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Tunneling in a cavity

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The mechanism of coherent destruction of tunneling found by Grossmann *et al.* [Phys. Rev. Lett. **67**, 516 (1991)] is studied from the viewpoint of quantum optics by considering the photon statistics of a single mode cavity field which is strongly coupled to a two-level tunneling system (TS). As a function of the interaction time between TS and cavity the photon statistics displays the tunneling dynamics. In the semiclassical limit of high photon occupation number n, coherent destruction of tunneling is exhibited in a slowing down of an amplitude modulation for certain parameter ratios of the field. The phenomenon is explained as arising from interference between displaced number states in phase space which survives the large n limit due to identical $n^{-1/2}$ scaling between orbit width and displacement. [S1050-2947(96)02712-6]

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In recent years the idea of modulation and therefore control of tunneling by a monochromatic electromagnetic field has been a subject of considerable interest. The typical Hamiltonian describes a particle in an isolated double-well potential (DWP) which is periodically driven by an external force

$$H(t)=H_{\mathrm{DWP}}(x)+Sx\cos(\omega_L t). \qquad (1)$$

Here $H_{\mathrm{DWP}}(x)$ is the Hamiltonian of the DWP, S is the amplitude, and ω_L the driving frequency. The attention has mainly focused on a possible enhancement or suppression of coherent tunneling: Lin and Ballentine [1] have demonstrated that the tunneling probability is highly enhanced due to periodic modulation for high-field strengths and driving frequencies close to the classical oscillation frequency at the bottom of each well. In the opposite limit Grossmann and co-workers [2] found complete suppression of tunneling such that a particle initially localized in one of the two wells will never escape to the other well. They termed this effect ‘‘coherent destruction of tunneling.’’ It has since been of continuing interest [3–9]. The most surprising feature is the *periodicity* of the destruction of tunneling for certain parameter ratios of S and ω_L. So far there is no clear understanding of this phenomenon.

By using the Floquet formalism it has been shown that many characteristic features of the tunneling suppression can already be described in a two-level approximation of the DWP [3,4]. Many different aspects of the effect have been illuminated in this framework: Makarov [5] and Plata and Gomez Llorente [6] quantized the electromagnetic field and recovered the effect in the limit of a large number of photons in the field. Wang and Shao [7] mapped the driven two-level dynamics to a classical one of a charged particle moving in a harmonic potential plus a magnetic field in a plane. Kayanuma [8] explained the suppression of tunneling as an effect arising from interference at periodic level crossings.

In the present paper, an alternative explanation is proposed which uses the concept of *phase-space interference* known from squeezed states [10,11] and displaced number states [12,13] in quantum optics.

Let us start with a description of the physical situation we have in mind. At $t=0$ a single two-level tunneling system (TS) prepared in a state localized in say the left well $|L\rangle$ is injected in a cavity and starts to tunnel between its left and right state $|L\rangle$ and $|R\rangle$. The cavity contains a single mode which has been prepared in a number state $|n\rangle$. We consider an ideal cavity, i.e., we neglect any kind of dissipation. After an interaction time t_{in} the TS leaves the cavity and the photon number distribution is measured irrespective of the state of the TS.

Thus our aim will be to calculate the transition probability from the product state $|L,n\rangle=|L\rangle|n\rangle$ at $t=0$ to another product state $|i,l\rangle$ for any $i=L,R$ at time $t=t_{in}$,

$$P_{ln}(t_{in})=\sum_{i=L,R}|\langle i,l|\Psi(t_{in})\rangle|^2, \qquad (2)$$

where $|\Psi(t_{in})\rangle=e^{-iHt_{in}/\hbar}|L,n\rangle$. For fixed t_{in} this is the photon number distribution with $P_{ln}(0)=\delta_{ln}$. For fixed l, $P_{ln}(t_{in})$ describes the dynamics of the cavity mode interacting with a TS.

An important aspect of the present problem is the strong coupling between the cavity mode and the TS, and the separation of time scales between the *slow* tunneling motion and the *fast* field oscillations. If we denote the coupling energy by g and the tunneling frequency by Δ, this means that we are interested in the limit $\Delta\ll\omega_L$ and $g\sim\hbar\omega_L$ for low and $\sqrt{n}g\sim\hbar\omega_L$ for high number of photons in the field.

In this regime the field strongly dresses the TS and both TS + field have to be treated as a single unit. In contrast to the situation where $\omega_L\approx\Delta\gg g/\hbar$, we are confronted in the present case with a situation in which the rotating-wave approximation is not applicable. Thus instead of using the Jaynes-Cummings Hamiltonian [14], we must include counter-rotating terms so that our Hamiltonian reads

$$H=-\frac{\hbar\Delta}{2}\sigma_x+g\sigma_z(a^\dagger+a)+\hbar\omega_L a^\dagger a+\frac{g^2}{\hbar\omega_L}, \qquad (3)$$

where $a^\dagger$, a are the bosonic creation and annihilation operators of the cavity mode. We have identified $\sigma_x=|L\rangle\langle R|+|R\rangle\langle L|$ and $\sigma_z=|L\rangle\langle L|-|R\rangle\langle R|$. Expressing the spin operator in the eigenstates of the TS, $|0\rangle=2^{-1/2}(|L\rangle+|R\rangle)$ and $|1\rangle=2^{-1/2}(|L\rangle-|R\rangle)$, the usual formulation of the Hamiltonian in quantum optics is recovered. We also have added a constant energy shift for later convenience. It has to be noted that suppression of tunneling does not occur in the Jaynes-Cummings model [6]. Hence the situation here is fundamentially different from the one usually encountered in quantum optics.

Owing to the separation of time scales between the tunneling and the oscillation dynamics, $P_{ln}(t_{in})$ can be calculated in perturbation theory in Δ/ω_L by introducing dressed states

$$|j(n)\rangle=U|n\rangle\frac{1}{\sqrt{2}}(|L\rangle+(-1)^j|R\rangle) \tag{4}$$

with $j=0,1$ and

$$U=\exp[-\sigma_z\alpha(a^\dagger-a)], \tag{5}$$

where $\alpha=g/\hbar\omega_L$. Equivalently, one may perform the polaron transformation $H\rightarrow U^\dagger HU$ and continue to use the product state instead of the dressed state basis. In the dressed state basis the Hamiltonian can be written as

$$\begin{aligned}H&=H_D+V\\&=\sum_{m,j}|j(m)\rangle\left(m\hbar\omega_L-(-1)^j\frac{\hbar\Delta_m}{2}\right)\langle j(m)|\\&\quad-\frac{\hbar\Delta}{4}\sum_{m,m',j,j'}|j(m)\rangle\{(-1)^{j'}[1+(-1)^{j-j'+m-m'}]\\&\quad\times D_{mm'}(2\alpha)(1-\delta_{jj'}\delta_{mm'})\}\langle j'(m')|,\end{aligned} \tag{6}$$

with

$$\Delta_n=\Delta D_{nn}(2\alpha) \tag{7}$$

and

$$D_{ln}(\alpha)=\langle l|D(\alpha)|n\rangle=\left(\frac{n!}{l!}\right)^{1/2}\alpha^{l-n}e^{-(1/2)|\alpha|^2}L_n^{l-n}(|\alpha|^2), \tag{8}$$

where $D(\alpha)=\exp[\alpha(a^\dagger-a)]$ is the shift operator and $L_n^{l-n}(x)$ an associate Laguerre polynomial $(l\geqslant n)$. The diagonal part H_D builds a ladder of tunneling doublets with intradoublet spacing $\hbar\Delta_m$ and interdoublet spacing $\hbar\omega_L$. Because $\langle 1(m)|V|0(m)\rangle=0$, V induces only mixing between dressed states belonging to *different* doublets. Hence, corrections to the dressed states are $O(\Delta/\omega_L)$. Neglecting V for this reason we find for the transition probability if $l\geqslant n$

$$\begin{aligned}P_{ln}(t_{in})=&\left|\sum_m D_{lm}(\alpha)D_{mn}(-\alpha)e^{-im\omega_Lt_{in}}\cos\left(\frac{1}{2}\Delta_mt_{in}\right)\right|^2\\&+\left|\sum_m D_{lm}(\alpha)D_{mn}(\alpha)e^{-im\omega_Lt_{in}}\sin\left(\frac{1}{2}\Delta_mt_{in}\right)\right|^2\end{aligned} \tag{9}$$

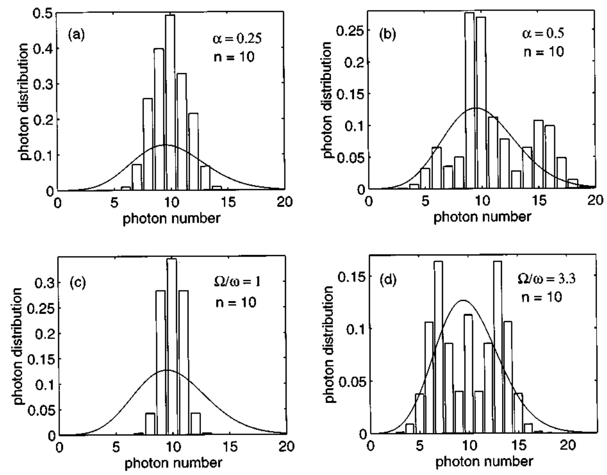

FIG. 1. Characteristic oscillations in the photon-number distribution of dynamically displaced number states for $\omega_Lt_{in}=250$, $\Delta/\omega_L=0.2$, and $|n\rangle=|10\rangle$. In (a) and (b) the quantum limit, Eq. (9), is displayed for different values of $\alpha\equiv g/\hbar\omega_L$. In (c) and (d) the semi-classical limit, Eq. (10), is shown for different values of $\Omega/\omega_L\equiv2\sqrt{n}\alpha$. The full line shows the Poisson distribution with $\overline{n}=10$.

and $P_{ln}(t_{in})=P_{nl}(t_{in})$ if $l<n$. *We conclude that the tunneling dynamics can be seen in the spectrum of the transition probability of the cavity field.* In addition to harmonics of ω_L its power spectrum also contains resonances at Δ_m arising from the tunneling motion. This behavior strongly depends on the initial preparation of the TS. Injecting it in its ground state $|0\rangle$, for instance, yields only resonances at $m\omega_L+(-)^\xi\frac{1}{2}\Delta_m$ where $\xi=0,1$ depending on whether $n-m$ is even or odd, respectively, and hence only rapid oscillations.

The effect of a strong coupling between the cavity mode and the TS is to mix a coherent amplitude $\alpha\equiv g/\hbar\omega_L$ with the intial number state of the mode. This happens by displacing the oscillator wave function $\varphi_n(x)=\langle x|n\rangle$ like $x\rightarrow x-\sqrt{2}\alpha$ in dimensionless coordinates $x=(\mu\omega_L/\hbar)^{1/2}q$. Thus by injecting a TS which strongly couples to the cavity field, it is possible to realize displaced number states [12,13]. The statistical properties of displaced number states have been discussed in Ref. [13]. The photon-number distribution is simply given by $P_{DNS}(l)=|D_{ln}(\alpha)|^2$ because $D_{ln}(\alpha)$ is the probability amplititude of finding l photons in a displaced number state $|\alpha,n\rangle=D(\alpha)|n\rangle$.

The photon distribution (9) for number states which are *dynamically* displaced by the tunneling process resembles $P_{DNS}(l)$. The displaced number states can either be shifted into the same well [first term in Eq. (9)] or opposite wells [second term in Eq. (9)]. As expected for $\alpha=0$, $P_{ln}(t_{in})=\delta_{ln}$ independent of the interaction time.

In Figs. 1(a) and 1(b) we have plotted $P_{ln}(t_{in})$ for $n=10$, $\omega_Lt_{in}=250$, and $\alpha=0.25$ and 0.5. Whereas a coherent state with $\overline{n}=10$ obeys the familiar Poisson distribution [11] shown by the line, the dynamically displaced number states exhibit oscillations shown by the histogram. Increasing α results in further oscillations. This effect is independent of the specific value of ω_Lt_{in} for $t_{in}\neq0$, though the absolute value of $P_{ln}(t_{in})$ depends on ω_Lt_{in}. These modulations are clearly exhibited in the asymptotic expansion of Eq. (9) in the semiclassical limit (Bohr's correspondence principle): $l,n\rightarrow\infty$, $n/l\rightarrow 1$ with $l-n\geqslant 0$ finite. If we scale the coupling constant between the field and the TS as $g\propto n^{-1/2}$ and note that associate Laguerre polynomials asymptotically approach Bessel functions [15] one finds in the semiclassical limit

$$P_{l-n}(t_{in})=\cos^2\left(\frac{1}{2}\tilde{\Delta}t_{in}\right)J^2_{l-n}\left(\frac{2\Omega}{\omega_L}\sin(\omega_L t_{in})\right)+\sin^2\left(\frac{1}{2}\tilde{\Delta}t_{in}\right)J^2_{l-n}\left(\frac{2\Omega}{\omega_L}\cos(\omega_L t_{in})\right). \quad (10)$$

Here

$$\tilde{\Delta}=\Delta|J_0(2\Omega/\omega_L)| \quad (11)$$

is the renormalized tunneling frequency, and

$$\Omega=2\sqrt{n}g/\hbar \quad (12)$$

is the Rabi frequency. In Figs. 1(c) and 1(d), Eq. (10) is plotted for $\Omega/\omega_L=1$ and 3.3. It shows the same oscillations as the exact expression (9).

The expression (10) is easily understood. The cosine and the sine factors represent the probability for the tunneling particle to stay in the well where it has been prepared initially, or to escape to the other well, respectively [3]. The Bessel functions represent the probability for the corresponding displaced number state to contain l photons if it had n initially [note that $P_{DNS}(l)\rightarrow J^2_{l-n}(\Omega/\omega_L)$ in the semiclassical limit]. Hence the Bessel functions represent the photon statistics of displaced number states with an effective displacement $(2\Omega/\omega_L)\sin(\omega_L t_{in})$ or $(2\Omega/\omega_L)\cos(\omega_L t_{in})$. From Eq. (11) and the cosine and sine factors in Eq. (10) we further see that for specific parameter values of the driving field—where $2\Omega/\omega_L$ hits the roots of the zero order Bessel function—the tunneling process is completely suppressed. Note that localization cannot occur for small photon number in the field because there is no value of α which is simultaneously a root of all Laguerre polynomials. We conclude that in the large n limit *the tunneling dynamics is displayed in the photon statistics by an amplitude modulation of the cavity mode oscillations. Coherent destruction of tunneling manifests itself in the slowing down of this amplitude modulation* as depicted in Fig. 2.

We now show how the present picture can give a quantitative understanding of this effect. First recall that the oscillation in the photon-number statistics in Fig. 1 originate from phase-space interference [13]: the nth number state can be associated with a circular band of width $(2n)^{-1/2}$ around its orbit with radius $\sqrt{2(n+1/2)}$ and centered around the origin. Analogously so can the lth displaced number state which is shifted by $\sqrt{2}\alpha$. According to the area-of-overlap concept [10], the transition amplitude between two states is governed by the sum of all possible overlap areas weighted with appropriate phases. This results in the interference between contributions from different overlaps which can be constructive or destructive, thus giving rise to oscillations in the photon-number distribution. Another way of understanding the oscillations is that they arise from the possibility of a

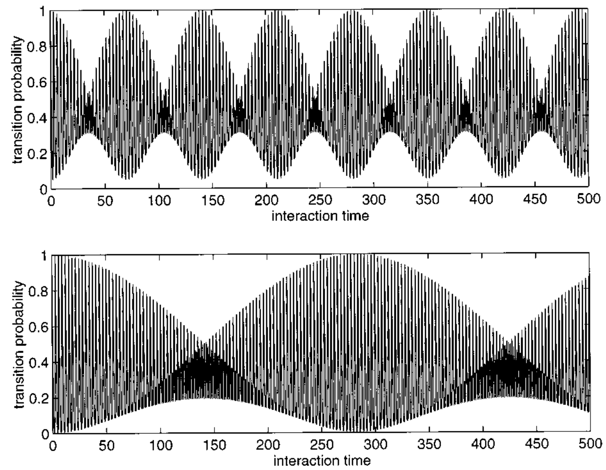

FIG. 2. Coherent destruction of tunneling monitored in the cavity field. Displayed is Eq. (10) for $l=n$, $\Delta/\omega_L=0.2$ as a function of $\omega_L t_{in}$: upper figure, $2\Omega/\omega_L=2$; lower figure, $2\Omega/\omega_L=2.3$. The first root of $J_0(2\Omega/\omega_L)$ occurs at $2\Omega/\omega_L\approx 2.405$. The effect is exhibited by a decrease of the amplitude modulation in the cavity mode oscillations.

constructive and destructive overlap between displaced harmonic-oscillator wave functions by noting that $D_{lm}(\alpha)$ is a polynomial in l of order $\min(l,m)$, i.e., has $\min(l,m)$ roots.

From this consideration it becomes apparent that varying the relative radius of the two bands by changing l vs n, or varying the relative displacement by changing α for fixed l and n should result in similar effects. If one notices further that the factor $J_0(2\Omega/\omega_L)$ in Eq. (11) is the probability amplitude of finding in a number state displaced by $x\rightarrow x-2[\Omega/(\omega_L\sqrt{2n})]$, again exactly the same number of photons, one expects that the periodic suppression of tunneling arises from destructive and constructive interference of displaced harmonic-oscillator wave functions in phase space.

To verify this argument, the $n\rightarrow\infty$ limit has to be considered with care. First take the number of photons in the cavity mode to be finite. The factor $D_{nn}(2\alpha)$ which scales the doublet splitting (7) is the overlap between a number state $|n\rangle$ shifted into the right well with the same number state shifted into the left well. If we note that $D_{nn}(2\alpha)$ is a polynomial in $|\alpha|^2$ of degree n, and consequently has n zeros, we expect $n+1$ oscillations of $D_{nn}(2\alpha)$ as a function of α. The n zeros between the maxima result from the n possible ways of destructive overlap between displaced harmonic oscillator wave functions. This explains Fig. 2 in Ref. [5]. However, contrary to the claim there, this picture remains also valid if the oscillator energy is larger than the reorganization energy $(n+\frac{1}{2})\hbar\omega_L>g^2/\hbar\omega_L$, and even in the limit $n\rightarrow\infty$. The simple reason for this is that in the semiclassical limit $n\rightarrow\infty$ with $2\sqrt{n}g/\hbar\rightarrow\Omega$ fixed, both the distance between the nodes of the harmonic-oscillator wave function (as a function of x) *and* the displacement $x\rightarrow x-2[(\Omega/(\omega_L\sqrt{2n})]$ scale as $n^{-1/2}$. Based on this argument, we expect oscillation to become noticable as soon as the displacement exceeds the bandwidth, i.e., $2\Omega/\omega_L>1$. Finally, we note that with the scaling chosen above both displaced orbits will always intersect no matter how large Ω/ω_L is. This results in the infinite number of oscillations seen in the zero-order Bessel function (11).

Summarizing, coherent destruction of tunneling is explained as a *quantum effect arising from the destructive interference of displaced harmonic-oscillator wave functions in phase space*. The effect is strictly speaking only observable in the semiclassical limit for the reasons mentioned above, and survives the large n limit since both *the bandwidth of the orbit and the displacement scale in the same way to zero as* $n\to\infty$.

In closing, we note that experimental study of the effects described here is presently still out of range. In a microcavity the driving frequency ω_L is $O(\mathrm{GHz})$ whereas typical values of the one-photon Rabi frequency $g/\hbar$ are $O(\mathrm{kHz})$ [16]. Hence a number state with large n is needed which is difficult to realize experimentally. Possibly a trap is more suited because of its lower driving frequency $\omega_L\sim O(\mathrm{MHz})$ [17]. Damping of the cavity mode which is too strong will also render an observation impossible. Furthermore, the tunneling dynamics must still be coherent. Finally the superposition state $|L\rangle=2^{-1/2}(|0\rangle+|1\rangle)$ is difficult to prepare because the particle experiences strong electromagnetic fields when it enters and leaves the cavity [17] (for a discussion of this point if one uses quantum wells, see Ref. [18]).

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