Proposal for a mesoscopic cavity QED realization of the Greenberger-Horne-Zeilinger state

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(Received 8 March 1996)

A proposal is made for obtaining a mesoscopic realization of the Greenberger-Horne-Zeilinger (GHZ) ''all or nothing'' contradiction between quantum mechanics and local realism in the context of cavity QED. The proposed experiment involves three separated micromaser cavities, each initially prepared in a coherent state, which are then entangled by dispersive atom-field interactions followed by selective ionization of a single atom that has passed through all three cavities. The cavities are then probed individually by single atoms manipulated to obtain a measurement of the parity of the cavity fields. Using an isomorphism between coherent-state cavity fields and a spin-$\frac{1}{2}$ system, we show how this experiment gives rise to violations of local realism in the manner of GHZ. [S1050-2947(96)50410-5]

PACS number(s): 03.65.Bz, 32.80.−t

In recent years, there has been much interest in a new form of Bell's theorem, described by Greenberger, Horne, and Zeilinger (GHZ) [1], in which contradictions between quantum mechanics and local realism can be exposed, not statistically as in the usual formulation of Bell's theorem [2], but in one experimental run or set of measurements. This form of Bell's theorem requires states of three or four entangled particles. A typical GHZ state involving three spin-$\frac{1}{2}$ particles has the simple form

$$|\psi_{\rm GHZ}\rangle=\frac{1}{\sqrt{2}}[|+\rangle_1|+\rangle_2|+\rangle_3-|-\rangle_1|-\rangle_2|-\rangle_3], \qquad (1)$$

where $|+\rangle$ or $|-\rangle$ are spin-up or spin-down states along some axis. Usually this axis is taken to be the z axis; however, for reasons that will become clear below, we shall take it to be the x axis [3]. The state (1) is an eigenstate of the set of commuting operators

$$\sigma_z^1\sigma_y^2\sigma_y^3, \quad \sigma_y^1\sigma_z^2\sigma_y^3, \quad \sigma_y^1\sigma_z^2\sigma_y^3 \qquad (2)$$

where the $\sigma_j^i(i=1,2,3,\ j=x,y,z)$ are spin operators (apart from the factor $\hbar/2$) for particles 1, 2, and 3. But the operator

$$\sigma_z^1\sigma_z^2\sigma_z^3 \qquad (3)$$

commutes with all the operators in Eq. (2) and therefore also has $|\psi_{\rm GHZ}\rangle$ as an eigenstate. The eigenvalues of the operators of Eq. (2) are $+1$. Local realistic theory predicts that the eigenvalue of the operator of Eq. (3) should also be $+1$, but a straightforward quantum-mechanical calculation yields -1 instead. Thus a single set of measurements of the z component of all three spins is sufficient to demolish the assumption of local reality [4].

However, it has been difficult to find a source of three entangled spins, and to date there has been no experimental realization of the GHZ state. A number of proposals have been made, aside from those given in Ref. [1]. In the context of cavity QED, Cirac and Zoller [5], Haroche [6], and Gerry [7] have presented various proposals for entangling three or more ''two-level'' Rydberg atoms (these atoms being the ''spins''). Reid and Munro [8] and Klyshko [9] have proposed optical realizations of the GHZ state, the former actually involving a large number of particles, such that violations of local realism can be manifested on a macroscopic scale. Yet another approach is to find a system that in some way is isomorphic to a set of three entangled spins. An example of this has been discussed again in the context of cavity QED, by Wódkiewicz *et al.* [10]. However, the system is complicated and involves four cavity modes and a single two-state atom interacting with the cavity fields through Raman transitions.

In this Rapid Communication we propose another possible realization of the GHZ state, again in the context of cavity QED, but this time involving entangled mesoscopic field states (coherent states) in three separate micromaser cavities. As we shall show, a field in a coherent state of sufficiently large amplitude is isomorphic to a spin system where the role of the spin-up and spin-down states along the z axis is played by the parity of the even and odd coherent states (examples of Schrödinger-cat states [11]). Detection of the parity of a cavity field is easily accomplished, and since this type of measurement involves the selective ionization of a circular Rydberg atom, state detection is nearly 100% efficient, a distinct advantage over proposals involving the detection of photons [12].

First we present the isomorphism between coherent states and spin states. Actually we need to consider two coherent states of identical amplitude but shifted in phase by 180°. These we denote as $|\alpha\rangle$ and $|-\alpha\rangle$, where

$$|\alpha\rangle=e^{-|\alpha|^2/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \qquad (4)$$

and where $|\alpha|$ is large enough so that they are orthogonal states: $\langle-\alpha|\alpha\rangle=e^{-2|\alpha|^2}\approx 0$. (This is valid for, say, $|\alpha|^2=10$, an average of ten photons.) From these states one can form the even and odd coherent states,

$$|\alpha+\rangle=\frac{1}{\sqrt{2}}(|\alpha\rangle+|-\alpha\rangle), \qquad (5)$$

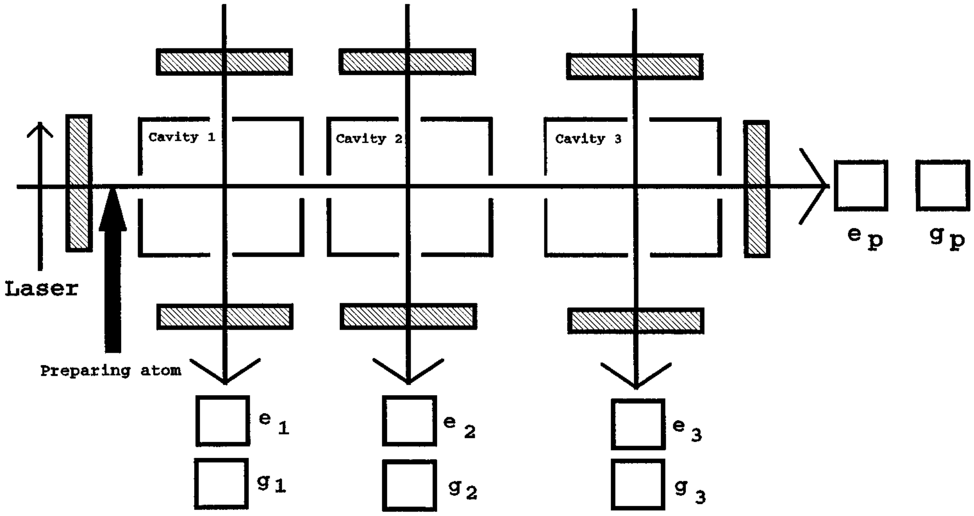

FIG. 1. Proposed experimental setup for preparing and detecting a GHZ state. Three identical cavities are prepared in coherent states by classical sources (not shown). The preparing atom passes through all three cavities. The atoms 1, 2, and 3 are for measuring the parity of the cavity fields. The small squares represent ionization detectors and the shaded regions the Ramsey zones.

$$|\alpha-\rangle=\frac{1}{\sqrt{2}}(|\alpha\rangle-|-\alpha\rangle),$$

respectively, where $\langle\alpha+|\alpha-\rangle=0$. These are eigenstates of the parity operator $\Pi=(-1)^{a^\dagger a}$ with respective eigenvalues ±1. We now interpret these parity eigenstates as the eigenstates of the ''z'' component of a ''spin'' operator given as $\Sigma_z=|\alpha+\rangle\langle\alpha+|-|\alpha-\rangle\langle\alpha-|$, where obviously $\Sigma_z|\alpha\pm\rangle=\pm|\alpha\pm\rangle$. Evidently, from Eq. (4), we can write

$$|\alpha\rangle=\frac{1}{\sqrt{2}}(|\alpha+\rangle+|\alpha-\rangle),$$
$$|-\alpha\rangle=\frac{1}{\sqrt{2}}(|\alpha+\rangle-|\alpha-\rangle), \tag{6}$$

which are the analogs of the spin-x eigenstates where the x-component ''spin'' operator here is $\Sigma_x=|\alpha+\rangle\langle\alpha-|+|\alpha-\rangle\langle\alpha+|$. Similarly, for the ''$y$'' component we have the states

$$\frac{1}{\sqrt{2}}(|\alpha+\rangle+i|\alpha-\rangle),$$
$$\frac{1}{\sqrt{2}}(|\alpha+\rangle-i|\alpha-\rangle), \tag{7}$$

which are eigenstates of $\Sigma_y=i(|\alpha+\rangle\langle\alpha-|-|\alpha-\rangle\langle\alpha+|)$. Obviously, the Σ operators satisfy the spin algebra $[\Sigma_i,\Sigma_j]=2i\epsilon_{ijk}\Sigma_k$ and $\{\Sigma_i,\Sigma_j\}=\delta_{ij}$ $(i,j,k=x,y,z)$, where $\{\}$ is an anticommutator.

Our goal is to generate a GHZ state consisting of an entanglement of coherent states in three separated micromaser cavities. We propose to accomplish this by extending, to the three-cavity situation, techniques already under development for situations involving one cavity, particularly those for the generation of Schrödinger-cat states [12,13]. The states will be mesoscopic in the sense that the fields of each cavity may contain, on average, about ten photons.

We now consider the experimental setup pictured in Fig. 1. Three identical micromaser cavities are coupled in series by a single two-level atom that passes through all three cavities. In addition, after passage of this preparing atom, each cavity receives another atom, which, as discussed below, is to be used to detect the parity of the cavity field. Also shown are several classical microwave fields (Ramsey zones) important for preparing, manipulating, and analyzing the atomic states outside the cavities. Each atom is subjected to atomic-state detection by selective ionization.

We assume that the cavities are initially prepared, by classical currents, in coherent states $|\alpha_1\rangle$, $|\alpha_2\rangle$, and $|\alpha_3\rangle$ for cavities 1, 2, and 3, respectively. The amplitudes α_i $(i=1,2,3)$ need not be identical. The interaction between the atoms and cavity fields is assumed to be dispersive. We assume that the atoms have the same energy-level configuration as that given in Fig. 2. The $|f\rangle\leftrightarrow|e\rangle$ transition is detuned from the cavity frequency ω_c enough that only virtual transitions occur between these levels. On the other hand, the $|e\rangle\leftrightarrow|g\rangle$ transition is highly detuned from the cavity frequency, so that it is essentially decoupled. Inside the cavities, the atom-field interaction is given by the effective Hamiltonian [14]

$$H_{Ii}=\hbar\chi a_i^\dagger a_i(|f\rangle\langle f|-|e\rangle\langle e|), \tag{8}$$

where a_i $(a_i^\dagger)$ $(i=1,2,3)$ are the cavity field annihilation (creation) operators and χ is a parameter depending on the

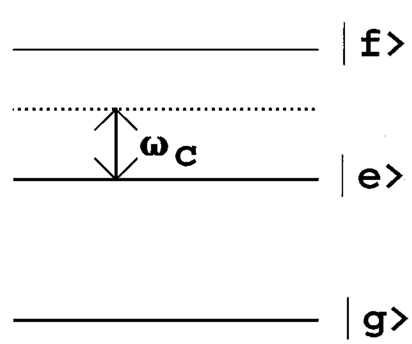

FIG. 2. Rydberg-atom energy-level diagram. The ''ground'' state $|g\rangle$ corresponds to a circular Rydberg state of principal quantum number $n=50$, and $|e\rangle$ corresponds to $n=51$.

atomic dipole moment and the detuning. Outside the cavities, the classical fields in the Ramsey zones are resonant with the $|e\rangle \leftrightarrow |g\rangle$ transition, and generate $\pi/2$ pulses having the effects $|e\rangle \rightarrow (|e\rangle + |g\rangle)/\sqrt{2}$ and $|g\rangle \rightarrow (|g\rangle - |e\rangle)/\sqrt{2}$. The atomic states $|e\rangle$ and $|f\rangle$ could be of principal quantum numbers 50 and 51, respectively, with the cavity tuned near the 50–51 circular-to-circular state transition at a frequency of about 50 GHz.

Now let $|e_p\rangle$ and $|g_p\rangle$ represent the states of the preparing atom that passes through all three cavities. Laser excitation boosts the atom into state $|e_p\rangle$, and a subsequent $\pi/2$ pulse creates the superposition $(|e_p\rangle + |g_p\rangle)/\sqrt{2}$. With the cavities prepared in coherent states, the initial atom-field state vector is

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|e_p\rangle + |g_p\rangle)|\alpha_1\rangle|\alpha_2|\alpha_3\rangle. \tag{9}$$

Using the relation

$$e^{+i\Lambda a^\dagger a}|\alpha\rangle = |\alpha e^{+i\Lambda}\rangle, \tag{10}$$

it is easy to see that after the atom emerges from the third cavity the state vector is

$$\begin{aligned}|\psi(\tau)\rangle &= e^{-iH_{I3}t/\hbar}e^{-iH_{I2}t/\hbar}e^{-iH_{I1}t/\hbar}|\psi(0)\rangle \\ &= \frac{1}{\sqrt{2}}[|e_p\rangle|\alpha_1 e^{i\delta}\rangle|\alpha_2 e^{i\delta}\rangle|\alpha_3 e^{i\delta}\rangle \\ &\quad + |g_p\rangle|\alpha_1\rangle|\alpha_2\rangle|\alpha_3\rangle],\end{aligned} \tag{11}$$

where $\delta = \chi t$ and where we have assumed that the atom transits each cavity in the same time t, so that $\tau = 3t$. We further assume that the atom can be velocity selected such that $\delta = \pi$. This requires a velocity of about 100 m/s for centimeter-size cavities. Upon achieving this, our state vector is then

$$|\psi(\tau)\rangle = \frac{1}{\sqrt{2}}[|e_p\rangle|-\alpha_1\rangle|-\alpha_2\rangle|-\alpha_3\rangle + |g_p\rangle|\alpha_1\rangle|\alpha_2\rangle|\alpha_3\rangle]. \tag{12}$$

After exiting the cavity, the atom is subjected to a second $\pi/2$ pulse, which results in

$$\begin{aligned}|\psi(\tau)\rangle' = \tfrac{1}{2}[&|g_p\rangle(|\alpha_1\rangle|\alpha_2\rangle|\alpha_3\rangle + |-\alpha_1\rangle|-\alpha_2\rangle|-\alpha_3\rangle) \\ &- |e_p\rangle(|\alpha_1\rangle|\alpha_2\rangle|\alpha_3\rangle - |-\alpha_1\rangle|-\alpha_2\rangle|-\alpha_3\rangle)].\end{aligned} \tag{13}$$

If, upon selective ionization, the atom is detected in state $|e_p\rangle$, the cavity fields are reduced to the state

$$|\psi_F\rangle = \frac{1}{\sqrt{2}}[|\alpha_1\rangle|\alpha_2\rangle|\alpha_3\rangle - |-\alpha_1\rangle|-\alpha_2\rangle|-\alpha_3\rangle]. \tag{14}$$

Clearly, Eq. (14) has the form of the GHZ state of Eq. (1) and is, in fact, an entanglement of the ''spin-*x*'' eigenstates of Eq. (5). This being the case, all that is required to demonstrate a violation of local realism is a measurement of the ''spin-*z*'' component Σ_z or, in other words, the parity of each cavity. Again, local realistic theory predicts that the product of the three measured values of Σ_z yields $+1$, whereas quantum mechanics predicts the product to always be -1.

It remains to indicate how the parity of each cavity is to be measured [15]. The procedure is exactly the same as that used to prepare the cavities, but applied to each cavity. Let $|\beta_i\rangle$ be $|\alpha_i\rangle$ or $|-\alpha_i\rangle$ for the ith cavity. An atom prepared in the superposition $(|e_i\rangle + |g_i\rangle)/\sqrt{2}$ passes through the cavity to produce the entanglement state

$$|\psi_i\rangle = \frac{1}{\sqrt{2}}[|e_i\rangle|-\beta_i\rangle + |g_i\rangle|\beta_i\rangle], \tag{15}$$

where again it is assumed that the velocity of the atom is selected to give the field a phase shift of π. A second $\pi/2$ pulse produces

$$|\psi_i\rangle' = \tfrac{1}{2}[-|e_i\rangle(|\beta_i\rangle - |-\beta_i\rangle) + |g_i\rangle(|\beta_i\rangle + |-\beta_i\rangle)]. \tag{16}$$

Detection of state $|e_i\rangle$ is correlated with odd parity and that of state $|g_i\rangle$ with even parity. In essence, the Rydberg atoms along with the classical microwave fields and selective ionization detectors play the role of Stern-Gerlach magnets.

Finally, a brief discussion of the feasibility of the proposed experiment is warranted. The required technology is identical to that being developed to generate and observe the Schrödinger-cat states of a single cavity field [12,13]. The essential ingredients consist of high-Q niobium superconducting micromaser cavities supporting single-mode fields and Rydberg atoms of long radiative lifetimes. The cavity lifetime is given by $T_{\text{cav}} = Q/2\pi\nu$. If the frequency $\nu \sim 50$ GHz, then $T_{\text{cav}} \sim 3\times10^{-12}Q$. For a $Q = 10^8$, as in the recently reported experiments on the quantum Rabi oscillations [16], we obtain a cavity lifetime of $T_{\text{cav}} = 0.3$ ms, which would be too short for the present purposes. However, cavities with a Q factor of 10^{12} have been reported in the literature [17]. Such a Q factor would yield $T_{\text{cav}} \sim 3$ s, more than enough time, since the radiative lifetime (T_{rad}) of the circular Rydberg states is on the order of 10^{-2} s. The required atomic velocities are about 100 m/s. Assuming that the total distance traveled by each atom before ionization is about 0.3 m, the required time is about 1.2×10^{-2} s, on the order of the radiative lifetime of the atom. Of course, T_{rad} could also be improved by using higher-n circular Rydberg states as for these atoms $T_{\text{rad}} \sim n^5$. The preparation of atomic circular Rydberg states has been described by Nussenzweig *et al.* [18]. The atoms are easily ionized leading to a near 100% efficiency in detection. Finally, the decoherence time for three cavities containing ten photons each is on the order of $T_{\text{dec}} = T_{\text{cav}}/30$, which for $Q = 10^{12}$ gives 0.3 s, which is a longer than T_{rad} by an order of magnitude. One improvement in the experimental procedure has been conveyed to the author [19]. Instead of requiring an additional three sets of Ramsey zones and ionization detectors, as shown in Fig. 1, it would be much easier to use only the one set used to prepare the cavities where the three parity measurement atoms pass through each cavity where two of the cavities are effectively widely detuned by an applied electric field for a given atom.

With the above numbers taken into account, it appears that the cavities can have macroscopic separations (of a few centimeters), and that the cavities can have a mesoscopic average photon number (around ten). Even the detectors—the Rydberg atoms—have a mesoscopic size (1 μm). Thus with technology that is currently or soon to be available, it should be possible eventually to experimentally produce a mesoscopic realization of a GHZ state in cavity QED.

I wish to thank P. K. Aravind and S. Haroche for discussions and encouragement.

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