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Quantum statistics of fundamental and higher-order coherent quantum solitons in Raman-active waveguides

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The quantum dynamics of coherent optical pulses are studied using a nondiagonal coherent-state generalized P representation including photon-phonon interactions. Photon-number squeezing of coherent quantum solitons using spectral filtering is theoretically predicted. It is shown that Raman noise does not significantly reduce photon-number squeezing produced by spectral filtering of 1-ps fundamental coherent quantum solitons in optical fibers. Coherent Nsech pulses with $N>1$ can show a larger reduction in photon-number fluctuations even at room temperature. The reduction in quantum noise for $N>1$ is not restricted to photon number and an improvement of more than 3 dB is also found for the quadrature-phase squeezed soliton experiments using a fiber Sagnac interferometer at 77 K. [S1050-2947(96)50210-6]

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I. INTRODUCTION

Experiments have used pulses from mode-locked lasers to demonstrate quantum effects in *coherent* optical solitons. The first experiment was by Rosenbluh and Shelby at IBM [1], which produced quadrature-phase squeezed quantum solitons. The second was by Friberg and co-workers at NTT [2], which showed quantum correlations between the photon number and phase of quantum solitons after collision. These latter two experiments are discussed in a review of coherent quantum solitons [3].

A recent experiment by Friberg and co-workers [4] has demonstrated sub-Poissonian photon statistics of spectrally filtered optical pulses in the anomalous dispersion regime of silica fibers. Spectral filtering has been investigated previously as a means of reducing the Gordon-Haus effect in soliton communication systems [5]. The fiber input was a chirped coherent pulse with dimensionless chirp-free temporal envelope $N\operatorname{sech}(t/t_0)$, where t_0 is the fundamental classical soliton time scale and was the first demonstration of quantum effects in an $N>1$ soliton. Previous theoretical work on sub-Poissonian fundamental solitons of the quantum nonlinear Schrödinger equation used the technique of mixing the output with a strong coherent field [6] and not spectral filtering. This paper presents a theoretical prediction of photon-number squeezing from spectral filtering of $N\geqslant 1$ 1-ps coherent solitons. In addition, it is shown that quadrature-phase squeezing of $N>1$ coherent solitons can be initially larger than that of fundamental solitons, including the Raman effects.

A quantum field-theoretic treatment for the propagation of optical fields with pulse widths $\gtrsim 100$ fs in single-mode polarization-preserving fibers was presented by Carter and Drummond [7]. That work was described in more detail recently by Carter [8]. Here we use the positive-P dynamical equations for the photon flux amplitude to propagate the quantum fields through a dispersive nonlinear fiber and then calculate the photon-number statistics after spectral filtering. Similarly, we calculate the quadrature-phase squeezing produced in a fiber Sagnac interferometer.

The Raman-modified stochastic nonlinear Schrödinger equation for the normalized photon flux field $\phi(\xi,\tau)$ is given by [7]

$$\frac{\partial\phi}{\partial\xi}=-\frac{i}{2}\left[1\pm\frac{\partial^2}{\partial\tau^2}\right]\phi+if\phi^\dagger\phi^2+\sqrt{i}\phi\Gamma_e+i\phi\left[\int_{-\infty}^{\tau}d\tau' h(\tau-\tau')\phi^\dagger(\tau')\phi(\tau')+\Gamma_v\right], \quad (1)$$

where the length and time variables (ξ,τ) in the comoving frame at speed ω' (group velocity at the carrier frequency) in the laboratory frame (x,t) are $\tau=(t-x/\omega')/t_0$, $\xi=x/x_0, x_0=t_0^2/|k''|$. The characteristic time scale t_0 will be chosen to be the pulse width later, and the soliton period is $\pi/2$ times longer than the dispersion length x_0 determined by t_0 and the second-order dispersion k''. The Raman noise correlations are given by the autocorrelation

$$\langle\Gamma_v(\xi,\omega)\Gamma_v(\xi',\omega')\rangle=\frac{1}{\bar{n}}\delta(\xi-\xi')\delta(\omega+\omega')\{-i\sqrt{2\pi}h(\omega)+[n_{th}(|\omega|)+\theta(-\omega)]\alpha(|\omega|)\}, \quad (2)$$

where $n_{\rm th}$ is the thermal photon occupation number, $\theta(\omega)$ is a step function with value unity for positive arguments,

$$\sqrt{2\pi}h(\omega)=\int_0^\infty d\Omega\,\frac{\alpha(\Omega)\Omega}{\pi(\Omega^2-\omega^2)}+i\,\frac{\operatorname{sgn}(\omega)}{2}\alpha(|\omega|), \quad (3)$$

and the cross correlation

$$\langle\Gamma_v(\xi,\omega)\Gamma_v^\dagger(\xi',\omega')\rangle=\frac{1}{\bar{n}}\delta(\xi-\xi')\delta(\omega+\omega')[n_{th}(|\omega|)+\theta(-\omega)]\alpha(|\omega|). \quad (4)$$

The quantum noise from the electronic nonlinearity Γ_e is a real δ-correlated Gaussian noise, with variance given by the product of the electronic fraction f and inverse photon-number scale $1/\bar{n}=\eta_0 t_0/k''\omega'^2$, where η_0 is the nonlinearity over the time scale of interest [9]. The parameter $\alpha(\omega)$ is the Raman gain, which has a peak near 13 THz for silica fiber. Raman inhomogeneous model parameters used here correspond to the Raman gain curve in Ref. [7]. An effect at low frequencies is guided acoustic wave Brillouin scattering (GAWBS) noise [10], which differs significantly depending on the fiber used. For this reason no attempt is made to include GAWBS contributions explicitly, although one of the advantages of the approach used here is that such details can be included easily for specific fibers.

II. PHOTON-NUMBER SQUEEZING OF FUNDAMENTAL SOLITONS

Neglecting Raman effects, $N\mathrm{sech}(\tau)$ pulses with N a positive integer are Nth order solitons of the nonlinear Schrödinger equation. The $N=1$ fundamental solitons have an intensity profile that is unchanged by propagation. An initial quantum field comprising a coherent $\mathrm{sech}(\tau)$ pulse can be approximately described as a soliton whose phase and position diffuse with propagation. The soliton parameters associated with photon number and momentum have a variance that is unaltered by propagation according to the quantum nonlinear Schrödinger equation. However, initial coherent $\mathrm{sech}(\tau)$ pulses have nontrivial quantum dynamics, giving rise to quantum correlations that can produce sub-shot-noise statistics. For example, it is already known that phase diffusion with propagation produces a quadrature-phase squeezed soliton whose spectral correlations indicate quadrature-phase squeezing over the bandwidth of the pulse [3]. Here we show that spectral filtering of a coherent fundamental soliton can lead to sub-Poissonian statistics.

The techniques to study the propagation of the quantum field equations are to be discussed elsewhere [12,9]; however, a brief outline of similar methods has been given by Carter [8]. By using a normally ordered representation, no additional noise sources need be added to compensate for the effect of the spectral filter $f(\omega)$ on vacuum fluctuations. In the positive-P representation, the photon number is taken to be

$$\langle n\rangle=\overline{n}\int d\omega|f(\omega)|^2\langle\phi^\dagger(-\omega)\phi(\omega)\rangle. \tag{5}$$

Formally, this also corresponds to taking into account the photodetector response, which is usually assumed to be $Ze\delta(t)$. However, this paper only considers the effect of the simplest filter–a symmetric square bandpass in the Fourier domain. Filter optimization is still an open problem.

The scaled normally ordered variance of the spectral intensity of an initial coherent fundamental quantum soliton after propagating two soliton periods is shown in Fig. 1. In order to compare the bandwidth of the squeezing with the pulse bandwidth, the spectral intensity is also shown along with the noise variance. This clearly shows that, although the photon-number statistics are Poissonian, the quantum noise produced by the nonlinear interaction generates sub-shot-noise intensity fluctuations in a significant bandwidth about the carrier frequency. By filtering the outlying components of the spectrum, one obtains photon-number squeezed optical solitons. Narrowing the filter to remove all the frequencies with intensity fluctuations above shot noise does not

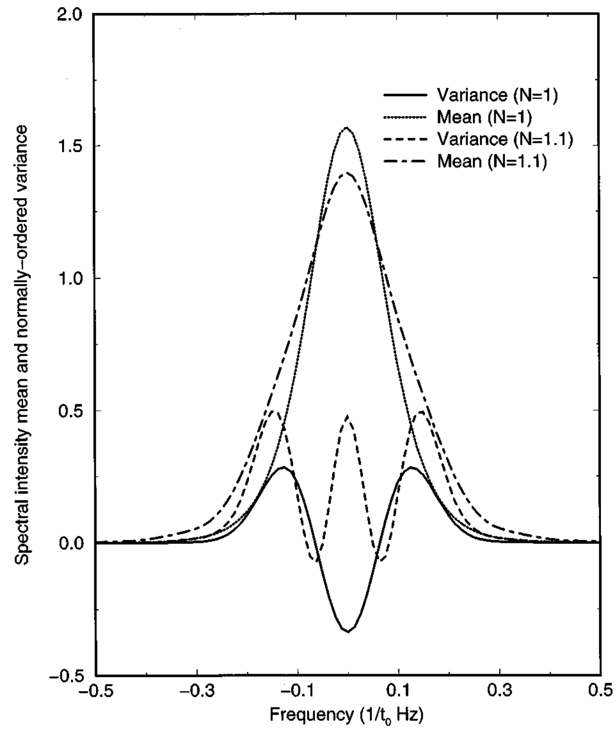

FIG. 1. Normally ordered spectral intensity variance after propagating two soliton periods and the spectral intensity for an initial coherent $N\mathrm{sech}(\tau)$ pulse for $N=1,1.1$ with $\overline{n}=10^8$.

maximize the photon-number squeezing in this case, indicating that intermode correlations play a role.

Fundamental solitons of 1-ps duration in silica fiber are immune to Raman self-frequency shifts over propagation distances of a few soliton periods, but are affected by the temperature-dependent Raman noise. An example of the photon-number variance versus propagation length along the fiber for a fixed filter is given in Fig. 2. The maximum fiber length in Fig. 2 corresponds to four soliton periods and shows that for the particular filter used the maximum squeezing occurs around four dispersion lengths ($\sim$2.5 soliton periods). The temperature-dependent Raman noise decreases the squeezing by 0.9 dB from the 5.2-dB below-shot-noise result for no Raman contribution. The filter has a bandwidth of 300 GHz and corresponds to an 11% reduction in photon number for the optical pulse. The filter bandwidth can be larger and still obtain significant photon-number noise reductions, depending on the fiber length. This means that using a stable, low-noise, mode-locked laser producing 1-ps pulses it should be relatively easy, even at room temperature, to produce photon-number squeezed fundamental solitons. By lowering the phonon temperature, the expected photon-number squeezing will increase.

III. SQUEEZING OF $N>1$ HIGHER-ORDER SOLITONS

For initial classical pulses with $0.5<N<1.5$, classical inverse scattering theory tells us that pulses with $N\neq 1$ contain a soliton component and a nonsoliton contribution. The pulses asymptotically evolve into a fundamental soliton [11]. Here we are mainly concerned with the behavior for $N>1$. Over the short propagation distances considered here, the nonsoliton contribution is an important part of the dynamics.

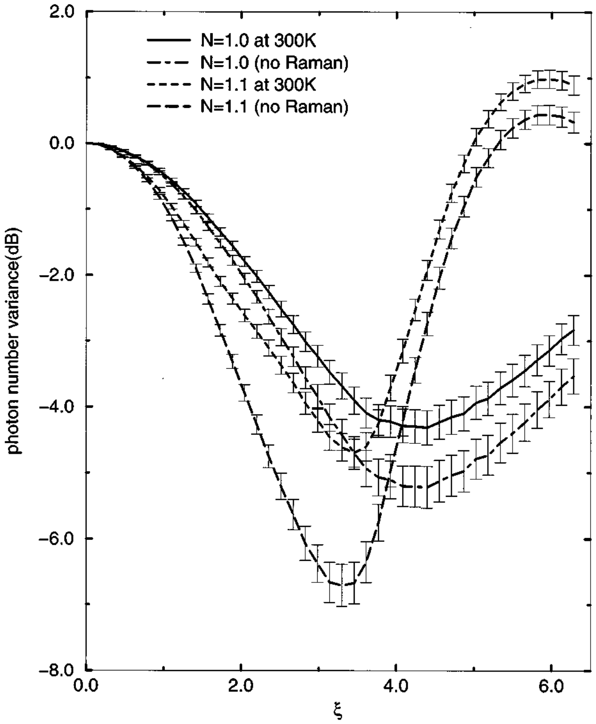

FIG. 2. Photon-number variance versus propagation distance for an initial coherent $N\mathrm{sech}(\tau)$ pulse using $N=1.0,1.1$ with $\overline{n}=10^8$ with and without Raman effects at a phonon reservoir temperature of 300 K.

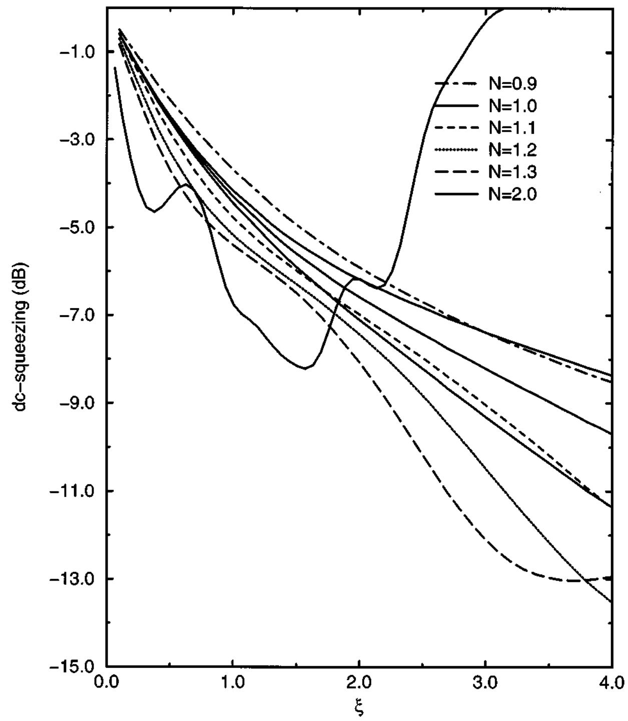

FIG. 3. dc squeezing $[1+S(0,\xi)]$ in dB versus propagation distance for an initial coherent $N\mathrm{sech}(\tau)\exp(-i\frac{1}{2}C\tau^2)$ pulse using $N=0.9,1.0,1.1,1.2,1.3,2.0$ with $\overline{n}=10^9$, including Raman effects at a phonon reservoir temperature of 77 K. The chirp is nonzero only for $N=1$ where $C=-0.1,0.0,0.1$. The upper $N=1$ trace corresponds to $C=-0.1$, while the lower one is for $C=0.1$.

An initial pulse with $1<N<1.5$ has a maximum amplitude that oscillates with a period that decreases with N. This implies that the output photon number after spectral filtering will contain the same oscillations, since the pulse bandwidth oscillates as it travels down the waveguide with a period that decreases with N. The scaled spectral intensity variance for $N=1.1$ after propagating two soliton periods is shown in Fig. 1. An important feature is the above-shot-noise intensity fluctuations near the carrier frequency. This typically causes photon-number fluctuations to be more than 3 dB above shot noise as the filter bandwidth is decreased.

Ignoring Raman effects, an initial coherent pulse with $N>1$ can exhibit stronger photon-number squeezing than a fundamental soliton for the same filter. However, initial 1-ps pulses with $N>1$ spectrally broaden due to self-phase modulation and therefore experience higher Raman gain than a fundamental soliton. Such pulses can clearly exhibit an asymmetric spectrum after a few soliton periods. Here we wish to investigate to what extent the Raman effects can negate any advantage in using $N>1$ pulses.

To illustrate the general behavior for photon-number squeezing, we consider the same filter as before, and one finds that the photon-number squeezing versus N has its first local minimum about $N=1.1$ for a propagation distance of two soliton periods. The photon-number squeezing versus propagation distance for $N=1.1$ is given in Fig. 2 and is reduced from 6.8 dB to 4.7 dB by the Raman effect for $t_0=1$ ps at room temperature. Clearly, cooling the fiber will tend to improve the noise reduction toward the result with no Raman effects included. There are a number of important features shown in Fig. 2. Spectral filtering can produce super-Poissonian statistics just as easily as sub-shot-noise statistics. Also, for longer propagation distances it is seen that sub-Poissonian statistics can be turned into super-Poissonian statistics by the Raman effects. Although these results are only for a specific filter bandwidth, the positive result is that the increased Raman gain for $N>1$ solitons is not necessarily enough to destroy the advantage of using $N>1$ to obtain a larger amount of photon-number squeezing of 1-ps pulses. For pulse widths less than 1 ps, the Raman self-frequency shift will come into play for these $N\sim1$ pulses. The subpicosecond pulse regime is currently being studied and will be reported elsewhere.

The increased noise reduction seen in the photon-number squeezing results also appears in other measurements, such as quadrature-phase squeezing. The squeezing spectrum in the positive-P representation is defined as [14]

$$S(\omega,\xi)=\frac{2\pi\overline{n}\langle X(-\omega,\xi)X(\omega,\xi)\rangle}{\int_{-T/2}^{T/2}d\tau\phi^{\dagger}(\tau,\xi)\phi(\tau,\xi)}, \tag{6}$$

where $\hat{X}=\hat{\phi}_{LO}^{\dagger}\hat{\phi}+\mathrm{H.c.}$ and $\hat{\phi}_{LO}$ is a local oscillator (which is the bright pulse from the Sagnac fiber loop [1]) for the balanced homodyne detector with a measurement period of T. Neglecting Raman effects, one finds that the squeezing initially increases as N increases due to the self-phase modulation. However, after a few dispersion lengths the squeezing typically decreases with propagation distance due to the group-velocity dispersion. Similar behavior has been noted previously by Lai and Yu [13] for $N=1$ pulses, where it was found to be due to the self-Raman effect, and has also been predicted in traveling-wave parametric amplifiers [14], where it was counteracted by choosing an appropriately chirped local oscillator. The results in Fig. 3 give the dc squeezing $[1+S(0,\xi)]$ in dB versus propagation distance for the Sagnac interferometer arrangement using $N = 0.9,1.0,1.1,1.2,1.3,2.0$, including Raman effects for a temperature of 77 K. Using a pulse width of 100 fs, it is found that more than a 3-dB improvement over a fundamental soliton can be obtained by using $N>1$. In particular, for $N=1.2$, after propagating four dispersion lengths, one finds an improvement of 3.8 dB. The 100-fs fundamental soliton has maximum squeezing after approximately 8.5 soliton periods. Neglecting Raman effects, a classical $N=2$ pulse is a soliton with a periodicity of $\xi=\pi/2$. It is seen that the $N=2$ coherent pulse has a local minimum in the amount of squeezing in the region of $\xi=\pi/4$ due to pulse self-compression. The Raman effect is quite significant for squeezing of 100-fs $N=2$ coherent pulses. The noise level returns to the shot-noise level after about two soliton periods, due primarily to the soliton self-frequency shift. Results for chirped $N=1$ pulses in the anomalous dispersion regime are also given in Fig. 3 and show that positive chirp tends to increase the squeezing initially compared to unchirped pulses of the same energy due to self-compression from self-phase modulation. Negative chirp has the opposite effect.

In summary, spectral filtering of coherent quantum solitons can lead to both sub-Poissonian and super-Poissonian statistics. While the photon-number fluctuations of the soliton are stationary during propagation the intensity spectral correlations are produced by propagation. Spectral filtering probes the internal structure of the soliton not apparent in the total photon-number fluctuations. Raman noise tends to increase the super-Poissonian character of the filtered pulses. Using a fixed filter bandwidth, the photon-number squeezing varies with the propagation distance along the fiber, even for a fundamental coherent quantum soliton. Spectrally filtered $N>1$ solitons exhibit strong variation of their photon statistics with N, as demonstrated experimentally by Friberg and co-workers [4]. Despite the higher Raman gain experienced by $N>1$ pulses, there can be an advantage in using pulses with $N>1$ for achieving a larger reduction in photon-number fluctuations, given a particular filter bandwidth, for 1-ps pulses, even at room temperature. The possibility of increased noise reduction for $N>1$ is more general than the particular photon-number squeezing arrangement discussed here. For example, it was shown that quadrature-phase squeezed coherent solitons for $N>1$ can initially exhibit larger noise reduction than their fundamental counterpart.

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