# Existence of an Ericson regime in stretched helium

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The method of complex rotation is used to establish the existence of a regime of strongly overlapping resonances (the Ericson regime) in stretched helium, a collinear model of the helium atom. The existence of an Ericson regime in helium is conjectured and a method for proving its existence experimentally is suggested. [S1050-2947(96)03011-9]

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Classically, the helium atom is a chaotic system with a mixed phase space [1]. This means that not all of the quantum states of the helium atom can be grouped into regular sequences of states and we should be prepared to discover new quantum dynamical regimes in the helium atom. One consequence of a mixed phase space is the existence of regular islands in the chaotic sea. If the islands are big enough, they can support new types of regular sequences of states, not anticipated on the basis of traditional atomic physics techniques. An example is the recent discovery of a class of long-lived helium states associated with a prominent regular island in the helium phase space [2]. This discovery has already found an important technical application as a trap for antiprotons [3]. The occurrence of chaos in the helium atom means that already the simplest ''nontrivial atom'' is far from trivial. Several models have been suggested to investigate the classical and quantum mechanics of the helium atom in the classically chaotic regime. The use of simple models in atomic physics is recommended whenever the aim of the research is not the reproduction of high accuracy experimental atomic physics data, but the qualitative discussion of known phenomena or the search for qualitatively new dynamical regimes. Some of these models are surprisingly realistic. *S*-wave helium, e.g., a model based on breathing helium shells, was successful in predicting good approximations for the energies of whole series of helium states [4]. Another model is stretched helium [5]. Its Hamiltonian is very similar to the Hamiltonian of *S*-wave helium. The stretched model was investigated recently [1,6–10] as an excellent model for the investigation of the chaotic properties of two-electron atoms and ions. In this paper we are not concerned with high accuracy calculations of helium states and resonances. The aim is to provide evidence for the existence of a quantum dynamical regime in the helium atom, the ''Ericson regime'' of strongly overlapping autoionizing resonances. Therefore in the spirit of the above discussion we use a simple model, stretched helium, to provide evidence for the existence of this regime.

If the two electrons and the helium nucleus are all restricted to move on a straight line, we obtain the collinear helium model. There are two versions. (i) The two electrons are on the same side of the nucleus. This model was successfully employed by Richter and Wintgen to establish the existence of the class of helium states mentioned above. (ii) The two electrons are on opposite sides of the nucleus. This model defines stretched helium. In *Z* scaled units the coordinate of electron 1 is defined as $x_1 = -\xi_1/Z$, where $\xi_1$ is the position of electron 1 in atomic units. The coordinate of electron 2 is defined as $x_2 = \xi_2/Z$. If $\hat{h}$ is the Hamiltonian of stretched helium in atomic units, we define the Hamiltonian in scaled units as $\hat{H} = \hat{h}/Z^2$ and obtain

$$\hat{H} = \frac{\hat{p}_1^2}{2} + \frac{\hat{p}_2^2}{2} - \frac{1}{x_1} - \frac{1}{x_2} + \frac{\epsilon}{x_1 + x_2}, \tag{1}$$

where $\epsilon = 1/Z$. A rough approximation to the spectrum of (1) is given by

$$E_{mn} = -\frac{1}{2m^2} - \frac{1}{8n^2}, \quad n \geqslant m, \quad m,n = 1,2,\ldots$$

$$E_{mk} = -\frac{1}{2m^2} + \frac{k^2}{8}, \quad k \geqslant 0. \tag{2}$$

The basis of (2) is an independent particle picture based on the assumption that electron 1 is tightly bound in a state with quantum number *m* whereas electron 2 is more loosely bound (or unbound) and ''sees'' a nuclear charge screened by one unit by electron 1. The spectrum (2) is qualitatively correct, i.e., it agrees in all essential features with the exact spectrum of (1) computed and discussed below. A schematic sketch of (2) is shown in Fig. 1. Apparently (1) contains a Rydberg series of bounded states for $m=1$, and a doubly infinite series of autoionizing resonances for $m>1$. Figure 1(a) shows that for small *m* the resonance series are isolated from each other, overlapping only with lower *m* continua. The helium atom shows the same qualitative behavior. The source of complication in the spectrum of (1) is the existence of intruder states, i.e., states of an *m* series intruding into an *M* series with $M<m$. Figure 1(b) illustrates this case for $M=21$, $m=22$ ($M=21,22$, $m=23$). For (2) it is easy to show that the number of intruders grows with growing *m* such that for sufficiently large *m* an *m* series can contain any number of intruder states. Because of the coupling term $\epsilon/(x_1+x_2)$ in (1) the intruders perturb the *M* series strongly for sufficiently large *M* such that the $(m,n)$ classification scheme breaks down, resulting in a ''chaotic'' jumble of states. We show now that the above physical picture, developed on the basis of the independent particle model, is in fact qualitatively correct for the exact spectrum of (1).
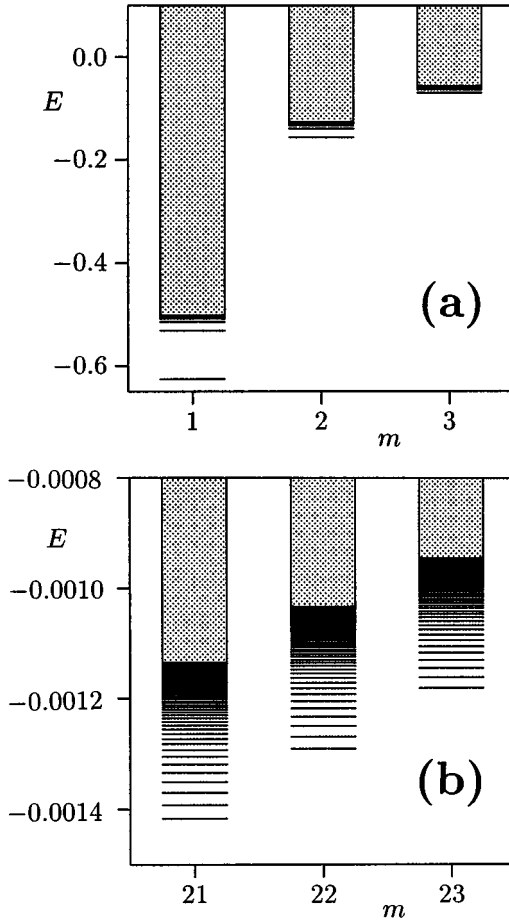
FIG. 1. Spectrum of the independent particle model of stretched helium. (a) Isolated series of states for $m = 1, 2, 3$. (b) Series with intruder states for $m = 21, 22, 23$.

We use the method of complex rotation to compute numerically the spectrum of (1). Dilating the coordinates in (1) according to $x \rightarrow \exp(i\theta)x$, we obtain the complex Hamiltonian

$$\hat{H}(\theta) = \frac{e^{-2i\theta}\hat{p}_1^2}{2} + \frac{e^{-2i\theta}\hat{p}_2^2}{2} - \frac{e^{-i\theta}}{x_1} - \frac{e^{-i\theta}}{x_2} + \frac{e^{-i\theta}}{(x_1 + x_2)}. \quad (3)$$
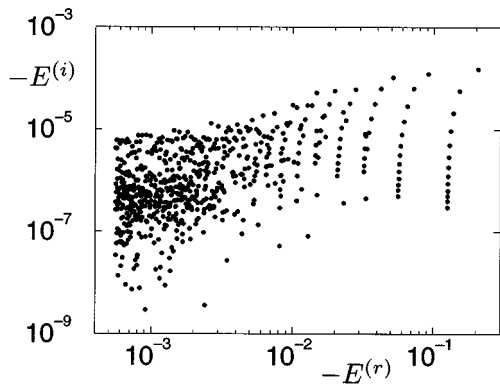


FIG. 2. Resonance positions $E_j = E_j^{(r)} + iE_j^{(i)}$, $j = 1, \ldots, 650$ of the one-dimensional stretched helium atom in the complex energy plane. The resonances were computed numerically using complex rotation.
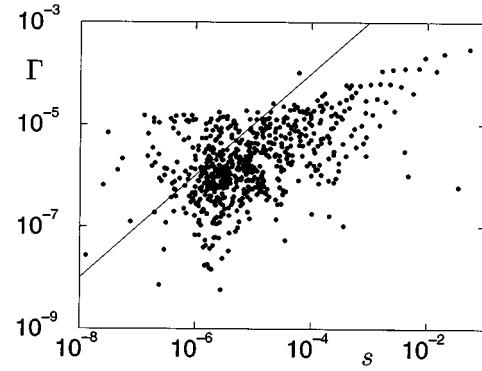


FIG. 3. Width versus spacing of the 650 resonances shown in Fig. 2. Data points to the left and above the straight line $\Gamma = s$ correspond to overlapping resonances with $\Gamma > s$.

We computed only the positive parity spectrum of (1) diagonalizing (3) in a square normalizable basis consisting of 3240 two-body states constructed from properly symmetrized products of the first 80 one-body Sturmian functions given by

$$\varphi_n^\alpha(x) = \left[\frac{2}{\alpha n(n+1)}\right]^{1/2} \left(\frac{2x}{\alpha}\right) L_{n-1}^{(2)}\left(\frac{2x}{\alpha}\right) \exp(-x/\alpha). \quad (4)$$

The functions (4) are derived from the eigenfunctions of the surface state electron Hamiltonian [11]. Convergence of the results was assured by optimizing the Sturmian label $\alpha$ and the rotation angle $\theta$. As predicted by the simple independent particle model, we obtain a single strictly bounded series of Rydberg states for $m = 1$ and a doubly infinite series of autoionizing resonances with energies $E = E^{(r)} + iE^{(i)}$. We computed 650 resonances with energies corresponding to the $m = 30$th threshold or lower. The result is shown in Fig. 2. We see that for small $E^{(r)}$ the resonances form regular Rydberg series which are progressively broken up for higher excitation energy eventually resulting in a "chaotic" jumble of resonance positions in the complex energy plane. This is an indication that a systematic and physically useful classification scheme of resonances probably does not exist in the "chaotic" regime of resonances. Here a statistical description of resonances may be more useful. However, embedded
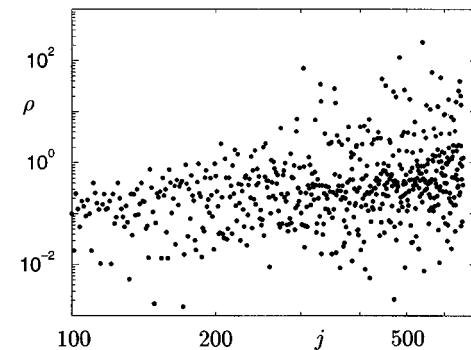


FIG. 4. The ratio $\rho = \Gamma/s$ for stretched helium resonances with $100 \leq j \leq 650$. Ratios well in excess of $\rho = 10$ are observed, proving the existence of an Ericson regime of strongly overlapping resonances ($\Gamma \gg s$) in the stretched helium model.

in the chaotic jumble, regular progressions of Rydberg series do exist. They are confined to the immediate neighborhood of $E^{(r)} = -1/2m^2$, $m = 1,2, \ldots$ , and have very small imaginary parts. Apart from the low $m$ series we did not search for these states. They play no role in the context of our search for the Ericson regime.

We turn now to the main point of this paper, the existence of a regime of strongly overlapping resonances. First we order the resonances shown in Fig. 2 according to the size of their real parts, $\{E_j = E_j^{(r)} + iE_j^{(i)}\}_{j=1}^{650}$, $E_j^{(r)} > E_{j'}^{(r)}$ for $j > j'$. Then, we define the spacing of resonances, $s_j = E_{j+1}^{(r)} - E_j^{(r)}$, and the widths $\Gamma_j = -2E_j^{(i)}$ The Ericson regime is defined as the regime of strongly overlapping resonances [12,13], i.e., $\Gamma \gg s$. Figure 3 shows the resonance widths $\Gamma_j$ versus the spacings $s_j$ of the 650 resonances of Fig. 2. The straight line denotes the boundary $\Gamma = s$ which separates the regime of overlapping resonances ($\Gamma > s$) from the regime of isolated resonances ($\Gamma < s$). Resonances to the left and above the straight line in Fig. 3 are overlapping with $\Gamma > s$. Figure 4 shows directly the ratio $\rho = \Gamma/s$ for the reso-nances with $100 \leq j \leq 650$. We see (i) that an appreciable fraction of the overlapping resonances is actually strongly overlapping with $\Gamma \gg s$, and (ii) that the trend in Fig. 4 is to higher values of $\rho$ for higher excitation energy. This indi-cates that the Ericson regime is even more pronounced for $m > 30$.

In summary, we established the existence of an Ericson regime for the stretched model of the helium atom. Since stretched helium emulates many features of the helium atom [1,6–10] we conjecture that an Ericson regime should also exist in the helium atom. Experimentally the existence of an Ericson regime in helium may be established by measuring the scattering cross section of slow electrons on He$^+$ Ryd-berg states. As observed by Ericson [12,13] in a nuclear physics context this cross section is expected to fluctuate strongly as a function of the energy of the projectile electron despite the fact that the resonances have large widths and are strongly overlapping.

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