Propagation and amplitude correlation of pairs of intense pulses interacting with a double-Λ system

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The propagation of intense laser pulses through a four-level atomic system in a double-Λ scheme is examined. Under conditions of adiabatic perturbation of the atomic quantum state, paired pulses with arbitrary shapes establish a correlation between their amplitudes and reach a quasiform-stable regime of propagation. The amplitude correlation is a feature of pulse matching, while the propagation presents the same properties as the so-called adiabatons, predicted and observed in single-Λ systems. We show that in the double-Λ scheme the phenomena of pulse matching and adiabaton-type propagation are associated with two distinct propagation normal modes. [S1050-2947(96)07212-5]

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I. INTRODUCTION

A Λ-type atomic medium can be pumped, by a pair of strong resonant electromagnetic fields, into a coherent superposition of the lower-energy states, where the atomic population is trapped because of destructive interferences between two different absorption paths. This phenomenon, known as coherent population trapping (CPT) [1], leads to the suppression of the total absorption of the resonant fields and renders the atomic system transparent, even with most population remaining in the lower-energy states. This type of cancellation of the absorption is currently termed electromagnetically-induced transparency (EIT). In Λ systems, the EIT is obtained as a direct consequence of the coherent trapping. In general, the EIT is produced by quantum coherences and interferences and can be achieved in different multilevel systems, including V-type systems where the CPT does not occur [2].

EIT has attracted much interest for its application to the amplification without inversion (AWI) [3,4] and several authors have focused their attention on the implications of the EIT on the total transmission of resonant light [5]. Dispersion properties [6] and spatial consequences [7–9] of the EIT have also been investigated, as well as applications to nonlinear optics [10]. In [11], the analysis of the phenomena of CPT, EIT, and AWI has been extended to autoionizing transitions.

The absorption and dispersion features of the EIT have important consequences on the interaction of time-dependent electromagnetic fields with an atomic medium and different aspects of the propagation of pulsed excitations through three-level systems have been discussed in several theoretical papers [12–22]. In [12], the process of pulse matching has been predicted as a result of the nonlinear interaction of two laser pulses with a Λ system, with the coherence between the lower-energy states fixed by an external preparation. This process generates a correlation between the Fourier components of the two pulses, and hence a shape matching of their temporal profiles. After the correlation is established, the matched pulses propagate without losses and without group velocity reduction and dispersion. These stable pulses can have arbitrary shape, determined by the initial pulse shapes and by the preparation conditions of the medium. As pointed out in [13], pulse matching originates by the selective absorption of a well-defined superposition of the two applied laser fields interacting with the coherently prepared Λ system, while the orthogonal superposition propagates without attenuation. The propagation of those field superpositions, termed as ''dressed fields,'' has been discussed in [14,15], as well. Matched fields represent a steady state solution to the problem of the propagation of a pair of time-dependent fields through a Λ system. This steady state is stable against small fluctuations of the intensity and phase-difference of the two fields, as explicitly shown in [17–19]. The propagation of matched pulses in the absence of initial coherent preparation has been investigated in [13,20]. It has been shown that the preparation of the atoms is performed by the leading edge of the applied pulses, if sufficiently intense, through the CPT process, so that the atomic medium results transparent to the pulse trailing edge.

In [21], new form-stable pulses, named adiabatons, have been predicted under specific conditions of adiabatic excitation of a Λ system. The adiabatons develop as a pair of complementary pulses interacting with two different transitions and propagate with reduced group velocity. The invariance properties of this type of pulse have been discussed and specified in [22] and their essential features described experimentally in [7]. The formation of the adiabatons is understood in terms of adiabatic following of the instantaneous nonabsorbing CPT superposition and is related to the process of stimulated Raman adiabatic passage (STIRAP) [23].

Interesting propagation features have also been predicted for four-component fields interacting, in strong-coupling–weak-probe configurations, with four-level atomic systems. As shown in [24,25], these systems can be prepared coherently, by applying a pair of coupling pulses to two different atomic transitions, so that paired probe pulses, coupled to other transitions, experience shape matching and propagate without losses. Different configurations of interaction, such as the double-Λ [24] and the double-V [25], lead to the transparency of the four-level medium for weak probe fields. In [26], the refractive properties of the coupling-probe double-

Λ system, associated to the process of pulse matching, have been investigated.

In this work, we report on additional properties of the dynamics of the double-Λ system, when all of the driving-field components have a high intensity. We focus on the case of adiabatic perturbation of the initial coherent atomic state and use, to describe the system under these conditions, the dressed-basis representation introduced in [22]. We demonstrate, both analytically and numerically, that the adiabatons originate also in four-level systems and evidence the correlation that, in such systems, is established between the amplitudes of these pulses. Under adiabatic conditions, the field propagation is governed by two spatial normal modes. One of these modes, mainly absorptive and associated to the transient regime of pulse matching, is characterized by a short extinction length. Its absorption determines the establishment of a correlation between the amplitudes of pulses acting on different transitions. The second mode, essentially dispersive and traveling with reduced group velocity and very small losses, describes the propagation of pairs of adiabatons. Our analysis clarifies that pulse matching and adiabatons represent two distinct transient phases towards the stationary state of the atom-field system, which is represented by matched fields and coherently trapped atoms. New effects of mutual interaction between the different field components and interplay between the process of pulse matching and the formation of the adiabatons are predicted.

The organization of the paper is as follows: In Sec. II we derive the Maxwell-Bloch equations for the double-Λ system in the basis of the adiabatic dressed states. In Sec. III these equations are solved under quasiadiabatic conditions, in terms of propagation normal modes. The normal-mode solution is discussed in Sec. IV, where numerical simulations that support and extend the analytical description are also presented.

II. THE FOUR-LEVEL SYSTEM IN THE ADIABATIC DRESSED STATE BASIS

A four-level system in the double-Λ configuration is shown in the diagram of Fig. 1(a). The interaction scheme is composed by two Λ subsystems that share the lower-energy states $|1\rangle$ and $|2\rangle$. For simplicity, the dipole allowed $|1\rangle \to |i\rangle$ and $|2\rangle \to |i\rangle$ transitions ($i = c,p$) of both Λ subsystems are supposed symmetric, i.e., with equal frequency, dipole moment, and spontaneous emission rate. We describe the interaction of this system with slowly-varying four-component fields and assume that each component is only coupled to one of the allowed transitions and has its carrier frequency exactly resonant with the transition frequency. We disregard the transverse distribution of the fields and the inhomogeneous broadening of the medium. These assumptions allow us to obtain a straightforward analytical description of the spatio-temporal evolution of the system.

We indicate the angular frequency of the transitions to the upper state $|i\rangle$ ($i = c,p$) as ω_i and the natural decay rate of $|i\rangle$ as Γ_i. The population of $|i\rangle$ is supposed to decay by spontaneous emission into $|1\rangle$ and $|2\rangle$ with equal rates, given by $\Gamma_i/2$. Nonradiative decays of the lower states are neglected. A symmetric double-Λ system is provided, for instance, by two $F=1 \to F=1$ atomic transitions with com-

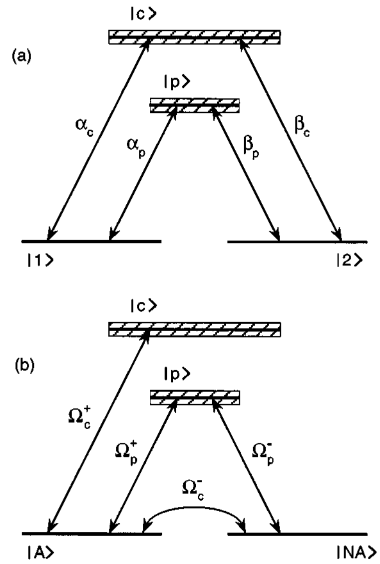

FIG. 1. Double-Λ system in the (a) bare-state and (b) dressed-state bases.

mon lower state, driven by circularly polarized laser fields: the $m_F = 1$ and $m_F = -1$ Zeeman substates of the lower state are coupled to the $m_F = 0$ substate of either upper state by resonant σ^- and σ^+ field components, respectively.

We describe the electromagnetic fields through the space-time–dependent Rabi frequencies $\alpha_c(z,t) = d_c E_{c1}(z,t)/\hbar$, $\beta_c(z,t) = d_c E_{c2}(z,t)/\hbar$, $\alpha_p(z,t) = d_p E_{p1}(z,t)/\hbar$, and $\beta_p(z,t) = d_p E_{p2}(z,t)/\hbar$, where $E_{ij}(z,t)$, for $i = c,p$ and $j = 1,2$, is the slowly-varying envelope of the field component interacting with the transition $|j\rangle \to |i\rangle$ and $d_i = (3\hbar c^3 \Gamma_i / 8\omega_i^3)^{1/2}$ is the dipole moment matrix element of the symmetric transitions from the lower states to the upper state $|i\rangle$. The Maxwell-Bloch equations of the double-Λ system in the basis of the bare atom are written explicitly in [27]. Here we introduce the basis change to the dressed states $|NA\rangle$ and $|A\rangle$,

$$|NA\rangle = \frac{\beta_c|1\rangle - \alpha_c|2\rangle}{\sqrt{|\alpha_c|^2 + |\beta_c|^2}}, \tag{1a}$$

$$|A\rangle = \frac{\alpha_c|1\rangle + \beta_c|2\rangle}{\sqrt{|\alpha_c|^2 + |\beta_c|^2}}. \tag{1b}$$

If we consider $\alpha_p, \beta_p \equiv 0$, then the system in Fig. 1(a) is reduced to the Λ system composed by the states $|1\rangle$, $|2\rangle$, and $|c\rangle$: the state $|NA\rangle$ represents the nonabsorbing CPT superposition [1] associated to this system, while $|A\rangle$ is the absorbing orthogonal superposition. When the fields α_c and β_c are time dependent, unless they have matched time-profiles, $|NA\rangle$ and $|A\rangle$ are explicit functions of the time, as well. The field components α_p and β_p could be chosen as coefficients of the superpositions in Eqs. (1) in the place of α_c and β_c: the following analysis is independent of such a choice.

We describe the spatio-temporal evolution of the atom-field system in the moving frame of the coordinates $\zeta = z$ and $\tau = t - z/c$. Under exact resonance conditions, it follows from the Maxwell-Bloch equations that, if all field components are in phase at the entry surface of the medium, and the initial atomic polarization is $\pi/2$ out of phase with respect to the field, then the phase of the field, as well as that of the atomic variables, remains unchanged during the interaction. Thus, in the following, we can assume the Rabi frequencies α_c, β_c, α_p, and β_p real for any τ and ζ. We define the new field variables $\Omega_c^{\pm}(\zeta,\tau)$ and $\Omega_p^{\pm}(\zeta,\tau)$, with dimensions of frequencies, as

$$\Omega_c^{+}(\zeta,\tau)=\sqrt{\alpha_c^2(\zeta,\tau)+\beta_c^2(\zeta,\tau)}, \tag{2a}$$

$$\Omega_c^{-}(\zeta,\tau)=\frac{\beta_c(\zeta,\tau)\frac{\partial}{\partial\tau}\alpha_c(\zeta,\tau)-\alpha_c(\zeta,\tau)\frac{\partial}{\partial\tau}\beta_c(\zeta,\tau)}{\Omega_c^{+2}(\zeta,\tau)}, \tag{2b}$$

$$\Omega_p^{+}(\zeta,\tau)=\frac{\alpha_c(\zeta,\tau)\alpha_p(\zeta,\tau)+\beta_c(\zeta,\tau)\beta_p(\zeta,\tau)}{\Omega_c^{+}(\zeta,\tau)}, \tag{2c}$$

$$\Omega_p^{-}(\zeta,\tau)=\frac{\beta_c(\zeta,\tau)\alpha_p(\zeta,\tau)-\alpha_c(\zeta,\tau)\beta_p(\zeta,\tau)}{\Omega_c^{+}(\zeta,\tau)}. \tag{2d}$$

A pair of field variables of the form of $\Omega_c^{\pm}$ has been introduced in [22] to describe the atom dynamics and the loss-free propagation of laser fields in a three-level Λ scheme under STIRAP conditions. In terms of $\Omega_c^{\pm}$ and $\Omega_p^{\pm}$, the original Rabi frequencies are expressed by

$$\alpha_c(\zeta,\tau)=\Omega_c^{+}(\zeta,\tau)\sin\left[\int_{-\infty}^{\tau}\Omega_c^{-}(\zeta,\tau')d\tau'+\text{const}\right], \tag{3a}$$

$$\beta_c(\zeta,\tau)=\Omega_c^{+}(\zeta,\tau)\cos\left[\int_{-\infty}^{\tau}\Omega_c^{-}(\zeta,\tau')d\tau'+\text{const}\right], \tag{3b}$$

$$\alpha_p(\zeta,\tau)=\frac{\alpha_c(\zeta,\tau)\Omega_p^{+}(\zeta,\tau)+\beta_c(\zeta,\tau)\Omega_p^{-}(\zeta,\tau)}{\Omega_c^{+}(\zeta,\tau)}, \tag{3c}$$

$$\beta_p(\zeta,\tau)=\frac{\beta_c(\zeta,\tau)\Omega_p^{+}(\zeta,\tau)-\alpha_c(\zeta,\tau)\Omega_p^{-}(\zeta,\tau)}{\Omega_c^{+}(\zeta,\tau)}. \tag{3d}$$

Equations (3a) and (3b) are easily obtained from the relation $\Omega_c^{-}=(d/d\tau)[\arctan(\alpha_c/\beta_c)]$, with the condition $\lim_{\tau\to-\infty}\arctan[\alpha_c(\zeta,\tau)/\beta_c(\zeta,\tau)]=\text{const}$.

In the basis of the dressed states defined in Eqs. (1), the equations for the elements of the density matrix ρ in the interaction picture read, under exact resonance conditions,

$$\frac{\partial\rho_{c(NA)}}{\partial\tau}=-\gamma_c\rho_{c(NA)}-\Omega_c^{-}\rho_{cA}-i\Omega_p^{-}\rho_{pc}+i\Omega_c^{+}\rho_{A(NA)}, \tag{4a}$$

$$\frac{\partial\rho_{cA}}{\partial\tau}=-\gamma_c\rho_{cA}-i\Omega_c^{+}(\rho_{cc}-\rho_{AA})+\Omega_c^{-}\rho_{c(NA)}-i\Omega_p^{+}\rho_{pc}, \tag{4b}$$

$$\frac{\partial\rho_{p(NA)}}{\partial\tau}=-\gamma_p\rho_{p(NA)}-i\Omega_p^{-}(\rho_{pp}-\rho_{(NA)(NA)})+i\Omega_p^{+}\rho_{A(NA)}, \tag{4c}$$

$$\frac{\partial\rho_{pA}}{\partial\tau}=-\gamma_p\rho_{pA}-i\Omega_p^{+}(\rho_{pp}-\rho_{AA})-i\Omega_c^{+}\rho_{pc}+i\Omega_p^{-}\rho_{A(NA)}, \tag{4d}$$

$$\frac{\partial\rho_{pc}}{\partial\tau}=-\gamma_{pc}\rho_{pc}-i\Omega_p^{-}\rho_{c(NA)}-i\Omega_p^{+}\rho_{cA}-i\Omega_c^{+}\rho_{pA}, \tag{4e}$$

$$\frac{\partial\rho_{A(NA)}}{\partial\tau}=\Omega_c^{-}(\rho_{(NA)(NA)}-\rho_{AA})+i\Omega_c^{+}\rho_{c(NA)}+i\Omega_p^{+}\rho_{p(NA)}+i\Omega_p^{-}\rho_{pA}, \tag{4f}$$

$$\frac{\partial\rho_{cc}}{\partial\tau}=-\Gamma_c\rho_{cc}-2i\Omega_c^{+}\rho_{cA}, \tag{4g}$$

$$\frac{\partial\rho_{pp}}{\partial\tau}=-\Gamma_p\rho_{pp}-2i(\Omega_p^{-}\rho_{p(NA)}+\Omega_p^{+}\rho_{pA}), \tag{4h}$$

$$\frac{\partial\rho_{(NA)(NA)}}{\partial\tau}=\frac{\Gamma_c}{2}\rho_{cc}+\frac{\Gamma_p}{2}\rho_{pp}+2i\Omega_p^{-}\rho_{p(NA)}-2\Omega_c^{-}\rho_{A(NA)}, \tag{4i}$$

$$\frac{\partial\rho_{AA}}{\partial\tau}=\frac{\Gamma_c}{2}\rho_{cc}+\frac{\Gamma_p}{2}\rho_{pp}+2i(\Omega_c^{+}\rho_{cA}+\Omega_p^{+}\rho_{pA})+2\Omega_c^{-}\rho_{A(NA)}, \tag{4j}$$

with

$$\gamma_c=\frac{\Gamma_c}{2}, \tag{5a}$$

$$\gamma_p=\frac{\Gamma_p}{2}, \tag{5b}$$

$$\gamma_{pc}=\frac{1}{2}(\Gamma_c+\Gamma_p). \tag{5c}$$

Here all terms proportional to the field variable Ω_c^{-} arise from the explicit dependence of $|NA\rangle$ and $|A\rangle$ on time (cf. the equations derived in [22] for the single-Λ system). Equations (4) hold for real fields. We notice that $\Omega_c^{\pm}$ and $\Omega_p^{\pm}$ can be consistently considered real if the ijth density matrix elements, with $i=A,NA$ and $j=c,p$, are considered purely imaginary and the other elements real.

Equations (4) must be solved in a self-consistent way with the Maxwell equations that, in the slowly varying envelope approximation and in terms of the field variables $\Omega_c^{\pm}$ and $\Omega_p^{\pm}$, are

$$\frac{\partial\Omega_c^{+}}{\partial\zeta}=i\kappa_c\rho_{cA}, \tag{6a}$$

$$\frac{\partial\Omega_c^{-}}{\partial\zeta}=i\kappa_c\frac{\partial}{\partial\tau}\left(\frac{\rho_{c(NA)}}{\Omega_c^{+}}\right), \tag{6b}$$

$$\frac{\partial\Omega_p^{+}}{\partial\zeta}=i\kappa_p\rho_{pA}+i\kappa_c\frac{\Omega_p^{-}}{\Omega_c^{+}}\rho_{c(NA)}, \tag{6c}$$

$$\frac{\partial \Omega_p^-}{\partial \zeta} = i\kappa_p \rho_{p(NA)} - i\kappa_c \frac{\Omega_p^+}{\Omega_c^+} \rho_{c(NA)}, \tag{6d}$$

where the coupling coefficient κ_i, for $i=c,p$, is given by

$$\kappa_i = \frac{8\pi\omega_i N|d_i|^2}{c\hbar}, \tag{7}$$

with N the atomic density.

Formally, Eqs. (4) and (6) describe a four-level system driven by the fields $\Omega_c^{\pm}$ and $\Omega_p^{\pm}$. As sketched in Fig. 1(b), Ω_c^+ and Ω_p^+ couple the state superposition $|A\rangle$ to the upper states $|c\rangle$ and $|p\rangle$, respectively, Ω_p^- couples $|NA\rangle$ to $|p\rangle$, and Ω_c^- connects $|NA\rangle$ and $|A\rangle$. Some interesting conclusions can be directly drawn from the scheme in Fig. 1(b). Let us observe that, when the original fields α_c and β_c have matched time profiles, i.e., have the same time dependence and only differ from each other by a constant scaling factor, then the transformed field Ω_c^- is identically equal to zero. On the other hand, Ω_p^- vanishes when α_p and β_p are, at any time, in the same ratio as α_c and β_c,

$$\frac{\alpha_p(\zeta,\tau)}{\beta_p(\zeta,\tau)} = \frac{\alpha_c(\zeta,\tau)}{\beta_c(\zeta,\tau)}. \tag{8}$$

When α_c and β_c have matched profiles, so that their ratio is independent of time, this equation establishes the condition of pulse matching for α_p and β_p. Provided that both Ω_c^- and Ω_p^- are equal to zero, the state superposition $|NA\rangle$ is decoupled from the fields. Then, if the atomic system is coherently prepared in $|NA\rangle$, all the population remains there indefinitely, while the transformed fields Ω_c^+ and Ω_p^+ interact with the remaining empty states and propagate freely. The conservation of Ω_c^+ and Ω_p^+ corresponds, in the bare-state representation, to the stable propagation of two pairs of matched pulses, whose amplitudes satisfy the correlation condition given in Eq. (8). Thus, in the dressed state basis it is immediately seen that pairs of arbitrarily strong and arbitrarily shaped pulses, with matched profiles and correlated amplitudes, maintain the atomic population coherently trapped in a nonabsorbing state. Atoms in the coherent trapping superposition $|NA\rangle$ and matched pulses correspond to a stationary state of the atom-field system [28].

The application of the fields Ω_c^- and Ω_p^- represents a perturbation to the stationary state described above. In this work, the spatio-temporal dynamics of the system is examined in the hypothesis of weak perturbations. This assumption does not require, in general, that the original fields α_c, β_c, α_p, and β_p are weak, since both Ω_c^- and Ω_p^- can be rendered small by properly choosing the relative amplitudes of those fields. As shown in the following, with strong original fields new nonlinear phenomena arise in the transient dynamics of the four-level system, not observed in strong-coupling–weak-probe configurations as those considered in Refs. [24–26].

III. PULSE PROPAGATION UNDER QUASIADIABATIC CONDITIONS

We now describe the transient dynamics of the atom-field system under conditions of weak coupling of the state $|NA\rangle$ to the upper states, i.e., for Ω_c^- and Ω_p^- much weaker than Ω_c^+ and Ω_p^+. When this requirement is satisfied, the evolution of the state $|NA\rangle$ is quasiadiabatic [15]. We consider, for any position ζ, the state $|NA\rangle$ fully occupied at the initial time, as an effect of a coherent preparation of the atomic sample. In fact, the double-Λ system can be prepared into any superposition of the lower-energy bare states by means of the application of a pair of sufficiently long and intense matched pulses, with proper amplitude ratio, to either Λ subsystem [20,24].

From Eqs. (4) and (6) we can see that, with all the population initially in $|NA\rangle$, Ω_c^- and Ω_p^- remain small during the interaction, if they are both small at the entry into the medium, in $\zeta=0$. For $|\Omega_c^-|,|\Omega_p^-| \ll |\Omega_c^+|,|\Omega_p^+|$, the atomic population always remains, at first order, in the state $|NA\rangle$ and the coherences between initially empty states are never excited,

$$\rho_{(NA)(NA)}(\zeta,\tau) = 1, \tag{9a}$$

$$\rho_{AA}(\zeta,\tau) = \rho_{cc}(\zeta,\tau) = \rho_{pp}(\zeta,\tau) = 0, \tag{9b}$$

$$\rho_{cA}(\zeta,\tau) = \rho_{pA}(\zeta,\tau) = \rho_{pc}(\zeta,\tau) = 0. \tag{9c}$$

Under these conditions, the equations of motion for the remaining atomic variables become

$$\frac{\partial}{\partial \tau}\rho_{c(NA)}(\zeta,\tau) = -\gamma_c \rho_{c(NA)}(\zeta,\tau) + i\Omega_c^+ \rho_{A(NA)}(\zeta,\tau), \tag{10a}$$

$$\frac{\partial}{\partial \tau}\rho_{p(NA)}(\zeta,\tau) = -\gamma_p \rho_{p(NA)}(\zeta,\tau) + i\Omega_p^-(\zeta,\tau) + i\Omega_p^+ \rho_{A(NA)}(\zeta,\tau), \tag{10b}$$

$$\frac{\partial}{\partial \tau}\rho_{A(NA)}(\zeta,\tau) = i\Omega_c^+ \rho_{c(NA)}(\zeta,\tau) + i\Omega_p^+ \rho_{p(NA)}(\zeta,\tau) + \Omega_c^-(\zeta,\tau). \tag{10c}$$

Moreover, at first order, the driving terms on the left-hand side of Eqs. (6a) and (6c) vanish, so that the fields Ω_c^+ and Ω_p^+ are conserved along ζ,

$$\Omega_c^+(\zeta,\tau) = \Omega_c^+(0,\tau), \tag{11a}$$

$$\Omega_p^+(\zeta,\tau) = \Omega_p^+(0,\tau). \tag{11b}$$

We further assume that the decay rates γ_c and γ_p are sufficiently large that the coherences $\rho_{c(NA)}$ and $\rho_{p(NA)}$ instantaneously follow the evolution of the fields. This adiabaticity hypothesis simplifies the analysis but does not change, in substance, the results presented in the following. By eliminating $\rho_{c(NA)}$ and $\rho_{p(NA)}$ from Eqs. (10a) and (10b), and substituting them in Eqs. (10c), (6b), and (6d), the Maxwell-Bloch equations are reduced to

$$\frac{\partial}{\partial\tau}\rho_{A(NA)}(\zeta,\tau)=-\left[\frac{\Omega_c^{+2}(0,\tau)}{\gamma_c}+\frac{\Omega_p^{+2}(0,\tau)}{\gamma_p}\right]\rho_{A(NA)}(\zeta,\tau)+\Omega_c^-(\zeta,\tau)-\frac{\Omega_p^+(0,\tau)}{\gamma_p}\Omega_p^-(\zeta,\tau), \quad (12a)$$

$$\frac{\partial}{\partial\zeta}\Omega_c^-(\zeta,\tau)=-\frac{\kappa_c}{\gamma_c}\frac{\partial}{\partial\tau}\rho_{A(NA)}(\zeta,\tau), \quad (12b)$$

$$\frac{\partial}{\partial\zeta}\Omega_p^-(\zeta,\tau)=-\frac{\kappa_p}{\gamma_p}\Omega_p^-(\zeta,\tau)+\left(\frac{\kappa_c}{\gamma_c}-\frac{\kappa_p}{\gamma_p}\right)\Omega_p^+(0,\tau)\rho_{A(NA)}(\zeta,\tau). \quad (12c)$$

By choosing the amplitudes of the incident fields Ω_c^+ and Ω_p^+ as constants, Eqs. (12) become a set of linear differential equations with constant coefficients, easily solved analytically in the frequency domain. This particular case allows us to determine the basic mechanisms underlying the evolution of the system.

By Fourier-transforming Eq. (12a) with respect to τ, we obtain

$$\hat\rho_{A(NA)}(\zeta,\omega)=\frac{1}{\Gamma_c'+\Gamma_p'-i\omega}\left[\hat\Omega_c^-(\zeta,\omega)-\frac{\Omega_p^+}{\gamma_p}\hat\Omega_p^-(\zeta,\omega)\right], \quad (13)$$

where the Fourier-transformed variables are marked by a circumflex accent and the effective decay rates $\Gamma_i'=\Omega_i^{+2}/\gamma_i$, with $i=c,p$, are introduced. By substitution of Eq. (13), the propagation equations for $\hat\Omega_c^-$ and $\hat\Omega_p^-$ become

$$\frac{\partial}{\partial\zeta}\hat\Omega_c^-(\zeta,\omega)=-A(\omega)\hat\Omega_c^-(\zeta,\omega)-B(\omega)\hat\Omega_p^-(\zeta,\omega), \quad (14a)$$

$$\frac{\partial}{\partial\zeta}\hat\Omega_p^-(\zeta,\omega)=-C(\omega)\hat\Omega_c^-(\zeta,\omega)-D(\omega)\hat\Omega_p^-(\zeta,\omega), \quad (14b)$$

with

$$A(\omega)=\eta_c^B\frac{\omega^2-i\omega(\Gamma_c'+\Gamma_p')}{\omega^2+(\Gamma_c'+\Gamma_p')^2}, \quad (15a)$$

$$B(\omega)=-\eta_c^B\frac{\omega^2-i\omega(\Gamma_c'+\Gamma_p')}{\omega^2+(\Gamma_c'+\Gamma_p')^2}\frac{\Omega_p^+}{\gamma_p}, \quad (15b)$$

$$C(\omega)=(\eta_p^B-\eta_c^B)\frac{\Gamma_c'+\Gamma_p'+i\omega}{\omega^2+(\Gamma_c'+\Gamma_p')^2}\Omega_p^+, \quad (15c)$$

$$D(\omega)=\frac{(\eta_p^B\Gamma_c'+\eta_c^B\Gamma_p')(\Gamma_c'+\Gamma_p')+\omega^2\eta_p^B-i\omega(\eta_p^B-\eta_c^B)\Gamma_p'}{\omega^2+(\Gamma_c'+\Gamma_p')^2}, \quad (15d)$$

where $\eta_i^B=\kappa_i/\gamma_i$, for $i=c,p$, denotes the Beer's absorption coefficient of the symmetric transitions from the lower states to the upper state $|i\rangle$. We notice that, if Ω_p^+ is equal to zero, or very small with respect to Ω_c^+ and to the upper state decay rates, then the coefficients B and C vanish and the evolution of $\hat\Omega_c^-$ and $\hat\Omega_p^-$ is diagonal. Thus, only for large values of Ω_p^+ a mutual interaction between the perturbation fields Ω_c^- and Ω_p^- is produced. This interaction will be discussed in the next section.

The solutions to Eqs. (14) are combinations of two normal modes $\exp\{-\eta_1\zeta\}$ and $\exp\{-\eta_2\zeta\}$,

$$\hat\Omega_c^-(\zeta,\omega)=a_1(\omega)e^{-\eta_1(\omega)\zeta}+a_2(\omega)e^{-\eta_2(\omega)\zeta}, \quad (16a)$$

$$\hat\Omega_p^-(\zeta,\omega)=b_1(\omega)e^{-\eta_1(\omega)\zeta}+b_2(\omega)e^{-\eta_2(\omega)\zeta}, \quad (16b)$$

with the propagation coefficients η_1 and η_2 satisfying the condition

$$\eta^2-(A+D)\eta+AD-BC=0\,. \quad (17)$$

The coefficients a_i and b_i ($i=1,2$) are determined by imposing that $\hat\Omega_c^-$ and $\hat\Omega_p^-$ satisfy Eqs. (14) with the boundary conditions assigned in $\zeta=0$.

By solving Eq. (17), we obtain the following expressions for η_1 and η_2:

$$\eta_1=\frac{1}{2}[A+D+\sqrt{(A+D)^2-4(AD-BC)}], \quad (18a)$$

$$\eta_2=\frac{1}{2}[A+D-\sqrt{(A+D)^2-4(AD-BC)}]. \quad (18b)$$

We assume that all the relevant Fourier frequencies are sufficiently small, such that $|\omega|\ll\Gamma_c'$. At first order in ω/Γ_c', the propagation coefficients are given by

$$\eta_1\simeq\frac{1}{\zeta_1}-\frac{i\omega}{u_1}+O\left[\left(\frac{\omega}{\Gamma_c'}\right)^2\right], \quad (19a)$$

$$\eta_2\simeq-\frac{i\omega}{u_2}+O\left[\left(\frac{\omega}{\Gamma_c'}\right)^2\right], \quad (19b)$$

where the length ζ_1 and the velocity parameters u_1 and u_2 are introduced as

$$\zeta_1=\frac{\Gamma_p'+\Gamma_c'}{\eta_p^B\Gamma_c'+\eta_c^B\Gamma_p'}, \quad (20a)$$

$$u_1=\frac{(\Gamma_p'+\Gamma_c')^2}{(\eta_p^B-\eta_c^B)^2}\left(\frac{\eta_p^B}{\Gamma_p'}+\frac{\eta_c^B}{\Gamma_c'}\right), \quad (20b)$$

$$u_2=\frac{\Gamma_p'}{\eta_p^B}+\frac{\Gamma_c'}{\eta_c^B}. \quad (20c)$$

The real and imaginary parts of η_1 and η_2 represent the absorption and dispersion coefficients, respectively, associated to the two propagation normal modes. At first order $\mathcal{R}\{\eta_1\}$ is constant in ω and describes a uniform damping, with characteristic length ζ_1, of all Fourier components, while $\mathcal{I}\{\eta_1\}$ is linear in ω and determines the slowing down of the group velocity: the more Beer's coefficients η_c^B and η_p^B differ from each other, the larger this dispersive term is. For $\zeta\gg\zeta_1$ only the second mode survives. At lowest order

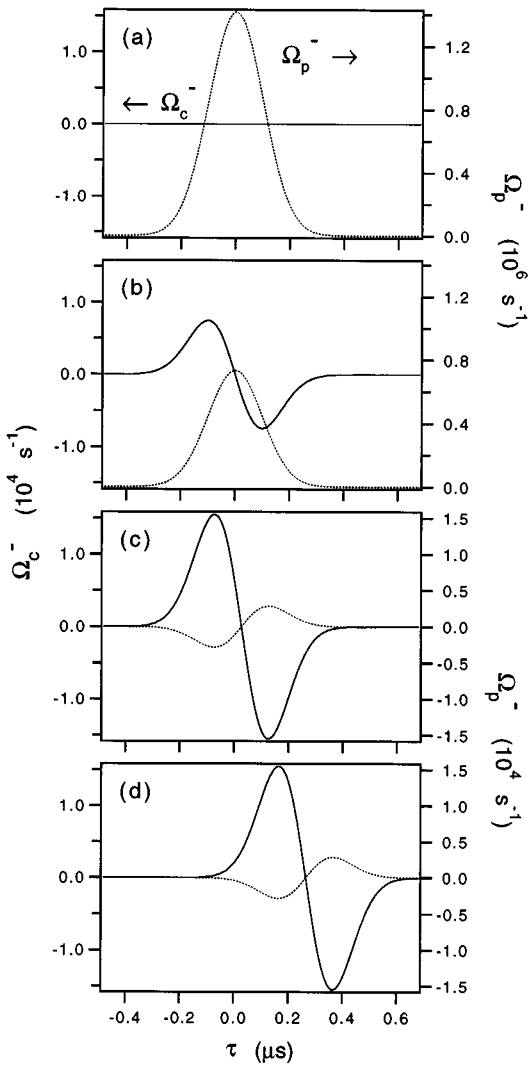

FIG. 2. Time dependence of the fields Ω_c^- (solid curves, left axes) and Ω_p^- (dotted curves, right axes) from Eqs. (21), for different penetration lengths within the medium: (a) $\zeta\eta_p^B=0$, (b) $\zeta\eta_p^B=1$, (c) $\zeta\eta_p^B=50$, and (d) $\zeta\eta_p^B=500$. The parameter values used in the calculation are $\Gamma_c=1.0\times10^8\ \mathrm{s}^{-1}$, $\Gamma_p=9.3\times10^7\ \mathrm{s}^{-1}$, $\kappa_c=3.9\times10^8\ \mathrm{cm}^{-1}\ \mathrm{s}^{-1}$, $\kappa_p=1.5\times10^9\ \mathrm{cm}^{-1}\ \mathrm{s}^{-1}$. For the fields Ω_c^+ and Ω_p^+ the constant value $\Omega_c^+=\Omega_p^+=1.41\times10^8\ \mathrm{s}^{-1}$ has been assumed. The corresponding extinction length for the first normal mode is $\zeta_1\eta_p^B=1.51$. In (c) and (d) the scale on the right axes is expanded by two orders of magnitude with respect to (a) and (b).

η_2 is purely imaginary and linear in ω and describes a dispersive and shape-invariant propagation. In the laboratory frame, the group velocity v_i associated to the ith mode, for $i=1,2$, is defined by $1/v_i=1/c+1/u_i$. The higher-order terms in the expansions of Eqs. (19) represent nonlinear corrections to the absorption and dispersion coefficients and are responsible for small effects of group-velocity dispersion and selective absorption of high-frequency Fourier components.

IV. AMPLITUDE CORRELATION AND FORMATION OF ADIABATONS

In this section we illustrate the normal-mode solution derived above and show how it accounts for phenomena of correlation between the amplitudes of the different field components and for the formation and propagation of adiabatons. The solution is discussed, in Sec. IV A, in the dressed-atom representation, i.e., for the fields Ω_c^- and Ω_p^-, and in Sec. IV B for the original fields α_c, β_c, α_p, and β_p in the bare-atom representation. The analytical results obtained in Sec. III only apply for a limited choice of time distributions of the amplitudes of the input fields, namely, when the field variables Ω_c^+ and Ω_p^+ are independent of time. An example of time distributions that do not meet this requirement is considered in Sec. IV C. In that case, the field evolution is computed numerically, but is still understood in terms of the mechanisms previously described analytically.

A. Field evolution in the dressed-atom representation

We first use Eqs. (16) to describe the evolution of input fields $\Omega_c^-(0,\tau)$ and $\Omega_p^-(0,\tau)$ of the form shown in Fig. 2(a): $\Omega_c^-(0,\tau)$ is taken equal to zero, so that $\Omega_p^-(0,\tau)$ represents the only nonvanishing but small perturbation to the CPT steady state of the system. The fields Ω_c^+ and Ω_p^+ are assumed independent of time. For the assigned boundary conditions, the solution given in Eqs. (16), transformed back into the time domain, reads

$$\Omega_c^-(\zeta,\tau)=-\mathcal{F}\left[\frac{\partial}{\partial\tau}\Omega_p^-\left(0,\tau-\frac{\zeta}{u_1}\right)\exp\left\{-\frac{\zeta}{\zeta_1}\right\}-\frac{\partial}{\partial\tau}\Omega_p^-\left(0,\tau-\frac{\zeta}{u_2}\right)\right], \tag{21a}$$

$$\Omega_p^-(\zeta,\tau)=\Omega_p^-\left(0,\tau-\frac{\zeta}{u_1}\right)\exp\left\{-\frac{\zeta}{\zeta_1}\right\}+\mathcal{G}\left[\frac{\partial}{\partial\tau}\Omega_p^-\left(0,\tau-\frac{\zeta}{u_1}\right)\exp\left\{-\frac{\zeta}{\zeta_1}\right\}-\frac{\partial}{\partial\tau}\Omega_p^-\left(0,\tau-\frac{\zeta}{u_2}\right)\right], \tag{21b}$$

with

$$\mathcal{F}=\frac{\eta_c^B}{\eta_p^B\Gamma'_c+\eta_c^B\Gamma'_p}\,\frac{\Omega_p^+}{\gamma_p}, \tag{22a}$$

$$\mathcal{G}=\frac{\eta_c^B(\eta_p^B-\eta_c^B)\Gamma'_p}{(\eta_p^B\Gamma'_c+\eta_c^B\Gamma'_p)^2}. \tag{22b}$$

This solution is plotted as a function of τ, for three different positions ζ, in Figs. 2(b)–2(d). For strong values of Ω_p^+, the spatial evolution of the perturbation fields Ω_c^- and Ω_p^- is determined by a combination of the two normal modes. This renders the two fields coupled to each other. For instance, in the situation examined here, we see from Eqs. (21) and from Fig. 2 that Ω_c^-, initially equal to zero, builds up from the absorption of Ω_p^- during the transient of decay of the first mode. In general, for $\zeta\gg\zeta_1$ the ratio between the fields Ω_c^- and Ω_p^- becomes independent of ζ. In the bare-atom picture this corresponds, as exemplified in Sec. IV B, to a correlation between the amplitudes of the different field components. In Figs. 2(c) and 2(d) the first propagation mode is extinguished and the field evolution, purely determined by the second one, corresponds to a dispersive and shape-invariant propagation.

In the adiabatic limit, when the rate of change of the perturbation fields is completely negligible with respect to Ω_c^+ and Ω_p^+, Eqs. (21) simplify to $\Omega_c^-(\zeta,\tau)=0$ and $\Omega_p^-(\zeta,\tau)=\Omega_p^-(0,\tau)\exp\{-\zeta/\zeta_1\}$. This also applies when the field Ω_p^+ is negligibly small. In these cases, for $\zeta \gg \zeta_1$ both perturbation fields Ω_c^- and Ω_p^- are equal to zero so that, as shown in Sec. II, a steady propagation regime with matched pulses is established. In this sense the exponential decay of the first normal mode, with characteristic length ζ_1, corresponds to the spatial transient of pulse matching.

The second mode, corresponding to the last term on the right-hand side of both of Eqs. (21), describes the form-stable and delayed propagation typical of the adiabatons, introduced in [21] for the Λ system. As first pointed out in [22], the invariance of the pulse shapes, in this kind of propagation, is an approximate result, which holds when the relevant Fourier frequencies of the perturbation fields are sufficiently small. In the present analysis, apart from the condition $|\Omega_c^-|,|\Omega_p^-| \ll |\Omega_c^+|,|\Omega_p^+|$, the shape invariance follows from the first-order truncation in the expansion of Eqs. (19). Actually, in the propagation of the adiabatons, absorption and group velocity dispersion are small effects, but not negligible over very long propagation distances. These effects cannot be observed within the distances considered in Fig. 2.

B. Field evolution in the bare-atom representation

From the solution for Ω_c^- and Ω_p^- given in Eqs. (21) and displayed in Fig. 2, by applying the formulas of Eqs. (3), we find the corresponding solution for the original fields α_c, β_c, α_p, and β_p, shown in Fig. 3. In Fig. 3(a), the fields α_c and β_c in $\zeta=0$ are independent of time and equal to each other, while, in Fig. 3(b), α_p and β_p are given by opposite-signed modulations superimposed to strong and equal continuous components: such time distributions for the original fields correspond to those of Fig. 2(a) for the transformed fields Ω_c^- and Ω_p^-, with Ω_c^+ and Ω_p^+ constant in τ. Figures 3(c) and 3(d) show that, as Ω_c^- develops at the expense of Ω_p^- [cf. Fig. 2(b)], the modulations of the incident fields α_p and β_p are transmitted to the initially flat fields α_c and β_c. In Figs. 3(e)–3(h), the behavior of the fields is shown after the first propagation mode has died out. From Eqs. (21) we see that the second mode in the evolution of both Ω_c^- and Ω_p^- is described by terms proportional to the time derivative of Ω_p^- evaluated in $\zeta=0$. In the present case, those terms are of comparable strength and, at any time τ, much smaller than the values assumed by Ω_p^- in $\zeta=0$. This appears from Figs. 2(c) and 2(d), where the fields Ω_c^- and Ω_p^- are shown after the full absorption of the first mode, when only the contribution of the second one is present: in those figures the amplitudes of Ω_c^- and Ω_p^- are two orders of magnitude smaller than the initial amplitude of Ω_p^-, in Fig. 2(a). Nevertheless, as shown in Fig. 3(e), the second mode in Ω_c^- results in a relatively strong modulation of the fields α_c and β_c, dependent on Ω_c^- through a time integral [cf. Eqs. (3a) and (3b)]. On the contrary, the second mode in Ω_p^- affects α_p and β_p very weakly. Thus, for $\zeta \gg \zeta_1$, Ω_p^- is negligible and Eq. (8) is approximately satisfied. However, that equation does not describe, here, a condition of shape matching for the fields α_p and β_p, as the ratio α_c/β_c is time dependent. Instead, it is nearly obeyed with $\alpha_c(\zeta,\tau)\approx\alpha_p(\zeta,\tau)$ and $\beta_c(\zeta,\tau)\approx\beta_p(\zeta,\tau)$. The conservation of Ω_c^+ and Ω_p^+ along ζ forces the modulations in the field pairs α_c-β_c and α_p-β_p to have opposite signs. In the bare-atom representation, the formation of modulations with complementary amplitudes in the time profiles of the fields is a feature of the adiabatons [21,22]. The comparison between the curves in Figs. 3(e) and 3(f) and those in Figs. 3(g) and 3(h) evidences that, after the absorption of the first mode, such modulations propagate simultaneously with reduced group velocity, nearly preserving their shapes for long penetration distances.

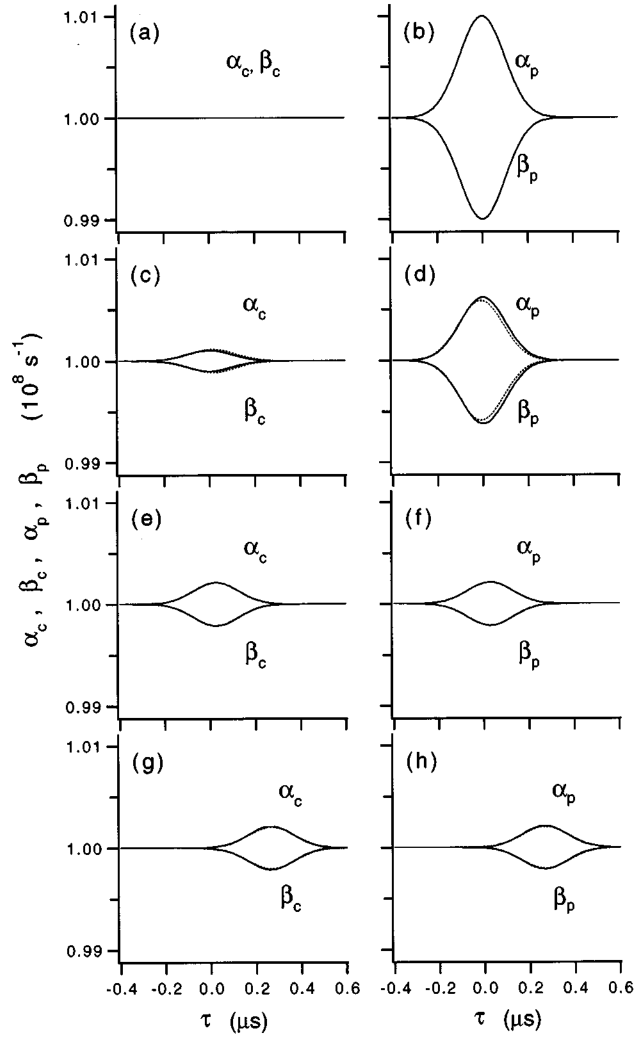

FIG. 3. Spatio-temporal evolution of the original fields α_c-β_c (left column) and α_p-β_p (right column) corresponding to that of the transformed fields Ω_c^- and Ω_p^- in Fig. 2. The time profiles of the fields are shown in (a) and (b) $\zeta\eta_p^B=0$, (c) and (d) $\zeta\eta_p^B=1$, (e) and (f) $\zeta\eta_p^B=50$, and (g) and (h) $\zeta\eta_p^B=500$. Exact numerical results (dotted curves) are shown together with the analytical results (solid curves).

The generation of the field Ω_c^- and, in general, the coupling between the two perturbation fields can be regarded as a phenomenon of nonlinear mixing between the ''pump

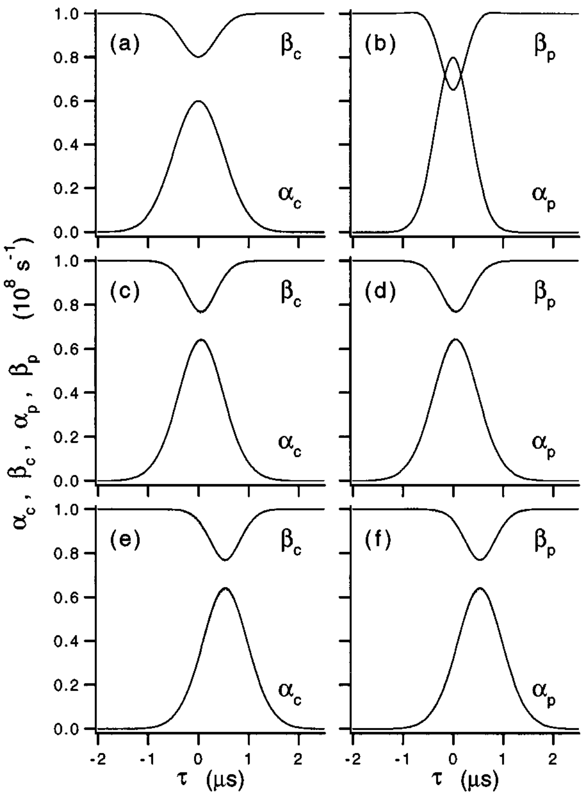

FIG. 4. Time dependence of the field pairs α_c-β_c (left column) and α_p-β_p (right column) in (a) and (b) $\zeta\eta_p^B=0$, (c) and (d) $\zeta\eta_p^B=50$, and (e) and (f) $\zeta\eta_p^B=500$. Numerical (dotted curves) and analytical (solid curves) results are almost indistinguishable. Parameters as in Fig. 3.

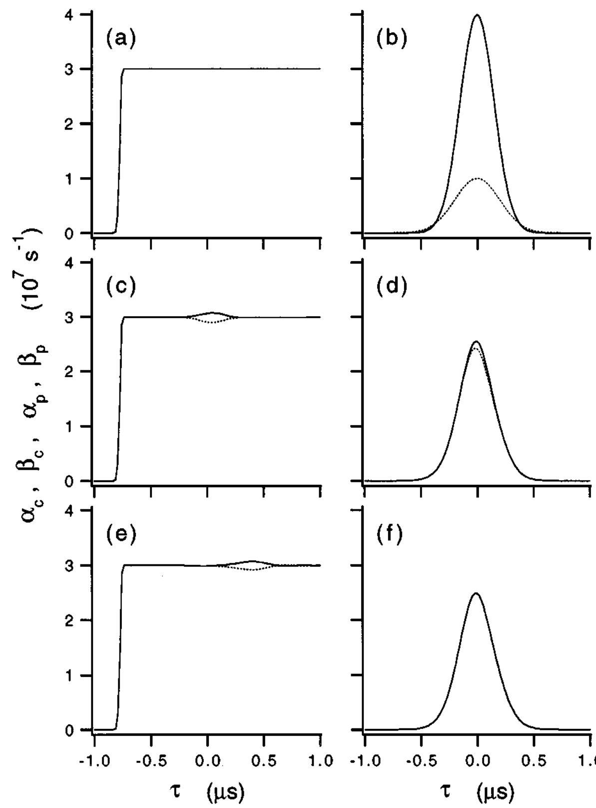

FIG. 5. Time profiles of the fields α_c, β_c (left column, solid and dotted curves, respectively), α_p, and β_p (right column, solid and dotted curves) from the numerical solution of the Maxwell-Bloch equations, at different penetration lengths: (a) and (b) $\zeta\eta_p^B=0$, (c) and (d) $\zeta\eta_p^B=50$, and (e) and (f) $\zeta\eta_p^B=500$.

waves'' Ω_c^+ and Ω_p^+ and the ''probe waves'' Ω_c^- and Ω_p^-. In the bare-atom picture, a consequence of this mixing is the possibility to transfer amplitude modulations from one pair of fields to another. We remind the reader that, in the considered $F=1\rightarrow F=1$ interaction scheme, the field components α_c and β_c, as well as α_p and β_p, interacting with frequency-degenerate transitions from a pair of lower states to an upper state, must have an opposite circular polarization. In the situation shown in Figs. 3(a) and 3(b), α_c and β_c have flat time profiles, so that the polarization of the total field at the frequency of the transitions to the upper state $|c\rangle$ is constant in time, while the complementary modulations in the profiles of α_p and β_p correspond to a modulation in the polarization of the total field at the frequency of the transitions to the state $|p\rangle$. Thus, the adiabatons shown in Fig. 3, in the form of complementary-shaped amplitude modulations in the field pairs α_c-β_c and α_p-β_p, correspond to ''polarization adiabatons'' in the total fields at the two transition frequencies.

To check the validity of our approximate analysis, we have examined the evolution of the input fields shown in Figs. 3(a) and 3(b), using the whole set of Maxwell-Bloch equations. Numerical and analytical solutions, plotted together in Figs. 3(c)–3(h), show an excellent agreement.

In Figs. 4(a) and 4(b) different time distributions for the input fields in the bare-atom picture, still satisfying the conditions $\Omega_c^+(0,\tau)=\Omega_c^+(0,0)$ and $\Omega_p^+(0,\tau)=\Omega_p^+(0,0)$, are shown. With these input fields both Ω_c^- and Ω_p^- result, in $\zeta=0$, different from zero. Moreover, as the field α_c vanishes for $\tau\rightarrow-\infty$, while β_c assumes a constant value different from zero, the nonabsorbing state $|NA\rangle$ of Eq. (1a) coincides, at the beginning of the interaction, with the bare atomic state $|1\rangle$. The spatio-temporal evolution of the field amplitudes, evaluated both analytically and numerically, is illustrated in Figs. 4(c)–4(f). As noticed above, the absorption of the first propagation normal mode establishes a condition of correlation between the field components, in the form of a constant ratio between Ω_c^- and Ω_p^-. Also in this case, such a condition results in a negligibly small amplitude for Ω_p^-, compared to the amplitudes of all original fields, so that Eq. (8) is nearly satisfied, for $\zeta\gg\zeta_1$, with $\alpha_c(\zeta,\tau)\approx\alpha_p(\zeta,\tau)$ and $\beta_c(\zeta,\tau)\approx\beta_p(\zeta,\tau)$. Thus, the correlation between the field amplitudes arising from the extinction of the first mode leads again to adiabatons with approximately matched shapes.

C. Propagation of fields with a finite duration

Here we consider, in the bare-atom representation, the propagation of the input fields shown in Figs. 5(a) and 5(b). At the beginning of the interaction the field components α_c and β_c, in Fig. 5(a), are matched and, apart from their rising edge, constant in time, while α_p and β_p, in Fig. 5(b), have different time profiles and finite lengths. The analytical results derived in the preceding section do not apply in this case, since the variables Ω_c^+ and Ω_p^+ are time dependent and the condition $|\Omega_p^-(0,\tau)| \ll |\Omega_c^+|,|\Omega_p^+|$ is not strictly fulfilled. We have calculated the evolution of the fields and the atomic variables numerically from the Maxwell-Bloch equations, with the assumption that the atoms have been prepared, for any ζ, in the pure state $|NA\rangle$. Parameter values relative to the double-Λ system formed by the states $|1\rangle = |5\,^2S_{1/2}F = 1\, m_F = 1\rangle$, $|2\rangle = |5\,^2S_{1/2}F = 1\, m_F = -1\rangle$, $|p\rangle = |5\,^2P_{3/2}F = 1\, m_F = 0\rangle$, and $|c\rangle = |6\,^2P_{3/2}F = 1\, m_F = 0\rangle$ of ^{87}Rb atoms have been assumed: $\Gamma_c = 8.93\times10^6$ s^{-1}, $\Gamma_p = 3.77\times10^7$ s^{-1}, $\omega_c = 4.5\times10^{15}$ s^{-1}, $\omega_p = 2.4\times10^{15}$ s^{-1}, and $N = 10^{10}$ cm^{-3}, corresponding to $\kappa_c = 3.74\times10^7$ cm^{-1} s^{-1} and $\kappa_p = 5.54\times10^8$ cm^{-1} s^{-1}.

To a certain extent, the situation considered here is similar to that illustrated in Fig. 3: at the entry into the medium the field variable Ω_c^- is equal to zero, α_c and β_c being identical to each other, while Ω_p^- is different from zero because of the shape mismatch between α_p and β_p. However, an important difference from the case of Fig. 3 is that here the fields α_p and β_p are taken as pulses with a finite duration, so that Ω_p^+ is different from zero only within a certain time interval. The time profiles of the field components are shown in Figs. 5(c)–5(f), for different penetration depths within the medium. Also in this case the basic mechanisms pointed out in our previous analysis govern the evolution of the system. The absorptive mode in the field propagation is rapidly extinguished. As a consequence, the mismatch between the field components α_p and β_p is strongly reduced, while modulations build up in the initially flat profiles of α_c and β_c, giving rise to a pair of complementary-shaped adiabatons. This behavior is illustrated in Figs. 5(c) and 5(d) and is analogous to that shown in Figs. 3(c) and 3(d). The remaining dispersive mode, whose group velocity is slower than the velocity c of the light in the nonresonant medium, determines the further evolution of the generated adiabatons. In terms of the transformed fields, in the dressed representation, the adiabatons typically appear, as seen in Figs. 2(c) and 2(d), as time-dependent structures, localized in time, in the profiles of the fields Ω_c^- and Ω_p^- that are, elsewhere, equal to zero. In the present case, such adiabatons accumulate longer and longer delay with respect to Ω_p^+, which has a finite length along τ and travels at velocity c. Eventually, both Ω_c^- and Ω_p^- vanish within the whole time interval of interaction of Ω_p^+. As a consequence, as predicted by the relation in Eq. (8), the original fields α_p and β_p become exactly matched in shape. Moreover, with Ω_c^- and Ω_p^- equal to zero, the fields α_p and β_p become proportional to the field variable Ω_p^+ and travel, like Ω_p^+, at velocity c without any further absorption or dispersion. On the other hand, the fields α_c and β_c, having an infinite length, can support the delayed propagation of the pair of adiabatons developed on their profiles. This situation, with a pair of matched pulses on one pair of transitions and a pair of adiabatons on the other, is depicted in Figs. 5(e) and 5(f). If the fields α_c and β_c are regarded as pulses with long but finite duration, then it turns out that the adiabatons slip through the entire length of these pulses and vanish after reaching the falling edge. Thus, finally, both of the pulse pairs α_c-β_c and α_p-β_p become matched and the CPT steady state of the atom-field system, perturbed by the initial nonzero value of the field Ω_p^-, is restored.

V. SUMMARY AND CONCLUSIONS

We have analyzed the propagation of strong resonant fields through a double-Λ four-level atomic system. We have shown that the basis of adiabatic dressed states used in [22] for the single-Λ is also convenient to describe the double-Λ system. In that basis it is immediately seen that, when both pairs of allowed transitions are driven by a pair of matched fields, then the atomic system is clamped, by CPT, in a coherent superposition of lower-energy states. Therefore, if the medium is initially prepared in such a superposition, the matched fields propagate freely at the speed of light in the nonresonant medium, and represent a stationary state for the atom-field system. Under conditions of quasiadiabatic perturbation of this steady state, the spatio-temporal evolution of the system has been investigated analytically. Our approximate analysis is confirmed by the exact numerical solution of the Maxwell-Bloch equations.

All the dynamics of the system is determined by the evolution of the field variables Ω_c^- and Ω_p^- that, in the dressed basis, represent the weak couplings of the CPT state. A perturbation approach has been used to linearize the propagation equations of such field components. Those equations are easily solved in terms of two normal modes. One of these modes, which corresponds, in the adiabatic limit, to the transient of pulse matching, is mainly absorptive and is extinguished after a relatively short penetration length of the fields inside the medium. The second mode survives the first one, because its absorption losses are very small. It describes a quasiform-stable and dispersive propagation, typical of the adiabatons and, in general, of EIT.

It may be supposed that an experimental realization of pulse propagation in media with very-many absorption lengths could be complicated due to the transverse distribution of the fields, which has been ignored throughout this work. Note, however, that the superposition $|NA\rangle$ is not dipole-connected to the excited states. As a consequence, when the population is trapped in $|NA\rangle$, saturation effects caused by the intensity-dependent atomic susceptibility, which usually arise with intense laser fields tuned near transition resonances, are eliminated [8]. Thus, the coherent trapping allows laser beams with a transverse spatial structure to propagate without distortions. In effect, an experimental demonstration that the CPT can be used for suppressing optical self-focusing and defocusing has been given in [8]. Moreover, high quality beam propagation in a CPT configuration has been reported in [7]. Nonlinear effects like bleaching and self-focusing may be important in the phase of preparation of the state $|NA\rangle$. Different methods of preparation are required, depending on the initial conditions of the atomic medium. If the population lies initially in an incoherent superposition of both states $|1\rangle$ and $|2\rangle$, then the preparation is achieved by optical pumping [24]. In this case, nonlinear distortions of the preparing fields can be minimized by rendering the characteristic time for the preparation of the superposition $|NA\rangle$ as short as possible, that is, by choosing very fast relaxation rates for the upper states. On the other hand, if all atoms are initially in a unique ground state, then the coherent trapping can be attained by employing the technique of the stimulated Raman adiabatic passage. In Refs. [7,8], it has been shown that, under STIRAP conditions, the phase of preparation of the trapping state does not modify the transverse profile of the propagating pulses. A complete analysis of the different processes that occur during the transient of preparation, taking into account the transverse distributions of the fields, is still needed.

In conclusion, we have obtained an approximate propagation law for time-dependent fields interacting with double-Λ systems, under quasiadiabatic conditions. It includes and generalizes the description of the process of pulse matching and of the propagation of the adiabatons. Both of these phenomena have been individually studied in Λ systems, in several papers [12,13,16,19,21,22]. Our propagation law, expressed by Eqs. (14), can be easily adapted to a single-Λ system, by equating the proper field variables, along with the corresponding coupling coefficients, to zero: in this way most results derived in those papers are recovered. For instance, the well-known features of the pulse matching in the propagation of a pair of weak pulses, say α_p and β_p, through a Λ system externally prepared in a coherent superposition of lower states [13,16], can be obtained from Eqs. (14) if α_c and β_c are regarded as constant coefficients, i.e., Ω_c^- is considered identically equal to zero, and if κ_c is also taken equal to zero. On the other hand, if we consider $\Omega_p^+,\Omega_p^-\equiv 0$ and $\kappa_p=0$, we obtain, for the propagation of adiabatons through a Λ system, the same description as in Ref. [22].

Our analysis predicts correlation phenomena, peculiar of the double-Λ system, arising from the interaction between the different field components. These phenomena take place because the evolution of the field couplings is determined by a combination of the ''pulse-matching mode'' and the ''adiabaton-type mode.''

Finally, it has been pointed out that, as concerns the polarization dynamics, the four-level schemes are more flexible and versatile than the three-level ones. The examples illustrated in Figs. 3 and 5 show that, in the double-Λ configuration, it is possible to generate a quasiform-stable modulation in the polarization of the total field at one transition frequency, by modulating the polarization of the total field at another frequency. This effect cannot be achieved in a single-Λ scheme.

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[28] The pulse matching is not necessarily required to satisfy the conditions $\Omega_c^-,\Omega_p^-\equiv 0$. A stationary solution is also obtained in the trivial situation when either α_c and α_p or β_c and β_p are equal to zero and the state $|NA\rangle$ coincides with one of the two low-lying states of the bare atoms.