Quantum Zeno effect on atomic excitation decay in resonators

A. G. Kofman and G. Kurizki
Chemical Physics Department, Weizmann Institute of Science, Rehovot 76100, Israel
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A unified theory of two-level atom coupling to vacuum field reservoirs with *arbitrary* mode-density spectra is used to demonstrate that the quantum Zeno effect on excitation decay of the atom (and, correspondingly, inhibition of spontaneous emission) is observable in open cavities and waveguides, using a sequence of evolution-interrupting pulses on a nanosecond scale. [S1050-2947(96)51111-X]

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The ''watchdog'' or quantum Zeno effect (QZE) is a spectacular manifestation of the influence of continuous measurements on the evolution of a quantum system. The original QZE prediction has been the inhibition of exponential decay of an excited state into a reservoir, by repeated interruption of the system-reservoir coupling by measurements [1–3]. The essential requirement for the QZE is that the measurements of the system state, which cause the interruption, be more closely spaced in time than the reservoir correlation (memory) time. This implies that the uninterrupted decay must deviate from the exponential law over a time interval comparable to the spacing between successive measurements (see below). Atomic excited-state decay by spontaneous emission into the electromagnetic field vacuum in open space cannot fulfill this requirement, as the relevant correlation time is then $<10^{-15}$ s. Although in resonators this correlation time is undoubtedly longer, the issue of QZE observability in resonators has not been studied in detail. Instead, an experiment [4] (based on a theoretical suggestion [5]) and ensuing theoretical studies [6] have focused on the QZE in the Rabi-oscillations regime of a coupled field-atom system, and, lately, in the evolution of a decaying field-driven multilevel atom [7] or in parametric down-conversion [8].

It is our purpose here to demonstrate that the inhibition of *nearly exponential* excited-state decay by the QZE in two-level atoms, in the spirit of the original suggestion [1], is amenable to experimental verification in resonators. Although this task is widely believed to be very difficult, we show, by further developing our unified theory of spontaneous emission into reservoirs with arbitrary mode-density spectra [9], that several realizable configurations based on two-level emitters in cavities [10,11] or in waveguides [12] are in fact adequate for QZE observation. The possibilities for such observation are examined in various regimes that can arise in resonators. Finally, we address the issue of QZE suitability for decoherence-error prevention, which has been brought up recently [13].

We start with a general analysis of the evolution of an initially excited two-level atom coupled to an *arbitrary* density-of-modes (DOM) spectrum $\rho(\omega)$ of the electromagnetic field in the vacuum state. At time τ this evolution is interrupted by a short optical pulse, which serves as a quantum measuring device [4–7]. Its role is to break the evolution coherence, by transferring the populations of the excited state $|e\rangle$ to an auxilliary state that then decays back to $|e\rangle$ *incoherently* [6]. As in our previous treatment [14], the atomic response, i.e., the emission rate into this reservoir at frequency ω, which is $|g(\omega)|^2\rho(\omega)$, $\hbar g(\omega)$ being the field-atom coupling energy, is divided into two parts,

$$G(\omega)=G_s(\omega)+G_b(\omega). \tag{1}$$

Here $G_s(\omega)$ stands for the sharply varying (nearly singular) part of the DOM distribution, associated with narrow cavity-mode lines, the frequency cutoff in waveguides, or photonic band edges. The complementary part $G_b(\omega)$ stands for the broad portion of the DOM distribution (the ''background'' modes), which always coincides with the free-space DOM $\rho(\omega)\sim\omega^2$ at frequencies well above the sharp spectral features. In an open structure (see below), $G_b(\omega)$ represents the atom coupling to the unconfined free-space modes. We cast the excited-state amplitude in the form $\alpha_e(\tau)e^{-i\omega_a\tau}$, where ω_a is the atomic resonance frequency. Then, for arbitrary DOM spectra and coupling strengths, one can reduce the equations for spontaneous decay [9] to the following evolution equation, up to the interruption time τ:

$$\dot{\alpha}_e(\tau)=-\int_0^\tau dt[\Phi_s(t)+\Phi_b(t)]e^{i\Delta t}\alpha_e(\tau-t). \tag{2a}$$

Here $\Delta=\omega_a-\omega_s$, ω_s is a characteristic frequency corresponding to the maximum or the singularity of the sharp spectral feature, whereas $\Phi_s(t)$ and $\Phi_b(t)$ are the time-domain Fourier transforms of $G_s(\omega)$ and $G_b(\omega)$, respectively,

$$\Phi_{s(b)}(t)=\int_0^\infty d\omega G_{s(b)}(\omega)e^{-i(\omega-\omega_s)t}. \tag{2b}$$

Restricting ourselves to a sufficiently short interruption interval τ, such that $\alpha_e(\tau)\simeq1$, yet long enough to allow the rotating-wave approximation (RWA), Eq. (2) yields

$$\alpha_e(\tau)\simeq1-\int_0^\tau dt(\tau-t)\Phi_s(t)e^{i\Delta t} +(\Delta_b'-i\gamma_b'/2-\gamma_b\tau/2). \tag{3}$$

The terms within the parentheses in Eq. (3) are the contribution of the background DOM, simplified according to the Weisskopf-Wigner approximation [9]. Here $\Delta_b'=d\Delta_b/d\omega_a$, where Δ_b is the Lamb shift due to the background modes, $\Delta_b = P\int_0^\infty d\omega G_b(\omega)/(\omega_a - \omega)$. Likewise, $\gamma_b' = d\gamma_b/d\omega_a$ is the frequency derivative of $\gamma_b = 2\pi G_b(\omega_a)$, the effective rate of spontaneous emission into the background modes. Equation (3) is obtained to first order in the atom-field interaction. To the same accuracy, the excited-state probability after n interruptions (measurements), $W(t=n\tau)=|\alpha_e(\tau)|^{2n}$, can be written as

$$W(t=n\tau)\approx[2\ \mathrm{Re}\alpha_e(\tau)-1]^n \approx e^{-\kappa t}, \tag{4}$$

where

$$\kappa = 2\ \mathrm{Re}[1-\alpha_e(\tau)]/\tau. \tag{5}$$

One can estimate that typically [9] $|\Delta_b'| \sim |\Delta_b|/\omega_a \sim \gamma_f/\omega_a$, γ_f being the free-space rate of spontaneous emission. Hence, Δ_b' is utterly negligible in Eq. (4), unless $\gamma_b = 0$, as in a *perfect* photonic band gap. In most structures, however, γ_b is comparable to γ_f and gives rise to an *exponential* decay factor in the excited-state probability, regardless of how short τ is, i.e., $\kappa = \kappa_s + \gamma_b$, where κ_s is the contribution to κ from the sharply varying modes.

Thus the background-DOM effect cannot be modified by QZE. Only the sharply varying DOM portion allows for QZE, provided that

$$\kappa_s = (2/\tau)\ \mathrm{Re}\int_0^\tau dt(\tau - t)\Phi_s(t)e^{i\Delta t} \tag{6}$$

rises with τ for sufficiently short τ. This is essentially a condition on the correlation (or memory) time of the field reservoir. If $G_s(\omega)$ falls off faster than $1/|\omega-\omega_s|$ for $|\omega - \omega_s| \gg \Gamma_s$, where Γ_s is the width (or the reciprocal correlation time) of the sharply varying reservoir, then for $t \ll |\Delta|^{-1}, \Gamma_s^{-1}$ one can neglect the exponential in the integral expression (2b) for $\Phi_s(t)$, leading to $\Phi_s(t) \approx \int_0^\infty d\omega G_s(\omega) \equiv g_s^2$ This yields (neglecting the background contribution)

$$\alpha_e(\tau) \simeq 1-(g_s\tau)^2/2 \quad (\tau \ll |\Delta|^{-1}, \Gamma_s^{-1}, g_s^{-1}), \tag{7}$$

i.e., a parabolic-segment evolution. Correspondingly, Eq. (5) reduces to

$$\kappa = \kappa_s + \gamma_b, \quad \kappa_s = g_s^2 \tau. \tag{8}$$

Only the κ_s term decreases with τ, indicating the QZE inhibition of the smooth nearly exponential decay into the field reservoir as $\tau \to 0$.

First and foremost, we wish to apply the above analysis to the case of a two-level atom coupled to a near-resonant Lorentzian line centered at ω_s, characterizing a high-Q cavity mode or a ''defect'' mode in a photonic band structure [9]. In this case, $G_s(\omega) = g_s^2\Gamma_s/\{\pi[\Gamma_s^2 + (\omega-\omega_s)^2]\}$, where g_s is the resonant coupling strength and Γ_s is the linewidth (Fig. 1—inset). The evolution of $\alpha_e(t)$ [Eq. (2)] is then *exactly solvable* within the RWA, in the form

$$\alpha_e(\tau) = (1/2)e^{(i\Delta - \Gamma_s)\tau/2}(A_+ e^{D\tau} + A_- e^{-D\tau}), \tag{9}$$

where $A_\pm = 1 \pm (\Gamma_s - i\Delta)/(2D)$ and $D = [(\Gamma_s - i\Delta)^2/4 - g_s^2]^{1/2}$.

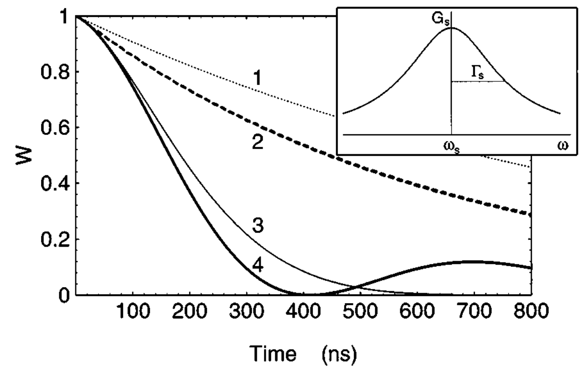

FIG. 1. Evolution of excited-state population W in a two-level atom coupled to a cavity mode with Lorentzian line shape (inset) in case (i) (on resonance, $\Delta = 0$): curve 1, decay to background-mode continuum at rate $\gamma_b \simeq \gamma_f = 10^6\ \mathrm{s}^{-1}$; curve 3, uninterrupted decay in cavity with $F \equiv (1-R)^{-2} = 10^5$, $L = 15$ cm, and $f = 0.02$; curve 4, idem, but with $F = 10^6$ (damped Rabi oscillations); curve 2, interrupted evolution along *both* curves 3 and 4, at intervals $\tau = 3\times 10^{-8}$ s.

We can henceforth draw a distinction between the *strong-coupling regime* of underdamped Rabi oscillations, corresponding to $2g_s \gg \Gamma_s + |\Delta|$, and the *weak-coupling regime* of overdamped Rabi oscillations, or *irreversible nearly exponential decay*, corresponding to $2g_s \ll \Gamma_s + |\Delta|$. However, we choose to underscore the *common character* of both regimes as regards the QZE. To this end, we introduce the short-time approximation to the solution (9), to the same accuracy as Eq. (3). Taking into account that the Fourier transform of the Lorentzian $G_s(\omega)$ is $\Phi_s(t) = g_s^2 e^{-\Gamma_s t}$, Eq. (3) yields (neglecting the background modes)

$$\alpha_e(\tau) \approx 1 - \frac{g_s^2}{\Gamma_s - i\Delta}\left[\tau + \frac{e^{(i\Delta - \Gamma_s)\tau} - 1}{\Gamma_s - i\Delta}\right]. \tag{10}$$

We can now distinguish among the following cases:

(i) $\tau \ll (\Gamma_s + |\Delta|)^{-1}, g_s^{-1}$. Obviously, it is easiest to satisfy this inequality *on resonance*, when $\Delta = 0$. Then Eq. (10) yields Eqs. (7) and (8). Since Γ_s and Δ have dropped out of Eq. (8), the decay inhibition is the *same* for both strong- and weak-coupling regimes (Fig. 1). Physically, this comes about since for $\tau \ll g_s^{-1}$ the energy uncertainty of the emitted photon is too large to distinguish between reversible and irreversible evolutions.

(ii) $\Gamma_s^{-1} \ll \tau \ll (\Gamma_s + |\Delta|)/g_s^2$. This is the weak-coupling regime with τ sufficiently long for exponential relaxation to begin, $\alpha_e(\tau) \approx 1 - g_s^2[(\Gamma_s - i\Delta)\tau - 1]/(\Gamma_s - i\Delta)^2$. The second term in the square brackets represents a small correction to the exponential decay. The evolution after n measurements now obeys Eqs. (4)-(6), with

$$\kappa = \kappa_s + \gamma_b = \frac{2g_s^2\Gamma_s}{\Gamma_s^2 + \Delta^2} + \frac{2g_s^2(\Delta^2 - \Gamma_s^2)}{(\Gamma_s^2 + \Delta^2)^2\tau} + \gamma_b, \tag{11}$$

which approximates the decay rate for the uninterrupted case, as one would expect. The second term in Eq. (11), arising from the above correction to exponential decay, reduces or *enhances* the decay rate for $|\Delta| < \Gamma_s$ or $|\Delta| > \Gamma_s$,

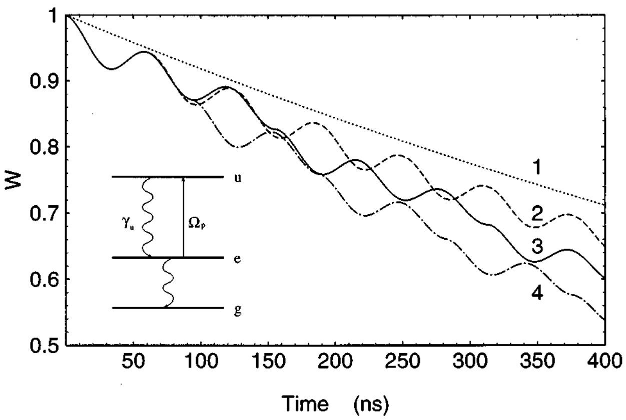

FIG. 2. Idem, in case (iii) of a two-level atom coupling to Lorentzian mode, with $\Delta = 10^8$ s^{-1}, $F = 10^6$, $L = 15$ cm, $f = 0.15$, and $\gamma_f = 10^6$ s^{-1}: curve 1, decay to background-mode continuum at rate γ_b; curve 2, interrupted free-evolution oscillations at intervals $\tau = 2\pi \times 10^{-8}$ s ($\Delta\tau = 2\pi$); curve 3, idem, for $\tau = 5\pi \times 10^{-8}$ s ($\Delta\tau = 5\pi$); curve 4, idem, for $\tau = 3\pi \times 10^{-8}$ s ($\Delta\tau = 3\pi$). Inset: the level scheme for all figures.

respectively. Though small, this term decreases with τ, which facilitates its observation.

(iii) $|\Delta|^{-1} \ll \tau \ll \Gamma_s^{-1}, |\Delta|/g_s^2$. This case corresponds to an intermediate regime of off-resonant evolution, where diminishing oscillations with amplitude $\lesssim g_s^2/\Delta^2 \ll 1$ take place, coinciding asymptotically (in the limit of large t) with exponential decay $1 - g_s^2 t/(\Gamma_s - i\Delta)$ [cf. Eq. (10)]. In this case $\alpha_e(t)$ is *sensitive* to the *free-evolution phase* $\Delta\tau$ between two consecutive interruptions (Fig. 2). When the measurements are performed at the maxima of the free-evolution oscillations, $\Delta\tau = 2\pi m$(m being an integer), then

$$\kappa = \kappa_s + \gamma_b = 4 g_s^2 \Gamma_s/\Delta^2 + \gamma_b, \tag{12}$$

the decay rate due to the sharply varying modes being twice that for the uninterrupted case [compare the first terms in (11) and (12)]. On the other hand, when the measurements are performed at the minima, $\Delta\tau = (2m+1)\pi$, the decay rate

$$\kappa = \kappa_s + \gamma_b = 4 g_s^2/(\Delta^2 \tau) + \gamma_b \tag{13}$$

is much larger than in Eq. (12) and decreases as τ grows, thus presenting a non-QZE behavior (Fig. 2).

The experimental scheme we envisage for observing the above effects is as follows. A fraction of an atomic beam oriented perpendicular to the axis of a confocal cavity is excited to state $|e\rangle$ by a laser outside the cavity. Within the cavity the atoms repeatedly interact with a pump laser, which is resonant with the $|e\rangle \rightarrow |u\rangle$ transition frequency. The resulting $|e\rangle \rightarrow |g\rangle$ fluorescence rate is collected as in Ref. [10] and monitored as a function of the pulse repetition rate. Each short, intense pump pulse of duration t_p and Rabi frequency Ω_p is followed by spontaneous decay (via fluorescence) from $|u\rangle$ back to $|e\rangle$, at a rate γ_u. The "measuring" pulse has to satisfy $t_p^{-1} \ll \gamma_u \ll \Omega_p$, so as to *destroy the coherence* of the system evolution, on the one hand, and *reshuffle the entire population* from $|e\rangle$ to $|u\rangle$ and back, on the other hand (Fig. 2—inset). By combining these requirements with the demand that the interval between measurements significantly exceed the measurement time, we infer the inequality $\tau \gg t_p$. The above inequality can be relaxed to require $\tau \gg \gamma_u^{-1}$ if the "measurements" are performed with π pulses: $\Omega_p t_p = \pi$, $t_p \ll \gamma_u^{-1}$. The only real constraint in case (i) is that $(\Gamma_s + |\Delta|)^{-1} \gg \tau \gg \gamma_u^{-1}$. This calls for choosing a $|u\rangle \rightarrow |e\rangle$ transition with a much shorter radiation lifetime than that of $|e\rangle \rightarrow |g\rangle$. The curves in Figs. 1 and 2 are calculated for such a choice; and for feasible cavity parameters, $\Gamma_s = (1-R)c/L, g_s = \sqrt{c f \gamma_f/(2L)}$, $\gamma_b = (1-f)\gamma_f$, where R is the geometric-mean reflectivity of the two mirrors, f is the fractional solid angle (normalized to 4π) subtended by the confocal cavity, and L is the cavity length.

We now extend the above analysis to DOM distributions of a more general form, including distributions characterized by a lower cutoff frequency, as in a waveguide or a photonic band edge [9]. For convenience, we leave out the background contribution in what follows. If there is a slowly decreasing tail in the reservoir, e.g.,

$$G_s(\omega) \approx C/(\omega - \omega_s)^\beta \quad (\omega - \omega_s \gg \Gamma_s), \tag{14}$$

with $0 < \beta < 1$, and $G_s(\omega)$ diminishes fast or is cut off for $\omega - \omega_s < \Gamma_s$, then one can show, by using the Fourier transform of (14), $\Phi_s(t)$, in Eq. (3), that for $\tau \ll \Gamma_s^{-1}, |\Delta|^{-1}, C^{-1/(\beta+1)}$

$$\alpha_e(\tau) \approx 1 - [\Gamma(-\beta)/(\beta+1)] C (i\tau)^{\beta+1}, \tag{15}$$

where $\Gamma(\ )$ is the Euler Γ function [15]. Here, instead of the parabolic-segment evolution of $\alpha_e(\tau)$ as the limiting form of Eq. (10) for a Lorentzian reservoir, we obtain a lower exponent $q = 1 + \beta$, in the range $1 < q < 2$, implying that the QZE exists, but is less pronounced than in the previous case, i.e.,

$$\kappa_s = \pi [\cos(\pi\beta/2) \Gamma(2+\beta)]^{-1} C \tau^\beta. \tag{16}$$

When there are slowly diminishing tails on both sides of $G_s(\omega)$, contributions from the tails add up independently in Eqs. (15) and (16).

A specific model for the spectral response of a DOM distribution with a cutoff is represented by [9] [Fig. 3—inset (a)]

$$G_s(\omega) = [C\sqrt{\omega - \omega_s}/(\omega - \omega_s + \Gamma_s)] \Theta(\omega - \omega_s), \tag{17}$$

where ω_s is the cutoff (or band-edge) frequency, Γ_s is the cutoff-smoothing parameter, C is the strength of the coupling of the atomic dipole to this reservoir, and $\Theta(\ )$ is the Heaviside step function. Upon computing the Fourier transform of Eq. (17), we find from Eq. (2) that the QZE condition is

$$\tau \ll \min\{\Gamma_s^{-1}, |\Delta|^{-1}, C^{-2/3}\}. \tag{18}$$

Under this condition, Eqs. (2a) and (6) yield $\alpha_e(\tau)$ of the form (15) with $\beta = 1/2$ and, correspondingly,

$$\kappa_s = (2^{5/2} \pi^{1/2}/3) C \tau^{1/2}. \tag{19}$$

As mentioned above, the QZE is now *less pronounced* (see Fig. 3, where we used the exact solution [Eq. (A11) in [9]] to compute $\alpha_e(\tau)$). This case is realizable for an active dipole

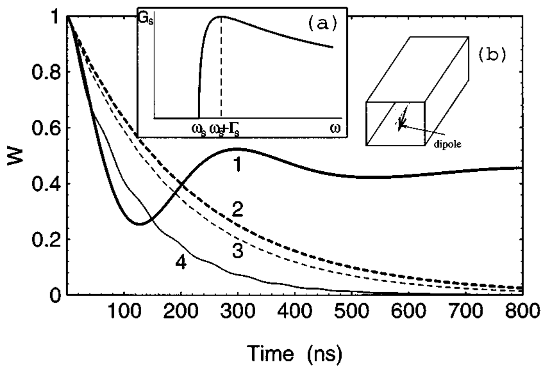

FIG. 3. Evolution of excited-state population W for a two-level atom ($\gamma_f = 10^6$ s^{-1}) coupled to the waveguide field, with coupling $C^{2/3} = 1.2\times 10^7$ s^{-1} and width $\Gamma_s = 0$: curve 1, uninterrupted evolution at cutoff frequency ($\Delta = 0$); curve 4, idem, $\Delta = 10^8$ s^{-1}; curve 2, interrupted evolution at intervals $\tau = 10^{-8}$ s for $\Delta = 0$; curve 3, idem, for $\Delta = 10^8$ s^{-1}. Insets: (a) DOM with cutoff [Eq. (17)]; (b) dipole in a waveguide.

layer embedded in a dielectric waveguide [12] [Fig. 3—inset (b)], using a level scheme similar to that of Fig. 2.

By contrast, if, instead of Eq. (18), τ satisfies $\Gamma_s^{-1} \ll \tau \ll |\Delta|^{-1}, \Gamma_s^2/C^2$, one obtains $\alpha_e(\tau) \approx 1 - (2C/\Gamma_s)\sqrt{i\pi\tau}$ and

$$\kappa_s = (2^{3/2}\pi^{1/2}C/\Gamma_s)\tau^{-1/2}. \tag{20}$$

In this case the measurements *enhance* the decay rate, contrary to QZE behavior. This regime can be realized for a sufficiently weak coupling, $C \ll \Gamma_s^{3/2}$. Finally, for $\tau \gg |\Delta|^{-1}$ the exponential behavior [9] prevails, irrespective of the values of C and Γ_s.

Our unified analysis of two-level atom coupling to field reservoirs has revealed (a) the most general optimal conditions for observing the QZE in various field-confining structures (cavities, waveguides, and three-dimensional photonic band structures); (b) the common character of QZE in both strong-coupling (reversible) and weak-coupling (irreversible) evolution regimes; and (c) the importance of the free-evolution phase accrued between consecutive interruptions, which can significantly affect the rate of decay and can set non-QZE behavior. We note that the wave-function collapse notion is not involved here, since the measurement is explicitly described as an act of coherence breaking [6]. This analysis also clarifies that the QZE cannot combat the background-mode contribution to exponential decay, and is therefore inadequate for decoherence error prevention [13]. The best way to achieve such prevention is by switching off the entire density of modes, i.e., placing the atomic resonance well within an ideal photonic band gap.

[1] B. Misra and E. C. G. Sudarshan, J. Math. Phys. (N.Y.) **18**, 756 (1977).
[2] J. Maddox, Nature (London) **306**, 111 (1983).
[3] A. Peres, Phys. Rev. D **39**, 2943 (1989).
[4] W. M. Itano *et al.*, Phys. Rev. A **41**, 2295 (1990).
[5] R. Cook, Phys. Scr. **T21**, 49 (1988).
[6] P. L. Knight, Nature (London) **344**, 493 (1990); T. Petrosky, S. Tasaki, and I. Prigogine, Phys. Lett. A **151**, 109 (1990); E. Block and P. R. Berman, Phys. Rev. A **44**, 1466 (1991); L. E. Ballentine, *ibid.* **43**, 5165 (1991); V. Frerichs and A. Schenzle, *ibid.* **44**, 1962 (1991).
[7] M. B. Plenio, P. L. Knight, and R. C. Thompson, Opt. Commun. **123**, 278 (1996).
[8] A. Luis and J. Peřina, Phys. Rev. Lett. **76**, 4340 (1996).
[9] A. G. Kofman, G. Kurizki, and B. Sherman, J. Mod. Opt. **41**, 353 (1994).
[10] D. J. Heinzen *et al.*, Phys. Rev. Lett. **58**, 1320 (1987).
[11] F. DeMartini *et al.*, Phys. Rev. Lett. **59**, 2995 (1987); Y. Yamamoto, S. Machida, and G. Björk, Opt. Quantum Electron. **24**, S215 (1992).
[12] R. G. Hulet, E. S. Hilfer, and D. Kleppner, Phys. Rev. Lett. **55**, 2137 (1985); H. Yokoyama *et al.*, Opt. Quantum Electron. **24**, S245 (1992).
[13] L. Vaidman, L. Goldenberg, and S. Wiesner, Phys. Rev. A **54**, R1745 (1996); S. L. Braunstein (unpublished).
[14] G. Kurizki, A. G. Kofman, and V. Yudson, Phys. Rev. A **53**, R35 (1996).
[15] *Handbook of Mathematical Functions*, Natl. Bur. Stand. Appl. Math. Ser. No. 55, edited by M. Abramowitz and I. A. Stegun (U.S. GPO, Washington, DC, 1964).