Nonlinear theory of noninversion lasers of an open three-level system

Shi-Yao Zhu,[1] De-Zhong Wang,[1] and Jin-Yue Gao[2]
[1]*Department of Physics, Hong Kong Baptist University, Kowloon Tong, Hong Kong*
[2]*Physics Department, Jilin University, Changchun, Jilin 130023, China*
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A nonlinear theory of a noninversion laser of an open three-level system based on electromagnetically induced transparency is given. In this noninversion laser model, a nonselective pumping mechanism is used for obtaining lasing gain such that the population in the lower lasing level is always larger than a half of the total population no matter whether the driving field is on or off. In the lasing region, there is no direct pumping to the upper level. The conditions for the existence of a nontrivial and stable laser field are obtained. The dependence of the laser intensity on the intensity of the driving field and the ratio of the two decay rates from the upper level has been obtained, and the ways to obtain a large laser field have been studied. [S1050-2947(96)07112-0]

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I. INTRODUCTION

Electromagnetically induced transparency (EIT), which reduces or even cancels the stimulated absorption while keeping the stimulated emission unaffected, can result in lasing without inversion (LWI), which is important in the generation of high-frequency lasers, where the upper lasing level is difficult to populate. Many theoretical [1–10] and some experimental [11,12] studies have been carried out during the past several years on EIT and EIT-based LWI. In a closed three-level system, the absorption from the lower level to the lasing upper level can be totally canceled by using the EIT effect under certain conditions. However, incoherent pumping to the upper level is still required in order to have a gain to overcome the cavity and other losses. Furthermore, the incoherent pumping will destroy some atomic coherence which is responsible for the reduction or cancellation of the absorption. That is to say, the incoherent pumping will decrease the absorption reduction, and the absorption cannot be totally canceled due to the incoherent pumping. Therefore, a certain amount of pumping is necessary for having a gain to overcome the absorption as well as the cavity and other losses. In the early papers [2], the problem of how to pump the upper level was not discussed. Later, selective incoherent pumping to the upper lasing level was introduced to get the gain [4]. The selective pumping was optical, which means we already have an incoherent light at the same frequency of the laser to be generated. In order to get sufficient pumping, the incoherent light needs to reach a certain intensity. In the generation of high-frequency lasers via LWI, this incoherent pumping is still the main obstacle. Another method to get the gain was suggested to pump the third level (not the upper and lower lasing levels) from the lower lasing level [6]. However, this is a forbidden transition, and the scheme to pump this level from the lower level was not discussed.

In the EIT-based LWI systems, noninversion is defined to mean that the population in the upper and the third levels (say levels a and c) is less than the population in the lower level (say level b), $\rho_{aa}+\rho_{cc}<\rho_{bb}$. However, the population is dependent on the Rabi frequency of the driving field. The commonly used definition of noninversion is $\rho_{aa}+\rho_{cc}<\rho_{bb}$, when the driving field is on (Rabi frequency not equal to zero) [2,4]. However, if the driving field is switched off (Rabi frequency equal to zero) and other parameters are kept the same, it can be shown that the population in the upper and the third levels would be larger than the population in the lower level (inversion $\rho_{aa}+\rho_{cc}>\rho_{bb}$) [13]. Another definition [6] of inversion is $\rho_{aa}+\rho_{cc}<\rho_{bb}$, when the driving field is off (Rabi frequency equal to zero). Under the second definition, a pumping to the third level (level c) is necessary. The transition from the lower level to the third level is forbidden, and this pumping is quite difficult to realize without other influences in a closed three-level system. In addition, when the driving field is on (Rabi frequency not equal to zero), the population in the upper and the third level could be larger than the population in the lower level (inversion $\rho_{aa}+\rho_{cc}>\rho_{bb}$) [13]. Note that the two definitions are contradictory to each other.

Quite a few LWI experiments have been reported. Now it is time to consider how to use the LWI to generate high-frequency lasers. The key problem, in addition to the cavity, is the incoherent pumping. How do you use nonselective pumping to populate the upper or third level without destroying the atomic coherence which is needed for the absorption cancellation?

A recent study [14] shows that a positive gain can be obtained with the population in the lower level being always larger than a half, no matter whether the driving field is on or off, in an open three-level system. Moreover, in such a system nonselective pumping can be used for getting the gain. A linear theory was given there. In this paper, we would like to extend the previous work to get the nonlinear theory for such a LWI system.

II. EQUATION OF MOTION

Consider an atomic beam composed of three-level atoms (see Fig. 1), which is injected into a cavity. Before entering the cavity, the atoms are populated in the lower lasing level $|b\rangle$ and another level $|c\rangle$. The transition between level $|b\rangle$ and level $|c\rangle$ is forbidden. In the cavity the atoms are driven by a coherent field on resonance at the transition between

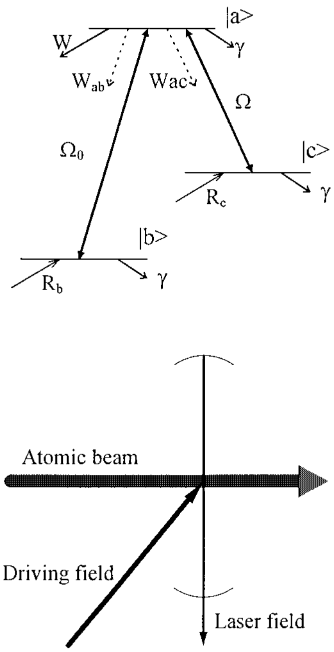

FIG. 1. The three-level atom.

levels $|a\rangle$ and $|c\rangle$, and interact with the laser field resonantly on the transition between levels $|a\rangle$ and $|b\rangle$. After the interaction the atoms exit from the cavity at a rate γ. The interaction Hamiltonian of the system (the atom, the driving field, and the laser field) is

$$H_I=-\Delta_0|b\rangle\langle b|-\hbar\Delta|c\rangle\langle c|+\hbar\Omega|a\rangle\langle c| +\hbar\Omega^*|c\rangle\langle a|+\hbar\Omega_0|a\rangle\langle b|+\hbar\Omega_0^*|b\rangle\langle a|, \tag{1}$$

where $\Delta_0=\omega_{ab}-\omega_0$ and $\Delta=\omega_{ac}-\omega$ are detunings of the laser field (frequency ω_0) and the driving field (ω), respectively, and Ω_0 and Ω are the associated Rabi frequencies.

The equations of motion for the elements of the atomic density operator are

$$\dot{\rho}_{aa}=-i\Omega_0\rho_{ba}-i\Omega\rho_{ca}+i\Omega_0^*\rho_{ab}+i\Omega^*\rho_{ac} -(W_{ab}+W_{ac}+W+\gamma)\rho_{aa}, \tag{2a}$$

$$\dot{\rho}_{bb}=i\Omega_0\rho_{ba}-i\Omega_0^*\rho_{ab}+W_{ab}\rho_{aa}-\gamma\rho_{bb}+R_b, \tag{2b}$$

$$\dot{\rho}_{cc}=i\Omega\rho_{ca}-i\Omega^*\rho_{ac}+W_{ac}\rho_{aa}-\gamma\rho_{cc}+R_c, \tag{2c}$$

$$\dot{\rho}_{ab}=-i\Omega_0\rho_{bb}-i\Omega\rho_{cb}+i\Omega_0\rho_{aa}-i\Delta_0\rho_{ab}-\gamma_{ab}\rho_{ab}, \tag{2d}$$

$$\dot{\rho}_{ac}=-i\Omega_0\rho_{bc}-i\Omega\rho_{cc}+i\Omega\rho_{aa}-i\Delta\rho_{ac}-\gamma_{ac}\rho_{ac}, \tag{2e}$$

$$\dot{\rho}_{bc}=-i\Omega_0^*\rho_{ac}+i\Delta_0\rho_{bc}+i\Omega\rho_{ba}-i\Delta\rho_{bc}-\gamma_{bc}\rho_{bc}, \tag{2f}$$

where W_{ab} (or W_{ac}) is the population decay rate from $|a\rangle$ to $|b\rangle$ (or to $|c\rangle$), γ_{ij} is the off diagonal decay rate between levels i and j, R_b (or R_c) is the atomic injection rate for levels $|b\rangle$ (or $|c\rangle$). Here, we introduced W for the population decay rate from the upper level to other levels, which may exist in a real atomic system. The equation of motion for the laser field is determined by

$$\dot{\Omega}_0=i(\omega_c-\nu)\Omega_0-\frac{\Gamma}{2}\Omega_0-ig^2\rho_{ab}, \tag{3}$$

where Γ is the cavity loss rate, g is the coupling constant of the atom for the lasing transition (assuming real), ω_c and ν are the frequency of an empty cavity and the frequency of the laser field, respectively.

For simplicity, we let $\Delta=0$ and $\Delta_0=0$. In the steady case, we get

$$\rho_{ab}=i\Omega_0\alpha[(\gamma_{bc}\gamma_{ac}+|\Omega_0|^2)(\rho_{aa}-\rho_{bb})-|\Omega|^2(\rho_{aa}-\rho_{cc})], \tag{4}$$

where

$$\alpha=(\gamma_{ab}\gamma_{bc}\gamma_{ac}+\gamma_{ab}|\Omega_0|^2+\gamma_{ac}|\Omega|^2)^{-1}. \tag{5}$$

The populations in the three levels are given by

$$\begin{pmatrix}\rho_{aa}\\ \rho_{bb}\\ \rho_{cc}\end{pmatrix}=\begin{pmatrix}A_{11} & A_{12} & A_{13}\\ A_{21} & A_{22} & A_{23}\\ A_{31} & A_{32} & A_{33}\end{pmatrix}^{-1}\begin{pmatrix}0\\ -R_b\\ -R_c\end{pmatrix}, \tag{6}$$

where

$$A_{11}=2\alpha[I_0(I-\gamma_{bc}\gamma_{ac}-I_0)+I(I_0-\gamma_{ab}\gamma_{bc}-I)] -W_{ac}-W_{ab}-W-\gamma, \tag{7a}$$

$$A_{12}=2\alpha(I_0\gamma_{bc}\gamma_{ac}+I_0^2-II_0), \tag{7b}$$

$$A_{13}=2\alpha(-I_0I+I\gamma_{ab}\gamma_{bc}+I^2), \tag{7c}$$

$$A_{21}=-2\alpha I_0(I-\gamma_{bc}\gamma_{ac}-I_0)+W_{ab}, \tag{7d}$$

$$A_{22}=-2\alpha I_0(\gamma_{bc}\gamma_{ac}+I_0)-\gamma, \tag{7e}$$

$$A_{23}=2\alpha I_0I, \tag{7f}$$

$$A_{31}=-2\alpha I(I_0-\gamma_{ab}\gamma_{bc}-I)+W_{ac}, \tag{7g}$$

$$A_{32}=2\alpha II_0, \tag{7h}$$

$$A_{33}=-2\alpha I(\gamma_{ab}\gamma_{bc}+1)-\gamma, \tag{7i}$$

and

$$I_0=|\Omega_0|^2, \quad I=|\Omega|^2. \tag{8}$$

On substituting the above expressions for the populations ρ_{ii} into Eq. (4), we obtain

$$\rho_{ab}=-i\Omega_0\frac{A_2I_0^2+A_1I_0+A_0}{B_3I_0^3+B_2I_0^2+B_1I_0+B_0}, \tag{9}$$

where

$$A_2=R_b(W_{ac}+W_{ab}+W+\gamma), \tag{10a}$$

$$A_1=R_b[I(W_{ac}+3W_{ab}+3W+5\gamma) +2\gamma_0\gamma(W_{ac}+W_{ab}+W+\gamma)] -R_cI(W_{ac}-W_{ab}+W+3\gamma), \tag{10b}$$

$$A_0 = R_b(\gamma_0\gamma + I)[I(2W_{ab} + 2W + 4\gamma) + \gamma_0\gamma(W_{ac} + W_{ab} + W + \gamma)] - R_c I(\gamma_0\gamma + I)(W_{ac} - W_{ab} + W + 3\gamma), \tag{10c}$$

$$B_3 = 2(W_{ac} + W + 2\gamma), \tag{10d}$$

$$B_2 = 2I(2W_{ac} + W_{ab} + 2W + 4\gamma) + \gamma_0\gamma(5W_{ac} + W_{ab} + 5W + 9\gamma), \tag{10e}$$

$$B_1 = 2(\gamma_0\gamma + I)[I(W_{ac} + 2W_{ab} + 3W + 6\gamma) + \gamma_0\gamma(2W_{ac} + W_{ab} + 2W + 3\gamma)], \tag{10f}$$

$$B_0 = (\gamma_0\gamma + I)^2[2I(W_{ab} + W + 2\gamma) + \gamma_0\gamma(W_{ac} + W_{ab} + W + \gamma)], \tag{10g}$$

with $\gamma_0 = \gamma_{ac} = \gamma_{ab}$. From Eq. (9) we know that the condition of having a positive gain is A_0 negative. Because the numerator on the right-hand side of Eq. (9) is proportional to I_0^2, while its denominator is proportional to I_0^3, the lasing process would be saturated as the laser intensity increases (in a complicated way). Equations (9) and (10) are the basic equations for the following discussion. However, it is quite difficult to pick up important physics features by looking at them. In order to have some physical understanding of this problem let us consider two special cases, $I \gg \gamma_{ac}\gamma$ or $W_{ac} \gg W_{ab}, \gamma$ with low laser intensity and $W=0$. At low laser intensity, Eq. (9) can be rewritten as $\rho_{ab} = -i\Omega_0 A_0/(B_1 I_0 + B_0)$. First, considering $I \gg \gamma_{ac}\gamma$, we have

$$\rho_{ab} = i\Omega_0 R_c \frac{(W_{ac} + 3\gamma) - S(2W_{ab} + 4\gamma) - W_{ab}}{2W_{ab} I}, \tag{11a}$$

with $S = R_b/R_c$. From this equation we see that the gain condition is $R_c(W_{ac} - W_{ab} + 3\gamma) > R_b(2W_{ab} + 4\gamma)$. The noninversion condition $R_c < R_b$ will result in the requirement of the relation between W_{ac} and W_{ab}. For the case of $W_{ac} \gg W_{ab}$ and γ we can write

$$\rho_{ab} = i\Omega_0 \frac{IR_c - \gamma_{ac}\gamma R_b}{\gamma_{ac}\gamma(\gamma_{ac}\gamma + I)}, \tag{11b}$$

which yields the gain condition $I > S\gamma_{ac}\gamma$ for the intensity of the driving field.

In these two specific cases, we saw that the intensity of the driving field and atomic parameters need to meet certain criteria in order to have noninversion lasing. In the following sections, we will study the general dependence of linear gain and laser intensity on various parameters

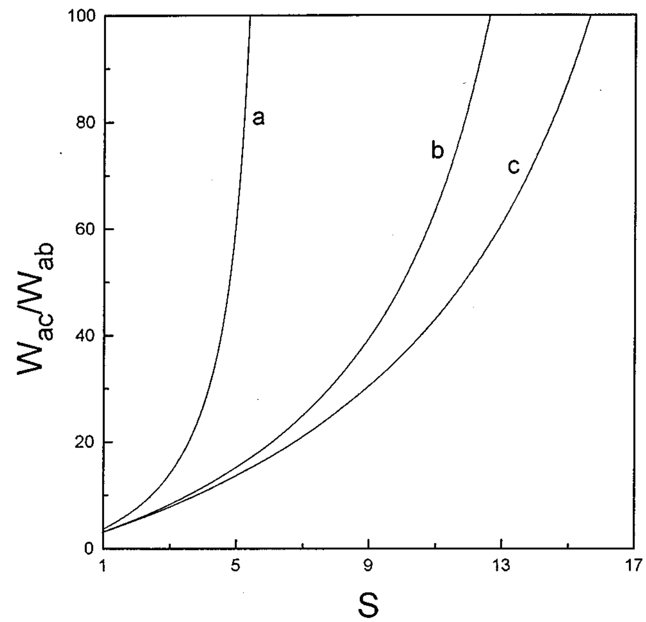

FIG. 2. The ratio of decay rates vs the injection rates to levels $|b\rangle$ and $|c\rangle$ with $W=0$, $\gamma=0.01W_{ac}$, and different driving fields (a) $\Omega=0.2W_{ac}$, (b) $\Omega=0.5W_{ac}$, (c) $\Omega=W_{ac}$, respectively.

III. LINEAR GAIN

The linear gain of the system is proportional to the imaginary part of the density element ρ_{ab} and is given by

$$G = -g^2 \frac{A}{B}, \tag{12a}$$

where

$$A = R_b[I(2W_{ab} + 2W + 4\gamma) + \gamma_0\gamma(W_{ac} + W_{ab} + W + \gamma)] - R_c I(W_{ac} - W_{ab} + W + 3\gamma), \tag{12b}$$

$$B = (\gamma_0\gamma + I)[2I(W_{ab} + W + 2\gamma) + \gamma_0\gamma(W_{ac} + W_{ab} + W + \gamma)]. \tag{12c}$$

It can be seen from Eq. (12c) that B is always positive. The condition of having a positive gain is that A should be negative. From the expression for A, i.e., Eq. (12b), we can see that A is a first-order function of I. For $I=0$ we have a positive A (loss for the laser field). By increasing the intensity of the driving field, we can have a negative A. For an intensive driving field ($I \gg \gamma_0\gamma$), the condition of having a net gain becomes

$$W_{ac} > (4S-3)\gamma + (2S+1)W_{ab} + (2S-1)W. \tag{13}$$

It can be seen from Eq. (13) that W_{ac} needs to be at least three times larger than W_{ab} for a gain without inversion ($S > 1$). In general, the condition for having a gain is determined by $A<0$. In Fig. 2, we plot the relation between W_{ac}/W_{ab} and S for having a noninversion gain. Larger W_{ac}/W_{ab} will lead to larger noninversion (larger S) with a positive gain.

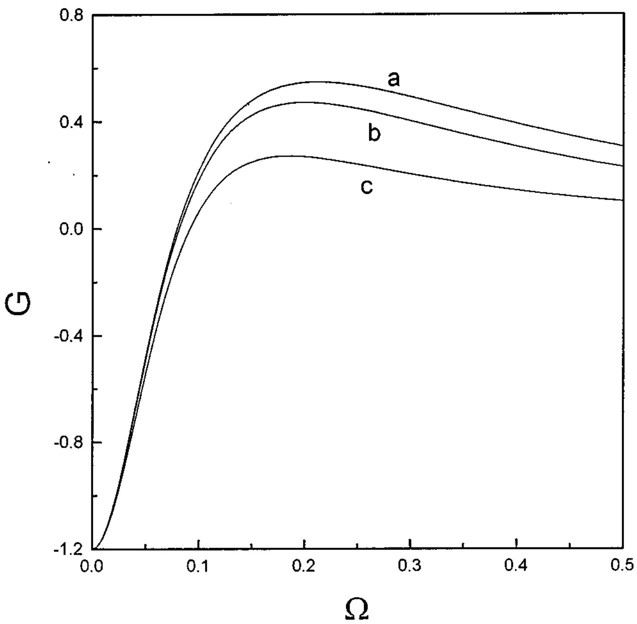

FIG. 3. Dependence of the gain (in the unit of $g^2 R_c / \gamma_{ac}\gamma$) on the Rabi frequency with $S=1.2$, $W=0$, $\gamma=0.01W_{ac}$. Curves (a), (b), (c) are corresponding to $W_{ab}=0.001W_{ac}$, $W_{ab}=0.01W_{ac}$, and $W_{ab}=0.05W_{ac}$, respectively.

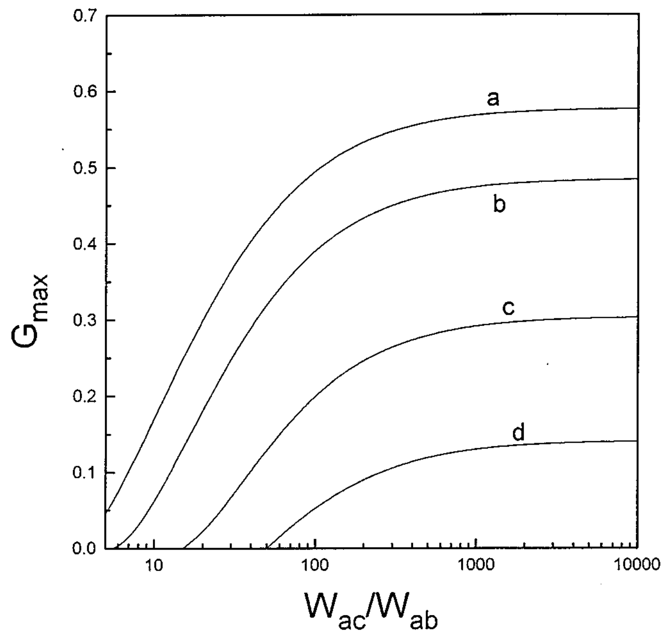

FIG. 4. Maximum gain (in the unit of $g^2 R_c / \gamma_{ac}\gamma$) vs the ratio W_{ac}/W_{ab} with $\gamma=0.01W_{ac}$, $W=0$. Curves (a), (b), (c), (d) are for $S=1$, $S=2$, $S=5$, $S=10$, respectively.

According to Eq. (13), we also plot G in the unit of $g^2 R_c / \gamma_{ac}\gamma$ as a function of the Rabi frequency with $S=1.2$, $W=0$, and $\gamma=0.01W_{ac}$, and with different $W_{ab}=0.001W_{ac}$, $W_{ab}=0.01W_{ac}$, $W_{ab}=0.05W_{ac}$, as shown in Fig. 3. There is a threshold for the Rabi frequency of the driving field. Above this threshold we have a positive gain. In Fig. 3 we can see that there exists a maximum gain. It is because A is a first-order function of I, and B is a second-order function of I [see Eqs. (12)]. At the point where the system has a maximum gain, we have

$$\frac{dG}{dI}=0. \tag{14}$$

From Eqs. (11), we can get the condition for having the maximum gain,

$$aI^2+bI+c=0, \tag{15}$$

where

$$a=2(W_{ab}+W+2\gamma)[2(W_{ab}+W+2\gamma)R_b - (W_{ac}+W_{ab}+W+\gamma)R_c], \tag{16a}$$

$$b=4\gamma_0\gamma(W_{ab}+W+2\gamma)(W_{ac}+W_{ab}+W+\gamma)R_b, \tag{16b}$$

$$c=\gamma_0^2\gamma^2(W_{ac}+W_{ab}+W+\gamma)[(W_{ac}+W_{ab}+W+\gamma)R_b + (W_{ac}-W_{ab}+W+3\gamma)R_c]. \tag{16c}$$

Under the condition of Eq. (13), we have $a<0$, $b>0$, and $c>0$, which means that Eq. (15) has a positive and a negative root. Neglecting the unreasonable negative solution, we obtain the driving field intensity for a maximum gain,

$$I_1=\frac{-b-\sqrt{b^2-4ac}}{2a}. \tag{17}$$

Substituting the above solution into Eq. (12), we can get the maximum gain. In Fig. 4, the dependence of the maximum gain on W_{ac}/W_{ab} is plotted with $\gamma=0.01W_{ac}$ and $W=0$ for different $S=1$, 2, 5, 10, respectively. It is clear that the larger the ratio of W_{ac} and W_{ab}, the higher the maximum gain. In Fig. 5, the dependence of the maximum gain on S is given with $\gamma=0.01W_{ac}$ and $W=0$ for different $W_{ab}=0.001W_{ac}$, $W_{ab}=0.01W_{ac}$, $W_{ab}=0.05W_{ac}$, respectively. As expected, the maximum gain increases as the ratio W_{ac}/W_{ab} increases or the ratio S decreases (equivalently the increasing of the population in upper levels).

IV. STABILITY ANALYSIS

For the stability analysis we need to consider the dynamic equation of the laser field, Eq. (3). Assuming resonance, $\omega_c=\nu$, we have from Eqs. (3) and (9)

$$\dot{I}_0=2\left(-\frac{\Gamma}{2}-g^2\frac{A_2I_0^2+A_1I_0+A_0}{B_3I_0^3+B_2I_0^2+B_1I_0+B_0}\right)I_0. \tag{18}$$

In the steady state ($\dot{I}_0=0$), it can be seen that $I_0=0$ always is one of the solutions of Eq. (18). There will exist a positive solution if certain conditions are satisfied. The positive solution for the laser intensity will be given by

$$-\frac{\Gamma}{2}-g^2\frac{A_2I_0^2+A_1I_0+A_0}{B_3I_0^3+B_2I_0^2+B_1I_0+B_0}=0, \tag{19a}$$

which can be written as

$$F(I_0)=C_3I_0^3+C_2I_0^2+C_1I_0+C_0=0, \tag{19b}$$

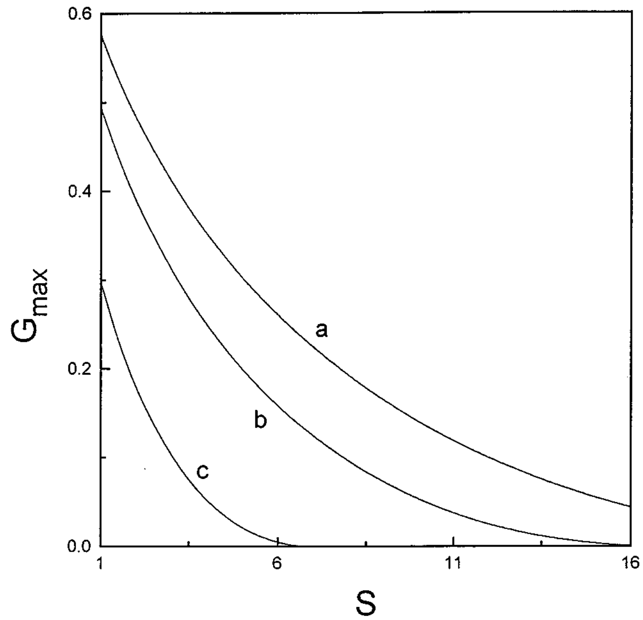

FIG. 5. Maximum gain (in the unit of $g^2R_c/\gamma_{ac}\gamma$) plotted as a function of S with $\gamma=0.01W_{ac}$, $W=0$. Curves (a), (b), (c) are for $W_{ab}=0.001W_{ac}$, $W_{ab}=0.01W_{ac}$, and $W_{ab}=0.05W_{ac}$, respectively.

where

$$C_3=\Gamma(W_{ac}+W+2\gamma), \tag{20a}$$

$$\begin{aligned}C_2=&R_bg^2(W_{ac}+W_{ab}+W+\gamma)\\&+\Gamma[I(2W_{ac}+W_{ab}+3W+6\gamma)\\&+\tfrac{1}{2}\gamma_0\gamma(5W_{ac}+W_{ab}+5W+9\gamma)],\end{aligned} \tag{20b}$$

$$\begin{aligned}C_1=&g^2R_b[I(W_{ac}+3W_{ab}+3W+5\gamma)\\&+2\gamma_0\gamma(W_{ac}+W_{ab}+W+\gamma)]\\&-g^2R_cI(W_{ac}-W_{ab}+W+3\gamma)+\Gamma(\gamma_0\gamma+I)\\&\times[I(W_{ac}+2W_{ab}+3W+6\gamma)\\&+\gamma\gamma_0(2W_{ac}+W_{ab}+2W+3\gamma)],\end{aligned} \tag{20c}$$

$$\begin{aligned}C_0=&g^2R_b(\gamma_0\gamma+I)[2I(2\gamma+W_{ab}+W)\\&+\gamma\gamma_0(W_{ac}+W_{ab}+W+\gamma)]\\&-g^2R_cI(\gamma_0\gamma+I)(W_{ac}-W_{ab}+W+3\gamma)\\&+\frac{\Gamma}{2}(\gamma_0\gamma+I)^2[2I(W+W_{ab}+2\gamma)\\&+\gamma\gamma_0(W_{ac}+W+W_{ab}+\gamma)].\end{aligned} \tag{20d}$$

As $C_3, C_2, C_1>0$, $F(I_0)$ monotonously increases with I_0, and, consequently, there is only one possible positive solution. The condition for existing a positive solution is $C_0<0$.

For the stability analysis, we assume that there is a fluctuation from the steady state, ΔI_0, and then examine the evolution of this fluctuation. Replacing I_0 in Eq. (18) with $I_0+\Delta I_0$, we get

$$\begin{aligned}\frac{1}{\Delta I_0}\frac{d\Delta I_0}{dt}=&2\left(-\frac{\Gamma}{2}-g^2\frac{A_2I_0^2+A_1I_0+A_0}{B_3I_0^3+B_2I_0^2+B_1I_0+B_0}\right)\\&+\frac{g^2I_0}{(B_3I_0^3+B_2I_0^2+B_1I_0+B_0)^2}\\&\times[B_3(A_2I_0^2+A_0)I_0^2+2B_3(A_1I_0+A_0)I_0^2\\&-A_2B_1I_0^2+B_2(A_1I_0+A_0)I_0\\&+(A_0B_2-2A_1B_0)I_0+(A_0B_1-A_1B_0)].\end{aligned} \tag{21}$$

For the solution of a zero laser field, the second term is equal to zero. If the linear gain is larger than $\Gamma/2$, the first term is positive, we will have

$$\frac{1}{\Delta I_0}\frac{d\Delta I_0}{dt}>0.$$

Consequently the zero solution is unstable. If the linear gain is smaller than $\Gamma/2$, the first term is negative. In this case, the zero solution is stable because of

$$\frac{1}{\Delta I_0}\frac{d\Delta I_0}{dt}<0.$$

Now let us consider the nonzero solution. For a nonzero solution we have Eq. (19a), i.e., the first bracket on the right-hand side of Eq. (21) is equal to zero. From Eq. (19a) we can obtain $(A_2I_0^2+A_1I_0+A_0)<0$. Here the fact that A_2, A_1, B_3, B_2, B_1, and B_0 are always positive [see Eqs. (10)] has been taken into account. Consequently, three inequalities $A_0<0$, $A_2I_0^2+A_0<0$, and $A_1I_0+A_0<0$ can be obtained (note positive A_2 and A_1). From the three inequalities and positive A_2, A_1, B_3, B_2, B_1, and B_0, we can conclude that the second bracket on the right-hand side of Eq. (21) is negative and

$$\frac{1}{\Delta I_0}\frac{d\Delta I_0}{dt}<0.$$

Therefore, any fluctuation will die out, and the nonzero intensity is stable when the linear gain is larger than the loss.

V. LASER FIELD AMPLITUDE (INTENSITY)

After being sure that we can have a nonzero stable laser field when $C_0<0$, now we analyze the requirement for the driving field and the atom in order to have $C_0<0$. From Eq. (20.4), we find

$$\begin{aligned}D(I)=&I^2\Gamma(W+W_{ab}+2\gamma)+I[2g^2R_b(2\gamma+W_{ab}+W)\\&-g^2R_c(W_{ac}-W_{ab}+W+3\gamma)]\\&+I\frac{\Gamma}{2}\gamma\gamma_0(W_{ac}+3W+3W_{ab}+5\gamma)\\&+g^2R_b\gamma\gamma_0(W_{ac}+W_{ab}+W+\gamma)]\\&+\frac{\Gamma}{2}\gamma^2\gamma_0^2(W_{ac}+W+W_{ab}+\gamma)\\&<0.\end{aligned} \tag{22}$$

From the minimum value of $D(I)$, we can obtain the maximum Γ for having a nonzero laser field

$$\Gamma < \frac{2g^2(R_c a_1 a_2 - R_b a_3 a_4) - 4g^2\sqrt{2R_c a_1 a_3 a_5 (R_c a_1 - R_b a_4)}}{\gamma\gamma_0 a_4^2}. \tag{23a}$$

For Γ less than its maximum value, we can find a region for the driving field intensity, within which a nonzero laser field can be established

$$\frac{a_7 - \sqrt{a_6}}{2a_3\Gamma} < I < \frac{a_7 + \sqrt{a_6}}{2a_3\Gamma}. \tag{23b}$$

In the above, we have defined

$$a_1 = W_{ac} - W_{ab} + W + 3\gamma, \tag{24a}$$

$$a_2 = W_{ac} + 3W_{ab} + 3W + 5\gamma, \tag{24b}$$

$$a_3 = W_{ab} + W + 2\gamma, \tag{24c}$$

$$a_4 = -W_{ac} + W_{ab} + W + 3\gamma, \tag{24d}$$

$$a_5 = W_{ac} + W_{ab} + W + \gamma, \tag{24e}$$

$$a_6 = \frac{\Gamma^2}{4}\gamma_0^2\gamma^2 a_4^2 + 2\Gamma\gamma_0\gamma R_b a_3 a_4 - \Gamma\gamma_0\gamma g^2 R_c a_1 a_2 + g^4[2R_b a_3 - R_c a_1]^2, \tag{24f}$$

$$a_7 = \frac{\Gamma}{2}\gamma_0\gamma a_3 + g^2 R_c a_1 - 2g^2 R_b a_3. \tag{24g}$$

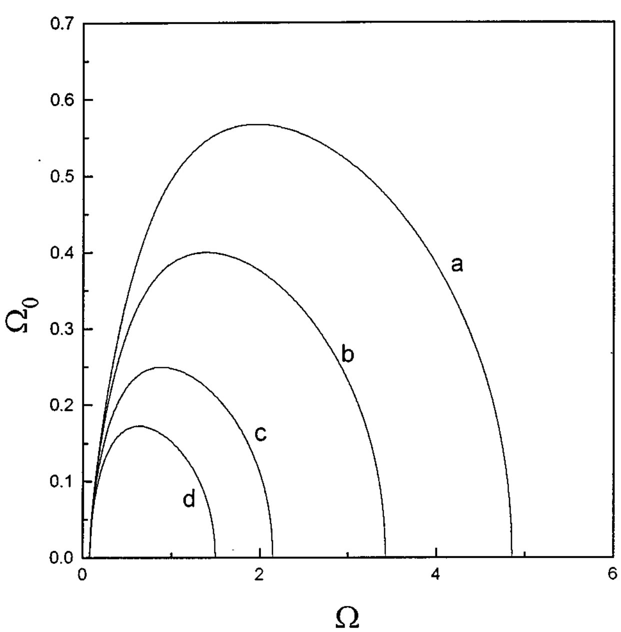

FIG. 6. The dependence of the stable solution of the laser field Ω_0 (in unit of W_{ac}) on the driving field Ω (in unit of W_{ac}) with $S=1.2$, $W_{ab}=0.001W_{ac}$, $W=0$, $\gamma=0.01W_{ac}$. Curves (a), (b), (c), (d) are for $\Gamma=0.01$, $\Gamma=0.02$, $\Gamma=0.05$, and $\Gamma=0.1$, respectively (Γ is in the unit of $g^2R_c/\gamma_{ac}\gamma$).

From Eqs. (23a) and (23b), it can be seen that not only a cavity loss below a certain level but also a suitable intensity for the driving field are needed if we want to get a laser output. That is to say the driving field should be strong enough but not too strong. In Fig. 6 we plot Ω_0 as a function of Ω with $W_{ab}=0.001W_{ac}$, $\gamma=0.01W_{ac}$, and $W=0$, $S=1.2$ for different losses $\Gamma=0.01g^2R_c/\gamma_{ac}\gamma$, $\Gamma=0.02g^2R_c/\gamma_{ac}\gamma$, $\Gamma=0.05g^2R_c/\gamma_{ac}\gamma$, $\Gamma=0.1g^2R_c/\gamma_{ac}\gamma$, respectively. It shows that there is a region for the Rabi frequency of the driving field Ω where we can have a nonzero laser field Ω_0, and there exists a maximum laser field. When the Rabi frequency of the driving field is below a threshold, the intensity of the laser field is zero due to negative gain or the gain is smaller than the loss. When the Rabi frequency of the driving field is above a certain value, the laser field becomes zero again. This is due to the decrease of the gain into a value below the loss for a large Rabi frequency of the driving field (see Fig. 3). Here we would like to point out that under the conditions shown by Eqs. (23), the zero solution is unstable.

When the loss is smaller than the maximum linear gain, there exist two critical Rabi frequencies for the driving field at which the linear gain is equal to the loss. When the Rabi frequency is out of the region determined by the two critical Rabi frequencies, the linear gain is smaller than the loss, and we have a stable zero laser field. When the Rabi frequency is located within the region, the linear gain is larger than the loss, and we have a stable nonzero laser field.

In Fig. 7, the dependence of Ω_0 on S is shown with $\Gamma=0.10g^2R_c/\gamma_{ac}\gamma$, $\gamma=0.01W_{ac}$, $W=0$, $\Omega=0.3W_{ac}$, and

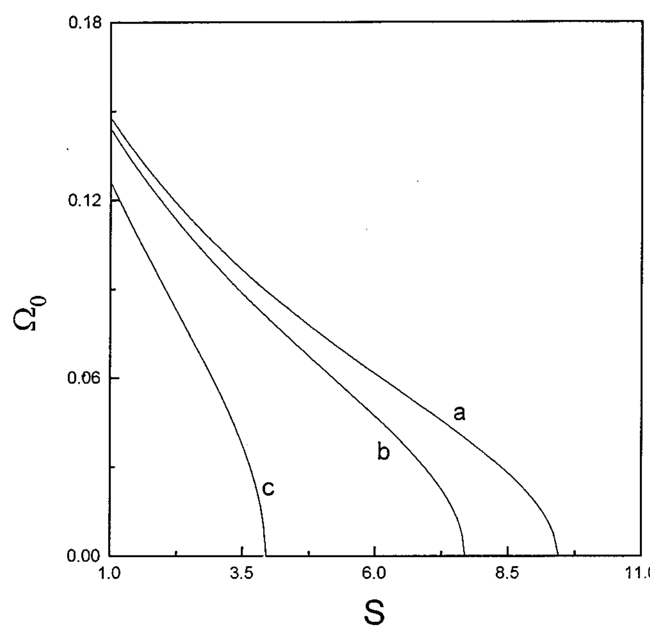

FIG. 7. The stable solution of the laser field Ω_0 (in unit of W_{ac}) against S with $W=0$, $\gamma=0.01W_{ac}$, $\Omega=0.3W_{ac}$, $\Gamma=0.1g^2R_c/\gamma_{ac}\gamma$. Curves (a), (b), (c) are for $W_{ab}=0.001W_{ac}$, $W_{ab}=0.01W_{ac}$, and $W_{ab}=0.05W_{ac}$, respectively.

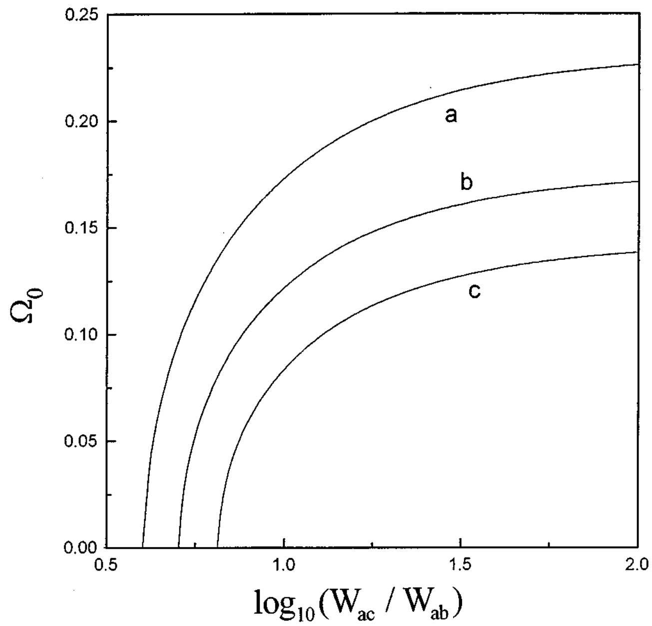

FIG. 8. The stable solution of the laser field Ω_0 (in unit of W_{ac}) against W_{ac}/W_{ab} with $W=0$, $\gamma=0.01W_{ac}$, $\Omega=0.3W_{ac}$, $S=1.2$. Curves (a), (b), (c) are for $\Gamma=0.01$, $\Gamma=0.05$, and $\Gamma=0.1$, respectively (Γ in the unit of $g^2R_c/\gamma_{ac}\gamma$).

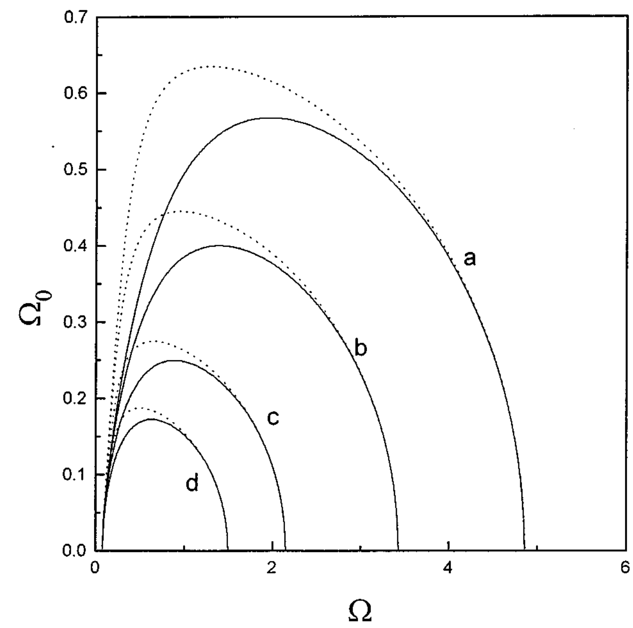

FIG. 9. The dependence of the stable solution of the laser field Ω_0 (in unit of W_{ac}) on the driving field Ω (in unit of W_{ac}) with $S=10$, $W_{ab}=0.001W_{ac}$, $W=0$, $\gamma=0.01W_{ac}$. Curves (a), (b), (c), (d) are for $\Gamma=0.01$, $\Gamma=0.02$, $\Gamma=0.05$, and $\Gamma=0.1$, respectively (Γ in the unit of $g^2R_c/\gamma_{ac}\gamma$). For each couple of curves, the higher one is the approximate solution given by Eq. (25), the other one is same with the corresponding curve in Fig. 6.

with $W_{ab}=0.05W_{ac}$, $W_{ab}=0.01W_{ac}$, and $W_{ab}=0.001W_{ac}$. It can be seen that small S and large ratio W_{ac}/W_{ab} yields a high laser field, and the laser field decreases sharply with the increase of S. In Fig. 8, the dependence of Ω_0 on W_{ac}/W_{ab} is shown with $S=1.2$, $\gamma=0.01W_{ac}$, $W=0$, $\Omega=0.3W_{ac}$, and with different cavity losses ($\Gamma=0.1g^2R_c/\gamma_{ac}\gamma$, $\Gamma=0.05g^2R_c/\gamma_{ac}\gamma$, $\Gamma=0.01g^2R_c/\gamma_{ac}\gamma$). There is a threshold for W_{ac}/W_{ab}. Below this threshold the intensity of the laser field is zero. Above this threshold, with the increase of the ratio W_{ac}/W_{ab}, the laser field increases promptly first and, then slowly when the ratio W_{ac}/W_{ab} reaches about 20. When the cavity loss is small and the ratio W_{ac}/W_{ab} is large enough, we can have high laser intensity.

Assuming that the laser field is weak, we can neglect the second- and third-order terms in Eq. (19b), and have an approximate solution (first-order result)

$$I_0=\frac{-C_0}{C_1}. \tag{25}$$

In Fig. 9, we have compared the approximate solution of the laser field Ω_0 given by Eq. (25) and the exact solution as a function of the driving field Ω with the same parameters as Fig. 6. We find that the two solutions fit well for a higher driving field and poorly for a lower driving field. In Eqs. (20) we can find that C_3, C_2, C_1, and C_0 are the functions of zeroth, first, second, and third orders of I, respectively, and naturally our approximation is nice for a higher driving field.

VI. DISCUSSION AND CONCLUSION

Here we would like to consider the conceptual possibility of a noninversion lasing experiment based on this model in a beam of He atoms (or heliumlike ions) [14]. The ground level $1\,^1S_0$ is the lower lasing level ($|b\rangle$) and the triplet state $2\,^3P_1$ is the upper lasing level ($|a\rangle$), while the $2\,^3S_1$ serves as the third (nonlasing) level ($|c\rangle$). The laser transition is from $2\,^3P_1$ to $1\,^1S_0$ with a wavelength of 59.1 nm. The decay rates from $2\,^3P_1$ to $1\,^1S_0$ and $2\,^3S_1$ (W_{ab} and W_{ac}) are 1.76×10^2 sec^{-1} and 10^7 sec^{-1}, respectively. Before the beam is injected into the cavity, the atoms are excited to level $2\,^3S_1$. This excitation can be realized by a discharge followed by a free flight (free decay). In the discharge the atoms are excited to all levels, and in the free decay process the atoms decay to $1\,^1S_0$ and $2\,^3S_1$. In the text, we used [15]

$$g^2R_c/\gamma_{ac}\gamma=g^2N/\gamma_{ac}(1+S)=\frac{12\pi c^3}{\omega_{ab}}\frac{W_{ab}}{W_{ac}}\frac{N/V}{(1+S)} \tag{26}$$

as the unit for the gain and cavity loss, W_{ac} for the unit of the Rabi frequencies of the driving field and laser field. In obtaining Eq. (26) we have noticed $R_b+R_c=N\gamma$ for the system studied above, where N is the atom number in the cavity. We assume a low pressure of 0.2 torr for the atomic beam, which yields $N/V=7.1\times10^{21}$ m^{-3}. The coupling constant (also the decay rate W_{ab}) can be increased by using a dc electric field due to the mixture with $2\,^1P_1$. The decay rate γ_{ac} is 0.5×10^7 sec^{-1}. Consequently, we obtain $g^2R_c/\gamma_{ac}\gamma=5.0\times10^{10}$ sec^{-1}, where $S=1.2$ has been assumed. In the text we took $\Gamma=0.01$ to $0.1g^2R_c/\gamma_{ac}\gamma$ (or $\Gamma=5.0\times10^8$ to 5.0×10^9 sec), which corresponds to approximately 96.5% to 71.5% mirror losses with 1 cm cavity length. Therefore, if we have a cavity with a loss of 10% (corresponding $\Gamma=0.02g^2R_c/\gamma_{ac}\gamma$), we can have a laser field of Rabi frequency more than 2.0×10^6 sec^{-1} (in the cavity) at 59.1 nm (vuv).

The nonlinear theory of noninversion lasers in an open three-level system based on electromagnetically induced transparency has been obtained. As shown by our analysis and calculation, this model can give a stable nontrivial laser intensity. In order to realize a stable laser output, the Rabi frequency of the driving field must be within a certain region, which is mainly determined by the cavity loss. The dependence of linear gain and the laser intensity on different parameters (the ratio of the pumping rates to the lower level and the third nonlasing level (S) and the ratio of the two decay rates from the upper level (W_{ac}/W_{ab}), and the Rabi frequency of the driving field) is studied. Large W_{ac}/W_{ab} value and small S value (still $S>1$) are favorable to the laser oscillation. However, the Rabi frequency of the driving field, in order to have a laser oscillation, must be not only larger than a threshold, but also smaller than a certain value.

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