Vibrational amplification by stimulated emission of radiation

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(Received 19 July 1995)

We propose the use of a single trapped and laser-cooled ion for vibrational amplification assisted by stimulated emission of radiation. This quantum-mechanical counterpart of the micromaser allows one to modify the fundamental vibronic coupling externally. In particular, a vibrational number state can be prepared in a coherent interaction regime, whereas incoherent interaction may yield a sub-Poissonian, binomial number distribution. [S1050-2947(96)06407-4]

PACS number(s): 42.50.Dv, 32.80.Pj, 03.65.−w

Single trapped ions exhibit many interesting features. One of them is the almost undamped mechanical motion in the trap potential. When being prepared in a quantum state of vibrational motion, the ion remains stable on a long time scale [1]. Moreover, laser cooling is a powerful method of preparing the vacuum state of the vibrational motion of the ion in the trap potential [2,3]. This state may serve as a well-defined start for further preparations of nonclassical states of the motion. These prospects are encouraging for performing fundamental experiments with quantum-mechanical states of trapped ions.

Consider a weak electronic transition of the ion (e.g., an electric quadrupole line), whose vibrational sidebands are well resolved. Laser-controlled vibronic coupling is obtained when resonantly irradiating the ion by laser light on the first vibrational sideband. For the ion being localized very well within half a wavelength, in the so-called Lamb-Dicke regime, the well known Jaynes-Cummings interaction is found [4]. More generally, beyond the Lamb-Dicke regime even a strongly nonlinear dynamics of Jaynes-Cummings type can be realized [5].

The Jaynes-Cummings interaction has been demonstrated in micromaser experiments [6,7], where a low-density atomic beam is used for the active medium. The relevant electronic transition is inverted by prepumping the atoms before they enter the interaction region. When one injects the atoms at a low rate, the dominant maser dynamics is due to the interaction of a single atom with a single cavity mode. Thus, the evolution of the system is expected to be strongly nonclassical. Micromaser theory even predicts how to prepare photon-number states or trapping states of the cavity field [8]. Experimentally sub-Poissonian statistics of the field has been demonstrated by detecting the statistics of the atoms after having passed through the cavity [9]. The production of trapping states in the micromaser, however, is rather difficult in view of disturbances, such as thermal effects, cavity damping, velocity dispersion of the atoms, and nonvanishing probability for simultaneous interaction of more than one atom with the field.

Here we propose the realization of a quantum-mechanical counterpart to the maser or laser based on a single trapped ion, where the vibrational motion of the ion in the trapping potential replaces the cavity field. The basic mechanism is vibrational amplification assisted by stimulated emission of radiation, which we call VASER. As for the demonstration of trapping states, the scheme allows us to eliminate the above-mentioned perturbations with the use of presently available experimental techniques. Moreover, the laser control of the vibronic coupling allows one to pass from coherent to incoherent interaction by varying the noise properties of the laser or the interaction time in order to allow for phase relaxation. Even in the limit of completely incoherent coupling we predict nonclassical vibrational statistics. The realization of such a regime in a micromaser would require one to establish noisy dipole coupling.

The VASER operates on an electronic three-level system as shown in Fig. 1, with two strong (dipole) transitions $|1\rangle \leftrightarrow |3\rangle$ and $|3\rangle \leftrightarrow |2\rangle$, and a weak (e.g., quadrupole) transition $|1\rangle \leftrightarrow |2\rangle$. The ion is prepared in a vibrational state of low excitation [2,10]. A pump pulse of Rabi frequency

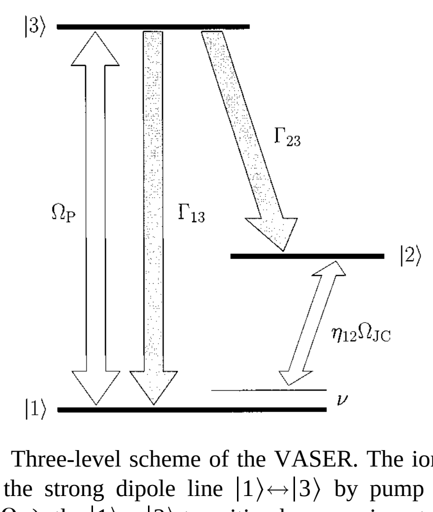

FIG. 1. Three-level scheme of the VASER. The ion being irradiated on the strong dipole line $|1\rangle \leftrightarrow |3\rangle$ by pump light (Rabi-frequency Ω_P), the $|1\rangle \leftrightarrow |2\rangle$ transition becomes inverted. The latter is a weak (quadrupole) transition with well-resolved vibrational sidebands (vibrational frequency ν). Appropriately driving this transition (effective Rabi frequency $\eta_{12}\Omega_{\rm JC}$) yields Jaynes-Cummings interaction.

$\Omega_P(t)$ is used to invert the weak transition. The pump pulse is followed by a pulse of Rabi-frequency $\Omega_{\rm JC}(t)$ tuned to the lower well-resolved vibrational sideband of the weak line. This second pulse switches on the Jaynes-Cummings interaction of the electronic transition and the quantized center-of-mass motion. Repeating the pumping and Jaynes-Cummings cycles, as considered in the context of stochastic cooling of a trapped ion [11], yields a dynamics similar to that of the micromaser. The system is described by the master equation

$$\dot{\hat{\rho}}=\frac{1}{i\hbar}[\hat{H}_0+\hat{H}_{\rm int}(t),\hat{\rho}]+\hat{\mathcal{L}}_{13}[\hat{\rho}]+\hat{\mathcal{L}}_{23}[\hat{\rho}], \qquad (1)$$

where $\hat{H}_0$ describes the free evolution of the electronic and mechanical subsystems, and $\hat{H}_{\rm int}(t)$ represents both the electronic preparation and the Jaynes-Cummings dynamics,

$$\hat{H}_{\rm int}(t)=\hat{H}_P(t)+\hat{H}_{\rm JC}(t). \qquad (2)$$

The Liouvillean parts $\hat{\mathcal{L}}_{13}$ and $\hat{\mathcal{L}}_{23}$ [12],

$$\hat{\mathcal{L}}_{ij}[\hat{\rho}]=\frac{\Gamma_{ij}}{2}\left(\int_{-1}^{1}ds\,\frac{3(1+s^2)}{4}e^{ik_{ij}s\hat{x}}\hat{A}_{ij}\hat{\rho}\hat{A}_{ji}e^{-ik_{ij}s\hat{x}}-\hat{A}_{jj}\hat{\rho}-\hat{\rho}\hat{A}_{jj}\right), \qquad (3)$$

describe the electronic relaxations from level $|3\rangle$ to levels $|1\rangle$ and $|2\rangle$, respectively, including all possible vibrational transitions. The probabilities for these transitions are determined by the corresponding Lamb-Dicke parameters η_{13} and η_{23}, where $k_{ij}\hat{x}=\eta_{ij}(\hat{a}+\hat{a}^\dagger)$, with $\hat{a}$ being the annihilation operator of a vibrational quantum of the center-of-mass motion; $\hat{A}_{ij}=|i\rangle\langle j|$ $(i,j=1,2,3)$.

The inversion is generated by resonantly irradiating the ion on the $|1\rangle\leftrightarrow|3\rangle$ line by a traveling-wave pulse, such as

$$\hat{H}_P(t)=\hbar\Omega_P(t)e^{ik_{13}\hat{x}}\hat{A}_{13}+\text{H.c.} \qquad (4)$$

In view of the large linewidth of this line both relaxation and pump-laser interaction could modify the vibrational statistics of the ion to some extent, since the laser simultaneously couples the vibrationless transition (carrier) and the vibrational sidebands. To suppress these perturbations, the pump light should be kept weak such that it is far from saturating the sidebands. In this case the perturbing effects due to relaxation can be made small to the order of magnitude η_{ij}^2, $i=1,2$ and $j=3$. As an alternative way of reducing unwanted vibrational coupling, the pumping may be achieved via electronic Raman-Stokes scattering, with the pump laser detuned from the $|1\rangle\leftrightarrow|3\rangle$ resonance.

The pump light having been switched off, the Jaynes-Cummings dynamics is immediately switched on by a laser pulse resonant with the first lower vibrational sideband of the weak $|1\rangle\leftrightarrow|2\rangle$ transition, as shown in Fig. 1. This dynamics is based on the Jaynes-Cummings interaction,

$$\hat{H}_{\rm JC}(t)=\hbar\,\eta_{12}\Omega_{\rm JC}(t)\hat{a}^\dagger\hat{A}_{12}+\text{H.c.} \qquad (5)$$

Note that we take into account the laser bandwidth $\Gamma_{\rm JC}$, which can be included in the explicit time dependence of the Rabi frequency $\Omega_{\rm JC}$ in terms of a stochastic process. The explicit time dependence of the Rabi frequency also includes the time T of the interaction of the light with the $|1\rangle\leftrightarrow|2\rangle$ transition. The Jaynes-Cummings dynamics might be driven also by two light fields whose beat frequency is in resonance with the lower sideband of transition $|1\rangle\leftrightarrow|2\rangle$, i.e., by stimulated electronic Raman-Stokes excitation of the vibration. Deexcitation by Raman processes has been observed before [10,13].

Consider first the coherent situation with a small laser bandwidth $\Gamma_{\rm JC}T\ll 1$. In this case the time of interaction can be set so as to fulfill the trapping-state condition [8], which for the number state $|n_{\rm tr}\rangle$ reads as

$$T=\frac{\pi}{\eta_{12}\Omega_{\rm JC}\sqrt{n_{\rm tr}+1}}. \qquad (6)$$

In Fig. 2 we give an example for the evolution of the vibrational statistics as a function of the number N of sequential pulses that have irradiated the ion on the sideband of the weak transition $|1\rangle\leftrightarrow|2\rangle$. The parameters used are realistic for an experiment on a Ba^+ ion, the trapping-state condition being chosen for $n_{\rm tr}=2$. After 15 interaction cycles the vibrational statistics peaks close to that of number state $|2\rangle$. It is important to note that the trapping state is not stationary [14]. As shown in Fig. 3, due to relaxational perturbations and laser phase diffusion the relative variance $\langle\Delta n^2\rangle/\langle n\rangle$ increases with the number of sequential pulses.

With a sufficiently large bandwidth $\Gamma_{\rm JC}$ of the light driving the weak transition, i.e., vibration frequency $\nu\gg\Gamma_{\rm JC}\gg\eta_{12}\Omega_{\rm JC}$, the vibronic coupling is incoherent. Now the trapping-state condition (6) does not hold. In the master equation (1) we may adiabatically eliminate the (electronic) off-diagonal density matrix elements of the weak transition.

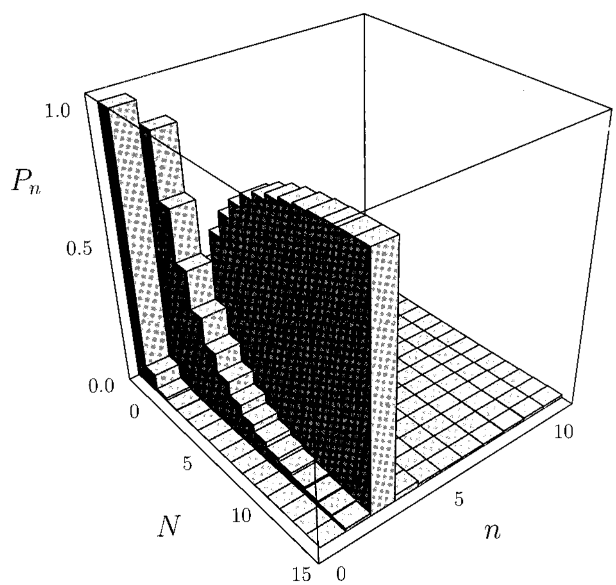

FIG. 2. Evolution of the vibrational number distribution of the VASER in the coherent regime for the trapping-state condition $n_{\rm tr}=2$, corresponding to $(\eta_{12}\Omega_{\rm JC})T=\pi/3^{1/2}$. Parameters for a typical experiment with a Ba^+ ion: $\Gamma_{\rm JC}/(\eta_{12}\Omega_{\rm JC})=0.01$, $\eta_{12}\Omega_{\rm JC}=10^4\ \text{s}^{-1}$, $\Gamma_{13}/\nu=9.5$, $\Gamma_{23}/\nu=3.3$, $\Omega_P/\nu=1.0$, $\nu=10^7\ \text{s}^{-1}$, $\eta_{13}=0.06$, and $\eta_{23}=0.046$.

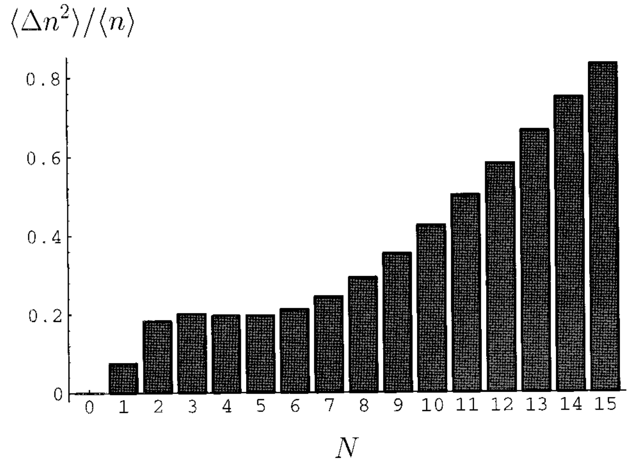

FIG. 3. Evolution of the relative variance $\langle \Delta n^2 \rangle / \langle n \rangle$ for the same conditions as in Fig. 2.

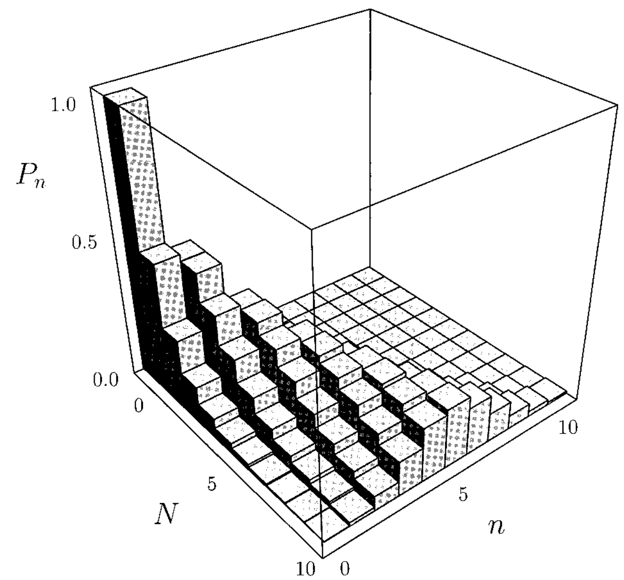

FIG. 4. Evolution of the vibrational number distribution of the VASER in the incoherent regime. Parameters for a typical experiment with a Ba^+ ion: $(\eta_{12}\Omega_{JC})T = 5\pi/3^{1/2}$, $\Gamma_{JC}/(\eta_{12}\Omega_{JC}) = 10$, $\eta_{12}\Omega_{JC} = 10^4$ s^{-1}, $\Gamma_{13}/\nu = 9.5$, $\Gamma_{23}/\nu = 3.3$, $\Omega_P/\nu = 1.0$, $\nu = 10^7$ s^{-1}, $\eta_{13} = 0.06$, and $\eta_{23} = 0.046$.

Thus the Jaynes-Cummings interaction is replaced by a simple recursion relation for the vibrational statistics,

$$P_n(t_{k+1}) = \frac{1}{2}\left[1 + \exp\left(-\frac{8\eta_{12}^2\Omega_{JC}^2 T}{\Gamma_{JC}}(n+1)\right)\right]P_n(t_k) + \frac{1}{2}\left[1 - \exp\left(-\frac{8\eta_{12}^2\Omega_{JC}^2 T}{\Gamma_{JC}}n\right)\right]P_{n-1}(t_k), \tag{7}$$

where t_k is the time after the kth cycle. For sufficiently large interaction times this recursion is further simplified:

$$P_n(t_{k+1}) = \tfrac{1}{2} P_n(t_k) + \tfrac{1}{2} P_{n-1}(t_k). \tag{8}$$

Disregarding the small influence of the pumping process on the vibrational statistics, we obtain the evolution of the statistics of the VASER in the incoherent regime simply by solving Eq. (8),

$$P_n(t_k) = \sum_{m=0}^{n} \binom{k}{n-m}\left(\frac{1}{2}\right)^k P_m(t_0). \tag{9}$$

This result shows that any initial number state $|m\rangle$ evolves into a binomial distribution with mean value $\langle n(t_k)\rangle = m + k/2$ and variance $\langle \Delta n(t_k)^2\rangle = k/4$. When the ion is cooled initially to the vacuum state of the mechanical motion, the statistics remains binomial for all times t_k,

$$P_n(t_k) = \binom{k}{n}\left(\frac{1}{2}\right)^k, \tag{10}$$

and the relative variance is given by $\langle \Delta n(t_k)^2\rangle / \langle n(t_k)\rangle = 1/2$. Thus, even in the incoherent regime the VASER can operate in a nonclassical manner at half the shot noise level. This result is easily understood. In the considered regime the weak electronic transition is saturated during each cycle. Accordingly, the probability for exciting a vibrational quantum is just one half per cycle. This intuitive picture for the emission of vibrational quanta is consistent with the generation of a number statistics of the binomial type.

A situation close to the solution (10) is shown in Fig. 4, which is obtained by numerically solving the full master equation under realistic conditions for an experiment on a Ba^+ ion. It is worth noting that in the VASER the Rabi-frequency Ω_{JC} corresponds to the dipole matrix element in the case of a maser or a laser. Thus, the incoherent, nonclassical VASER regime under consideration would correspond to a maser or laser with fluctuations in the dipole coupling. The VASER dynamics could be varied by externally modifying (via electro-optical modulation) the statistics of the laser light that drives the weak transition. This feature opens ways for externally controlling the quantum coupling, which are hardly achieved in masers or lasers.

In order to perform experiments of the type under consideration, a technique is needed that allows for the detection of the quantum state of the center-of-mass motion of the trapped ion. This problem has been solved recently for arbitrary quantum states [15]. However, the quantum states considered here are diagonal in the number representation and are simply reconstructed from the Jaynes-Cummings dynamics of the weak transition [16]. The latter is easily recorded via fluorescence detection according to the proposal in Ref. [13].

In summary we have proposed the realization of vibrational amplification assisted by stimulated emission of radiation with a single trapped ion irradiated by two sequences of laser pulses. The VASER, a quantum-*mechanical* counterpart of the micromaser or microlaser, affords approaches for fundamental experiments in single-particle quantum mechanics. When a well-stabilized laser is used for driving a narrow line of the ion, the ionic vibration may approach, after an appropriate pulse sequence, one of the trapping states known from micromaser theory. Moreover, the properties of the fundamental interaction in the VASER can be modified by externally controlling the laser noise. We have studied in some detail the situation for a large laser bandwidth, when we are allowed to describe the relevant vibronic coupling in the adiabatic approximation. In this case, the vibrational number statistics turns out binomial with its noise level close to half the shot-noise limit.

This work was supported by the Deutsche Forschungsgemeinschaft.

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