Synchronization of chaotic laser mode dynamics

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We show experimentally that under suitable conditions two globally coupled laser modes showing chaotic dynamics can spontaneously synchronize together. Under the synchronization the frequencies of the two modes lock together and the chaotic dynamics of each mode show similarities to those of the single-mode laser. [S1050-2947(96)02012-4]

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Due to its potential application in secure communications, chaotic synchronization of nonlinear dynamical systems has recently attracted great interest. The concept of synchronized chaos was first introduced by Pecora and Carroll [1]. They showed theoretically that the chaotic dynamics of two nearly identical nonlinear systems linked by a common signal can be synchronized. Using an electric circuit they also demonstrated the chaos synchronization of their concept [2]. A crucial requirement of their method of chaos synchronization is that one should be able to separate a chaotic system into two subsystems, one of which possesses a negative Liapunov exponent. For many chaotic dynamical systems, however, this condition cannot easily be fulfilled. Murali and Lakshmanan have shown that chaotic synchronization can also be achieved through one-way coupling of two identical chaotic systems [3], and in contrast this method does not need to construct any stable subsystem. Chaotic synchronization through mutual coupling of identical nonlinear systems has also been studied theoretically [4].

Laser systems are well-known nonlinear dynamical systems that show chaotic dynamics under a wide range of situations. The possibility of chaotic synchronization of laser dynamics was first studied theoretically by Winful and Rahman [5]. In semiconductor laser arrays they have found that under mutual interaction among the lasers, synchronized chaotic time series can be generated spontaneously. Chaotic synchronization of two mutually interacting lasers was first experimentally observed by Roy and Thornburg [6]. In their experiment two neodymium-doped yttrium aluminum garnet (Nd:YAG) lasers were used. Without mutual interaction each laser can exhibit steady or chaotic dynamics under the influence of the pump modulation. The two lasers were mutually coupled through their electrical field. Roy and Thornburg showed that depending on the strength of the coupling between the lasers, the chaotic dynamics of the two lasers can be synchronized or independent. Chaotic synchronization of two lasers has also been achieved through one-way coupling [7]. In this case the chaotic dynamics of one laser (slave laser) is driven by the output of another chaotic laser (master laser), and there exist no feedback of the slave laser dynamics to that of the master laser.

In this paper we report on another experimental observation of chaotic synchronization in laser systems. We show that under the mutual interaction between two longitudinal modes of a laser, the chaotic dynamics of each individual mode can spontaneously synchronize, despite the fact that both laser modes share the same population inversion and consequently strong-mode competition between them exists. In particular, we demonstrate that under the chaos synchronization, the chaotic dynamics of each mode, or equally of the two-mode laser system, show strong similarity to those of the single-mode laser.

Our experimental setup is the same as shown in [8]. It consists of an optically pumped $^{15}NH_3$ far-infrared (FIR) ring laser operating on the 153 μm transition. The cavity of the FIR laser is a triangular ring and has a perimeter of about 2 m. The output mirror of the FIR laser is made of a gold mesh. Utilizing mesh of different mesh constants changes the outcoupling of the FIR laser, so that the cavity loss of the laser can be changed while leaving all other parameters of the laser constant. The pump laser is an isotopic carbon dioxide ($^{13}CO_2$) laser, whose cavity has a Littrow structure. The pump frequency is finely tunable using a piezoceramic mount for the output cavity mirror. The pump-laser frequency is monitored with a CO_2 Lamp-dip cell. The pump-laser beam is mode matched to the fundamental mode of the FIR laser cavity through two lenses. The FIR laser emissions in the two directions of the ring laser are detected with two Schottky barrier diodes. The electrical signals are low-noise amplified and stored simultaneously with a digital acquisition signal processing card in a computer.

Because the absorption line profile of the NH_3 molecules is Doppler broadened and the optical pumping excites selectively molecules that have the same axial velocity, a special feature of the optically pumped NH_3 ring lasers is that through tuning the pump-laser frequency far away from the NH_3 absorption line center, one can separate the gain lines for the forward and backward modes of the laser sufficiently far that just one directional gain line can locate in the laser cavity line width. With this method one can achieve single-mode operation of the laser, respectively, in each direction without the influence of the other mode. The chaotic dynamics of the laser under this single-mode operation has been extensively investigated previously.

Depending on the laser operation conditions, a wide variety of chaotic dynamics such as period-doubling route to chaos [9,10], type-III intermittency [11], and Lorenz-like spiral chaos [12,13] have been revealed in the backward

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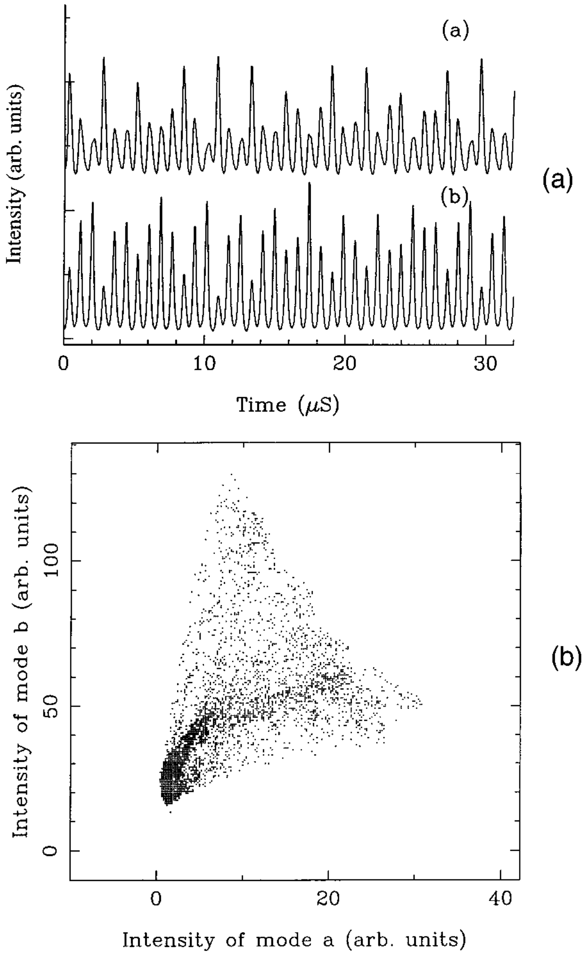

FIG. 1. Behavior of an in-phase collective state of the laser. Pump intensity is 4.3 W/cm^2, gas pressure is 6 Pa, and output mirror mesh constant is 51 μm. (a): Intensity evolutions of the two modes in the state. (a), Intensity evolution of the forward emission. (b), Intensity evolution of the backward emission. (b): X-Y plot of the two mode intensities.

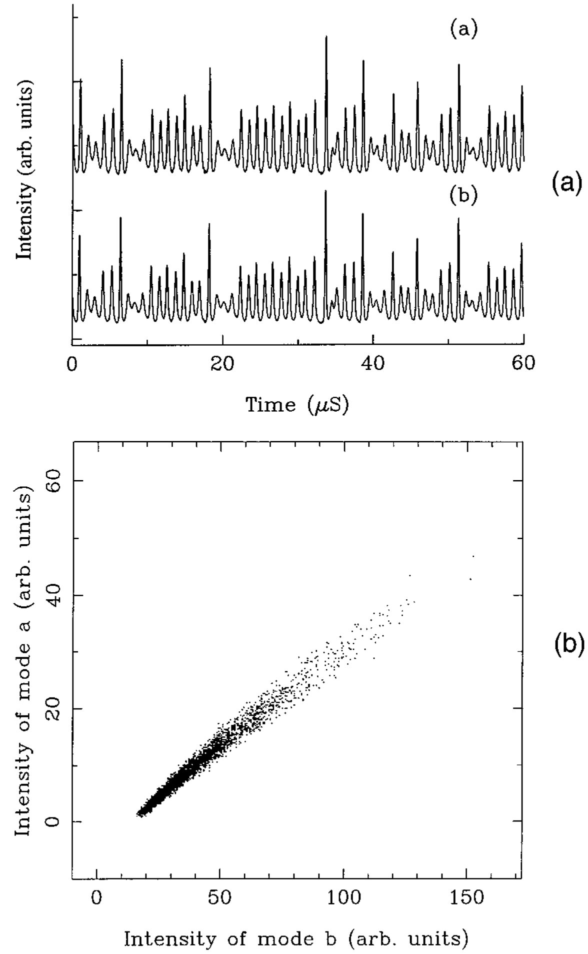

FIG. 2. Behavior of a chaotic synchronized state of the laser. Pump intensity is 3.5 W/cm^2, gas pressure is 3.5 Pa and output mirror mesh constant is 102 μm. (a): Intensity evolution of the two modes in the state. (a), Intensity evolution of the forward emission. (b), Intensity evolution of the backward emission. (b): X-Y plot of the two mode intensities.

emission of the laser. In particular, it was found that under suitable conditions, the optically pumped NH_3 single-mode ring lasers can exhibit chaotic dynamics that are very similar to those described by the Lorenz equations, one of the paradigms in nonlinear dynamical studies [14]. Since the optically pumped lasers are three-level laser systems in which pump coherence effects play a role, dynamics of these lasers is, in principle, different to that of the two-level laser systems. In order to understand the observed chaotic dynamics of the optically pumped NH_3 single-mode ring laser and its relation to the Lorenz chaotic dynamics, comprehensive theoretical modeling of the chaotic dynamics of the laser has also been conducted by Vilaseca and co-workers [15], and these studies have made a thorough understanding of the chaotic dynamics of these lasers possible. In contrast, the chaotic dynamics of the optically pumped bidirectional NH_3 ring lasers remains almost untouched. Recently it was shown that in multimode lasers, due to the coexistence of many laser modes and the nonlinear mode coupling between them, collective effects can arise, which manifest the spontaneous self-organization properties of nonlinear dynamical systems [16]. In order to experimentally explore the collective dynamics of a multimode laser, in particular to study the behavior of interactions between chaotic laser modes, we have extended the studies on the chaotic dynamics of optically pumped NH_3 single-mode ring lasers to the simplest multimode case: bidirectional operation. Experimentally, bidirectional operation of the laser is achieved by maintaining the pump-laser frequency close to the NH_3 absorption line center. Under strong pumping both modes of the laser can lase simultaneously and their dynamics are coupled, with the coupling strength between them depending on the pump-laser frequency offset to the NH_3 absorption line center and the FIR laser cavity frequency setting.

First we studied the dynamics of mode interaction between the forward and backward modes of the laser in the parameter range where without mode interaction each mode would lase in its steady state (the controlling parameters are laser gas pressure and pump power). Under these conditions we found that the interaction between the two modes is

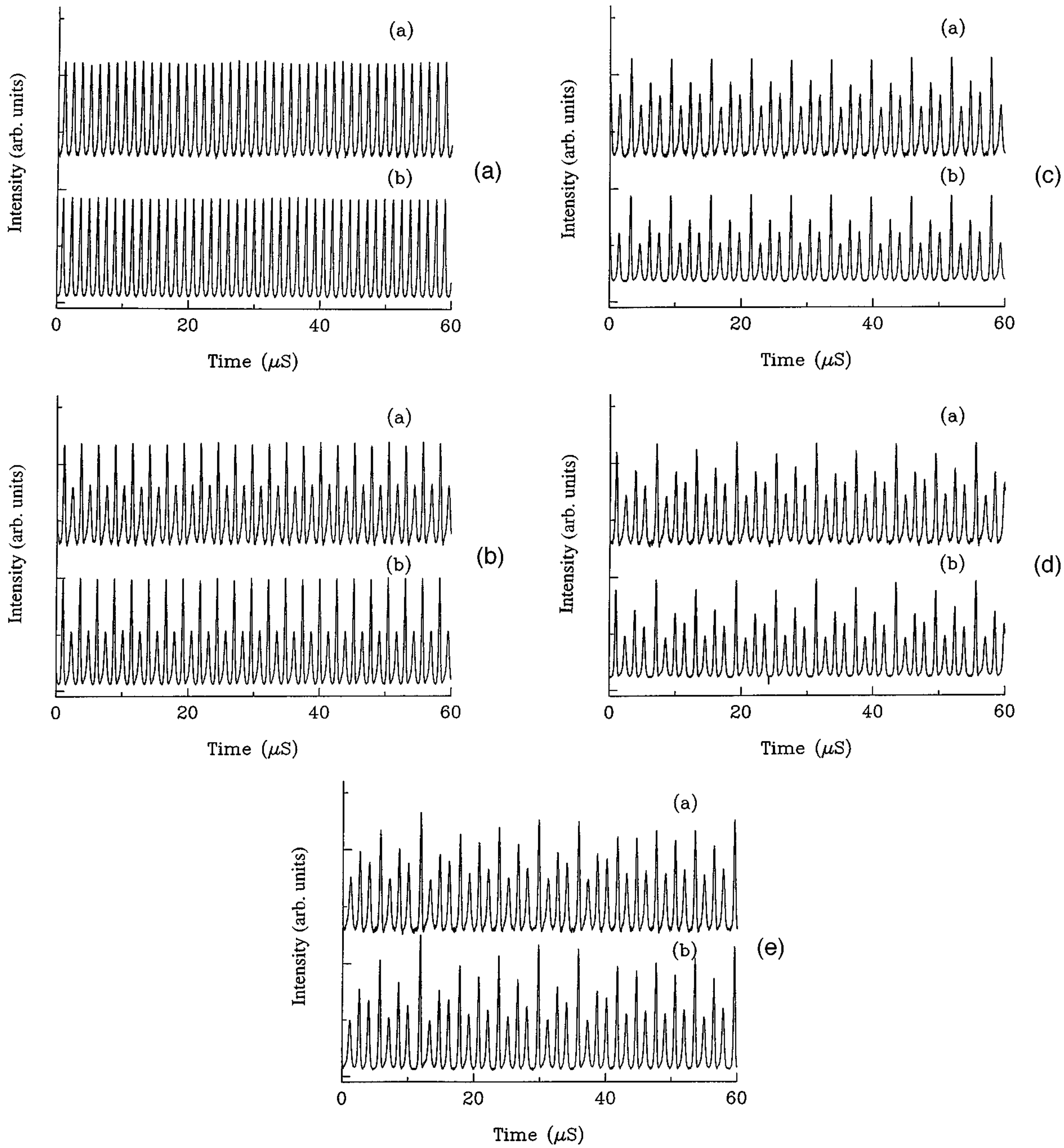

FIG. 3. Synchronized period-doubling route to chaos of the laser as the cavity detuning is reduced. Pump intensity is 3.5 W/cm^2, gas pressure is 4 Pa, and output mirror mesh constant is 102 μm. (a), Intensity evolution of the forward emission. (b), Intensity evolution of the backward emission. (a): period-one state; (b): period-two state; (c): period-four state; (d): period-eight; (e) chaotic state; From (a) to (e) the cavity detuning was reduced.

mainly the mode competition caused by sharing the same population inversion, and the mode coupling introduced by the spatial population inversion grating formed due to the counterpropagation of the two laser beams in the cavity. Under these mode interactions, antiphase dynamics of the laser was observed [17]. Next the laser was operated in the single-mode chaos parameter regime, where even without mode interaction, each mode would be intrinsically unstable. In this parameter regime, collective dynamics of the laser were still observed, which manifest now that the chaotic mode intensity pulsations of the two modes are found to be either in-phase or antiphase in the sense that the two modes pulse together or alternately, respectively [18]. However, we found that under normal laser operation conditions, even when in such a collective state, the dynamics of the two modes are not synchronized in that the trajectory of each mode in the phase space is not the same. As an example we show in Fig. 1 an observed in-phase collective state of the laser. Figure 1(a) shows the intensity evolutions of the two laser modes recorded simultaneously, Fig. 1(b) is an X-Y plot of the two

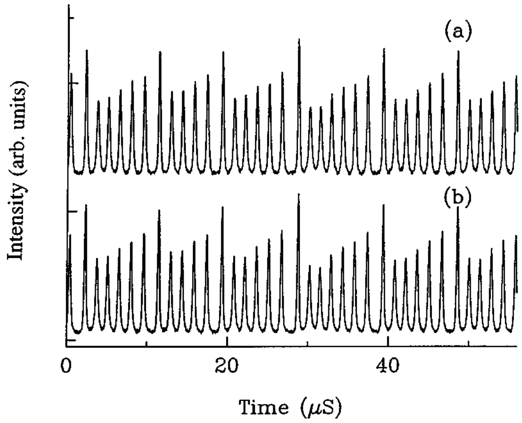

FIG. 4. Synchronized spiral chaos of the laser. Pump intensity is 3.5 W/cm^2, gas pressure is 4 Pa, and output mirror mesh constant is 102 μm. (a), Intensity evolution of the forward emission. (b), Intensity evolution of the backward emission.

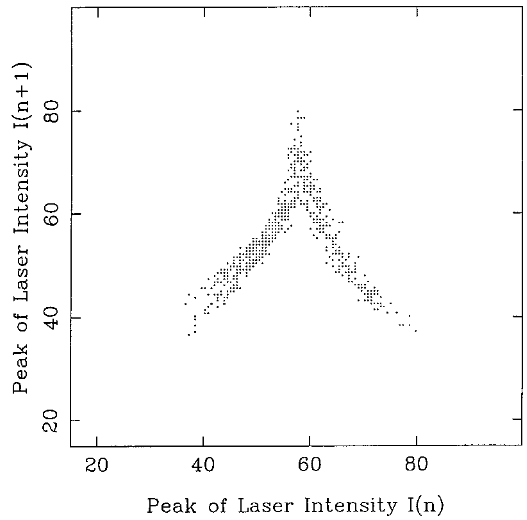

FIG. 5. First peak intensity return map constructed from the intensity evolution shown in Fig. 4(b).

mode intensities. We call the state shown in Fig. 1 an in-phase collective state, because in this state the fundamental intensity pulsation of each mode occurs at the same time. Figure 1(b) shows clearly that the chaotic dynamics of the two modes in this in-phase collective state is not synchronized. In collective laser states, although the intensity variations of the modes are correlated, the detailed intensity variations of each mode are quite different. Generally it was observed that under nonlinear laser-mode coupling, the chaotic dynamics of each mode becomes more complicated than its original one.

A great benefit of using a bidirectional ring laser to study the chaotic laser mode-mode interaction is that in this laser each mode propagates in a different direction, so one can easily measure the dynamics of each individual mode; the disadvantage is that due to the counterpropagation, the interaction between the modes becomes more complicated. Note that in the special case of an optically pumped NH_3 ring laser, by offsetting the pump-laser frequency by different amounts relative to the NH_3 absorption line center, one can also easily achieve single-mode operation of the laser in one direction or the other, which corresponds in fact to the chaotic dynamics of each mode without coupling with the other. The single-mode chaotic dynamics of the laser operating in each direction is already well understood. This behavior of the optically pumped NH_3 ring laser enables one practically to understand the dynamics of the laser in bidirectional operation as a result of the mutual interaction between the two individual modes. Instead of considering the laser as a single unit one could alternatively think of each mode as a subsystem. Once the concept of chaos synchronization through mutual coupling between two chaotic systems had been demonstrated both theoretically and experimentally, an intuitive question about the chaotic dynamics of the optically pumped bidirectional ring laser was the following: is it possible that under suitable experimental conditions, the chaotic dynamics of the individual modes could be synchronized through their mutual interaction? In order to answer this question, we have intensively investigated the chaotic-mode interaction of the laser under different laser operation conditions.

The phenomenon of frequency locking between two counterpropagating modes of ring-laser gyroscopes is well known and has been extensively studied [19]. When the frequency separation between the two counterpropagating laser modes is smaller than a critical value, due to the nonlinear mode interaction between them, their frequencies will spontaneously lock together, and consequently both modes lase on the same frequency. Although the situation for an optically pumped bidirectional ring laser is to some extent similar to that of a ring-laser gyroscope, there exists however an important difference between them, namely, that the optically pumped bidirectional ring laser now works in the single-mode chaos regime. Because of the intrinsic instability of each mode, in the normal case (e.g., small cavity loss) even when the pump-laser frequency detuning is very close

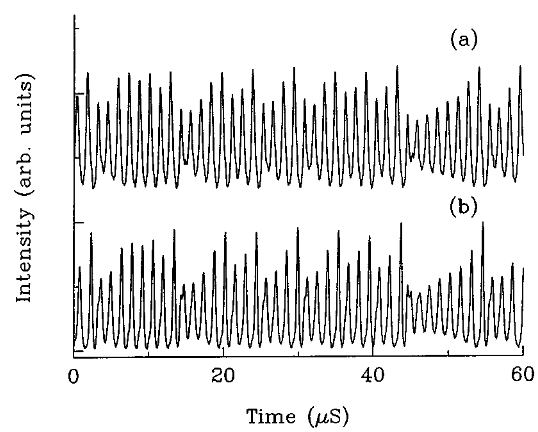

FIG. 6. Chaotic mode intensity variations of the two coupled laser modes observed. Note the intensity variations between the modes show clear tendency of chaos synchronization. Pump intensity is 2.4 W/cm^2, gas pressure is 5 Pa and output mirror mesh constant is 51 μm. (a), Intensity evolution of the forward emission. (b), Intensity evolution of the backward emission.

to the pump line center, it was found that the two modes cannot be frequency locked. We believe that when the laser mode dynamics is chaotic, because both its amplitude and phase varies with time, normally it becomes more difficult to frequency lock them. However, we found that for our laser operating in the single-mode chaos parameter regime, under conditions of very large laser cavity loss, no matter what the mode dynamics of each individual mode, the forward and the backward emissions of the laser can be frequency locked. This frequency locking exists in a wide range of cavity detuning and pump-laser frequency detuning relative to the pump line center. That the mode frequencies are locked is confirmed in the experiment by the disappearance of broad band beat signals between mode frequencies due to nonlinear coupling between the modes and very weak scattering of the counterpropagating wave to each detector.

A direct consequence of the frequency locking between the two laser modes is that their dynamics becomes synchronized, even when the dynamics of the individual modes are chaotic. As an example we show in Fig. 2 one of such synchronized chaotic states. Like in Fig. 1, Fig. 2(a) shows the two-mode intensity evolutions recorded, and Fig. 2(b) is the X-Y plot of the two-mode intensities. In comparison with the state shown in Fig. 1, the intensity variations of the two modes recorded by the two detectors are now exactly the same, and consequently their intensity $X-Y$ plot shows a straight line, characteristic of synchronized chaotic dynamics. Another characteristic of the observed synchronized states of the laser is that there is no time delay between the intensity variations of the two modes. This is clearly different than the result of laser chaos synchronization through one-way coupling [7], where depending on the response of the slave system, a time delay exists between the variations of the master system and the slave system.

We note the difference of chaos synchronization in our system from that described in other systems. In our laser the dynamics of the two modes are globally mutually coupled through gain competition and back scattering caused by spatial grating of the population inversion, in contrast to other systems where coupling has been introduced through just one system variable. This global coupling could lead to the interpretation that the two modes are not really separate systems. However, since they are physically separated at output coupler, since they can be excited separately, and since they are usually not synchronized, we find it instructive to interpret the synchronization of the two outputs when frequency locked as a form of chaotic synchronization. Our experimental result demonstrates that under such global coupling, chaotic dynamics of two systems can be spontaneously synchronized. We also note that in the present experiment, longitudinal optical pumping of the FIR laser is used. As shown in [20] for strong pump-laser intensity, this has the consequence that the gain profiles for the forward and backward modes of the laser become different, as there exists strong ac-Stark splitting in the gain profile of the forward emission. Also, in the optically pumped bidirectional ring lasers, due to the pump frequency detuning and the cavity detuning, the conditions of each mode are different. In fact, in spite of this, our experimental result demonstrates that their chaotic dynamics can be synchronized. This result indicates that chaotic synchronization through global mutual coupling is robust in the sense that the chaotic systems to be synchronized could be significantly different.

When the variations of mode intensity of the two laser modes are synchronized, it was further found that, despite the fact that the laser is in fact operating on two modes and in particular the modes propagate in opposite directions, the total laser system behavior is similar to that of a single-mode laser. When the mode dynamics is synchronized, with fixed pump intensity and pump-laser frequency detuning relative to the NH_3 absorption line center, as the FIR cavity detuning is reduced, the laser emission in each direction shows simultaneously an intensity period-doubling route to chaos as shown in Fig. 3. In all the laser states where the frequencies of the two modes are locked, their intensity variations are also synchronized. If the mode frequencies are not locked, no such period-doubling route to chaos was observed. In contrast, due to the competition and coupling between the two modes, complex quasiperiodicity and chaos was observed [18]. Another significant characteristic of single-mode ring-laser chaotic dynamics is that under suitable conditions, the laser emission shows spiral chaos similar to that calculated from the Lorenz equations. We have also found a spiral kind of chaos in the two-mode laser, which is shown in Fig. 4. To explore the possible relation of this spiral chaos to that of the single-mode laser, we have constructed the first intensity return map from the intensity evolution, as shown in Fig. 5. It has a similar cusp form to that of the Lorenz map. All this experimental evidence suggests that when the counterpropagating mode frequencies are locked, and consequently their dynamics are synchronized, the dynamics of the two-mode laser system approaches that of the single-mode laser.

The tendency toward chaos synchronization can also be observed in certain other cases. Figure 6 shows the intensity variations of the two modes in a case where the two mode frequencies are not locked together and the pulsations occur alternately. Although the two modes are not perfectly synchronized, the long-term correlation is very high, and considering that sensitivity to initial conditions would normally cause such chaotic trajectories to diverge rapidly, it is clear that an interaction is forcing them back together. The fact that the pulsations alternate emphasizes the separateness of the two modes and supports our interpretation of the phase-locked results in terms of two interacting modes rather than one single one. This result suggests strongly that the observed chaos synchronization is formed due to a self-organization between the coupled modes and it is, in fact, a special cooperative state of the laser under mutual chaotic mode interaction.

In conclusion, we have experimentally observed chaotic frequency locking between the two counterpropagating longitudinal laser modes of an optically pumped NH_3 bidirectional ring laser. We found that under the mode-frequency locking the dynamics of the two individual laser modes are synchronized, no matter whether the dynamics of each mode is periodic or chaotic. Furthermore, we showed that when the two mode frequencies of the laser are locked together, the dynamics of each mode show strong similarities to those of the single-mode laser. To our knowledge, this is the first experimental result showing mode frequency locking in a chaotic laser.

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