Evolution of a Bose-condensed gas under variations of the confining potential

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We discuss the dynamic properties of a trapped Bose-condensed gas under variations of the confining field and find analytical scaling solutions for the evolving coherent state (condensate). We further discuss the characteristic features and the depletion of this coherent state. [S1050-2947(96)50209-X]

The recent successful experiments on Bose-Einstein condensation (BEC) in trapped ultracold alkali atom gases [1–3] open a unique possibility to investigate dynamic properties of a Bose-condensed phase. Of particular interest is the response of the system to time-dependent variations of the confining field. In an interacting Bose-condensed gas these properties are nontrivial. For example, if initially almost all trapped atoms are in the condensate ($T=0$), then under adiabatically slow change of the trapping potential they remain in the condensate, which now corresponds to the ground state of the system in the instantaneous trapping field. At the other extreme, a fast change of the potential from the initial to final shape brings the system to an excited superpositional (ES) state, where the admixture of the (final) ground state can be small. Then, even assuming complete isolation of the system from the environment, there is the question of how the correlation properties change. Especially interesting are those responsible for the reduction of the probability of inelastic processes due to the presence of a Bose condensate [4].

Another question concerns trapped gases with negative scattering length. The trapping field stabilizes the condensate provided that the spacing between adjacent trap levels exceeds the interparticle interaction [5,6]. Will this hold in the ES state or will the system ''collapse''?

Of principal importance is the evolution of a completely isolated many-body system which proves to be in the ES state. Does the system undergo stochastization and imitate relaxation behavior, at least for a large number of particles and large interaction between them compared to the level spacing in the potential well? This question is related to the well-known problem of the appearance of irreversibility in a quantum system with a large number of particles.

To answer most of the above questions we first consider the evolution of a Bose condensate in a parabolic trapping potential $V(r)=m\omega^2 r^2/2$ (m is the atom mass) with frequency $\omega(t)$ varing from ω_0 to ω_1. We use ideas of the analysis of the quantum motion of a particle in a harmonic oscillator with time-dependent frequency (see [7]) and find the solution of the time-dependent nonlinear Schrödinger equation for the evolving coherent state. In certain cases our analytical results can be compared with numerical calculations, first performed in Ref. [5]. We further analyze the characteristic features of the evolving coherent state and discuss the problem of relaxation and the loss of coherence.

Let us consider a Bose gas with a fixed number of particles N in a symmetric harmonic potential with time-dependent frequency $\omega(t)$. We assume a pair interaction potential between atoms of the form $U(\mathbf{r})=\widetilde{U}\delta(\mathbf{r})$. In a three-dimensional (3D) gas $\widetilde{U}=4\pi\hbar^2 a/m$, where a is the scattering length and m the atom mass. The Schrödinger equation for the Heisenberg field operator of atoms, $\hat{\Psi}(\mathbf{r},t)$, reads

$$i\hbar\frac{\partial\hat{\Psi}}{\partial t}=-\frac{\hbar^2}{2m}\Delta\hat{\Psi}+\frac{m\omega^2(t)r^2}{2}\hat{\Psi}+\widetilde{U}\hat{\Psi}^\dagger\hat{\Psi}\hat{\Psi}. \quad (1)$$

The field operator can be represented as a sum of the above-condensate part and the condensate wave function $\Psi_0=\langle\hat{\Psi}\rangle$, which is a c number: $\hat{\Psi}=\hat{\Psi}'+\Psi_0$ (see [8]). The equation for $\Psi_0(\mathbf{r},t)$, obtained by averaging both sides in Eq. (1), in the mean-field approach has the form

$$i\hbar\frac{\partial\Psi_0}{\partial t}=-\frac{\hbar^2}{2m}\Delta\Psi_0+\frac{m\omega^2(t)r^2}{2}\Psi_0+\widetilde{U}[|\Psi_0|^2+2n']\Psi_0, \quad (2)$$

where $n'(\mathbf{r},t)=\langle\hat{\Psi}'^\dagger\hat{\Psi}'\rangle$. The mean-field equation of motion for $\hat{\Psi}'(\mathbf{r},t)$ follows from Eqs. (1) and (2):

$$i\hbar\frac{\partial\hat{\Psi}'}{\partial t}=-\frac{\hbar^2}{2m}\Delta\hat{\Psi}'+\frac{m\omega^2(t)r^2}{2}\hat{\Psi}'$$
$$+2\widetilde{U}\left[(|\Psi_0|^2+n')\hat{\Psi}'+\frac{\Psi_0^2\hat{\Psi}'^\dagger}{2}\right]. \quad (3)$$

In Eqs. (2) and (3), due to the condition $n|a|^3\ll1$ (n is the gas density), we omitted the terms containing anomalous averages $\langle\hat{\Psi}'\hat{\Psi}'\rangle$.

Frequency variations change the time and distance scales in Eqs. (2) and (3). Let us formally introduce new operators taking this into account [$\boldsymbol{\rho}=\mathbf{r}/b(t)$]:

$$\Psi_0(\mathbf{r},t)=[b(t)]^{-d/2}\chi_0(\boldsymbol{\rho},\tau(t))\exp[i\Phi(\mathbf{r},t)],$$
$$\hat{\Psi}'(\mathbf{r},t)=[b(t)]^{-d/2}\hat{\chi}'(\boldsymbol{\rho},\tau(t))\exp[i\Phi(\mathbf{r},t)], \quad (4)$$

where d is the dimension of the system and the phase

$$\Phi(\mathbf{r},t)=(mr^2/2\hbar)\{\dot{b}(t)/b(t)\}. \quad (5)$$

The equations of motion for the operators χ_0 and $\hat{\chi}'$ in new coordinate (ρ) and time (τ) variables take the form

$$i\hbar \frac{\partial \chi_0}{\partial \tau}\left(\frac{d\tau}{dt}b^2(t)\right) = -\frac{\hbar^2}{2m}\Delta_\rho \chi_0 + \frac{m}{2}\left[\omega^2(t) + \frac{\ddot{b}(t)}{b(t)}\right]\rho^2 b^4(t)\chi_0 + \frac{\widetilde{U}}{b^{d-2}}[|\chi_0|^2 + 2\widetilde{n}']\chi_0, \tag{6}$$

$$i\hbar \frac{\partial \hat{\chi}'}{\partial \tau}\left(\frac{d\tau}{dt}b^2(t)\right) = -\frac{\hbar^2}{2m}\Delta_\rho \hat{\chi}' + \frac{m}{2}\left[\omega^2(t) + \frac{\ddot{b}(t)}{b(t)}\right]\rho^2 b^4(t)\hat{\chi}' + \frac{2\widetilde{U}}{b^{d-2}}[|\chi_0|^2 + \widetilde{n}']\hat{\chi}' + \frac{\widetilde{U}}{b^{d-2}}\chi_0^2\hat{\chi}'^{\dagger}, \tag{7}$$

where $\widetilde{n}'(\boldsymbol{\rho},\tau) = \langle \hat{\chi}'^{\dagger}(\boldsymbol{\rho},\tau)\hat{\chi}'(\boldsymbol{\rho},\tau)\rangle$. Note that the choice of phase in the form (5) leads to the cancellation of terms proportional to $\nabla_\rho\hat{\chi}'$ and $\nabla_\rho\chi_0$. It is important that the phase be the same for both Ψ_0 and $\hat{\Psi}'$.

Let us first consider $d=2$. We choose $\tau(t)$ and $b(t)$ such that they are governed by the equations

$$\tau(t) = \int^t dt'/b^2(t'), \tag{8}$$

$$\ddot{b}(t) + \omega^2(t)b(t) = \omega_0^2/b^3(t), \tag{9}$$

where $\omega_0 = \omega(-\infty)$ is the initial frequency. Then Eqs. (6) and (7) are reduced to a universal form

$$i\hbar \frac{\partial \chi_0}{\partial \tau} = -\frac{\hbar^2}{2m}\Delta_\rho \chi_0 + \frac{m\omega_0^2\rho^2}{2}\chi_0 + \widetilde{U}[|\chi_0|^2 + 2\widetilde{n}']\chi_0, \tag{10}$$

$$i\hbar \frac{\partial \hat{\chi}'}{\partial \tau} = -\frac{\hbar^2}{2m}\Delta_\rho \hat{\chi}' + \frac{m\omega_0^2\rho^2}{2}\hat{\chi}' + 2\widetilde{U}[|\chi_0|^2 + \widetilde{n}']\hat{\chi}' + \widetilde{U}\chi_0^2\hat{\chi}'^{\dagger}. \tag{11}$$

Equations (10) and (11) are universal in the sense that in the variables ρ, τ the problem is reduced to an interacting Bose gas in a harmonic well with constant frequency. Once we find the solution in the initial potential well, we know the answer at any t. One should only solve the simple equation (9) with initial conditions $b(-\infty)=1$, $\dot{b}(-\infty)=0$. For example, the expression for $\Psi_0(\mathbf{r},t)$ reads

$$\Psi_0(\mathbf{r},t) = \frac{1}{b(t)}\overline{\Psi}_0\left(\frac{\mathbf{r}}{b(t)}\right)\exp\left(i\frac{mr^2}{2\hbar}\frac{\dot{b}(t)}{b(t)} - i\mu\tau(t)\right), \tag{12}$$

where μ is the initial chemical potential and $\overline{\Psi}_0(\mathbf{r})$ is the stationary solution for the condensate wave function at $t\to-\infty$. Using the Bogolyubov transformation, generalized for an inhomogeneous case (see, e.g., [9]), on the basis of Eqs. (10) and (11) we can describe the evolution of the spectrum and wave functions of elementary excitations.

Neglecting the excitations, Eq. (12) describes the 2D (radial) evolution of Ψ_0 in long samples (axial frequency is much smaller than the radial one) under variations of the radial frequency; as to a first approximation one may omit the dependence of Ψ_0 on the axial coordinate.

If the nonlinear interaction terms in Eqs. (6) and (7) can be omitted the above universal scaling takes place for any d. This requires at least a small ratio of interparticle interaction to the level spacing in the initial potential, $\eta = n_0\widetilde{U}/\hbar\omega_0 \ll 1$, where $n_0 = |\Psi_0(0,-\infty)|^2$ (see below).

In the 3D case for arbitrary η there is no universality. On the other hand, in the limit $\eta \gg 1$, neglecting the excitations, Eq. (6) can be again reduced to a universal form. In this case the kinetic energy term in Eq. (6) is comparatively small and can be omitted (see below). Then, introducing, instead of (8) and (9), the equations

$$\tau(t) = \int^t dt'/b^3(t'), \tag{13}$$

$$\ddot{b}(t) + \omega^2(t)b(t) = \omega_0^2/b^4(t), \tag{14}$$

Eq. (6) is transformed to

$$i\hbar \frac{\partial \chi_0}{\partial \tau} = \frac{m\omega_0^2\rho^2}{2}\chi_0 + \widetilde{U}|\chi_0|^2\chi_0. \tag{15}$$

For $t\to-\infty$ Eq. (15) has a solution $\chi_0(t) = \overline{\chi}_0(r)\exp(-i\mu t)$, where $\overline{\chi}_0$ is given by the well known expression [10,11]

$$\overline{\chi}_0(\mathbf{r}) = \frac{1}{\widetilde{U}^{1/2}}\left(\mu - \frac{m\omega_0^2}{2}r^2\right)^{1/2}, \quad r \leq \left(\frac{2\mu}{m\omega_0^2}\right)^{1/2} \tag{16}$$

and zero otherwise. As follows from Eq. (15), for any t the condensate wave function has the form

$$\Psi_0(\mathbf{r},t) = \frac{1}{b^{3/2}(t)}\overline{\chi}_0\left(\frac{\mathbf{r}}{b(t)}\right)\exp\left(i\frac{mr^2}{2\hbar}\frac{\dot{b}(t)}{b(t)} - i\mu\tau(t)\right), \tag{17}$$

where $b(t)$ is governed by Eq. (14) with $b(-\infty)=1$, $\dot{b}(-\infty)=0$.

Equations (12) and (17) conserve the norm $\int|\Psi_0(\mathbf{r},t)|^2 d^d r = N_0$, where N_0 is the initial number of particles in the condensate. Universal solutions of Eqs. (10) and (11) conserve the norm $\int[|\Psi_0(\mathbf{r},t)|^2 + n'(\mathbf{r},t)]d^d r = N$.

We should emphasize that Eqs. (12) and (17) describe a coherent evolution of Ψ_0. Generally speaking, it is very different from the condensate wave function corresponding to the ground state of the system in the potential well with an instantaneous value of ω or the final value ω_1, even if $\omega(t)$ returns to the initial frequency ω_0.

The time dependence of Ψ_0 and normal excitations for $d=2$ (or at any d for $\eta \ll 1$, neglecting the nonlinear interaction terms) is determined by the solution of Eq. (9). The latter can be found in a general case (see, e.g., [7]). Note that the classical equation for a harmonic oscillator with time-dependent frequency

$$\ddot{\xi}(t) + \omega^2(t)\xi(t) = 0, \tag{18}$$

leads to Eq. (9), if one sets $\xi(t) \propto b(t) \exp[\pm i\omega_0 \tau(t)]$. With $\xi(t) = \exp(i\omega_0 t)$ for $t \to -\infty$ and t interpreted as a "spatial coordinate," Eq. (18) is equivalent to the one-dimensional Schrödinger equation for the reflection of a particle with "energy" ω_0^2 from the "potential" $\omega_0^2 - \omega^2(t)$. At times where ω is already constant (ω_1) we have

$$b^2(t) \equiv |\xi(t)|^2 = \frac{\omega_0}{\omega_1} \left[\frac{1+R}{1-R} - \frac{2R^{1/2}}{1-R} \cos(2\omega_1 t + \delta) \right],$$
(19)

$$\tau(t) = \frac{1}{\omega_0} \arctan\left[\frac{1+\sqrt{R}}{1-\sqrt{R}} \tan(\omega_1 t + \delta/2) \right],$$
(20)

where R is the reflection coefficient and δ the phase.

For slowly changing frequency (on a time scale $\tau_0 \gg \omega_{0,1}^{-1}$) the coefficient R is exponentially small, and $b = \sqrt{\omega_0/\omega_1}$. In this case the initial condensate is adiabatically transformed to the ground state of the system in the final trapping field, without oscillations. If the condition $\omega_{0,1}\tau_0 \gg 1$ is not valid, for at least one of the frequencies, the scaling parameter $b(t)$ will oscillate with a constant amplitude given by Eq. (19). There is no damping of the oscillations of the condensate density, unless relaxation is included (see below).

The instantaneous size of the evolving condensate is related to the initial size by $r_0(t) = r_0 b(t)$ [$r_0 \equiv r_0(-\infty)$]. In the case of abrupt change of the frequency we have

$$R = (\omega_0 - \omega_1)^2/(\omega_0 + \omega_1)^2, \quad \delta = 0,$$
(21)

and the function $b(t)$ oscillates from 1 to ω_0/ω_1. For $\omega_0 \gg \omega_1$ there is a large expansion of the condensate and then compression to the initial shape. At times $t \ll \omega_1^{-1}$ the expansion is practically free, and Eqs. (19) and (20) yield

$$b(t) = (1 + \omega_0^2 t^2)^{1/2}, \quad \tau(t) = (1/\omega_0) \arctan(\omega_0 t). \quad (22)$$

If $\omega_1 = 0$ the compression does not occur. We have an expanding condensate described by Eq. (22) at any t.

At times $t \gg \omega_0^{-1}$ the characteristic velocity of free expansion $v_0 = \dot{r}_0(t) = \omega_0 r_0$. As follows from Eq. (16), for $\eta \gg 1$ the initial size of the condensate $r_0 = (2\mu/m\omega_0^2)^{1/2}$, and we have $v_0 = \sqrt{2\mu/m}$. Since in this case $\mu = n_0 \widetilde{U}$, the velocity is determined by the interaction between particles. For $\eta \ll 1$ the initial size $r_0 \approx l_0$, where $l_0 = (\hbar/m\omega_0)^{1/2}$ is the amplitude of zero-point oscillations in the initial potential, and $v_0 \approx \sqrt{\hbar\omega_0/m}$ is much smaller than for $\eta \gg 1$.

The same picture holds for the 3D evolution of a condensate in the opposite limit, where initially $\eta \gg 1$, although we should use the scaling transformation following from Eqs. (13) and (14). The latter has no analog in the quantum theory of scattering and should be solved directly. We again obtain a periodic function $b(t)$, and oscillations of the condensate density will be determined by Eq. (17) which was derived assuming a large ratio of interparticle interaction to the kinetic-energy term (IK ratio) in Eq. (6). For $\eta \gg 1$ the IK ratio varies as $\eta^2/b(t)$ and definitely remains large if the frequency increases [$b(t) \lesssim 1$]. As follows from Eq. (14), fast decrease of the frequency to $\omega_1 \ll \omega_0$ leads to large os-

cillations with $b_{\max} \approx \sqrt{2/3}\,\omega_0/\omega_1$. The solution (17) will be valid if $\omega_1 \gtrsim \omega_0/\eta^2$. At times $t \ll \omega_1^{-1}$, where the expansion is free, Eq. (14) yields $b(t) \approx \sqrt{2/3}\,\omega_0 t$ ($t \gg \omega_0^{-1}$), and the velocity of expansion $v_0 \approx \sqrt{4\mu/3m}$. For $\omega_1 \ll \omega_0/\eta^2$ the IK ratio can become small and the scaling (9), instead of (14), should be used. Then the profile of the condensate density will change, but the parameter $b(t)$ determining the characteristic size of the condensate will be very close to that following from Eq. (14).

Let us now describe the evolution of a 3D Bose-condensed gas with negative scattering length. The initial condensate will be stabilized by the trapping field if $\eta \ll 1$, i.e., the IK ratio is small [5,6]. The prime stabilization factors are the presence of the gap $\sim \hbar\omega_0$ for one-particle excitations and the existence of a large energy barrier for quantum fluctuations leading to collapse [6]. Both are related to small values of η. In the case of radial evolution of long samples the IK ratio in Eq. (10) remains constant. Hence, the evolving condensate with $a < 0$ and initially small η will be equally stable with respect to collapse as the initial condensate. For the 3D evolution with $\eta \ll 1$ the IK ratio in Eq. (6) varies as $\eta/b(t)$ and decrease of the frequency (expansion) makes the evolving condensate even more stable. On the other hand, with large and fast increase of the frequency (compression) to $\omega_1 \gtrsim \omega_0/\eta$, or adiabatic increase to $\omega_1 \gtrsim \omega_0/\eta^2$, the parameter $\eta(t)/b(t)$ strongly increases and becomes of order unity. This can lead to instability of the condensate with respect to collapse. The principal difference of the uniform 3D compression from the radial compression of long samples is attractive for comparative experiments.

The evolving coherent state described by the wave function $\Psi_0(\mathbf{r},t)$ is an ES state. For sufficiently large and fast change of the frequency, the admixture of the final ground state in Ψ_0 is very small, which raises two questions: What happens with correlations characteristic of the static condensate in the absence of irreversible processes, and how fast is the depletion of the evolving coherent state? Analyzing the first question we consider correlations responsible for the reduction of the probability of inelastic processes due to the presence of the condensate. The event rate of an m-body inelastic process in a homogeneous gas $\nu_m = \alpha_m Z_m \Omega$, where α_m is the rate constant, Ω the system volume, and $Z_m = \langle [\hat{\Psi}^\dagger(0,0)]^m [\hat{\Psi}(0,0)]^m \rangle$ the local density correlator [4]. For three-body recombination we have $m = 3$, and for spin-dipole relaxation $m = 2$. In the absence of condensate $Z_m = m!(\bar{n})^m$, where $\bar{n}$ is the average particle density. At $T \to 0$ in a stationary condensate the density fluctuations are suppressed and $Z_m = (\bar{n})^m$. Hence, ν_m in the condensate decreases by a factor $m!$ [4].

In the spatially inhomogeneous evolving Bose-condensed gas, generalizing the above expression for the event rate we have $\nu_m(t) = \alpha_m \int d^3\mathbf{r}\, Z_m(\mathbf{r},t)$. The structure of the field operators in the correlator $Z_m(\mathbf{r},t)$ is determined by the scaling transformation (4). If almost all atoms are in the coherent state $\Psi_0(\mathbf{r},t)$, then both Eq. (12) and Eq. (17) lead to $\nu_m(t) = \nu_m(-\infty)[\Omega(-\infty)/\Omega(t)]^{m-1}$, where $\nu(-\infty)$ is the inelastic rate in the initial static condensate and the appearance of the quantity $[(\Omega(-\infty)/\Omega(t)]^{m-1}$ is a trivial consequence of changing the system volume: $\Omega(t) = \Omega(-\infty) b^d(t)$. This result shows that coherent evolution

retains the effect of partial suppression of inelastic processes, characteristic of the static condensate. Since any loss of coherence will lead to an increasing inelastic rate, there is an interesting possibility to study the depletion of the evolving condensate through the measurement of the rates of intrinsic or light-induced inelastic collisional processes.

In an isolated system the relaxation of the evolving coherent state will be accompanied by the appearance of an effective temperature. Assuming zero initial temperature and $\omega_1 \ll \omega_0$, a complete depletion of the condensate would lead to the effective temperature $T_{\text{eff}} \sim \mu$ for $\eta \gg 1$ and $T_{\text{eff}} \sim \hbar \omega_0$ for $\eta \ll 1$. In the 3D case the BEC transition temperature in the final potential $T_c \approx \hbar \omega_1 N_0^{1/3}$ [12]. For $\eta \ll 1$ the condition of complete depletion of the condensate, $T_c / T_{\text{eff}} \sim (\omega_1 / \omega_0) N_0^{1/3} \lesssim 1$, is rather strong although there is an upper bound for the number of particles, $N_0 \ll l_0 / a$. For $\eta \gg 1$, with $\mu = \eta \hbar \omega_0$ and $\eta \sim (N_0 a / l_0)^{2/5}$ [6], we obtain a weaker condition which can easily be fulfilled:

$$T_c / T_{\text{eff}} \sim (\omega_1 / \eta \omega_0) N_0^{1/3}$$
$$\approx (\omega_1 / \omega_0)(l_0 / a)^{2/5} N_0^{-1/15} \lesssim 1.$$

The question of relaxation and the loss of coherence in a quantum system with a large number of particles has several nontrivial aspects, especially with regard to dynamic evolution of a completely isolated system. In the latter case we can discuss the imitation of stochastization and the relaxation picture, although there are a number of reasons for the real relaxation. Thus far our analysis has assumed the mean-field approach. The relaxation can only appear beyond this approach: One should at least consider the exact Hamiltonian and include in the analysis the interaction terms proportional to $\widetilde{U}^2$.

Fast decrease of the frequency to $\omega_1 \ll \omega_0$ brings the system to a high ES state of the final potential. In the case of a large interaction ($\eta \gg 1$) the characteristic energy of expansion is $\sim \mu$. For maximum expansion ($b \approx \omega_0 / \omega_1$) the evolution is almost quasistationary and the size $r_0(t)$ is close to the size of stationary states with energy $\sim \mu$ in the final potential. The corresponding one-particle density of states $g(\mu) \approx \mu^2 / 2(\hbar \omega_1)^3$ ($d = 3$) will be very large, ensuring that the spacing between adjacent levels $\delta \varepsilon \sim \mu (\hbar \omega_1 / \mu)^3$ is much smaller than the interparticle interaction at maximum expansion $n_{0\min} \widetilde{U} \sim \mu (\omega_1 / \omega_0)^3$. The many-particle density of states grows exponentially, with the exponent depending on the number of particles N. Under these conditions even a small external influence can lead to ''mixing'' of states and insert irreversibility.

The characteristic relaxation time in this case will be determined by the collisional time $\tau_c \sim [n \sigma v(\mu)]^{-1}$, where $\sigma = 8 \pi a^2$ is the elastic cross section, $v(\mu)$ the particle velocity at energy μ, and $n \sim n_{0\min}$. Strictly speaking, τ_c represents the minimum relaxation time and it would be interesting to find possibilities to observe a larger time of relaxation. One may assume that even in the absence of any external influence τ_c will be responsible for the formation of the state which imitates relaxation. The imitation can be promoted by the deviation of the external field from harmonicity or spherical symmetry.

The characteristic time of dynamic evolution is determined by ω_1^{-1}, and we obtain a dimensionless parameter characterizing the relaxation:

$$(\tau_c \omega_1) \sim (l_0 / a)(l_0 / N_0 a)^{3/5} (\omega_0 / \omega_1)^2. \tag{23}$$

This estimate shows the possibility of both fast and slow relaxation. For $\tau_c \omega_1 \gg 1$ one can expect to observe oscillations of the condensate.

For a small interaction ($\eta \approx N_0 a / l_0 \ll 1$) the characteristic energy of expansion $\sim \hbar \omega_0$, and we have $(\tau_c \omega_1) \sim (l_0 / a) \times (l_0 / N_0 a)(\omega_0 / \omega_1)^2 \gg 1$. In this limit the relaxation, and hence the loss of coherence, is always slower.

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