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Photorefractive phase-conjugate reflectivity enhancement due to linear absorption

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The undepleted-pump photorefractive four-wave mixing equations for the case of orthogonally polarized pumps are solved taking into account the linear absorption of the medium. A phenomenon of the enhancement of the phase conjugate reflectivity due to linear absorption is found to occur for a certain range of the phase conjugator parameters. This phenomenon is shown to occur as a consequence of the existence of the self-oscillatory modes in the presence of linear absorption. [S1050-2947(96)01912-9]

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Photorefractive phase conjugation has been extensively studied by many groups over the past decade due to its ability to provide high conjugate reflectivities with low-power lasers. In many practical situations, parallel polarized pump beams are used. However, this becomes a disadvantage as the parametric gain due to four-wave mixing in a photorefractive medium is not possible from the two pumps simultaneously, so that the phase conjugate reflectivities with 90° spatial phase shift are not so large as those with 0° spatial phase shift [1]. It was earlier suggested by Stepanov and Petrov [2] that this problem could be overcome by using orthogonally polarized pump beams in the sillenite crystal family. Later, Kong *et al.* [1] developed a theory by generalizing the four-wave mixing coupled equations in the sillenite crystal family and extended the results to the $\mathrm{BaTiO_3}$ and strontium barium niobate (SBN) crystal family, using the undepleted pumps approximation. In this paper, we further extend the theory of Ref. [1] by incorporating linear absorption of the medium. This extension has already been done for the case of parallel polarized pump beams [3]; however, it has not been considered by anyone for the orthogonally polarized pump beams. It is shown that the phase conjugate reflectivity is enhanced due to linear absorption for a certain range of the conjugator parameters. This enhancement due to linear absorption occurs as a consequence of the existence of self-oscillatory modes in the presence of linear absorption. The physical explanation of the predicted phenomenon of the reflectivity enhancement due to linear absorption is given. The words ''reflectivity enhancement due to linear absorption'' are understood to have the meaning that the reflectivity increases with increasing linear absorption.

We consider the four-wave mixing geometry as shown in Fig. 1 of Ref. [1]. Under the undepleted pump approximation, the probe and conjugate wave amplitudes, A_4 and A_3, for the transmission grating with orthogonally polarized pump beams are governed by the following coupled linear differential equations [1,4]:

$$\frac{dA_4^*}{dz}+\alpha A_4^*=\frac{\gamma_f}{I_0(z)}\,[|A_1(0)|^2e^{-2\alpha z}A_4^* +A_1^*(0)A_2^*(L)e^{-\alpha L}A_3], \tag{1}$$

$$\frac{dA_3}{dz}-\alpha A_3=\frac{\gamma_b}{I_0(z)}\,[A_1(0)A_2(L)e^{-\alpha L}A_4^* +|A_2(L)|^2e^{-2\alpha(L-z)}A_3], \tag{2}$$

where α and L designate, respectively, the linear absorption coefficient and the thickness of the photorefractive material, $I_0(z)$ is the total intensity of the pump waves, $I_0(z)=|A_1(z)|^2+|A_2(z)|^2$, and γ_f and γ_b are complex coupling coefficients for the forward and backward gratings, respectively. Their expressions are $\gamma_f=ia_fe^{i\phi_f}$ and $\gamma_b=ia_be^{i\phi_b}$. The real constants a_f,a_b and ϕ_f,ϕ_b represent, respectively, the amplitudes and phases of these gratings. For the $\mathrm{Bi_{12}SiO_{20}}$ crystal family, also known as the sillenite crystal family, the amplitudes a_f and a_b and the phases ϕ_f and ϕ_b of the coupling coefficients for the forward and backward gratings are related [1,4] by $a_f=a_b$ and $\phi_b=\phi_f+\pi$. For the $\mathrm{BaTiO_3}$ and SBN crystal family, these parameters are related [1,4] by $a_f\neq a_b$ and $\phi_f=\phi_b$. When the parallel polarized pumps are used [3,5], one has $a_f=a_b$ and $\phi_f=\phi_b$.

Equations (1) and (2) are to be solved subject to two boundary conditions: $A_4(0)$ equals the amplitude of the incident probe wave at the face $z=0$, of the photorefractive crystal, and $A_3(L)=0$. These equations are the coupled ordinary differential equations with variable coefficients. Their solutions have not been reported earlier in the literature. To solve these equations, it is expedient to cast them in the following form:

$$\frac{de^{\alpha z}A_4^*}{dz}=\frac{\gamma_f}{I_0(z)}\,[A_1(0)e^{-\alpha z}A_4^* +A_2^*(L)e^{-\alpha L}e^{\alpha z}A_3]A_1^*(0), \tag{3}$$

$$\frac{de^{-\alpha z}A_3}{dz}=\frac{\gamma_b}{I_0(z)}\,[A_1(0)e^{-\alpha L}A_4^* +A_2^*(L)e^{-\alpha L}e^{\alpha z}A_3]A_2(L)e^{-\alpha L}. \tag{4}$$

Because the right-hand sides of Eqs. (3) and (4) are proportional to each other, a simple algebraic relation between A_4^* and A_3 is implied,

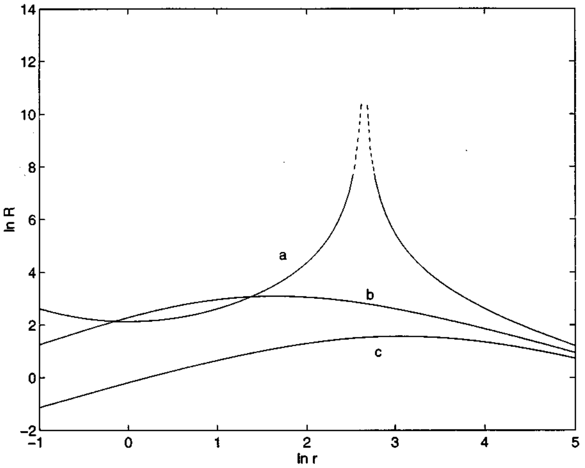

FIG. 1. Plots of $\ln(R)$ vs $\ln(r)$ with $\phi_b=\pi/2$ and $a_bL=3.05$. The normalized linear absorption coefficient $\alpha_0=0$. Curve a, $m=1$, $\phi_b-\phi_f=\pi$; curve b, $m=18$, $\phi_b=\phi_f$; curve c, $m=1$, $\phi_b=\phi_f$.

$$e^{\alpha z}A_4^*=\frac{A_1^*(0)}{A_2(L)e^{-\alpha L}}\,e^{-\alpha z}A_3\,\frac{\gamma_f}{\gamma_b}+C_1, \tag{5}$$

where C_1 is an integration constant. Using Eq. (5) in Eq. (2), a first-order inhomogeneous differential equation for $A_3'=A_3e^{-\alpha z}$ can be obtained,

$$\frac{dA_3'}{dz}+P(z)A_3'=Q(z), \tag{6}$$

where

$$P(z)=-\gamma_b\left[\frac{(\gamma_f/\gamma_b)+re^{-2\alpha L}e^{4\alpha z}}{1+re^{-2\alpha L}e^{4\alpha z}}\right], \tag{7}$$

$$Q(z)=\gamma_bC_1\,\frac{A_2(L)}{A_1^*(0)}\,\frac{e^{-\alpha L}}{1+re^{-2\alpha L}e^{4\alpha z}}. \tag{8}$$

The solution of Eq. (6) can be written as $A_3'(z)=e^{-I_1(z)}[I_2(z)+C_2]$, where $I_1(z)=\int P(z)dz$, $I_2(z)=\int Q(z)e^{I_1(z)}dz$, and C_2 is another integration constant. Using the boundary conditions and the following relation for $I_1(z)$,

$$I_1(z)=\left(\frac{\gamma_f-\gamma_b}{4\alpha}\right)\ln(1+re^{-2\alpha L}e^{4\alpha z})-\gamma_fz, \tag{9}$$

one obtains, after some algebraic manipulations, the following expressions for A_3 and A_4^*:

$$A_4^*(z)=A_4^*(0)e^{-(\alpha-\gamma_f)z}\,\frac{\left[\dfrac{J(z)}{F(z)}-e^{-\gamma_fz}\right]}{\left[\dfrac{J(0)}{F(0)}-1\right]}, \tag{10}$$

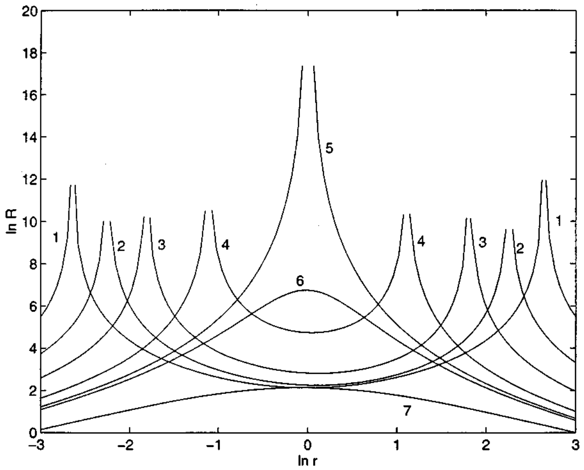

FIG. 2. Plots of $\ln(R)$ vs $\ln(r)$ with $\phi_b=\pi/2$ and $a_bL=3.05$ in the sillenite crystal family. The normalized linear absorption coefficient $\alpha_0=0$, 0.1, 0.2, 0.3, 0.345 19, 0.36, and 0.468, labeled as 1–7.

$$A_3(z)=A_4^*(0)e^{(\alpha+\gamma_f)z-\alpha L}\,\frac{A_2(L)}{A_1^*(0)}\,\frac{\gamma_b}{\gamma_f}\,\frac{\left[\dfrac{J(z)}{F(z)}\right]}{\left[\dfrac{J(0)}{F(0)}-1\right]}, \tag{11}$$

where $J(z)$ designates the definite complex integral,

$$J(z)=\int_{\gamma_fz}^{\gamma_fL}dx\,\frac{e^{-x}}{(1+re^{-2\alpha L}e^{4\hat{\alpha}x})^{1-\mu}}, \tag{12}$$

and the complex function $F(z)$ is $F(z)=(1+re^{-2\alpha L}e^{4\alpha z})^{\mu}$. In the above equations, r represents the pump intensity ratio, $|A_2(L)/A_1(0)|^2$, and the complex quantities μ and $\hat{\alpha}$ are $\mu=[1-(\gamma_b/\gamma_f)]/4\hat{\alpha}$ and $\hat{\alpha}=\alpha/\gamma_f$. The phase-conjugate intensity reflectivity R is given by

$$R=\left|\frac{A_3(0)}{A_4^*(0)}\right|^2=re^{-2\alpha L}m^2\left|\frac{J(0)}{J(0)-F(0)}\right|^2, \tag{13}$$

where $m\equiv a_b/a_f$. For $\gamma_f=\gamma_b$, $\mu=0$ and $F(0)=1$. Equation (13) then reduces to that for the parallel polarized pumps case [3].

In Fig. 2 of Ref. [1], the plots of $\ln(R)$ versus $\ln(r)$ are shown in the absorption-free medium for $\phi_b=\pi/2$ and $a_bL=3.05$ for the following three cases: (a) $m=1$ and $\phi_b-\phi_f=\pi$, (b) $m=18$ and $\phi_b=\phi_f$, and (c) $m=1$ and $\phi_b=\phi_f$. These cases correspond to orthogonally polarized pumps in the sillenite crystal family, orthogonally polarized pumps in $BaTiO_3$ and the SBN crystal family, and parallel polarized pumps, respectively. It should be mentioned here that curve a in this figure is plotted incorrectly. The correct plot of curve a is shown in Fig. 1 of this paper. Curve a exhibits an asymptotic behavior where $\ln(R)$ asymptotically approaches infinity (self-oscillation) as $\ln(r)$ approaches a value equal to 2.645 95. This value can be calculated by using Eq. (18a) of Ref. [1]. It should be noted that this equation does not change if we replace r by $1/r$, indicating that there are two values of r for which this condition is satisfied for each a_bL. These two r values are reciprocal of one another. Thus, for $\ln(r)=-2.645\,95$ also curve a of Fig. 1 will

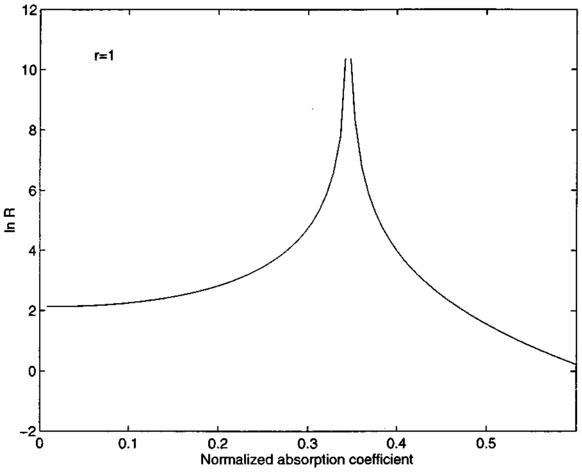

FIG. 3. Plot of ln(R) vs α_0 with $\phi_b=\pi/2$, $r=1$, and $a_bL=3.05$ in the sillenite crystal family.

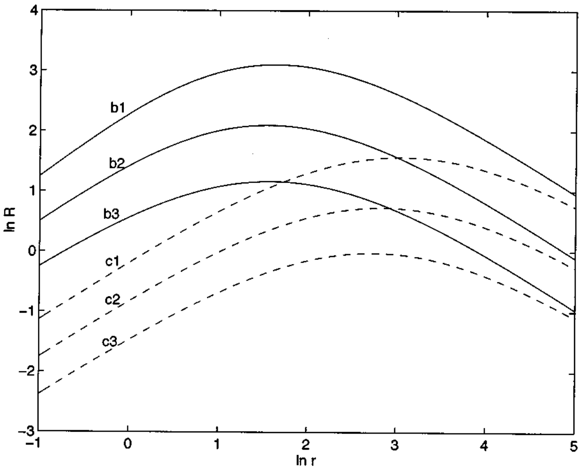

FIG. 4. Plots of ln(R) vs ln(r) with $\phi_b=\pi/2$ and $a_bL=3.05$. The normalized linear absorption coefficient $\alpha_0=0$, 0.1, and 0.2, labeled as 1–3. The labels b and c correspond to the $m=18$, $\phi_b=\phi_f$, and $m=1$, $\phi_b=\phi_f$ cases, respectively.

show the asymptotic behavior for ln(R). The enhancement of the phase-conjugate reflectivity (i.e., large reflectivity) in the sillenite crystal family for $\phi_b=\pi/2$ and $a_bL \geqslant 2$ occurs because of the presence of the self-oscillatory modes. If the conjugator parameters are in the neighborhood of the parameter regime for which the self-oscillation is possible, the reflectivity enhancement occurs. For case (a) of Fig. 1, the reflectivity enhancement occurs when the value of ln(r) is close to ±2.645 95. For cases (b) and (c) above, the large enhancement of the phase-conjugate reflectivity is not possible when $\phi_b=\pi/2$ since the self-oscillatory modes do not exist. However, for the nondegenerate four-wave mixing in these cases, the enhancement of the reflectivity will be possible since the nondegenerate self-oscillatory modes for the parallel polarized pumps and orthogonally polarized pumps in the $BaTiO_3$ and SBN crystal family with $\phi_b=\pi/2$ do exist [4,5]. In fact, it has already been previously shown by MacDonald and Feinberg [6] for the parallel polarized pumps that four-wave mixing in photorefractive crystals has the inherent property of producing enhanced phase-conjugate signals when the probe beam frequency is slightly detuned from the frequency of the pump beams. This happens because of the existence of the nondegenerate self-oscillatory modes for $\phi_b=\pi/2$ with parallel polarized pumps. Another example of the asymptotic behavior of ln(R) is shown in Fig. 3 of Ref. [3] for $\phi_b=\pi/6$, $r=6.13$, and $a_bL=3.627$ with parallel polarized pumps. In this case, if the value of r is close to 6.13 one would observe the phase-conjugate reflectivity enhancement.

In Fig. 2, we show the plots of ln(R) versus ln(r) for $\alpha_0 \equiv \alpha/a_b=0$, 0.1, 0.2, 0.3, 0.34 519, 0.36, and 0.468 with $\phi_b=\pi/2$ and $a_bL=3.05$. It is seen for case (a) in this figure that if α_0 is increased from 0 to 0.1, the phase-conjugate reflectivity is enhanced due to linear absorption (i.e., the reflectivity increases with increasing linear absorption) if $|\ln(r)|$ is approximately less than 2.453. If α_0 is increased from 0 to 0.2, the reflectivity is enhanced due to linear absorption if $|\ln(r)|$ is approximately less than 2.184. This shows that the range of ln(r) for which the reflectivity enhancement due to linear absorption occurs reduces if α_0 is increased from 0 to larger values. This range of ln(r) becomes zero (i.e., $r=1$) if α_0 is increased from 0 to approximately 0.468. However, if α_0 is increased beyond 0.345 19, the reflectivity enhancement due to linear absorption does not occur, i.e., the reflectivity decreases with increasing linear absorption. This is because for $\alpha_0>0.345\,19$, the self-oscillation is not possible. The self-oscillatory modes in the sillenite crystal family for $a_bL=3.05$ and $\phi_b=\pi/2$ exist only for α_0 between 0 and 0.345 19. It is to be noted here that linear absorption does not eliminate the existence of the self-oscillatory modes. In Fig. 3, we show the plot of ln(R) versus α_0 for $a_bL=3.05$, $\phi_b=\pi/2$, and $r=1$. The phase-conjugate reflectivity increases with increasing α_0 up to $\alpha_0=0.345\,19$. For $\alpha_0>0.345\,19$, the reflectivity decreases with increasing α_0. For $\alpha_0=0.345\,19$, ln(R) exhibits an asymptotic behavior (self-oscillation). Large reflectivity is possible if α_0 is close to 0.345 19. However, the reflectivity increases with increasing α_0 (i.e., the reflectivity enhancement due to linear absorption occurs) only if $\alpha_0<0.345\,19$. This type of large reflectivity and the reflectivity enhancement due to linear absorption do not occur for cases (b) and (c) shown in Fig. 4 since the self-oscillatory modes for these two cases do not exist. Larger reflectivity for case (b) compared to case (c) still occurs in Fig. 4 because of the large value of m for case (b).

The condition for self-oscillation is, from Eq. (13), $J(0)=F(0)$. This condition was solved numerically for the orthogonally polarized pumps in the sillenite crystal family taking $\phi_b=\pi/2$. The result of the computation is shown in Fig. 5. The presence of linear absorption only makes self-oscillation slightly more difficult in the sense that now a larger value of a_bL is required to obtain self-oscillation compared to that required in the absorption-free medium. Curve for $\alpha_0=0.345\,19$ is also shown in this figure. Any curve for $\alpha_0>0.345\,19$ will not intersect with the dotted horizontal line $a_bL=3.05$ implying that the self-oscillation is not possible. The reflectivity enhancement due to linear absorption that also occurs for $\alpha_0<0.345\,19$ is, therefore, a consequence of the existence of the self-oscillatory modes. The self-oscillation condition $J(0)=F(0)$ could not be satisfied for the parallel polarized pumps and orthogonally polarized pumps in $BaTiO_3$ and the SBN crystal family for $\phi_b=\pi/2$,

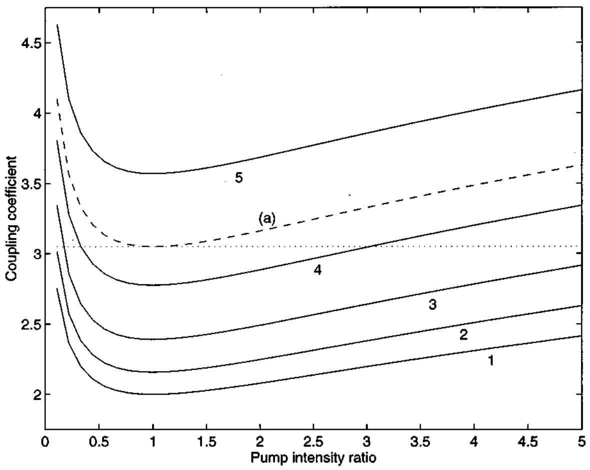

FIG. 5. Plots of a_bL vs r for the normalized linear absorption coefficient $\alpha_0=0$, 0.1, 0.2, 0.3, and 0.4, labeled as 1–5, and 0.345 19, labeled as (a), when degenerate self-oscillation takes place in the sillenite crystal family. The value of ϕ_b is equal to $\pi/2$. These plots were obtained by numerically solving the self-oscillation condition $J(0)=F(0)$. The dotted horizontal line is a plot of the line $a_bL=3.05$.

indicating that the linear absorption indeed does not change the situation concerning the self-oscillation in these two cases [7,4,5].

Physically, the enhancement of the four-wave mixing efficiency in the presence of linear absorption can be understood in a similar manner as for the case in which the enhancement occurs for a nonzero frequency shift [6]. From Fig. 5, we know that an increase in absorption coefficient requires a larger value of the coupling coefficient to obtain self-oscillation. This means that in the presence of linear absorption, the magnitude of energy coupling between the writing beams is decreased. This increases the average modulation of the interference fringes of the writing beams and the amplitude of the resulting refractive index grating. For small values of the linear absorption coefficient, the increase in modulation of the interference pattern more than compensates the decrease in the amplitude of the coupling coefficient, and the phase-conjugate signal increases. For larger values of the absorption coefficient, the decrease in magnitude of the coupling coefficient cannot be compensated by an increase in the modulation of the fringes, and the diffraction efficiency reduces giving reduced phase-conjugate signals. This is the reason why the reflectivity enhancement due to linear absorption does not occur for $\alpha_0>0.345\,19$ in Figs. 2 and 3. Figure 3 also indicates that the threshold condition for self-oscillation in the presence of linear absorption is attained when a decrease in magnitude of the coupling coefficient due to linear absorption is exactly balanced by an increase in the modulation of the fringes.

In conclusion, the reflectivity enhancement of a photorefractive phase conjugator is a consequence of the existence of the self-oscillatory modes and since linear absorption does not eliminate the existence of the self-oscillatory modes, the reflectivity enhancement due to linear absorption is indeed possible for a certain range of the conjugator parameters. This is an inherent feature of four-wave mixing in photorefractive materials similar to the one where the reflectivity enhancement occurs due to unequal frequencies of the pump and probe waves [6]. The two associated phenomena, namely, the reflectivity enhancement due to linear absorption and the self-oscillation in the presence of linear absorption, are being studied extensively by us for many different cases with both the parallel polarized pumps and orthogonally polarized pumps. Part of our results for the parallel polarized pumps case will appear elsewhere [8]. The results of other studies will also be published elsewhere.

Finally, it should be mentioned that the phenomenon of the reflectivity enhancement due to linear absorption is still present even if one removes the undepleted pump approximation [9]. The preliminary results of the numerical solution of the full set of four nonlinear equations [9] can be summarized as follows: When the pumps are initially strong compared to probe, they remain strong throughout the crystal in spite of the absorption loss. The numerical solution in this case agrees remarkably well with the results obtained using the undepleted pump approximation. When the probe is comparable to that of the pumps, the results differ from that obtained using the undepleted pump approximation. However, the reflectivity enhancement due to linear absorption is still present. Thus, the reflectivity enhancement due to linear absorption is not a physically unrealistic phenomenon. Its existence can even be predicted by the undepleted pump analysis presented in this paper. It should be experimentally observable. Its experimental observation can be made possible, for example, by varying the wavelength of the input pump and probe waves. This, in turn, will vary the absorption coefficient of the medium. For a crystal of the sillenite family with the normalized coupling coefficient $a_bL=3.05$ and pumped by two equal power counterpropagating orthogonally polarized pumps, the normalized absorption coefficient αL should range between 0 and 1.05 in order to observe the conjugate reflectivity enhancement due to linear absorption.

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