Amplification without population inversion in a V three-level system: A physical interpretation

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Amplification in a V system is studied both in the framework of the density matrix formalism and using the S matrix in the usual Hilbert space of the dressed-state vector. We show that it is possible to achieve amplification without inversion both in the bare- and in the dressed-state basis. In this case there is an asymmetry between amplification and absorption because of the occurrence of interferences between Feynman diagrams that can reduce the absorption. The comparison with the two-level atom shows a close analogy with the gain observed about the central resonance of the Mollow transmission spectrum. [S1050-2947(96)05606-5]

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I. INTRODUCTION

In recent years, quantum interference and coherence-induced effects have deeply modified the way we look at photon absorption and emission processes and at field propagation. In fact, such investigations were conducted by many groups working in different domains so that phenomena which are physically very similar are considered as basically different and, to our knowledge, very few attempts were made to clarify the relation between all these effects. This is the first purpose of this paper: to show for a particular example that the connection of ideas coming from these different domains can be used to get a better understanding of gain mechanisms in a three-level atom.

The dark resonance experiment of Alzetta *et al.* [1] and of Gray, Whitley, and Stroud [2] was one of the earliest examples of the fundamental importance of interference phenomena in atom-photon interactions. The seminal works of Kocharovskaya and Khanin [3], Harris [4], and Scully, Zhu, and Gavridiles [5] described quantum interference generated by classical fields. This was followed by a long series of papers which studied the properties of the simplest models. Reviews can be found in [6] and [7]. Among the models which have emerged, the most interesting seems to be the V-type *three-level* configuration in which one branch of the V is a strong pump field and the other branch is a weak probe field at a different (preferably higher) frequency. This model was referred to as the h scheme by Kocharovskaya and coworkers [8]. One can think of this scheme as arising from a two-level model where the upper level is split in two and each field connects one of the upper states to the lower state. The relevance of this model is that it was the three-level scheme in which *amplification without inversion* (AWI) was found with the absence of inversion both in the bare atomic state population and in the dressed atomic state population. In fact, no population inversion whatsoever was found to be necessary for AWI in suitable domains of parameters of the h scheme. Thus there does not seem to be *hidden inversion* as in the classical Λ configuration. This justified the continued interest in this particular scheme. The main purpose of this paper is to analyze and give a clear physical picture of the gain mechanisms which take place in the h scheme in AWI.

A distinction should be made between three broad categories of problems which all rely on the use of quantum interference to modify the response of the atom to a probe beam. The first is *electromagnetically induced transparency* (EIT), where the purpose is to reduce and eventually to cancel the absorption of a medium which is normally opaque at some frequency. Many experiments in EIT deal with pulse propagation and the latest experimental result showed an increase of the transmission from $\exp(-6000)$ to 90% in Pb vapor [9]. The second category is AWI, in which probe gain is achieved without inversion among the initial populations (i.e., before any coherent field is applied) *and* energy is extracted from the material medium. This probe gain vanishes when the upper lasing level population vanishes. The initial experiment was carried out in the pulsed and transient domain [10] but recent results indicate that AWI has been observed in steady state as well. Finally, *lasing without inversion* (LWI) is an extension of AWI when it takes place in a resonant cavity and when the gain overcomes the total losses [11].

The gain mechanisms we shall study in this paper are typical of three-level configurations. In *two-level atoms*, mechanisms leading to probe amplification have been successfully identified, beginning with the work of Rautian and Sobel'man [12], Marcuse [13], Holt [14], and Mollow [15]. These authors showed that when a two-level atom (resonant frequency ω_0) is driven by a strong nearly resonant field E of frequency ω, a probe field of frequency ω' can experience gain when $(\omega'-\omega)$ is close to the Rabi frequency. In the perturbative limit, the gain occurs for $\omega'=2\omega-\omega_0$ and can be explained as a hyper-Raman process with absorption of two photons of the driving field and stimulated emission of a photon in the probe field, the atom passing from the ground to the excited state. Such a gain process was indeed observed by Wu *et al.* [16] and a laser oscillation using this gain process was demonstrated by Lezama *et al.* [17]. However, when considered in the *dressed-atom basis*, this gain process is associated with a population inversion between the initial and final states of the transition [18]. This is thus an example of a *hidden inversion*. A more subtle gain process occurs for $\omega' \approx \omega \pm \Gamma$ (where Γ is the width of the upper state). In this case, the initial and final states of the process have the same population in the bare and in the dressed-state basis and the gain originates from an *asymmetry* between amplification and absorption, the absorption being *reduced* by an interference mechanism [19]. An oscillation using this gain mechanism was observed by Grandclément, Grynberg, and Pinard [20]. This stimulated Rayleigh process which starts and ends in the ground level does not require any external incoherent pumping of the excited level. This is the reason this process is often not considered as a LWI because no energy is extracted from the nonlinear medium. It should, however, be noticed that this Rayleigh gain process is strongly different from a *parametric* gain, also observed in atoms [21], because it involves a dissipative coupling, either through collisions or radiative relaxation [22], with the surrounding. In particular, contrary to the parametric oscillator, there is no relationship between the *phases* of the driving field and of the beam emitted by an oscillator driven by the stimulated Rayleigh gain mechanism.

Still another direction of research was initiated by the work of Bloembergen and co-workers on the pressure-induced extra resonances in four-wave mixing [23]. In one of their first papers [24], they noted that ''the conventional view is that no Raman-type resonances are observable between equally populated states because there is exact cancellation for processes starting from g and g', respectively. This is, however, no longer precisely true in the presence of collisions.'' The origin of the resonance occurring between equally populated states is the creation of an atomic coherence triggered by the relaxation mechanism (which can be different from collisions). Such a coherence is found in the usual density matrix formalism but its source is most easily understood using the dressed-atom picture [25]. The work of Bloembergen and co-workers was primarily associated with the four-wave mixing generation of a field at frequency $2\omega-\omega'$ for a medium interacting with two beams of frequencies ω and ω' but soon after it was realized that the same resonance could be observed on the transmission of each of the incident beams [26,27]. The resonance has a dispersive line shape and is centered around $\omega'-\omega=\omega_{gg'}$. Thus, on one side of the resonance, the presence of the field of frequency ω' leads to decrease of the absorption of the field of frequency ω. This resonance and the corresponding EIT were observed by Grynberg and co-workers [26,27]. It should be noticed that this effect is not necessarily related to the presence of a dark state because while the absorption of the field ω is reduced, there is a symmetric increase in the absorption of the field ω'. This kind of effect does not occur only for Raman resonances between ground-state sublevels. It is also found for the V-type system with excited states b and b'. Indeed, EIT due to such a two-beam coupling mechanism was predicted for $\omega-\omega'=\omega_{bb'}\pm\Gamma$ in a three-level system in the case of collisional damping and interpreted as resulting from a quantum interference [28,29].

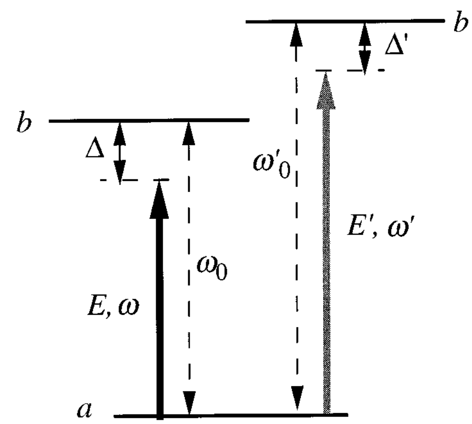

FIG. 1. Scheme of the energy levels.

It is the aim of this paper to show that AWI can occur in such a V (or h [8]) system and to give a physical interpretation of the amplification mechanism. The calculations are done in the *radiative limit* (i.e., in the absence of collisions). Using the standard density matrix formalism, we first show that amplification can be achieved when a small fraction of atoms are pumped in one of the upper levels. It is then shown that there is a range of parameters where this amplification occurs in a regime where there is *no population inversion*, either in the *bare*- or in the *dressed*-state basis. Finally, using an S-matrix approach similar to the one used in the case of a two-level atom [19], we show that the origin of AWI is the reduction of the absorption because of an *interference* mechanism that has no counterpart in the amplification. It must be noticed that the interference occurs between paths in the Hilbert space of *state vectors* and not between paths in the Liouville space of density matrix elements. Such an approach is obviously close to our understanding of quantum mechanics. As in the two-level atom case, the gain mechanism is thus associated with an asymmetry between absorption and amplification. A similar V-type system was recently studied by Wilson *et al.* [30] in a range of parameters such that the AWI condition coincides with inversion in the dressed-state basis. In this paper, we show that there exists another physically interesting range of parameters for the same system where the AWI condition is verified while there is no inversion in the dressed-state basis. In this case, gain is induced by quantum interferences due to multiple channels in the absorption processes, which reduces the overall absorption.

II. DENSITY MATRIX APPROACH

A. Probe transmission through a driven V system

We consider a set of three-level atoms. The ground state a is coupled to the excited states b and b' by electric dipole transitions (Fig. 1). For the sake of simplicity, we will assume in the following that a corresponds to a state having an angular momentum $J=0$, and that b and b' are states for which $J=1$, $m=0$. In this case, the coupling between a and b (or a and b') only occurs through z-polarized fields. The atom interacts with a strong driving field E of frequency ω

on the a-b transition and with a probe field E' of frequency ω' on the a-b' transition. These two fields are detuned from resonance by quantities $\Delta=\omega-\omega_0$ and $\Delta'=\omega'-\omega_0'$, respectively, which are assumed to be much larger than the natural widths of the excited levels ($|\Delta|, |\Delta'| \gg \Gamma, \Gamma'$). By contrast, the difference of detunings $\delta=\Delta-\Delta'$ can be small. Note that δ is the detuning for the Raman resonance between the two excited levels in the absence of the light shifts due to the driving field

$$\delta=\omega-\omega'-\frac{E_{bb'}}{\hbar}. \tag{1}$$

The resonance Rabi frequency for the a-b transition is $\Omega=dE/\hbar$ where d is the matrix element of the electric dipole moment D between the levels a and b.

The lifetime ($1/\Gamma'$) of level b' is assumed to be much longer than the lifetime ($1/\Gamma$) of the level b. We assume that there is an incoherent pumping of level b' from level a with a rate Λ'. To fix the notations, we present the master equation that is studied in the following:

$$\frac{d\rho}{dt}=\frac{1}{i\hbar}[H_0+H_l+H_l',\rho]+\left\{\frac{d\rho}{dt}\right\}, \tag{2a}$$

$$H_0=\hbar\omega_0|b\rangle\langle b|+\hbar\omega_0'|b'\rangle\langle b'|, \tag{2b}$$

$$H_I=-\left(\frac{dE}{2}\right)(e^{-i\omega t}|b\rangle\langle a|+e^{i\omega t}|a\rangle\langle b|), \tag{2c}$$

$$H_I'=-\left(\frac{d'E'}{2}\right)(e^{-i\omega' t}|b'\rangle\langle a|+e^{i\omega' t}|a\rangle\langle b'|). \tag{2d}$$

The term $\{d\rho/dt\}$ describes the relaxation and the pumping of the populations [Eq. (3)] and the coherences [Eq. (4)] of the density matrix ρ:

$$\left\{\frac{d}{dt}\rho_{bb}\right\}=-\Gamma\rho_{bb}, \quad \left\{\frac{d}{dt}\rho_{b'b'}\right\}=-\Gamma'\rho_{b'b'}+\Lambda'\rho_{aa},$$
$$\left\{\frac{d}{dt}\rho_{aa}\right\}=-\Lambda'\rho_{aa}+\Gamma\rho_{bb}+\Gamma'\rho_{b'b'}, \tag{3}$$

$$\left\{\frac{d}{dt}\rho_{ij}\right\}=-\Gamma_{ij}\rho_{ij}, \tag{4a}$$

with

$$\Gamma_{ba}=\frac{\Gamma+\Lambda'}{2}, \quad \Gamma_{b'a}=\frac{\Gamma'+\Lambda'}{2}, \quad \Gamma_{bb'}=\frac{\Gamma+\Gamma'}{2}. \tag{4b}$$

Equalities (4b) can be achieved in the radiative limit when there are no dephasing collisions [8]. In the absence of the probe field ($E'=0$), the solution $\rho^{(0)}$ of the master equation is

$$\rho_{ad}^{(0)}=\frac{A(1+x/2)}{1+(x/2)(1+A)}, \tag{5a}$$

$$\rho_{bb}^{(0)}=\frac{Ax/2}{1+(x/2)(1+A)}, \tag{5b}$$

$$\rho_{b'b'}^{(0)}=\frac{(1-A)(1+x/2)}{1+(x/2)(1+A)}, \tag{5c}$$

$$\rho_{ba}^{(0)}=\frac{i\Omega}{2(\Gamma_{ba}-i\Delta)}(\rho_{aa}^{(0)}-\rho_{bb}^{(0)})e^{-i\omega t}, \tag{5d}$$

$$\rho_{b'a}^{(0)}=0, \tag{5e}$$

with

$$A=\frac{1}{1+\Lambda'/\Gamma'}, \tag{6a}$$

$$x=\frac{\Gamma_{ba}}{\Gamma}\frac{\Omega^2}{\Delta^2+\Gamma_{ba}^2}. \tag{6b}$$

To first order in the probe field amplitude E', the density matrix can be written as $\rho^{(0)}+\rho^{(1)}$ where $\rho^{(1)}$ is linear in E'. The master equation (2) then become

$$\frac{d\rho^{(1)}}{dt}-\frac{1}{i\hbar}[H_0+H_I,\rho^{(1)}]-\left\{\frac{d\rho^{(1)}}{dt}\right\}=\frac{1}{i\hbar}[H_I',\rho^{(0)}]. \tag{7a}$$

In particular, we are interested in the coherence $\rho_{b'a}^{(1)}$ which gives the linear probe absorption. The solution of Eq. (7a) is

$$\rho_{b'a}^{(1)}=-\frac{\Omega'}{2\tilde{\Delta}'\tilde{\delta}_R}\left[(\rho_{aa}^{(0)}-\rho_{b'b'}^{(0)})\tilde{\delta}+(\rho_{aa}^{(0)}-\rho_{bb}^{(0)})\frac{\Omega^2}{4(\tilde{\Delta})^*}\right]\times e^{-i\omega' t}, \tag{7b}$$

with $\Omega'=d'E'/\hbar$ [d' is the matrix element of the electric dipole moment D between a and b' introduced in Eq. (2d)] and

$$\tilde{\Delta}=\Delta+i\Gamma_{ba}, \tag{8a}$$

$$\tilde{\Delta}'=\Delta'+i\Gamma_{b'a}, \tag{8b}$$

$$\tilde{\delta}=\delta-i\Gamma_{bb'}, \tag{8c}$$

$$\tilde{\delta}_R=\left(\delta+\frac{\Omega^2}{4\tilde{\Delta}'}\right)-i\Gamma_{bb'}. \tag{8d}$$

The quantity $\tilde{\delta}_R$ is the resonant denominator for the Raman process between levels b and b' which includes the light shift due to the driving field. In the limit considered in this paper ($|\Delta|\approx|\Delta'|\gg\Gamma_{ba}$, $\Gamma_{b'a}$, and $|\Omega/\Delta|^2\ll1$), we find $\tilde{\delta}_R \cong \delta'-i\Gamma_{bb'}$, with

$$\delta'=\delta+\frac{\Omega^2}{4\Delta}. \tag{9}$$

In Eq. (7b), it may be noticed that the effect of the driving field is obvious in the second term in square brackets but it is also present in the energy denominator $\tilde{\delta}_R$ and in the population difference $\rho_{aa}^{(0)}-\rho_{b'b'}^{(0)}$. In the following, we study the

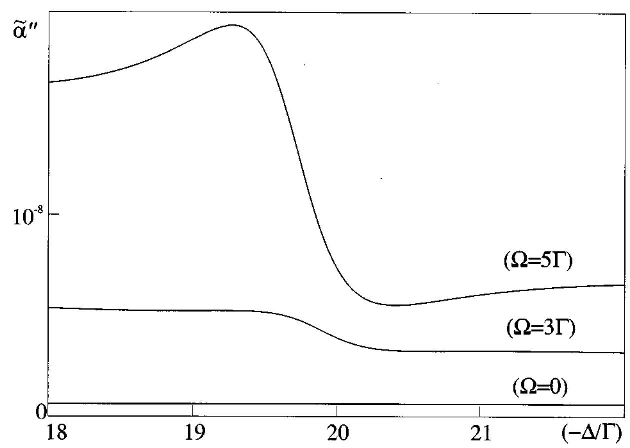

FIG. 2. Probe transmission versus $-\Delta/\Gamma$ for three values (0,3,5) of Ω/Γ. All the atoms are assumed to be initially in the ground state in the absence of any coherent field. These curves are obtained for $\Delta'=-20\Gamma$ and $\Gamma'=10^{-3}\Gamma$.

gain of the probe field per atom $\tilde{\alpha}''=\mathrm{Im}[\rho^{(1)}_{b'a}(\Gamma'/\Omega')e^{i\omega' t}]$ which is related to the absorption cross section by the relation

$$\sigma_{\mathrm{abs}}=\frac{3\lambda'^2}{2\pi}\,\tilde{\alpha}'', \tag{10}$$

where $\lambda'=2\pi c/\omega'$ is the wavelength of the probe beam. The coefficient $\tilde{\alpha}''$ is also proportional to the imaginary part of the atomic polarizability $\alpha=\langle D\rangle_{\omega'}/\varepsilon_0 E'$ (where $\langle D\rangle_{\omega'}$ is the component of the electric dipole moment oscillating at frequency ω'):

$$\tilde{\alpha}''=\frac{4}{3}\,\frac{\pi^2}{\lambda'^3}\,\mathrm{Im}(\alpha). \tag{11a}$$

For a medium of density $\mathcal{N}$ and length L the intensity gain G is equal to

$$G=\exp\left[\frac{-2\pi\mathcal{N}L\ \mathrm{Im}(\alpha)}{\lambda'}\right]=\exp(-\mathcal{N}\sigma_{\mathrm{abs}}L). \tag{11b}$$

B. Two-beam coupling

We first present in Fig. 2 the variation of $\tilde{\alpha}''$ versus $-\Delta/\Gamma$ for $\Lambda'=0$ (all the atoms are initially in the ground state in the absence of any coherent field) for three values (0, 3, and 5) of Ω/Γ. These curves are obtained for $\Delta'=-20\Gamma$ and $\Gamma'=10^{-3}\Gamma$. It can be noticed that the presence of the driving field leads to an *overall increase* of the probe field absorption. However, this increase is not uniform and a *dispersive* feature is observed around $\Delta'=\Delta$ (i.e., near the Raman resonance). Such a dispersive shape for the absorption is well known in two-beam coupling and its occurrence for a collisionally broadened V-type three-level atom was pointed out in a previous paper [29].

C. Amplification without inversion in the bare-state basis

We now study what occurs when there is some incoherent pumping of the level b'. Let B' be the population of the level b' in the absence of any coherent field. The probe beam frequency is kept fixed with $\Delta'=-20\Gamma$ and we plot $\tilde{\alpha}''$ versus $-\Delta/\Gamma$. We take $\Gamma'=10^{-3}\Gamma$ and the initial population $B'=1-A$ of the level b' is incremented from 6×10^{-4} to 7×10^{-4} with a step of 10^{-5} between two curves. Figures 3(a), 3(b), and 3(c) correspond to Ω/Γ=6, 8, and 10, respectively. Amplification of the probe beam occurs when $\tilde{\alpha}''$ is negative. This occurs for $B'\geqslant B'_{\mathrm{th}}$ with $B'_{\mathrm{th}}\approx(6.55\pm0.05)\times10^{-4}$ in the range of values for Ω/Γ considered in Fig. 3. The condition $B'_{\mathrm{th}}\ll A\approx 1$ indicates amplification without inversion in the *bare-atom* basis.

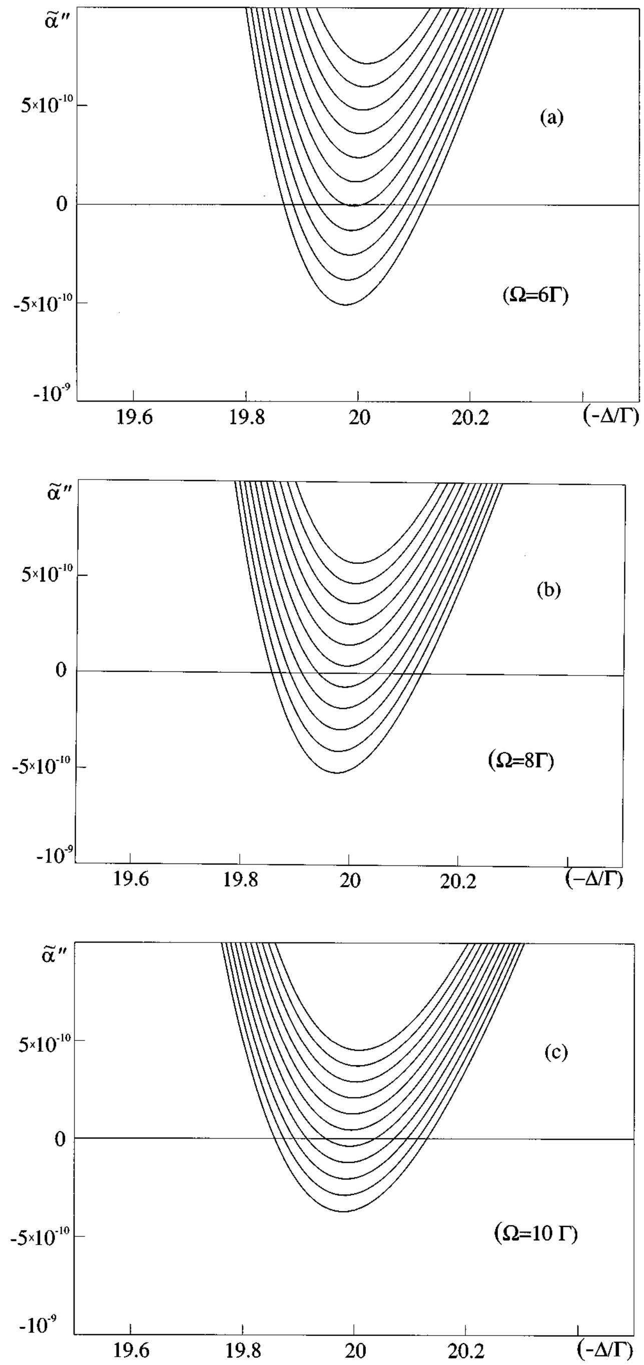

FIG. 3. Probe transmission versus $-\Delta/\Gamma$ for $\Gamma'=10^{-3}\Gamma$ and $\Delta'=-20\Gamma$. Ω/Γ is, respectively, equal to 6,8,10 [(a), (b), (c)]. For each curve, the population of level b' is incremented from 6×10^{-4} to 7×10^{-4} with a step of 10^{-5}.

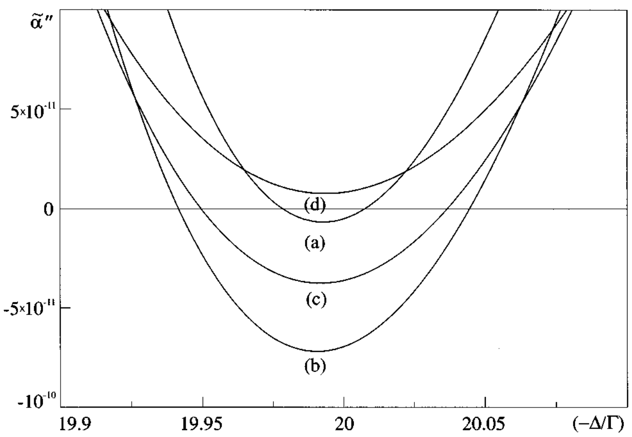

FIG. 4. Probe transmission versus $-\Delta/\Gamma$ for $\Gamma' = 10^{-3}\Gamma$, $\Delta' = -20\Gamma$, and $B' = 6.6\times10^{-4}$. The ratio Ω/Γ is, respectively, equal to 6, 8, 10, and 12 for the curves (a), (b), (c), and (d).

For a given value of B', there is also an optimum value for Ω/Γ. For example, we show in Fig. 4 the variation of the gain versus $-\Delta/\Gamma$ for $B' = 6.6\times10^{-4}$ and $\Delta' = -20\Gamma$ for increasing values of Ω/Γ. It can be seen that gain is expected for $5 \leqslant (\Omega/\Gamma) \leqslant 11$.

D. Amplification without inversion in the dressed-state basis

Following the idea of Wilson *et al.* [30], it is interesting to know whether there is population inversion in the dressed-atom basis. We consider the three-level atom dressed by the photons of the driving field. The dressed-atom Hamiltonian [18] is

$$H_0 = H_{at} + \hbar\omega\left(a^\dagger a + \frac{1}{2}\right) + \hbar\omega'\left(a'^\dagger a' + \frac{1}{2}\right) - d\left(\frac{\hbar\omega}{2\epsilon_0 V}\right)^{1/2}(S_+ a + S_- a^\dagger), \tag{12}$$

with $S_+ = |b\rangle\langle a|$ and $S_- = |a\rangle\langle b|$ and V the quantization volume. The eigenstates are $|1(N),N'\rangle$, $|2(N),N'\rangle$, and $|b',N,N'\rangle$ where N and N' are, respectively, the number of photons in the modes of the driving field and of the probe field. In the perturbative limit ($|\Omega/\Delta| \ll 1$) and for $\Delta < 0$, the expansions of the dressed eigenstates in terms of bare states are given through the relations

$$|1(N),N'\rangle = |b,N,N'\rangle - \frac{\Omega}{2\Delta}\,|a,N+1,N'\rangle, \tag{13a}$$

$$|2(N),N'\rangle = |a,N+1,N'\rangle + \frac{\Omega}{2\Delta}\,|b,N,N'\rangle. \tag{13b}$$

In the situation considered in this paper, the energies of the levels inside a multiplet are, respectively, equal to

$$E_{|1(N),N'\rangle} = N\hbar\omega + N'\hbar\omega' + \hbar\omega_0 - \frac{\Omega^2}{4\Delta}, \tag{14a}$$

$$E_{|2(N),N'\rangle} = (N+1)\hbar\omega + N'\hbar\omega' + \frac{\Omega^2}{4\Delta}, \tag{14b}$$

$$E_{|b',N+1,N'-1\rangle} = (N+1)\hbar\omega + (N'-1)\hbar\omega' + \hbar\omega_0'. \tag{14c}$$

In steady state, the ratio of the populations of the levels $|1(N),N'\rangle$ and $|2(N),N'\rangle$ is

$$\frac{\pi_1}{\pi_2} = \tan^4\theta, \tag{15}$$

with $\tan 2\theta = -\Omega/\Delta$.

Because $A \approx 1$ at the threshold of amplification in Fig. 3, the population of the levels $|1(N),N'\rangle$ is almost equal to $\tan^4\theta/(1+\tan^4\theta)$. Since the amplification of the probe beam can be attributed to a Raman process starting from b' and ending in b with stimulated emission of a photon ω' and absorption of a photon ω, *amplification without inversion occurs if*

$$B'_{\text{th}} \leqslant \frac{\tan^4\theta}{1+\tan^4\theta}. \tag{16}$$

The value of the right-hand side of Eq. (16) is equal to 4.6×10^{-4}, 13.7×10^{-4}, and 31×10^{-4} for Ω equal to 6Γ, 8Γ, and 10Γ, respectively (and $\Delta \approx -20\Gamma$). For this range of parameters we showed in Sec. II C that $B'_{\text{th}} \cong 6.5\times10^{-4}$. Whereas amplification with population inversion *in the dressed-state basis* occurs for $\Omega/\Gamma = 6$, *amplification without population inversion* is found for $\Omega/\Gamma = 8$ and 10. In these last cases, the gain cannot be explained as a standard Raman gain process due to population inversion in the dressed-state basis.

III. PHYSICAL INTERPRETATION USING INTERFERENCE BETWEEN FEYNMAN DIAGRAMS

We wish now to give a physical explanation of this gain without population inversion. For this purpose we follow closely the method introduced in the case of a two-level atom [19]. In particular, the assumptions for the perturbative expansion are the same: because the lifetime of b' is very long, it can be considered as an initial or a final state in a *scattering* process. We can thus calculate the matrix element of the T matrix between states such as $|b',N+1,N'-1\rangle$ and $|2(N),N'\rangle$ which is associated with the atomic ground state a in the situation considered here, i.e., in the perturbative limit ($\Omega \ll |\Delta|$) and $\Delta < 0$.

A. Amplification

We first consider the process in which the probe beam ω' is amplified. In the bare-state basis, this process is described by Fig. 5(a) and corresponds to a stimulated Raman transition from b' to b followed by the spontaneous emission of a photon ω_2, $\boldsymbol{\varepsilon}_2$ from b to a. In the dressed-state basis the same process corresponds to the spontaneous emission of a photon ω_2, $\boldsymbol{\varepsilon}_2$ from $|b',N+1,N'-1\rangle$ towards $|2(N-1),N'\rangle$ [Fig. 5(b)]. Because $|b',N+1,N'-1\rangle$ and $|2(N-1),N'\rangle$ are not directly coupled, such a process involves a virtual transition through a state $|i(p),p'\rangle$. Actually, the coupling of the atom with the probe field is described by a coupling Hamiltonian H_I' whose matrix elements are

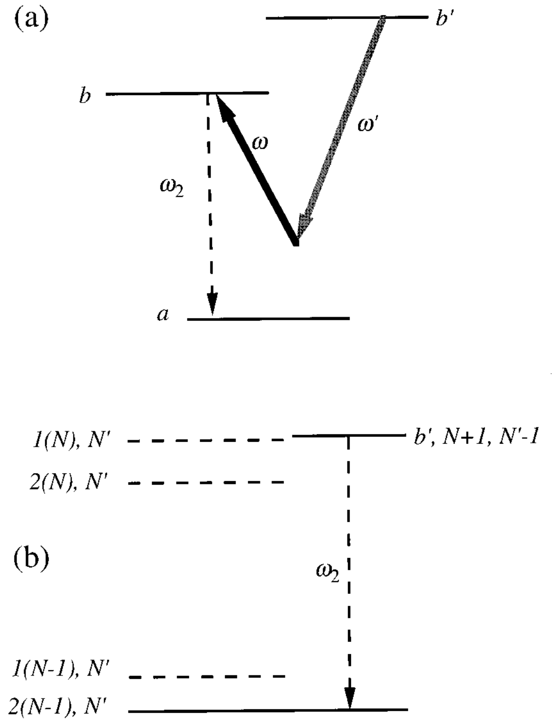

FIG. 5. Amplification process in the bare (a) and dressed-atom (b) pictures.

$$\langle 1(N),N'|H_I'|b',p,p'\rangle = d'\left(\frac{N'\omega'}{2\varepsilon_0 V}\right)^{1/2}\frac{\Omega}{2\Delta}\,\delta_{p,N+1}\delta_{p',N'-1}, \tag{17a}$$

$$\langle 2(N),N'|H_I'|b',p,p'\rangle = -d'\left(\frac{N'\omega'}{2\varepsilon_0 V}\right)^{1/2}\delta_{p,N+1}\delta_{p',N'-1}. \tag{17b}$$

The transition from $|i\rangle = |b',N+1,N'-1\rangle$ to $|f\rangle = |2(N-1),N',1\omega_2\rangle$ can follow two possible paths depending on the intermediate state, which can be either $|1(N),N'\rangle$ or $|2(N),N'\rangle$. However, the transition amplitude through $|1(N),N'\rangle$ is much larger because $|b',N'+1,N'-1\rangle$ and $|1(N),N'\rangle$ are nearly degenerate [and because the matrix element of $\mathbf{D}\cdot\boldsymbol{\varepsilon}_2$ between $|2(N),N'\rangle$ and $|2(N-1),N'\rangle$ is of a larger order in Ω/Δ than the matrix element of $\mathbf{D}\cdot\boldsymbol{\varepsilon}_2$ between $|1(N),N'\rangle$ and $|2(N-1),N'\rangle$ [18,19]]. More precisely, the transition amplitude through $|1(N),N'\rangle$ is equal to

$$T_{fi}^{(\mathrm{amp})} = \frac{\sqrt{N'\omega'\omega_2}}{2\varepsilon_0 V}\,\varepsilon_{2z}\,\frac{d'd}{\hbar(\delta'+i\Gamma/2)}\,\frac{\Omega}{2\Delta}, \tag{18}$$

where ε_{2z} is the component of the polarization of the emitted photon and δ' defined in Eq. (9) is the detuning from Raman resonance including the light shift. From the knowledge of the transition amplitude, one can calculate the transition rate towards the group of final states having the same energy as $|i\rangle$,

$$\sum_f w_{fi}^{\mathrm{amp}} = \frac{2\pi}{\hbar}\int d^3k_2\,\frac{V}{(2\pi)^3}\sum_{\varepsilon_2}|T_{fi}^{\mathrm{amp}}|^2\delta(E_{|b',N+1,N'-1\rangle} - E_{|2(N-1),N'\rangle} - \hbar\omega_2), \tag{19a}$$

and the amplification cross section

$$\sigma_{\mathrm{amp}} = \frac{V}{N'c}\sum_f w_{fi}^{\mathrm{amp}}. \tag{19b}$$

Using $d^2 = 3\pi\varepsilon_0\hbar\Gamma(c/\omega)^3$ and $d'^2 = 3\pi\varepsilon_0\hbar\Gamma'(c/\omega')^3$, we find

$$\sigma_{\mathrm{amp}} = \frac{3}{8\pi}\lambda'^2\frac{\Omega^2}{4\Delta^2}\frac{\Gamma\Gamma'}{\delta'^2+\Gamma^2/4}. \tag{20}$$

As expected, the Raman amplification is maximum when the condition for Raman resonance $\delta'=0$ is fulfilled.

B. Absorption

We now consider the absorption processes for the probe beam. The first process is the usual Rayleigh scattering whose cross section is [18]

$$\sigma_{\mathrm{abs}}^{(1)} = \frac{3}{8\pi}\lambda'^2\left(\frac{\Gamma'}{\Delta'}\right)^2. \tag{21}$$

This corresponds to the curve obtained for $\Omega=0$ in Fig. 2.

Let us now consider the nonlinear absorption processes that involve photons of the driving field. In the bare-atom basis there are two possible paths, shown in Figs. 6(a) and 6(b). These paths generalize to the case of radiative relaxation the processes considered earlier in the case of collisional relaxation [29]. It should be noticed that these two paths *interfere* because they correspond to the *same quantum numbers in the initial and final states*. In both cases, there is absorption of one photon of the probe field and of the driving field and spontaneous emission of one photon in the mode (ω_1,ε_1). In the dressed-atom picture, these two paths correspond to a transition from $|i'\rangle = |2(N),N'\rangle$ towards $|f'\rangle = |b',N,N'-1,1\omega_1\rangle$ [see Fig. 6(c)]. There are two possible intermediate states, $|2(N-1),N'\rangle$ and $|1(N-1),N'\rangle$. These two paths must be included in the calculation of the transition amplitude because the *nonresonant* path [through $|2(N-1),N'\rangle$] appears at a lower order of perturbation than the resonant path. We call $T_{f'i'}^{(a)}$ and $T_{f'i'}^{(b)}$ the transition amplitudes associated with the paths of Figs. 6(a) and 6(b),

$$T_{f'i'}^{(a)} = -\frac{\sqrt{N'\omega_1\omega'}}{2\varepsilon_0 V}\,\varepsilon_{1z}\,\frac{dd'}{\hbar\Delta}\,\frac{\Omega}{2\Delta}, \tag{22a}$$

$$T_{f'i'}^{(b)} = \frac{\sqrt{N'\omega_1\omega'}}{2\varepsilon_0 V}\,\varepsilon_{1z}\,\frac{dd'}{\hbar[\delta'+i\Gamma/2]}\left(\frac{\Omega}{2\Delta}\right)^3. \tag{22b}$$

In $T_{f'i'}^{(a)}$, the energy denominator for the virtual transition through $|2(N-1),N'\rangle$ is $-\hbar\Delta$ while $(\Omega/2\Delta)$ is associated with the matrix element of the dipole operator between $|2(N),N'\rangle$ and $|2(N-1),N'\rangle$. In $T_{f'i'}^{(b)}$, the energy denominator can be resonant and the factor $(\Omega/2\Delta)^3$ corresponds to the product of the matrix element of the dipole operator be-

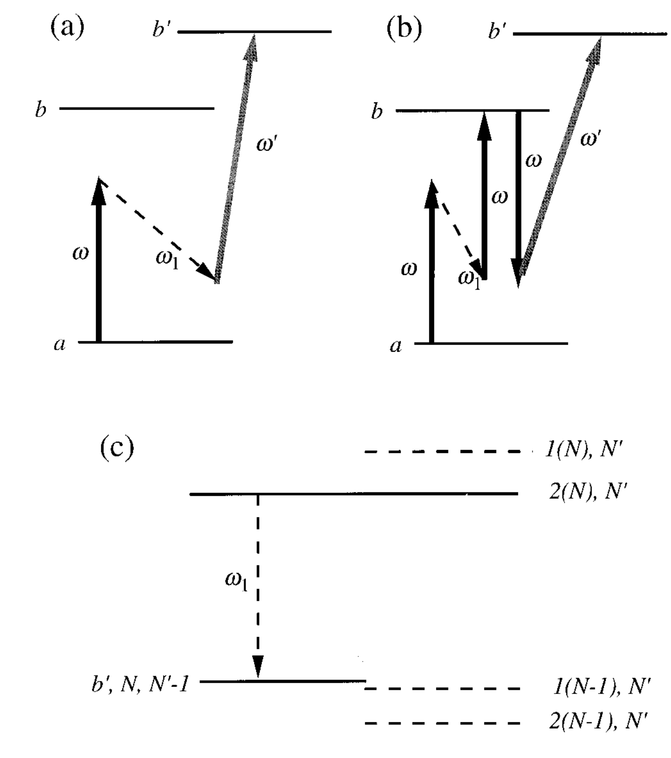

FIG. 6. Nonlinear absorption processes in the bare- [(a), (b)] and dressed-atom (c) pictures. The path that appears at lower order (a) is nonresonant while the higher-order term (b) is resonant. This is the reason these two terms should be included in the calculation of the transition amplitude. Note that these terms are described by just one diagram in the dressed-atom picture.

tween $|2(N),N'\rangle$ and $|1(N-1),N'\rangle$ which varies as $(\Omega/2\Delta)^2$ by the matrix element of H_I' between $|1(N-1),N'\rangle$ and $|b',N,N'-1\rangle$ which is proportional to $\Omega/2\Delta$ [18]. Finally, the transition amplitude $T^{(2)}_{f'i'}$ for these nonlinear absorption processes is

$$T^{(2)}_{f'i'}=T^{(a)}_{f'i'}+T^{(b)}_{f'i'}=\frac{\sqrt{N'\omega_1\omega'}}{2\varepsilon_0 V}\frac{dd'}{\hbar}\frac{\Omega}{2\Delta^2}\times\left[-1+\frac{\Omega^2}{4\Delta(\delta'+i\Gamma/2)}\right]. \tag{23}$$

As in the case of the two-level system [19], one finds the occurrence of two perturbation parameters: $|\Omega/\Delta|$ which is assumed to be small and $|\Omega^2/\Delta\Gamma|$ which is not necessarily small because one can simultaneously have $|\Omega/\Delta|\ll 1$ and $|\Omega/\Gamma|\gg 1$. Using the transition amplitude $T^{(2)}_{f'i'}$ and following the same steps that were used from Eq. (18) to Eq. (20), one finds the cross section $\sigma^{(2)}_{\rm abs}$ for nonlinear absorption of the probe beam:

$$\sigma^{(2)}_{\rm abs}=\frac{3}{8\pi}\lambda'^2\frac{\Omega^2}{4\Delta^2}\frac{\Gamma\Gamma'}{\Delta^2}\left|-1+\frac{\Omega^2}{4\Delta(\delta'+i\Gamma/2)}\right|^2$$
$$=\frac{3}{8\pi}\lambda'^2\frac{\Omega^2}{4\Delta^2}\frac{\Gamma\Gamma'}{\Delta^2}\left[1-\frac{\Omega^2\delta'}{2\Delta(\delta'^2+\Gamma^2/4)}+\frac{\Omega^4}{16\Delta^2(\delta'^2+\Gamma^2/4)}\right]. \tag{24}$$

The cross section $\sigma^{(2)}_{\rm abs}$ varies with the sign of δ'. This is a manifestation of the coherence in the dressed-state basis already mentioned by Luo and Xu [31]. This result, which is consistent with the curves of Fig. 2, corresponds to the interference between the paths of Figs. 6(a) and 6(b). As a result the total absorption exhibits a resonance around $\delta'=0$ (Raman resonance) with less absorption for $\Delta\delta'>0$ than for $\Delta\delta'<0$.

C. Outcome: Amplification without inversion

We now consider the case where a small fraction B' of the atoms ($B'\ll 1$) is pumped in the level b'. In that case the average absorption cross section is

$$\overline{\sigma_{\rm abs}}=\sigma^{(1)}_{\rm abs}+\sigma^{(2)}_{\rm abs}-B'\sigma_{\rm amp}, \tag{25a}$$

$$\overline{\sigma_{\rm abs}}=\frac{3}{8\pi}\lambda'^2\left\{\left(\frac{\Gamma'}{\Delta'}\right)^2+\Gamma\Gamma'\left(\frac{\Omega}{2\Delta}\right)^2\times\left[\frac{1}{\Delta^2}-\frac{\Omega^2\delta'}{2\Delta^3(\delta'^2+\Gamma^2/4)}+\frac{(\Omega/2\Delta)^4-B'}{\delta'^2+\Gamma^2/4}\right]\right\}. \tag{25b}$$

All the terms of Eq. (25b) have a simple physical interpretation. The first term corresponds to Rayleigh scattering. The first term inside the square brackets is associated with the nonlinear absorption of Fig. 6(a). The last term in the square brackets also has a particularly simple explanation. This term corresponds to the Raman resonance between the dressed states, the numerator being the difference of the population of the dressed level associated to b, which is equal to $(\Omega/2\Delta)^4$ in the perturbative limit ($\theta\ll 1$), and of the population of the level b' which is B'. This term corresponds to the naïve picture of absorption or amplification due to a Raman process. When $(\Omega/2\Delta)^4<B'$ it describes Raman amplification of the probe. However, this term does not describe everything around the Raman resonance because of the *interference term* (second term in the square brackets) which gives the dispersive line shape. The absorption being reduced for $\Delta\delta'>0$, amplification can occur even if there is no population inversion. Indeed, we have plotted in Fig. 7(a) the population $B'_{\rm in}$ of the level b' at the threshold of amplification ($\overline{\sigma_{\rm abs}}=0$) versus Ω/Γ for three values of δ' (and for $\Delta=15\Gamma$). The curves (1), (2), and (3), respectively, correspond to $\delta'=-\Gamma/2$, $\delta'=0$, and $\delta'=\Gamma/2$. The dashed curve corresponds to the population $(\Omega/2\Delta)^4$ in the dressed level corresponding to b. For large values of Ω/Γ, the dashed curve is above the curve (3). In that domain and for $\delta'=\Gamma/2$, amplification without population inversion in the dressed basis can be achieved. By contrast, the same crossing is not found with the curves (1) and (2): in these cases, amplification always occurs with population inversion in the dressed-state basis. Figure 7(a) also shows that amplification with population inversion may be easier to achieve than AWI. This is, for example, the case when the curve (2) is below the curve (3), i.e., for small values of Ω/Γ.

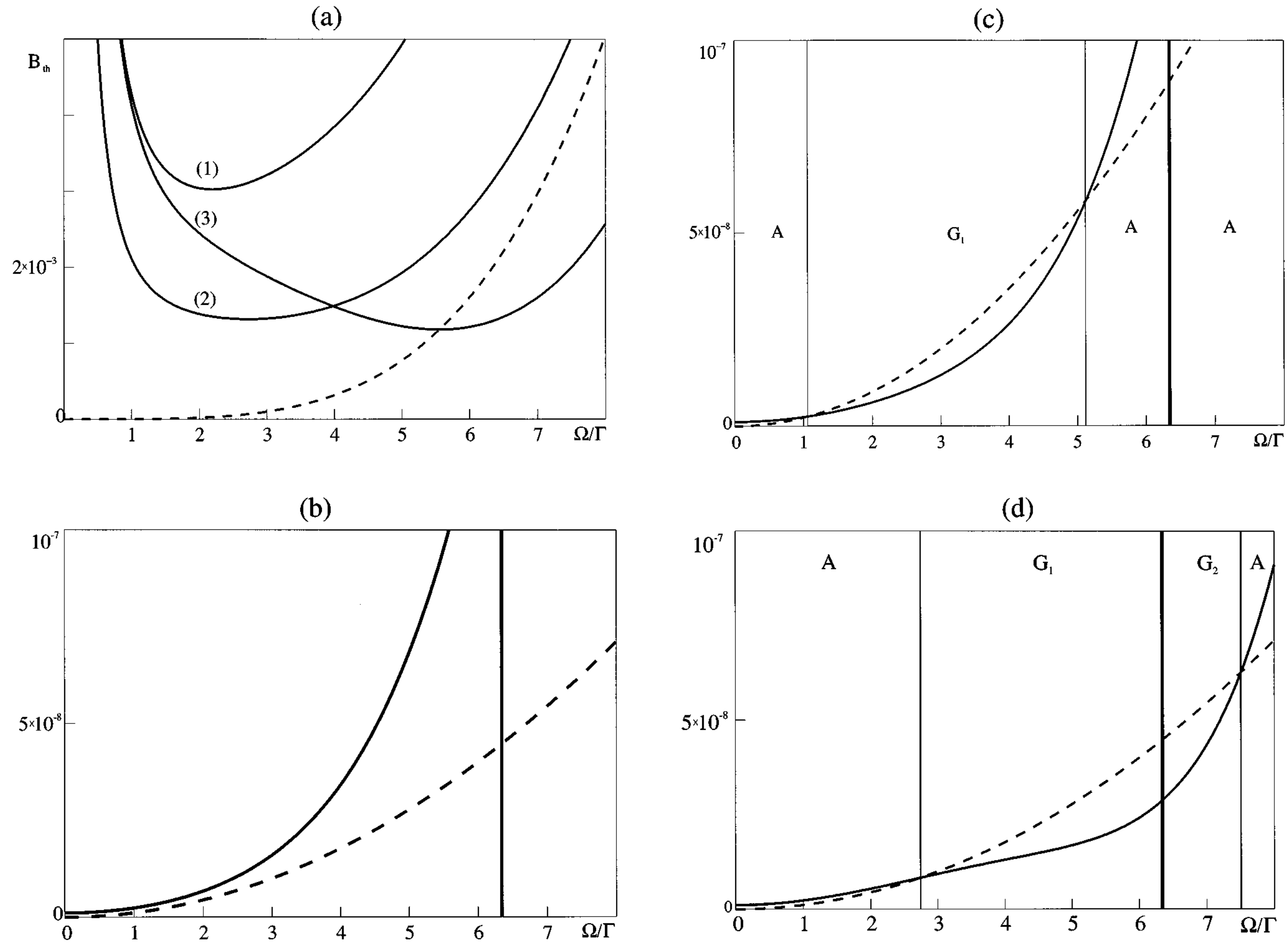

FIG. 7. (a) Variation of the population of the level b' at the threshold of amplification versus Ω/Γ for $\Delta=15\Gamma$ and for $\delta'=-\Gamma/2$, 0, and $\Gamma/2$ [curves (1), (2), and (3)]. The dashed curve corresponds to the population $(\Omega/2\Delta)^4$ in the dressed level associated with b. Amplification without population inversion in the dressed-state basis is achieved when $B'_{\mathrm{th}}<(\Omega/2\Delta)^4$. This condition is only fulfilled for the curve 3 when $\Omega/\Gamma>5.7$. (b) Variation of the absorption cross section (solid lines) and of the amplification cross section (dashed lines) versus Ω/Γ for $\delta'=-\Gamma/2$, $\Delta=15\Gamma$, and $B'=2\times10^{-3}$. The vertical unit is the resonant absorption cross section $3\lambda'^2/2\pi$. (c) Same cross sections but for $\delta'=0$. The bold vertical line located around $\Omega/\Gamma=6.3$ corresponds to the equality between B' and the population $(\Omega/2\Delta)^4$ of the dressed level. The gain domain (G_1) is surrounded by absorption domains (A). (d) Same cross sections but for $\delta'=\Gamma/2$. For the lower and the higher values of Ω/Γ the absorption cross section is larger than the amplification cross section. These domains correspond to probe absorption. The amplification cross section is larger than the absorption cross section in the domains G_1 and G_2. The boundary between these domains is determined by $(\Omega/2\Delta)^4=B'$. There is amplification with inversion in domain G_1 and AWI in domain G_2.

It should also be noticed that the driving field does not reduce the absorption of the probe field. On the contrary, probe absorption increases with the driving field intensity as shown in Fig. 2. However, there can be a range of parameter where the increase of the probe absorption is slower than the increase of the probe amplification and amplification can then be found. This point is illustrated in Figs. 7(b)–7(d) where we have plotted the absorption cross section (solid lines) and the amplification cross section (dashed lines) versus Ω/Γ for $\delta'=-\Gamma/2$ (b), $\delta'=0$ (c), and $\delta'=\Gamma/2$ (d) (the values of the other parameters are $\Delta=15\Gamma$ and $B'=2\times10^{-3}$). In the case of Fig. 7(b) ($\delta'=-\Gamma/2$), there is always probe absorption because the absorption cross section is larger than the amplification cross section. In the case of Fig. 7(c) ($\delta'=0$), two absorption domains (A) surround a gain domain (G_1). The bold vertical line located around $\Omega/\Gamma=6.3$ corresponds to the equality between B' and the population $(\Omega/2\Delta)^4$ of the dressed level. Because there are more atoms in the level b' than in the dressed level in the domain G_1, this domain corresponds to a situation of gain with population inversion in the dressed-state basis. In the case of Fig. 7(d) ($\delta'=\Gamma/2$), the gain domain is also surrounded by two absorption domains. However, the gain domain is now divided into two subdomains by the bold line corresponding to the equality of population $[(\Omega/2\Delta)^4=B']$. There is thus amplification *with inversion* in the domain G_1 and amplification *without inversion* in the domain G_2. Actually for the nonresonant V three-level system, the situation of Fig. 7(d) seems to be general for the occurrence of gain without population inversion: when we changed the parameters, we never found a situation where the domain of gain without inversion is just surrounded by domains of absorption.

We wish now to make the connection with the two-level atom case and we consider the simple case where the populations are equal, $B'=(\Omega/2\Delta)^4$. The formula for $\overline{\sigma_{\mathrm{abs}}}$ is then very close to the one found for a two-level atom [19]. In the two-level atom, there was no external pumping of the upper level, however, the combined effect of driving field and spontaneous emission brings some population in the upper level and this amount of population [18] which corresponds to B' is in fact exactly $(\Omega/2\Delta)^4$. The three-level atom appears thus to be simpler because the incoherent pumping and the reduction of absorption due to the driving field are well separated. However, one could also argue that the physics is basically identical to that of the two-level atom provided that one takes into account the fact that the incoherent pumping of the upper level is also associated with the driving field in the two-level atom case.

IV. CONCLUSION

In this paper, we have studied the gain condition for a V-type three-level atom where one branch of the V is driven by a strong pump field while the other branch is driven by a weak probe field. This is not the first paper to deal with this topic. The originality of the paper is that on one side the gain is calculated both in the bare- and in the dressed-state basis and on the other side that a clear physical picture is given for the amplification without population inversion through the interference between Feynman diagrams. In the bare-state basis, we use the density matrix formalism. Our result for the gain [Eqs. (7) and (10)] is identical to the gain condition of Eq. (35) of Ref. [8(b)], which, however, was discussed only when both fields are resonant with the atomic transition. On the contrary, the discussion here is focused on the off-resonant situation because the calculations are more easily done and understood in the dressed atom in this limit.

The main results which are obtained in this paper follow. (i) There is a domain of parameters for which gain is observed without population inversion in the bare-atom basis but with population inversion in the dressed-atom basis. This is in agreement with the result of Wilson *et al.* [30] but we have shown that this situation is not generic [32]. (ii) In fact there is a complementary domain of parameters in which there is gain but no population inversion either in the bare or in the dressed basis. We have calculated the Feynman diagrams corresponding to transitions between the dressed states and shown that the origin of amplification in this domain should be attributed to a destructive interference between the diagrams corresponding to absorption. This interference mechanism is almost identical to the one found earlier in the case of two-level atoms to explain the central resonance of the Mollow transmission spectrum [19]. Even if the physics of a three-level V system is, for this occasion, simpler than the physics of the two-level atom, there do not appear to be major differences between the processes that permit amplification or lasing without population inversion.

The essential limitation of this work is that the interpretation is restricted to the nonresonant driving field situation. However, this limitation can probably be overcome as in the two-level atom case where the probe gain for a resonant driving field excitation was recently studied by Szymanowski *et al.* [33]. They showed that in this case also, amplification without population inversion can occur and can be understood using interference processes.

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