Generation of atomic-squeezed states in an optical cavity with an injected squeezed vacuum

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We study the generation of atomic-squeezed states in an optical cavity. The cavity encloses a pair of two-level atoms and is coupled to a broadband squeezed vacuum. Using the bad-cavity limit, cavity field is adiabatically eliminated to obtain the equations of motion for collective atomic operators. These equations are then employed to obtain the atomic density-matrix elements and to study the generation of atomic-squeezed states in the steady state. To characterize atomic-squeezed states we use the squeezing parameter defined recently by Wineland *et al.* [Phys. Rev. A **46**, 6797 (1992)] for spectroscopic purposes. The aforementioned parameter is also compared with the squeezing parameter obtained from the uncertainty principle satisfied by the collective atomic operators. We show that in a cavity arrangement, unlike free space, atomic-squeezed states can be generated only for a restricted range of values of the parameters λ and N which characterize the cavity and the squeezed field driving the cavity, respectively. [S1050-2947(96)03112-5]

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I. INTRODUCTION

The interaction of a collection of two-level atoms with a broadband squeezed vacuum field has recently been studied by many authors and several interesting phenomena have been predicted. Palma and Knight [1] were the first to study the spontaneous emission dynamics of two two-level atoms embedded in a broadband squeezed vacuum. They showed that for a squeezed vacuum tuned to the atomic resonance, the steady state of the atomic system is far from thermal equilibrium. Specifically, for a minimum uncertainty squeezed vacuum the final atomic state is a highly correlated pure state. This state is a superposition of the collective ground state and the collective most excited state. These states are the atomic counterparts of the two-mode squeezed state of an electromagnetic field. Thus they have been termed as two-atom squeezed states [1] or pairwise atomic states [2] or atomic-squeezed states [3] (the name that we use in this paper). The work of Palma and Knight was further extended for many-atom systems by Agarwal and Puri [4] to include the effect of the coherent pumping of the atoms. They demonstrated that atomic-squeezed states can be generated only for certain discrete values of the pumping field strength.

It has already been pointed out [5] that the experimental verification of the above predictions is difficult since it requires squeezing of all the field modes that are interacting with the atoms. In free space this would correspond to having a squeezed field incident on the atoms from the entire 4π solid angle surrounding the atoms; this is impossible to achieve from the present sources of squeezed light. Moreover, this also does not leave any unsqueezed window which is essential for the observation of the fluorescence photons. Thus it has been suggested to perform such experiments by placing the atoms either in a microscopic Fabry-Pérot cavity [5] or a much bigger optical cavity (which is being used for experiments testing the predictions of cavity QED [6]), and injecting the squeezed vacuum through one of the lossy output coupling mirrors. We note that theoretical studies of the resonance fluorescence spectrum of a single atom in optical cavities with an injected squeezed vacuum indicate that all the features of free space still persist in the cavity environment [7,8]. In addition, the cavity environment also gives rise to many different phenomena.

In this paper we investigate the possibility of generating atomic-squeezed states in an optical cavity driven by a squeezed vacuum. For this purpose we study the dynamics of a pair of two-level atoms placed in a single mode optical cavity with a broadband squeezed vacuum being injected into the cavity through one of the lossy output mirrors. In particular, we use the over damped cavity model recently proposed by Rice and Pedrotti [8].

The atomic-squeezed states can be characterized in many different ways [3]. In our study we employ the squeezing parameter derived on the basis of usual consideration of the uncertainty principle (we denote this parameter by ξ_N), as well as its modified form (ξ_R) given by Wineland *et al.* [3] for population spectroscopy purposes. We discuss more about these squeezing parameters in Sec. II.

The organization of this paper is as follows. In Sec. II we discuss the model considered in this paper and present the equations of motion for atomic operator averages. The detail derivation of these equations are given in Appendix A. In Sec. III we discuss the results and the conclusion in Sec. IV.

II. DYNAMICS OF THE ATOM-CAVITY SYSTEM IN BAD-CAVITY LIMIT

A. Model

We consider a system of two identical two-level atoms placed in a single mode optical cavity. It is coupled to a broadband squeezed reservoir through the lossy output mirror of the cavity. The master equation for the atom plus cavity-mode density operator ρ is ($\hbar=1$)

$$\frac{d\rho}{dt}=-i[H,\rho]+L_a\rho+L_{sq}\rho. \quad (1)$$

The Hamiltonian H contains free atomic and cavity-mode evolution, and the interaction of the atoms with the cavity mode. It is given by

$$H=\omega_0 S_z+\omega a^\dagger a+g(S^+a+S^-a^\dagger). \tag{2}$$

In Eqs. (1) and (2), $S^\pm$ and S_z are collective atomic spin operators, $a^\dagger$ and a are the creation and annihilation operators for the cavity mode, ω_0 and ω denote resonant frequencies of the atoms and the cavity mode, respectively, and g is the atom-cavity-mode coupling constant. In the present study we will consider the resonant case $\omega=\omega_0$.

The term $L_a\rho$ and $L_{sq}\rho$ denote, respectively, the dissipation from the atoms via spontaneous emission in the background modes, and from the cavity mode which is coupled to a broadband squeezed vacuum. They are given as [7,8]

$$L_a\rho=\gamma(2S^-\rho S^+-S^+S^-\rho-\rho S^+S^-), \tag{3}$$

$$\begin{aligned}L_{sq}\rho=&\kappa(N+1)(2a\rho a^\dagger-a^\dagger a\rho-\rho a^\dagger a)+\kappa(N)(2a^\dagger\rho a\\&-aa^\dagger\rho-\rho aa^\dagger)+\kappa Me^{i\theta}(2a^\dagger\rho a^\dagger-a^{\dagger 2}\rho-\rho a^{\dagger 2})\\&+\kappa Me^{-i\theta}(2a\rho a-a^2\rho-\rho a^2).\end{aligned} \tag{4}$$

Here, γ denotes the atomic decay rate to all modes other than the privileged cavity mode, κ is the decay rate of the cavity electric field, N and $M=|M|e^{i\theta}$ characterize the broadband squeezed vacuum injected into the cavity, such that, $M^2\leqslant N(N+1)$. The equality condition holds for the minimum uncertainty squeezed states (MUS). The phase of the squeezed vacuum is denoted by θ.

The Hamiltonian above [Eq. (2)] is derived by assuming that the distance between the two atoms is much smaller than the resonant wavelength (Dicke model). Consequently, the cooperative decay rate is identical to that of the individual atomic decay rate. The collective atomic system is represented by the collective atomic states $|S,m\rangle$ which are eigenstates of the total spin operators $S^2=(1/2)(S^+S^-+S^-S^+)+S_z^2$ and S_z. In this representation the two-atom system Dicke model is equivalent to a three-level cascade system. These three levels correspond to the state $|S,m\rangle$ with $S=1$ and $m=0,\pm1$, and are defined as [9]

$$\begin{aligned}|2\rangle&=|1,1\rangle=|e_1\rangle|e_2\rangle,\\|1\rangle&=|1,0\rangle=\frac{1}{\sqrt{2}}\,(|e_1\rangle|g_2\rangle+|e_2\rangle|g_1\rangle),\\|0\rangle&=|1,-1\rangle=|g_1\rangle|g_2\rangle,\end{aligned} \tag{5}$$

where $|e_i\rangle$ and $|g_i\rangle$ denote the excited and ground state of the ith atom, respectively. Later we use these states to represent the density matrix of the atomic system at steady state.

B. Bad-cavity limit

We derive the equation of motion for atomic operators under the condition of the bad-cavity limit $\kappa\gg g$ and $\kappa\gg\gamma$. The bad-cavity condition implies that the cavity-mode response to the squeezed reservoir is much faster than to that produced by its interaction with the atoms. Consequently, the atoms always see the cavity mode in the state induced by the squeezed reservoir. Thus one can eliminate the cavity-mode variables adiabatically. For this purpose we follow the approach of Ref. [8] and obtain equations of motion for atomic variables. We also assume that the strength of the dipole-dipole interaction is much smaller than the cavity damping rate κ and we neglect the dipole-dipole interaction term in this study. The equation of motion for relevant atomic operators can be written in a matrix form as (the details are given in Appendix A)

$$\frac{dY}{d\tau}=AY+L, \tag{6}$$

where $\tau=2\Gamma t$ and $\Gamma=g^2/\kappa$ denote the cavity induced decay rate of atoms; Y is a column vector with components

$$\begin{aligned}Y_1&=\langle S_1^+S_1^-+S_2^+S_2^-\rangle,\\Y_2&=\langle S_1^+S_2^-+S_1^-S_2^+\rangle,\\Y_3&=\langle S_1^+S_2^+e^{i\theta}+S_1^-S_2^-e^{-i\theta}\rangle,\\Y_4&=\langle S_1^+S_1^-S_2^+S_2^-\rangle,\end{aligned} \tag{7}$$

A is a real 4×4 matrix given by

$$A=\begin{bmatrix}-x & -l & 0 & 0\\-(2x-l) & -x & 2|M| & 4x\\4|M| & 2|M| & -x & -8|M|\\(x-l)/2 & (x-l)/2 & -|M| & -2x\end{bmatrix}, \tag{8}$$

and inhomogenous vector term L has components

$$\begin{aligned}L_1&=(x-l),\\L_2&=(x-l),\\L_3&=-2|M|,\\L_4&=0.\end{aligned} \tag{9}$$

In Eqs. (8) and (9), $x=2N+1+\lambda$, $l=1+\lambda$ where λ denotes the inverse of a single atom cooperativity parameter $C=g^2/\kappa\gamma$. Moreover, for the model employed S^2 is a conserved quantity. To write this conservation condition, we express $\langle S^2\rangle$ in terms of the Y vector as

$$\langle S^2\rangle=2+2Y_4+Y_2-Y_1. \tag{10}$$

In the present case $\langle S^2\rangle=S(S+1)=2$. By putting this in Eq. (10) we get

$$Y_1-Y_2=2Y_4. \tag{11}$$

Thus conservation of S^2 leads to a constraint condition given by Eq. (11). As a result of this, the number of independent variables in Eq. (6) reduces by one. We solve this set of three coupled equations in the next section for the steady state to calculate the atomic density matrix and squeezing parameters.

C. Squeezing parameters

The commutation relations for angular momentum operators lead to uncertainty relations between them. For example, one of these is given by

$$\Delta S_x \Delta S_y \geq |\langle S_z \rangle|/2, \tag{12}$$

where

$$(\Delta S_i)^2 = \langle S_i^2 \rangle - \langle S_i \rangle^2 \tag{13}$$

and

$$S_x = \frac{S^+ + S^-}{2}$$

$$S_y = \frac{S^+ - S^-}{2i}. \tag{14}$$

From these relations, it is natural to define atomic-squeezed states or squeezed-spin states [3] as states where $(\Delta S_i)^2 \langle |\langle S_j \rangle/2|$ for $i \neq j$. So a squeezing parameter for this definition may be written as

$$\xi_N = (\Delta S_i)/|\langle S_j \rangle/2|^{1/2} \quad i \neq j \in (x,y,z). \tag{15}$$

The atomic-squeezing condition in terms of this parameter is given by the relation $\xi_N < 1$. As stated in the Introduction, Agarwal and Puri calculated this quantity to characterize the atomic squeezing in Ref. [4]. Recently Wineland *et al.* [3] have given an operational definition of squeezing parameter ξ_R in the context of Ramsey spectroscopy. This is given by

$$\xi_R = \sqrt{2S}(\Delta S_y)/|\langle S_z \rangle|. \tag{16}$$

The quantity ξ_R is the ratio of the statistical uncertainty in the measurement of the resonance frequency determined by correlated states and uncorrelated states. Here also $\xi_R < 1$ signifies atomic squeezing.

III. RESULTS AND DISCUSSION

A. Atomic density matrix

The steady-state solutions of Eq. (5) are given by

$$Y_1 = \frac{(3x^3 - 4x^2 l - 12|M|^2 x + l^2 x + 16|M|^2 l)}{xD},$$

$$Y_2 = \frac{x^2 - l^2 - 4|M|^2}{D},$$

$$Y_3 = \frac{-8Ml^2}{xD},$$

$$Y_4 = \frac{(x^3 - 2x^2 l - 4|M|^2 x + l^2 x + 8|M|^2 l)}{xD}, \tag{17}$$

where

$$D = 3x^2 + l^2 - 12|M|^2. \tag{18}$$

First we use these solutions to find the steady-state density-matrix elements. For this purpose we write density operator ρ in $|S,m\rangle$ representation as

$$\rho = \sum_{ij} \rho_{ij} |i\rangle\langle j|, \tag{19}$$

where density-matrix elements ρ_{ij} are

$$\rho_{ij} = \langle i|\rho|j\rangle \quad i,j = 0,1,2. \tag{20}$$

To calculate the elements of the density matrix we use following well known identities [10]:

$$\langle S_1^+ S_1^- + S_2^+ S_2^- \rangle = 2\rho_{22} + \rho_{11},$$

$$\langle S_1^+ S_1^- S_2^+ S_2^- \rangle = \rho_{22},$$

$$\langle S_1^+ S_2^+ \rangle = \rho_{02},$$

$$\langle S_1^- S_2^- \rangle = \rho_{20} = \rho_{02}^*. \tag{21}$$

Now using Eqs. (17) and (21) we find following expressions:

$$\rho_{22} = \frac{(x^3 - 2x^2 l - 4|M|^2 x + l^2 x + 8|M|^2 l)}{xD},$$

$$\rho_{11} = \frac{x^2 - l^2 - 4|M|^2}{D},$$

$$\rho_{00} = 1 - \rho_{22} - \rho_{11}, \tag{22}$$

for population and

$$\rho_{02} = \rho_{20}^* = \frac{-4Ml^2}{xD} \tag{23}$$

for steady-state coherences. The density-matrix elements ρ_{10} and ρ_{12} are zero for the present model. For free space, density-matrix elements can be determined by putting $\lambda = 0$ in the above expressions and these are

$$\rho_{22} = \frac{(n^3 - 2n^2 - 4|M|^2 n + n + 8|M|^2)}{nD'},$$

$$\rho_{11} = \frac{n^2 - 1 - 4|M|^2}{D'},$$

$$\rho_{02} = \rho_{20}^* = \frac{-4M}{nD'}, \tag{24}$$

where $n = 2N + 1$ and

$$D' = (3n^2 + 1 - 12|M|^2). \tag{25}$$

Before we discuss the results we note that if the free space decay rate is replaced by the cavity induced decay rate, $\lambda = 0$ also corresponds to a closed cavity or an open cavity which can inhibit spontaneous emission of the atoms out the sides of the cavity. The latter is possible for certain semiconductor geometries [11].

It can be seen from Eq. (24) that for $|M|^2 = (1/4)(n^2 - 1)$ (MUS), the intermediate state $|1\rangle$ is not populated ($\rho_{11} = 0$). This effect has been explained in terms of an absorption process of the correlated pair of photons from the squeezed vacuum field [12]. The population of the other two collective states are given by

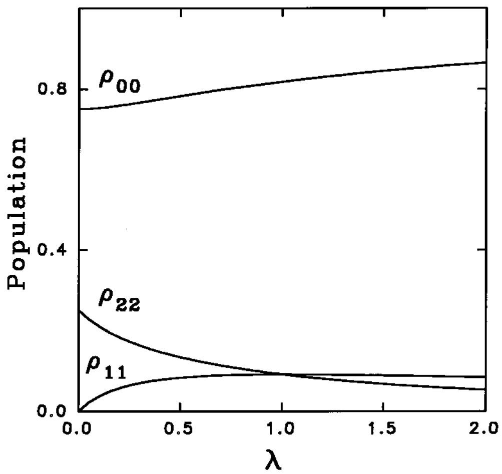

FIG. 1. Plot of population of collective atomic states vs λ for $N=0.5$.

$$\rho_{22}=\frac{N}{2N+1},$$

$$\rho_{00}=\frac{N+1}{2N+1}. \tag{26}$$

Thus the steady-state atomic population distribution in free space is nonthermal in nature as it does not satisfy the condition $\rho_{00}>\rho_{11}>\rho_{22}$. On the other hand, for atoms enclosed in an optical cavity ($\lambda\neq 0$), ρ_{11} is always nonzero signifying finite atomic population in the intermediate state. The nonzero population of the intermediate state is due to the inclusion of the extra decay channel of the atoms associated with the normal vacuum entering into the cavity from the open sides. However, we find that the nonthermal nature of population distribution is also present in the cavity arrangement. This is illustrated in Fig. 1 by plotting the population of the collective states as a function of λ for $N=0.5$ (the origin of Fig. 1 corresponds to the free space case). Figure 1 also shows that for $\lambda\neq 0$, nonthermal population distribution survives only if the parameter $\lambda\leqslant 1$. As λ increases, population of the lowest collective state ($|0\rangle$) approaches asymptotically unity and the population of the other two collective states ($|2\rangle,|1\rangle$) go to zero. This feature is characteristic of two atoms undergoing spontaneous emission in a normal vacuum. This observation is consistent with the fact that, $\lambda>1$ implies that the incoherent decay rate of atoms due to a normal vacuum is more than the decay rate due to coupling with a squeezed vacuum. Equation (23) and (24) also indicate that, both for free space and cavity arrangement the coherence term ρ_{02} is nonzero only if atoms are interacting with a squeezed vacuum. In fact, we find that finiteness of this term is essential for generations of atomic-squeezed states as this coherence term provides appropriate atomic correlation present in these states.

To characterize the final atomic state we also calculate $\mathrm{Tr}(\rho^2)$, which is defined as

$$\mathrm{Tr}(\rho^2)=\rho_{00}^2+\rho_{11}^2+\rho_{22}^2+2(|\rho_{01}|^2+|\rho_{12}|^2+|\rho_{02}|^2). \tag{27}$$

When $\mathrm{Tr}(\rho^2)=1$ the final atomic state is a pure state, whereas $\mathrm{Tr}(\rho^2)<1$ implies that the final atomic state is a mixed state.

Using Eqs. (22)–(25) with Eq. (27) we find that $\mathrm{Tr}(\rho^2)=1$ for $\lambda=0$ [12] and for $\lambda\neq 0$, $\mathrm{Tr}(\rho^2)<1$. Thus in free space or in a closed cavity the final atomic system is in a pure state in which only the ground and the most excited states are populated. On the other hand, in an open cavity the final atomic state is a mixed state with all three levels populated in accordance with Eq. (22). Nonetheless, we show that in a certain range of parameter space an appreciable amount of atomic squeezing can be generated in the cavity arrangement.

B. Squeezing

To calculate the squeezing parameter we first redefine S_x and S_y [Eq. (14)] as

$$S_x=\frac{S^+e^{i\theta/2}+S^-e^{i\theta/2}}{2}$$

$$S_y=\frac{S^+e^{i\theta/2}-S^-e^{i\theta/2}}{2i}. \tag{28}$$

This allows us to define quadrature atomic-squeezing operator with respect to the phase of the injected squeezed vacuum. We also define the rescaled squeezing parameter as

$$\eta_R=\xi_R^2-1 \tag{29}$$

and

$$\eta_N=\xi_N^2-1. \tag{30}$$

Now η_R and η_N less than zero signify collective atomic squeezing. Using steady-state solutions given by Eq. (17) we get η_R and η_N

$$\eta_R=[x(3x^2+l^2-12|M|^2)(4x^3-16|M|^2x-8|M|l^2) - D_1^2]/D_1^2 \tag{31}$$

and

$$\eta_N=[(4x^3-16|M|^2x-8|M|l^2)-D_1]/D_1, \tag{32}$$

where

$$D_1=4l(x^2-4|M|^2). \tag{33}$$

The detail derivation of Eqs. (31)–(33) is given in Appendix B.

We find from Eqs. (31) and (32) that for a normal vacuum ($N=0$, $|M|=0$), $\eta_{N,R}=0$ and for a thermal vacuum ($N\neq 0$, $|M|=0$), $\eta_{N,R}>0$. Thus a collection of two two-level atoms in a cavity with a normal or thermal vacuum being injected into it cannot produce atomic-squeezed states. Recall that, for $|M|=0$, $\rho_{02}=\rho_{20}=0$ and $Y_3=0$. Therefore, from Eq. (B4) and (B5) for the squeezing parameters η_R and η_N, we conclude that the nonzero value of these coherence terms is essential for generation of atomic-squeezed states, as mentioned earlier. Before discussing the squeezing characteristic for an arbitrary value of the parameter λ, we present the case

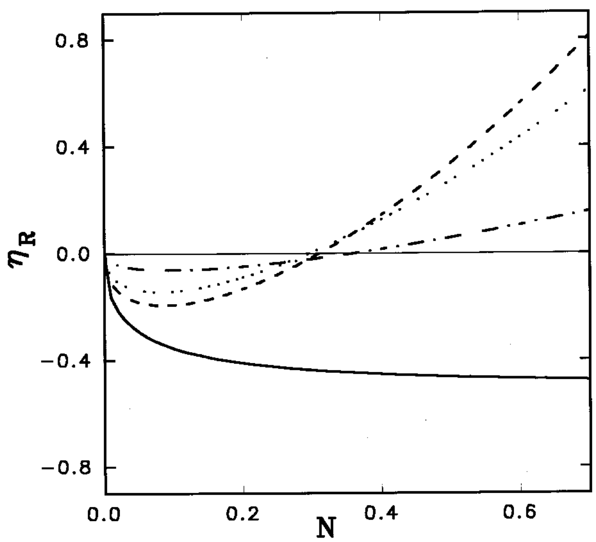

FIG. 2. Plot of η_N vs N for different values of λ: λ=0 (solid line), λ=0.5 (dashed line), λ=1.0 (dotted line), and λ=4.0 (dashed-dotted line). The horizontal line denotes zero squeezing.

for λ=0, which corresponds to free space or a closed cavity. For λ=0 and $|M|^2=(1/4)(n^2-1)$, Eq. (31) and Eq. (32) reduce to

$$\eta_R=\sqrt{n^2-1}(\sqrt{n^2-1}-n) \tag{34}$$

and

$$\eta_N=\sqrt{n-1}(\sqrt{n-1}-\sqrt{n+1}), \tag{35}$$

where $n=2N+1$. It is simple to verify from Eq. (34) and Eq. (35) that for allowed values of n (i.e., $n>1$) η_R and η_N are always negative signifying squeezing for all values of N. To study the dependence of atomic squeezing on the parameter λ, we plot in η_R and η_N in Figs. 2 and 3, respectively, as functions of the parameter N for several values of λ. The results shown are for minimum uncertainty squeezed states (MUS). In these figures $N=0$ corresponds to the case of a

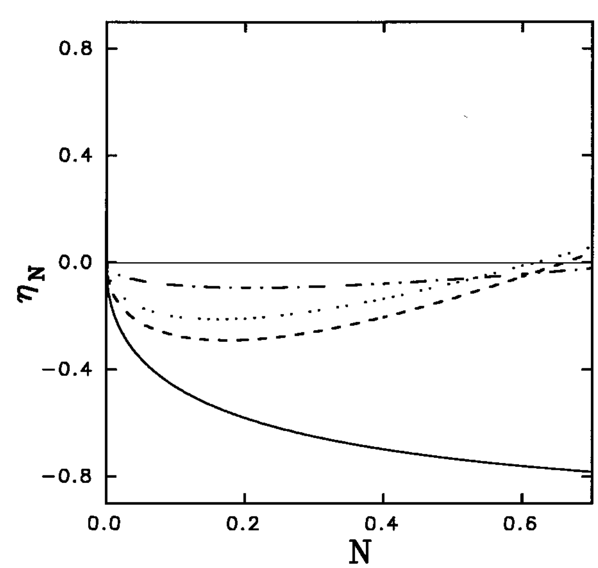

FIG. 3. Plot of η_R vs N with parameters the same as Fig. 2.

normal vacuum entering into the cavity. At this value of N both η_N and η_R are zero indicating no atomic squeezing. We observe that although qualitative features of atomic squeezing characterized by both η_N and η_R are similar, quantitatively they differ significantly. For finite values of the parameter λ, the magnitude of both η_N and η_R first increases, thereby indicating the increase in atomic squeezing with the increase in the value of N. It goes through a maximum and then goes to zero at a particular value of $N=N_c$. Here the quantitative difference between η_N and η_R become evident: the unsqueezing occurs at different values of N_c for the two squeezing parameters.

In contrast to this for λ=0 both η_N and η_R increase in magnitude with N and asymptotically approach the following values:

$$\eta_R \approx -\frac{1}{2}\left(1-\frac{1}{n^2}\right) \tag{36}$$

and

$$\eta_N \approx -\left(1-\frac{1}{n}\right). \tag{37}$$

This is shown in Figs. 2 and 3 by solid curves. Thus for this value of λ the collective atomic state is always squeezed irrespective of the squeezing parameters used to characterize the atomic-squeezed states at steady state. We find that the maximum value of atomic squeezing obtained decreases with the increase in the value of λ. Physically this can be explained by the fact that the increasing the value of λ corresponds to enhancement of incoherent spontaneous emission out the sides of the cavity. Because of this, the atomic correlations [13] required for the generation of atomic-squeezed states are destroyed.

IV. CONCLUSION

In this paper we have investigated the possibility of generating atomic-squeezed states or squeezed-spin states in a system of a pair of two-level atoms confined in a single mode optical cavity which is being driven by a squeezed vacuum field. The cavity is assumed to be in overdamped regime. This has allowed us to derive the equation of motion for atomic averages by adiabatically eliminating the cavity field mode. We have used solution of these equations at steady state to calculate atomic density-matrix elements and atomic-squeezing parameters. We have shown that unlike the free space case the intermediate state of the collective atomic states is always populated and the final atomic state is a mixed one. Nonetheless, the collective atomic population distribution shows a nonthermal characteristic similar to that of a free space case. This behavior, however, survives only for λ≤1. To characterize the atomic squeezing we have used a parameter which has been proposed recently in the context of Ramsey spectroscopy. The natural squeezing parameter defined in the context of uncertainty relations that exist between different components of angular momentum has also been studied and compared with spectroscopic squeezing. It is found that for a very small spontaneous emission rate of the atoms out the side of the cavity, atomic-squeezed states can be generated for all values of field squeezing parameter

N, provided that the injected squeezed vacuum is in minimum uncertainty state. For a finite value of the parameter λ atomic squeezing can be generated only if the value of N lies below some critical value N_c. Moreover, the value of N_c decreases with the increase in λ. Finally, to illustrate the feasibility of the scheme discussed in the present paper for the generation of atomic-squeezed, states we note that present cavity technology and sources of squeezed light can conveniently satisfy the conditions required for production of these states. For example, Polzik *et al.* [14] have demonstrated a frequency-tunable source of squeezed light exhibiting approximately 70% squeezing in a finite bandwidth and this corresponds to the value $N\approx 0.4$. The atom-cavity system used by Rempe *et al.* [15] for experimental investigation of optical bistability can provide cavity lying in the over damped regime.

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APPENDIX A

In this appendix we derive equations of motion for the atomic operators in the bad-cavity limit. For this purpose we follow the approach of Rice and Pedrotti [8] to adiabatically eliminate the cavity mode. Using Eq. (1) and commutation relation it can be shown that

$$\frac{dY_1}{dt}=-ig\langle a^\dagger(S_1^-+S_2^-)\rangle+\text{H.c.}-2\gamma Y_1-2\gamma Y_2, \tag{A1}$$

$$\frac{dY_2}{dt}=-2ig\langle(S_1^+S_2^+S_2^-+S_2^+S_1^+S_1^-+S_1^+/2+S_2^+/2)a\rangle+\text{H.c.} -2\gamma Y_2+4\gamma Y_4-2\gamma Y_1, \tag{A2}$$

$$\frac{dY_3}{dt}=ig\langle(S_1^-+S_2^--2S_2^+S_1^+S_1^--2S_1^-S_2^+S_2^-)a\rangle+\text{H.c.} -2\gamma Y_3, \tag{A3}$$

$$\frac{dY_4}{dt}=ig\langle(S_1^+S_2^+S_2^-+S_2^+S_1^+S_1^-)a\rangle\text{H.c.}-4\gamma Y_4. \tag{A4}$$

The coupling of atom and cavity field is manifested in terms like $\langle Aa\rangle$ and its conjugates in Eq. (A1)–(A4), where A is a function of atomic operators. To find these coupled expectation values, once again the above procedure is repeated and equations of motion are obtained. Here we write one of them

$$\frac{d\langle S_1^-a^\dagger\rangle}{dt}=-2ig\langle S_1^+S_1^-a^\dagger a+a^\dagger a/2+S_1^+S_1^-\rangle +2\gamma\langle S_1^+S_1^-S_2^-a^\dagger+S_2^-a^\dagger/2\rangle-(\gamma+\kappa)\langle S_1^-a^\dagger\rangle. \tag{A5}$$

In the bad-cavity limit expectation values of the coupled operators are solved in the steady state and then substituted in Eqs. (A1)–(A4). This procedure will further lead to higher-order joint atom field expectation values. However, in the bad-cavity limit this hierarchy of equation truncates at second order of the field mode. For example, when the cavity mode is coupled with a broadband squeezed vacuum it can be shown that

$$\langle a^\dagger aA\rangle=N\langle A\rangle,$$
$$\langle a^2A\rangle=|M|e^{2i\theta}\langle A\rangle,$$
$$\langle a^\dagger A\rangle=|M|e^{-2i\theta}\langle A\rangle. \tag{A6}$$

So by using Eq. (A6) and after doing some lengthy algebra we arrive at Eq. (6). Here we note that our Eqs. (6)–(9) correctly reduce to the equations of motion for an ordinary vacuum entering into the cavity [16].

APPENDIX B

In this appendix we derive Eqs. (31)–(33). For this purpose we first express the squeezing parameter η_R and η_N in terms of Y vector. It is easy to show that

$$\langle S_x^2\rangle=\tfrac{1}{2}[1+Y_2+Y_3],$$
$$\langle S_z\rangle=Y_1-1. \tag{B1}$$

Further, for the model employed here the value of $\langle S_1^+\rangle$ and $\langle S_2^+\rangle=0$ at steady state. As a result of this for our case

$$\langle S_x\rangle=0,$$
$$\langle S_y\rangle=0 \tag{B2}$$

and

$$(\Delta S_i)^2=\langle S_i^2\rangle. \tag{B3}$$

Then by using Eq. (B1) and definitions of η_R and η_N we get

$$\eta_R=\frac{(1+Y_2+Y_3)-(Y_1-1)^2}{(Y_1-1)^2} \tag{B4}$$

and

$$\eta_N=\frac{(1+Y_2+Y_3)-|(Y_1-1)|}{|(Y_1-1)|}. \tag{B5}$$

Now putting Y_2, Y_3, and Y_1 from Eq. (17) in Eqs. (B4) and (B5) we get an expression for η_R and η_N as given in Eqs. (31) and (32).

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