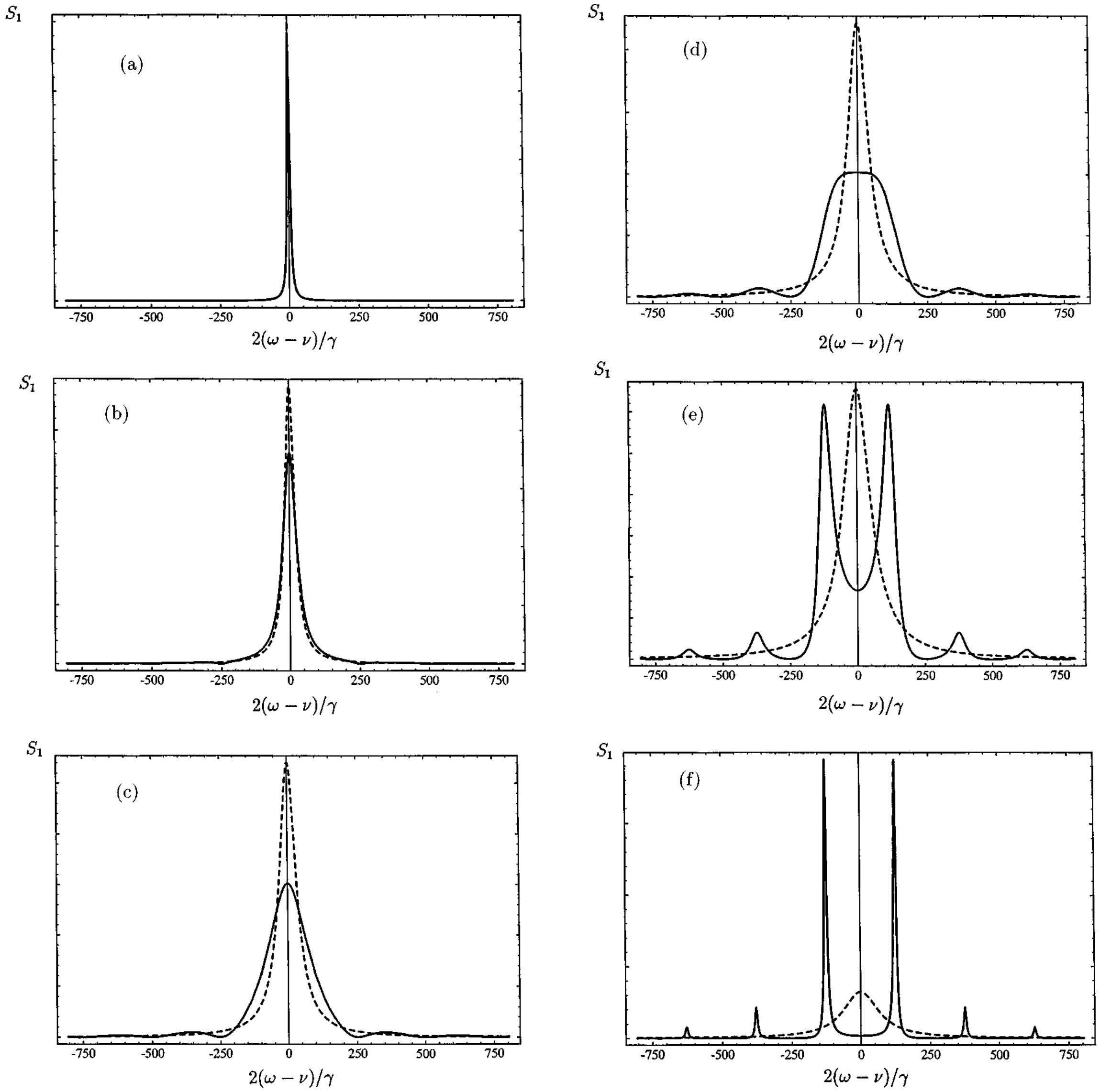

FIG. 2. One-photon spectrum of a regularly pumped micromaser with $p=1$ and $(\gamma T)^{-1}=20$ (full line) and of a Poissonian pumped micromaser with $r/\gamma=20$ (dashed line). The values of the parameter b_1 are (a) 0.9, (b) 0.5, (c) 0.1, (d) –0.1, (e) –0.5, and (f) –0.9.

for a micromaser with regular pumping the sawtoothlike decay of the field-field correlation functions at certain values of the interaction time and the corresponding spectral splitting into several equidistant peaks is caused by the fact that a single atom can truly reverse the phase of the entire cavity field due to the occurrence of Rabi oscillations with discrete frequencies, an effect which is closely related to the Jaynes-Cummings revival. Thus the detection of these equidistant spectral peaks could provide an independent means for the observation of one of the most pronounced features of the Jaynes-Cummings dynamics.

Note added. Since submitting this paper, we received a manuscript by Briegel *et al.* [18], which also deals with spectral micromaser properties.

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