Coherent quantum tunneling between two Bose-Einstein condensates

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We study an elementary model of two tunneling Bose-Einstein condensates with an atom detector in one of the traps. Stochastic simulations of atom detection in one trap show an oscillation of atom number between traps even though the condensates are initially in equal number Fock states. The oscillation has a random amplitude depending on the phase difference between the condensates, which coincides with that given by a spontaneously broken symmetry argument. [S1050-2947(96)50612-8]

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The recent experimental realization of Bose-Einstein condensation in a gas of alkali-metal atoms [1] has stimulated a number of theoretical studies on the properties of these condensates. One question that has received much attention is related to the phase of the condensate and how it is established [2–7]. One recent approach to the determination of the phase of a Bose-Einstein condensate is that of Javanainen and Yoo [2], who consider the interference of two condensates by dropping the condensates onto a measurement apparatus that measures the position of the atoms in space. Even if both the condensates are initially in number states an interference pattern is formed. A phase is established owing to the lack of knowledge as to in which condensate the detected atom originated. The interpretation is that a phase is created by the measurement process. This conclusion has also been reached by a number of other authors [3–6].

In this Rapid Communication we consider two Bose-Einstein condensates separated by a barrier that allows particles to tunnel between the two traps, analogous to the coherent tunneling of Cooper pairs in a Josephson junction. We shall show that by introducing a measurement device that continuously monitors the particle number in one of the wells a macroscopic quantum coherence or quantum phase is established between the two condensates that gives rise to quantum tunneling. Thus the same results as those with spontaneous symmetry-breaking arguments are obtained [8] without the need to introduce a small injected signal. We begin by briefly revising the usual spontaneous symmetry-breaking arguments for tunneling between two wells.

We assume that the symmetric double-well potential experienced by the atoms has two closely spaced low-energy states that are well separated from higher energy levels of the potential. This allows us to treat the system as two interacting modes [9–12]. In addition, we assume that the atoms in each condensate are noninteracting (collisions have been considered recently [6] in the interfering condensates case). The Hamiltonian,

$$H_{\rm sys}=\hbar\omega_0(a_r^\dagger a_l+a_l^\dagger a_r), \tag{1}$$

describes the tunneling between the two condensates, represented by the mode operators a_r and a_l for the right-hand and left-hand condensates, respectively. The Heisenberg equation of motion for the right-hand mode operator is $a_r(t)=a_r\cos\omega_0 t-ia_l\sin\omega_0 t$. The time variation of the expectation value of the atom number in the right-hand trap can then be written

$$\langle a_r(t)^\dagger a_r(t)\rangle=\frac{N}{2}(1+\alpha\cos2\omega_0 t+\beta\sin\phi\sin2\omega_0 t), \tag{2}$$

where $N=\langle a_r^\dagger a_r+a_l^\dagger a_l\rangle$ is the total number of atoms and we have defined $\alpha=\langle a_r^\dagger a_r-a_l^\dagger a_l\rangle/N$. β and ϕ are the modulus and phase of the complex number $\beta e^{i\phi}=2\langle a_r^\dagger a_l\rangle/N$.

Elementary spontaneously symmetry-breaking arguments [8] imply that each Bose-Einstein condensate is described by a coherent state with a distinct but random phase. We could then write $\langle a_j\rangle=\sqrt{N_j}e^{-i\phi_j}$ for $j=r,l$. Assuming we have the same number of atoms in each condensate the oscillation due to unequal numbers of atoms vanishes and we are left with an oscillation due only to the interference term,

$$\langle a_r^\dagger(t)a_r(t)\rangle=\frac{N}{2}[1+\sin(\phi_l-\phi_r)\sin2\omega_0 t]. \tag{3}$$

The phases ascribed to the condensates by the broken symmetry give rise to an oscillation.

Spontaneous symmetry breaking in a Bose-Einstein condensate assumes that the ground state is degenerate, the states being coherent states all of the same energy but differing in phase. The ground state is invariant under a gauge transformation that changes a coherent state to a coherent state with a different phase. The general argument goes that, because the particle number is very large, the ground state cannot exist in a superposition of coherent states; it must pick only one, breaking the gauge invariance of the system and giving the condensate a definite phase. The spontaneously broken symmetry argument relies on taking the thermodynamic limit, which here means approximating the atom number by infinity, $N\rightarrow\infty$ (the system then becomes essentially classical). However, in recent experiments [1] Bose-Einstein condensates have been formed with from 10^3 to 10^8 atoms, much smaller than this classical limit. The state of the condensate can of course be described as a statistical mixture of Fock states. If the condensates are in number states where there is no phase information at all, then

$\beta e^{i\phi}=\langle N_l,N_r|a_l^\dagger a_r|N_r,N_l\rangle=0$ and no oscillation occurs at all, a result incompatible with spontaneous symmetry breaking.

However, in the case of two interfering condensates, it has been shown [2] that, if an explicit measuring apparatus is assumed at every point in space, then an interference pattern is observed. For the case of tunneling between two Bose-Einstein condensates we consider an analogous situation where an atom detector is continuously measuring the atom number (measuring the atom number in the same sense as the photon number is measured by a photodetector) in the right-hand trap. In our atom detector each atom is removed from the trap upon detection, in analogy with photon detection.

By investigating a model similar to Javanainen and Yoo's but in the time domain, we are able to align this work with an increasing body of knowledge surrounding the *unraveling* of the master equation into stochastic trajectories for state vectors [13,14]. All these methods depend explicitly on the fact that the quantum state of the system is conditioned on the outcome of a measurement, i.e., on the value of a stochastic classical variable. In our case the system of two tunneling condensates interacting with a bath is conditioned on whether or not an atom is measured in the output of the system. One quantum trajectory (one set of m detections) will yield an oscillation, whereas a master equation treatment consists of an average over the times of the detections so that the oscillations will average to zero.

We wish to study the dynamics of the two coupled condensates conditioned on the measurement of the output of one of the condensates. The evolution of the state of the system and bath (assuming the rotating-wave approximation) in the interaction picture is given by [15]

$$\frac{d}{dt}|\psi(t)\rangle=\{-iH_{\rm sys}/\hbar+\sqrt{\gamma}(b_t^\dagger a_r-a_r^\dagger b_t)\}|\psi(t)\rangle,$$

where b_t is the bath operator in the Markov approximation, defined by $b_t=\int_{-\infty}^{\infty}b(\omega,t)d\omega$, which satisfies the commutation relation $[b_t,b_{t'}^\dagger]=\delta(t-t')$. $H_{\rm sys}$ is given by Eq. (1) and γ is the coupling constant between the bath and the condensate. The $b_t^\dagger a$ coupling describes the escape of an atom from the trap into the bath, where it is then detected. As we are continuously monitoring the output, in any infinitesimal interval dt there is a possibility that an atom is detected and also that an atom is not detected. The system evolution is thus conditioned on the outcome of the measurement. The probability of each outcome is governed by a classical probability density and will be given below. The unnormalized state of the system when a detection occurs is given by [15]

$$|\tilde{\psi}_{\rm sys}(t+dt)\rangle=\sqrt{\gamma}a_r|\psi_{\rm sys}(t)\rangle. \tag{4}$$

The wave function experiences a "jump" and one atom is removed from the condensate. When no atom is detected, the state is governed by an evolution equation of the form

$$|\tilde{\psi}_{\rm sys}(t+dt)\rangle=e^{-(i/\hbar)H_{\rm eff}dt}|\psi_{\rm sys}(t)\rangle, \tag{5}$$

where $H_{\rm eff}=H_{\rm sys}-i\hbar\gamma a_r^\dagger a_r/2$. This is a nonunitary evolution and arises because the output is being continuously monitored by the detector, so that even if a detection does not occur we still gain information about the state of the system [13,16].

To complete this scheme we need to find the probability that an atom is emitted during the time interval dt. The probability of a detection occurring is from Eq. (4), $p(1)=\langle\psi_{\rm sys}(t)|\gamma a_r^\dagger a_r|\psi_{\rm sys}(t)\rangle dt$; and for no detection, $p(0)=1-p(1)$.

We are interested in a number of detections m over a finite period of time T; in practice we evaluate this with an iterative process for each detection [14]. We first evolve the system via Eq. (5) for a finite time step $\Delta t=dt$, calculating $p(1)$ each time. As $p(1)$ is a classical probability we simply compare it with a random number R (uniformly distributed between 0 and 1) after each iteration of the time step. If $p(1)<R$ then we evolve the wave function for the next time step by Eq. (5); on the other hand, if $p(1)>R$ then we say a *jump* has occurred and the wave function collapses via Eq. (4). As the system is undergoing nonunitary evolution we normalize the wave function after each jump, i.e., each detection. The process then begins again with the free evolution. This process yields the times $t_1,\ldots,t_m$ at which an atom is detected.

This formulation is equivalent to the theory of continuous photon detection of Srinivas and Davies [16]. This can be seen if we write the probability density for m detections to occur at the times $t_1<t_2<\cdots<t_m$ during the interval $[0,t]$:

$$\begin{aligned}p^m(t_1,\ldots,t_m;[0,t])\\ &=\gamma^m\langle N_l,N_r|e^{iH_{\rm eff}^\dagger t_1/\hbar}a_r^\dagger e^{iH_{\rm eff}^\dagger(t_2-t_1)/\hbar}\\ &\quad\times a_r^\dagger\cdots a_r e^{-iH_{\rm eff}(t_2-t_1)/\hbar}a_r e^{-iH_{\rm eff}t_1/\hbar}|N_r,N_l\rangle.\end{aligned}$$

This is the temporal analog of Javanainen and Yoo's joint probability density for the positions of the atoms.

For our simple model the tunneling rate is much faster than the rate of detection and the condensates are initially in number states. The conditional probability density that the next detection will occur at time t given that atoms were emitted at times $t_1<t_2<\cdots<t_m$ and none in between is given by

$$p(t|t_1,\ldots,t_m) \tag{6}$$

$$\begin{aligned}&=\frac{p^{m+1}(t_1,..,t_m,t;[0,t])}{p^m(t_1,\ldots,t_m;[0,t])}\\ &=\gamma\langle\psi_{\rm sys}(t)|a_r^\dagger a_r|\psi_{\rm sys}(t)\rangle,\\ &=\gamma\frac{\langle\psi_{\rm sys}(t_m)|e^{(i/\hbar)H_{\rm eff}^\dagger\tau}a_r^\dagger a_r e^{(-i/\hbar)H_{\rm eff}\tau}|\psi_{\rm sys}(t_m)\rangle}{\langle\psi_{\rm sys}(t_m)|e^{(i/\hbar)(H_{\rm eff}^\dagger-H_{\rm eff})\tau}|\psi_{\rm sys}(t_m)\rangle},\end{aligned} \tag{7}$$

where $\tau=t-t_m$. We assume that the rate of detection is much slower than the rate of tunneling, i.e., $\gamma\ll\omega_0$. This assumption means that on average the atoms have plenty of time to mix between the two traps in between detections, allowing the state of the system to evolve due to the continuous detection, and yet not having it dominate the behavior. We can then write Eq. (7) as

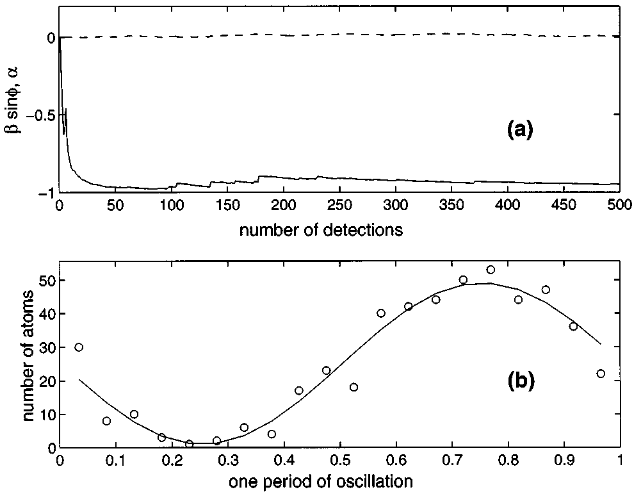

FIG. 1. Results of a numerical simulation of one run of 500 detections from a initial number of 1000 atoms. (a) shows the evolution of α (dashed line) and $\beta \sin\phi$ (solid line) over the detections. (b) is a histogram (given by the circles) of the number of atoms detected during 20 equal time intervals over one period of the oscillation. The solid line is a least-squares fit to the data.

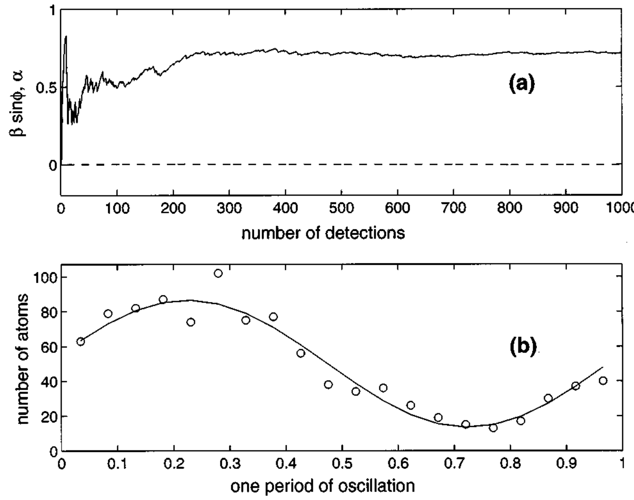

FIG. 2. The same as Fig. 1 but for one run of 1000 detections of 10 000 initial atoms.

$$p(t|t_1,\ldots,t_m) \simeq \gamma\langle \psi_{\rm sys}(t_m)|e^{-iH_{\rm sys}t_m/\hbar}a_r^{\dagger}(t)a_r(t)e^{iH_{\rm sys}t_m/\hbar}|\psi_{\rm sys}(t_m)\rangle,$$

where $a_r(t)=e^{(i/\hbar)H_{\rm sys}^{\dagger}t}a_r e^{(-i/\hbar)H_{\rm sys}t}$, and the system state vector is propagated back in time as if no detector were present. This is simply a unitary transformation of the system from a Schrödinger picture to a Heisenberg picture, and the motion is now in the operators instead of the state vector.

But our conditional probability density now has the same form as the average atom number, Eq. (2), with the initial state now $e^{iH_{\rm sys}t_m/\hbar}|\psi_{\rm sys}(t_m)\rangle$ and with $N\rightarrow\gamma(N-m+1)$. This equation has the necessary form for us to observe the oscillation, but we also need to simulate a run of detections with Eqs. (4) and (5) to determine that α, β, and ϕ tend to fixed values as more detections are made. Note that, for $m=1$, where we are simply detecting the number of atoms in the well, $e^{iH_{\rm sys}t/\hbar}|\psi_{\rm sys}(t)\rangle=|N_r,N_l\rangle$, there is no interference term ($\beta=0$), and only the oscillatory behavior between two traps with unequal initial atom number is observed.

This equation [Eq. (2)] should be compared to the equivalent conditional probability density given by Javanainen and Yoo [2] for the nontunneling case. Note that in our case the random phase ϕ determines the amplitude of the oscillation.

A histogram of the times of the detections of the atoms is given in Figs. 1(b) and 2(b) for 500 detections of an initial 1000 atoms and 1000 detections of 10 000 atoms, respectively. If we put $\omega_0=\pi$, then the period of the oscillations is simply 1 and the times of the detections can be rewritten $t_j'=t_j-k$, where k is an integer and $0\leqslant t_j'\leqslant 1$ is plotted on the interval $[0,1]$ to increase resolution. The solid line is a least-squares fit to the data. These plots show quite distinctly the oscillation of the atom number. In Figs. 1(a) and 2(a) we have also plotted the evolution of α and $\beta\sin\phi$ over the detections; $\beta\sin\phi$ becomes quite well defined by the time $\approx\sqrt{N}$ particles have been detected ($\beta\rightarrow 1$ as the number of detections is increased). α has a mean value of 0 due to our choice of initial state; it fluctuates about this mean value because for any detection we do not know which condensate the atom was originally from. Figure 1 is a ''good'' run, as in this case $|\sin\phi|\approx 1$. This requires the detection times to be very regular; in this case β goes to 1 at nearly the fastest possible rate; in general β rises at a slower rate. For an infinite number of atoms and completely regular detection times the values of β follow the sequence given by $\{0,1/2,3/4,\ldots,(m-1)/m\}$, where m is the number of detections; this describes the upper bound on the values of β.

The output atom intensity is then essentially of the same form as Eq. (3). If the experiment were repeated, the result would be the same except that the phase difference ϕ would vary randomly from one run to the next and depend only on the times of each atom detection. The interesting point here is that the oscillation has a random amplitude given by $\sin\phi$, which will be different for each experimental run. Obviously the more atoms detected the better the visibility of the histogram, but a better pattern is also found when the initial number of atoms is increased.

In summary we have demonstrated Josephson-like oscillations due to a phase difference between two tunneling Bose-Einstein condensates by using quantum trajectory techniques to model the continuous detection of the atom number in one of the condensates. We note that in this case the amplitude of the oscillations arises from a random phase, so that in one particular run of the experiment this amplitude will be random. We also note that the model outlined here may be more accessible to experiment than that of two nontunneling condensates in momentum eigenstates, for the reason given by Javanainen and Yoo. In recent experiments [1] the condensates were more accurately modeled by a large occupation number of the ground state of an atom trap than that of a momentum eigenstate. Our method simply requires two of these traps to be brought close enough together so that the atoms are able to tunnel between the traps.

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