Quantum logic using polarized photons

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We consider a realization of quantum computing using polarized photons. The information is encoded in two polarization directions of the photons and two-quantum-bit operations are performed using a conditional Faraday effect. We investigate the performance of the system as a computing device. [S1050-2947(96)05711-3]

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I. INTRODUCTION

After the early discussion of quantum computing [1–3], the field has attracted much attention because Shor [4] has shown that the famous problem of integer factorization can, in principle, be speeded up considerably by quantum data manipulation techniques. Quantum information is encoded on quantum bits (''qubits''), which can be in any superposition state of the classical bit values zero and one. Consequently registers consisting of several qubits can be encoded to superpositions of initial values, which will then be processed in parallel. Although the determination of all output values contained in the superposition would be no faster than obtaining them by classical computation, certain global properties of the output superposition can be efficiently extracted, and algorithms utilizing this information speeded up exponentially. The recent work on quantum computation has been reviewed, e.g., in [5,6].

Any two-state quantum system can serve as a qubit. To implement quantum logic, we have to be able to build a quantum gate which changes the state of a target qubit depending on the state of a control qubit. Once we know how a single quantum logic gate for two incoming bits can be realized, we can build up all necessary computations using this universal basic gate [7–11]. Many realizations of quantum computing have been suggested; at present the most promising ones seem to be ions trapped electrodynamically [12] or in a cavity [13]. Also the use of single atoms and cavity modes [8], solid state structures [14], and Ising systems [15] have been proposed. Experimental successes of an ion-trap quantum gate [16] and of conditional quantum logic with cavity photons [17] have been reported.

Polarized photons can be used to code quantum information, see, e.g., [18–20]. In recent work [21], one of us considered the possible use of photon polarization states to carry quantum information, and proposed a model to realize quantum logic by interaction via an atom. The advantage is that polarized photons provide a natural two-state basis with no additional Hilbert space components that may constitute losses of the coding. The polarization coding allows for easy gating on successful detection; in contrast, in the alternative coding to the zero and one photon Fock states, the information carried by the vacuum state is hard to distinguish from a failed detection. Photon coding in general allows long dephasing times and the possibility to transfer information from one device to another through fibers. The purpose of this paper is to investigate how realistic this suggestion is by numerical integration of the unitary time evolution of a semi-realistic situation. To make the system fully realistic, the effect of dissipative processes would have to be taken into account.

In the scheme considered in this paper, information is coded in the linear polarization directions of a photon, the x direction corresponding to the logical zero, the y direction to the logical one. To realize a basic two-qubit gate, we have to be able to affect rotations and phase shifts of the target qubit state conditioned on the control qubit state. A conditional rotation in the linear polarization basis corresponds to a conditional phase shift in the circular polarization basis. In this paper we work in the circular polarization basis and show how to get phase shifts by a conditional Faraday effect. This allows us to realize rotations in the linear basis. Furthermore, conditional phase shifts in the linear basis can be performed by switching between the linear and circular bases before and after the gate. This can be done with the help of retardation plates. For a more detailed explanation of the basics of the scheme, see [21].

The gating process considered in this paper is a Faraday phase rotation of the elliptically polarized photon state $(\alpha_+ a_+^\dagger + \alpha_- a_-^\dagger)|0\rangle$,[1] which is gated by the presence of a second photon $b_\pm^\dagger$. These are supposed to selectively transfer population from the atomic ground state $|0\rangle$ in Fig. 1 to the levels $|\pm 1\rangle$. Because each photon $a_\pm^\dagger$ is affected by only one transition $|\pm 1\rangle \rightarrow |2\rangle$ it becomes modified by the population transferred by the photons $b_\pm^\dagger$. If we keep the transition $|\pm 1\rangle \rightarrow |2\rangle$ off resonance, the atom acts as a dielectric only and hence the relative phases of $a_\pm^\dagger$ are modified; this implies a turning of the axis of the elliptically polarized state $(\alpha_+ a_+^\dagger + \alpha_- a_-^\dagger)|0\rangle$. As explained above, this allows the gated application of an arbitrary unitary transformation.

In this paper we are looking at two different cases. Case I corresponds to Fig. 1 when $\Delta_1 = 0$. Here both transitions

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[1] In our notation $|\nu\rangle \equiv |\nu\rangle_{\text{atom}} \otimes |\text{vacuum}\rangle_{\text{fields}}$, i.e., only the label for the atomic energy levels (ν) is written explicitly.

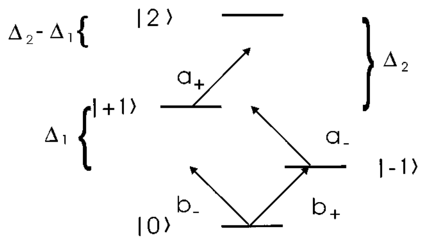

FIG. 1. The four-level system used in the gated Faraday effect.

$|0\rangle \rightarrow |\pm 1\rangle$ are in resonance and both photons $a_\pm^\dagger$ experience a modified phase. The situation is symmetric: if $b_+^\dagger$ is present alone we achieve a phase shift exactly opposite in sign to that caused by the presence of $b_-^\dagger$ only.

In case II we detune one of the transitions, $|0\rangle \rightarrow |+1\rangle$ say; see Fig. 1. Then only the presence of the resonant photon $b_+^\dagger$ affects the phase of the $a^\dagger$ photons. This corresponds to a gate where the presence of the state $b_-^\dagger|0\rangle$ does nothing. Most gates discussed earlier in the literature are of this type.

In this paper we are going to discuss the performance of the model above by direct integration of the time evolution, which allows us to compute the quantities characterizing its performance as a quantum gate. The model is introduced in detail in Sec. II, and the results of the numerical integrations are presented in Sec. III. In Sec. IV we discuss the results and show as a summarizing example how well our model can realize a gate where the state of the target bit is conditionally reversed.

II. THE QUANTUM GATE MODEL

A. Setting up the problem

The four-level system shown in Fig. 1 is described by the Hamiltonian

$$\begin{aligned}H=&\Omega_2(a_+^\dagger a_+ + a_-^\dagger a_-)+\Omega_1(b_+^\dagger b_+ + b_-^\dagger b_-)\\&+\omega_2|2\rangle\langle 2|+\omega_{1+}|+1\rangle\langle +1|+\omega_{1-}|-1\rangle\\&\times\langle -1|+\omega_0|0\rangle\langle 0|+\lambda_1(b_-|+1\rangle\\&\times\langle 0|+b_+|-1\rangle\langle 0|+\text{H.c.})\\&+\lambda_2(a_-|2\rangle\langle -1|+a_+|2\rangle\langle +1|+\text{H.c.}).\end{aligned} \tag{1}$$

In case I we assume that the states $|\pm 1\rangle$ are degenerate and that the transitions $|0\rangle \rightarrow |\pm 1\rangle$ are at resonance,

$$\Delta_1^{\mathrm{I}}=\omega_{1\pm}-\omega_0-\Omega_1=0. \tag{2}$$

The transitions $|\pm 1\rangle \rightarrow |2\rangle$ are assumed detuned, i.e.,

$$\Delta_2^{\mathrm{I}}=\omega_2-\omega_0-\Omega_1-\Omega_2=\omega_2-\omega_{1\pm}-\Omega_2 \tag{3}$$

is nonzero. These relations are shown in Fig. 1.

In case II we lift the degeneracy of the levels $|\pm 1\rangle$ by setting $\omega_{1+}\neq\omega_{1-}$. Then the transition $|0\rangle \rightarrow |-1\rangle$ is taken at resonance $\omega_{1-}-\omega_0=\Omega_1$ but the detunings

$$\Delta_1^{\mathrm{II}}=\omega_{1+}-\omega_0-\Omega_1, \tag{4}$$

$$\Delta_2^{\mathrm{II}}=\omega_2-\omega_0-\Omega_1-\Omega_2 \tag{5}$$

are nonzero. The transition $|+1\rangle \rightarrow |2\rangle$ is detuned by

$$\Delta_2'=\omega_2-\omega_{1+}-\Omega_2=\Delta_2^{\mathrm{II}}-\Delta_1^{\mathrm{II}}. \tag{6}$$

This is assumed well off resonance too.

The initial state is taken to be the disentangled form

$$|\Psi_{\mathrm{in}}\rangle=(\alpha_+a_+^\dagger+\alpha_-a_-^\dagger)(\beta_+b_+^\dagger+\beta_-b_-^\dagger)|0\rangle, \tag{7}$$

where $|0\rangle$ denotes the ground state of the atom and the vacuum of the fields (see Introduction). The coefficients are in general complex numbers normalized to unity.

We propagate the state vector (7) to the time t with the Hamiltonian (1) and write the final state as

$$|\Psi_{\mathrm{out}}\rangle=e^{-iHt}|\Psi_{\mathrm{in}}\rangle=\sum_{i=1}^{9}C_i|i\rangle, \tag{8}$$

where we have numbered the basis states according to the set

$$\begin{aligned}&\{|1\rangle,|2\rangle,|3\rangle,\ldots,|9\rangle\}\\&=\{|2\rangle,a_+^\dagger|+1\rangle,a_+^\dagger|-1\rangle,a_-^\dagger|+1\rangle,a_-^\dagger|-1\rangle,a_+^\dagger b_+^\dagger|0\rangle,\\&\quad a_+^\dagger b_-^\dagger|0\rangle,a_-^\dagger b_+^\dagger|0\rangle,a_-^\dagger b_-^\dagger|0\rangle\}.\end{aligned} \tag{9}$$

Initially the atom is in the ground state, i.e., the coefficients $\{C_6,C_7,C_8,C_9\}$ are prepared nonvanishing. Of these, the Hamiltonian couples in case I C_6 to C_3 ($a_+^\dagger b_+^\dagger|0\rangle$ to $a_+^\dagger|-1\rangle$) and C_9 to C_4 ($a_-^\dagger b_-^\dagger|0\rangle$ to $a_-^\dagger|+1\rangle$) only; in case II C_6 to C_3 only. In these subspaces the system can be solved exactly, and performing a rotating wave approximation with respect to the frequency $\omega_0+\Omega_1+\Omega_2$ we obtain in case I

$$C_9(t)=\cos(\lambda_1 t)C_9(0)+i\sin(\lambda_1 t)C_4(0), \tag{10}$$

$$C_6(t)=\cos(\lambda_1 t)C_6(0)+i\sin(\lambda_1 t)C_3(0). \tag{11}$$

In case II only (11) remains valid. Choosing the interaction time such that $\lambda_1 t=\pi$, we find that the probabilities are restored in these subspaces.

We are now left in case I with a five-dimensional subspace $\{|1\rangle,|2\rangle,|5\rangle,|7\rangle,|8\rangle\}=\{|2\rangle,a_+^\dagger|+1\rangle,a_-^\dagger|-1\rangle,a_+^\dagger b_-^\dagger|0\rangle,a_-^\dagger b_+^\dagger|0\rangle\}$ with the matrix elements

$$H_{11}=\omega_2-\omega_0-\Omega_1-\Omega_2, \tag{12}$$

$$\lambda_2=H_{12}=H_{15}=H_{21}=H_{51}, \tag{13}$$

$$\lambda_1=H_{27}=H_{58}=H_{72}=H_{85}. \tag{14}$$

In case II the subspace is seven dimensional containing in addition to the above the basis vectors $|4\rangle=a_-^\dagger|+1\rangle$ and $|9\rangle=a_-^\dagger b_-^\dagger|0\rangle$, with the matrix elements

$$H_{22}=\omega_{1+}-\omega_0-\Omega_1-\Omega_2, \tag{15}$$

$$\lambda_1=H_{49}=H_{94}. \tag{16}$$

The numerical calculations are performed in these five- and seven-dimensional subspaces. The final state is then projected to the subspace of interest, where the atom is in the ground state and both photons are present.

After the interaction, the state (8) is available for measurements. In the ideal situation, the initial photons would have been restored to the radiation field. This is desired because the information resides in these photons, and they should be available for subsequent computational operations. We can ensure that they have been returned by observing that the atom is back in its ground state $|0\rangle$ by projecting the final state on this. Namely, after the interaction, the atom is available for inspection; a measurement of its state no longer affects the outcome of the process. We write this state after an observation, $|\Psi_0\rangle=|0\rangle\langle 0|\Psi_{\mathrm{out}}\rangle$, as

$$|\Psi_0\rangle=\left(\frac{C_{++}}{\alpha_+\beta_+}e^{i\varphi_{++}}\alpha_+a_+^\dagger+\frac{C_{-+}}{\alpha_-\beta_+}e^{i\varphi_{-+}}\alpha_-a_-^\dagger\right)\beta_+b_+^\dagger|0\rangle + \left(\frac{C_{+-}}{\alpha_+\beta_-}e^{i\varphi_{+-}}\alpha_+a_+^\dagger+\frac{C_{--}}{\alpha_-\beta_-}e^{i\varphi_{--}}\alpha_-a_-^\dagger\right) \times\beta_-b_-^\dagger|0\rangle. \tag{17}$$

We have introduced the amplitudes and phases of the new coefficients as $C_{ij}e^{i\varphi_{ij}}$ $(i,j\in\{-,+\})$.

A measure of the efficiency of the process is the probability

$$P_0=|\langle 0|\Psi_{\mathrm{out}}\rangle|^2=|C_{++}|^2+|C_{+-}|^2+|C_{-+}|^2+|C_{--}|^2. \tag{18}$$

A small value of P_0 makes the process inefficient, but once the state $|0\rangle$ has been observed on the atom, the expressions in the brackets of (17) give the effect on the state $(\alpha_+a_+^\dagger+\alpha_-a_-^\dagger)|0\rangle$ conditioned on the presence of the photons $b_\pm^\dagger$ on the lower transitions. These expressions contain the effect of the gating action of the system. In all cases investigated in this paper, however, P_0 has been found to deviate from unity by less than 1%.

B. Gating performance

If the coefficients

$$\eta_{ij}=\frac{C_{ij}}{|\alpha_i\beta_j|} \tag{19}$$

in (17) are close to unity, the interaction only adds the phases φ_{ij}; the polarization of the $a^\dagger$ field has been changed by the interaction. If we define the initial phases $\varphi_\pm^a=\arg(\alpha_\pm)$ and $\varphi_\pm^b=\arg(\beta_\pm)$, we can denote the phase changes by

$$\overline{\varphi_\pm}=(\varphi_{+\pm}+\varphi_{-\pm}-\varphi_+^a-\varphi_-^a)/2-\varphi_\pm^b, \tag{20}$$

$$\Delta\varphi_\pm=(\varphi_{+\pm}-\varphi_{-\pm}-\varphi_+^a+\varphi_-^a)/2. \tag{21}$$

We now write the final state (17) in the form

$$|\Psi_0\rangle=\{e^{i\overline{\varphi_+}}(\eta_{++}e^{i\Delta\varphi_+}\alpha_+a_+^\dagger+\eta_{-+}e^{-i\Delta\varphi_+}\alpha_-a_-^\dagger)\beta_+b_+^\dagger + e^{i\overline{\varphi_-}}(\eta_{+-}e^{i\Delta\varphi_-}\alpha_+a_+^\dagger+\eta_{--}e^{-i\Delta\varphi_-}\alpha_-a_-^\dagger) \times\beta_-b_-^\dagger\}|0\rangle.$$

When we choose the initial coefficients $\alpha_\pm,\beta_\pm$ real, the phases (20) and (21) simplify; in Sec. III.B., we are going to discuss the influence of the phase on the gating performance.

In case I, the symmetry requires that $\overline{\varphi_+}=\overline{\varphi_-}$ and $\Delta\varphi_+=-\Delta\varphi_-\equiv\Delta\varphi$. In case II, we assert that $\varphi_{+-}\sim\varphi_{--}\simeq 0$, which implies $\overline{\varphi_-}\simeq 0$ and $\Delta\varphi_-\simeq 0$. We may consider the four-dimensional subspace $\{a_-^\dagger b_-^\dagger|0\rangle, a_+^\dagger b_-^\dagger|0\rangle, a_-^\dagger b_+^\dagger|0\rangle, a_+^\dagger b_+^\dagger|0\rangle\}$. Assuming now that all coefficients η_{ij} are unity, we obtain in the symmetric case the ideal transformation

$$U_{\mathrm{I}}=e^{i\overline{\varphi}}\begin{bmatrix} e^{i\Delta\varphi} & 0 & 0 & 0\\ 0 & e^{-i\Delta\varphi} & 0 & 0\\ 0 & 0 & e^{-i\Delta\varphi} & 0\\ 0 & 0 & 0 & e^{i\Delta\varphi}\end{bmatrix}. \tag{22}$$

In the detuned case II, we obtain

$$U_{\mathrm{II}}\sim\begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & e^{i(\overline{\varphi_+}+\Delta\varphi_+)} & 0\\ 0 & 0 & 0 & e^{i(\overline{\varphi_+}-\Delta\varphi_+)}\end{bmatrix}. \tag{23}$$

This is a phase transformation of the bit denoted by $a_\pm^\dagger$ induced by the presence of the photon $b_+^\dagger$.

III. NUMERICAL INTEGRATIONS

A. "Classical" input states

We are now going to consider the performance qualities of the model system as a gated bit transformation. The input to the calculation is the initial state (7). To begin we choose the "classical" case when only one of the input states is present. In the symmetric case I, the choice of state is not important, cf. U_{I}, but for case II, we need to look at the states $a_-^\dagger b_+^\dagger|0\rangle$ and $a_+^\dagger b_+^\dagger|0\rangle$. First we choose to discuss the single input state $a_-^\dagger b_+^\dagger|0\rangle$ with $\alpha_-=\beta_+=1$.

As stated above, the interaction time is chosen such that $t=\pi/\lambda_1$; in the calculations we choose $\lambda_1=1$. For large detunings

$$\lim_{\omega_2\to\infty}\eta_{-+}=\lim_{\omega_2\to\infty}\frac{C_{-+}}{|\alpha_-\beta_+|}=1, \tag{24}$$

but the phase shift $\Delta\varphi$ goes to zero. In case I, the numerical investigations show that we can retain $\eta_{-+}^2>0.9$ if we choose $\Delta_2^I>5$. For $\Delta_2^I=5$ we find $\Delta\varphi_+\simeq 10°$. This is achieved with $\lambda_2=1$; larger phases can be achieved by increasing λ_2, but the restoring of the population suffers. For $\lambda_2\leqslant 1.5$ we can achieve $\Delta\varphi_+\geqslant 15°$ and $\eta_{-+}^2>0.75$. The results can be illustrated in a graph plotting $\Delta\varphi_+$ as a function of η_{-+}^2 with the detuning as a parameter. For the symmetric case I, this is done in Fig. 2(a). As we can see, for $\Delta_2^{\mathrm{I}}>5$, no dependence on detuning is seen. The corresponding results for case II are shown in Fig. 2(b). Here the dependence on detuning is much stronger; however, for large values of detuning, $\Delta_1^{\mathrm{II}}=15$ and $\Delta_2^{\mathrm{II}}=30$, we can reach $\Delta\varphi_+\geqslant 43°$ with $\eta_{-+}^2\geqslant 0.9$. Thus the operation of this gate is

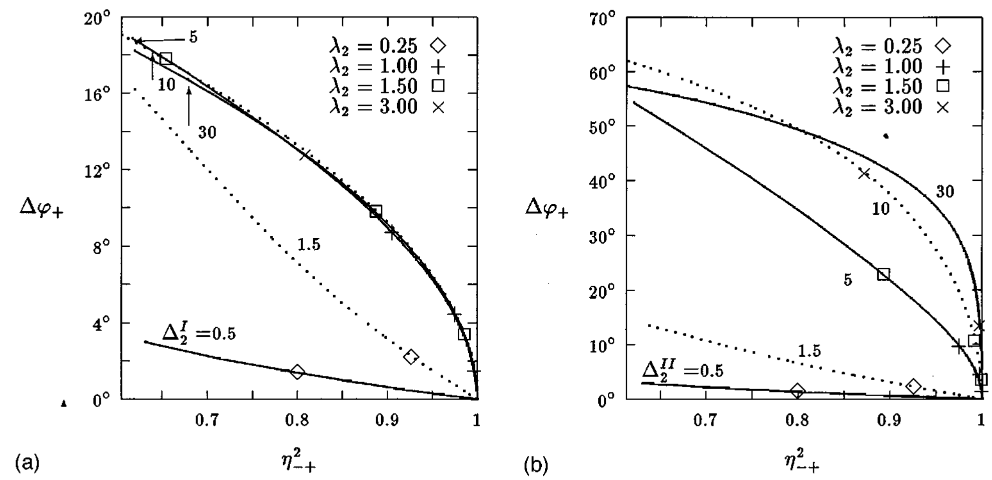

FIG. 2. The phase shift $\Delta\varphi_+$ as a function of η^2_{-+} for several values of the detunings Δ_2^{I}, Δ_2^{II}; some values of λ_2 used are marked. In all figures (a) corresponds to case I, (b) to case II where $\Delta_1^{\mathrm{II}} = \Delta_2^{\mathrm{II}}/2$.

much more efficient, as is to be expected. For larger values of Δ_1^{II} the results tend to become independent of the detuning.

We now choose to look at the case $\lambda_2 = 2.5$ and $\Delta_2 = 30$. For case I this gives $\Delta\varphi_+ \simeq 10°$ and $\eta^2_{-+} \simeq 0.90$. In case II it gives $\Delta\varphi_+ \simeq 10°$ and $\eta^2_{-+} \simeq 0.99$. In order to see where the missing population goes in case I, we plot the populations of the states $a_-^\dagger b_+^\dagger|0\rangle$, $a_-^\dagger|0\rangle$, $a_+^\dagger|0\rangle$, $a_+^\dagger b_-^\dagger|0\rangle$, and $|2\rangle$ in Fig. 3. At time $t=\pi$, the population of $a_-^\dagger b_+^\dagger|0\rangle$ is restored to 90% but the missing population is on the level $a_+^\dagger b_-^\dagger|0\rangle$. This is mediated through the off-resonant transition $|-1\rangle \rightarrow |2\rangle \rightarrow |+1\rangle$ which proceeds at the effective Rabi rate $(\lambda_2^2/\Delta_2^I) \sim 6.25/30$. With time, this increases the population of the state $a_+^\dagger|+1\rangle$, as can be seen in Fig. 3; this increase is modulated at the rate λ_1 by the population pulsations on level $a_-^\dagger|-1\rangle$. This effect can be decreased by increasing $\Delta_2 \gg \lambda_2^2$. In case II, the population of the level $a_-^\dagger b_+^\dagger|0\rangle$ is restored to better than 99% and the population in the states $a_+^\dagger|+1\rangle$ and $a_+^\dagger b_-^\dagger|0\rangle$ remains below 10^{-3}.

Here we have looked at the behavior for the single two-bit input state $a_-^\dagger b_+^\dagger|0\rangle$. In the symmetric case I, this state contains all the information necessary, see Eq. (22). For case II one should really examine also the input state $a_+^\dagger b_+^\dagger|0\rangle$ separately; its phase shifts are not the same, see Eq. (23). However, we are going to investigate the superposition quantum state next, and from that investigation all necessary information follows.

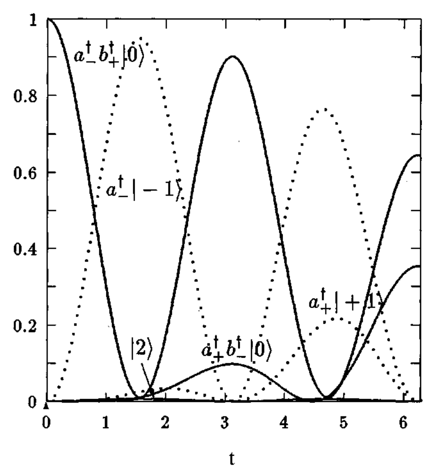

FIG. 3. The populations of the basis states as functions of time (case I).

B. Quantum input states

After having described the ''classical'' inputs, where each two-bit pure state has been treated separately, we now turn to consider the genuine quantum situation described by the input state (7). The performance of the system acting on this state is, of course, essential for its usefulness as a quantum computing device.

An input consisting of a pair of two-level systems contains four degrees of freedom: the four complex numbers involved lose two parameters to the over-all phase and two to the normalization conditions. It is still difficult to display the results of a four-parameter input space, and hence we start by considering only real coefficients in Eq. (7). The influence of the phases $\varphi_\pm^a$, $\varphi_\pm^b$ will be discussed below.

We are thus left with two real parameters, one for each input bit. We choose to display our results as functions of

$$\alpha_-^2 = 1 - \alpha_+^2 \tag{25}$$

for the two cases

$$|\beta_1\rangle = \frac{1}{\sqrt{2}}(b_+^\dagger + b_-^\dagger)|0\rangle, \tag{26}$$

$$|\beta_2\rangle = \left(\frac{\sqrt{3}}{2} b_+^\dagger + \frac{1}{2} b_-^\dagger\right)|0\rangle. \tag{27}$$

We want to introduce a quality factor for the use of a system like this in computations. The performance is close to ideal, when the parameter $\eta_{ij} \simeq 1$. However, when either one of the input parameters α_i, β_j becomes close to zero, any minute value in the corresponding coefficient C_{ij} is likely to cause a large value η_{ij}. Thus we want to consider the retention of that product $\alpha_i\beta_j$ which is the largest. A value close to unity here signals a good performance. To test this idea we consider the variables

$$\eta^2_{-+}(\alpha_-^2 \geq 0.5), \quad \eta^2_{+-}(\alpha_-^2 \leq 0.5). \tag{28}$$

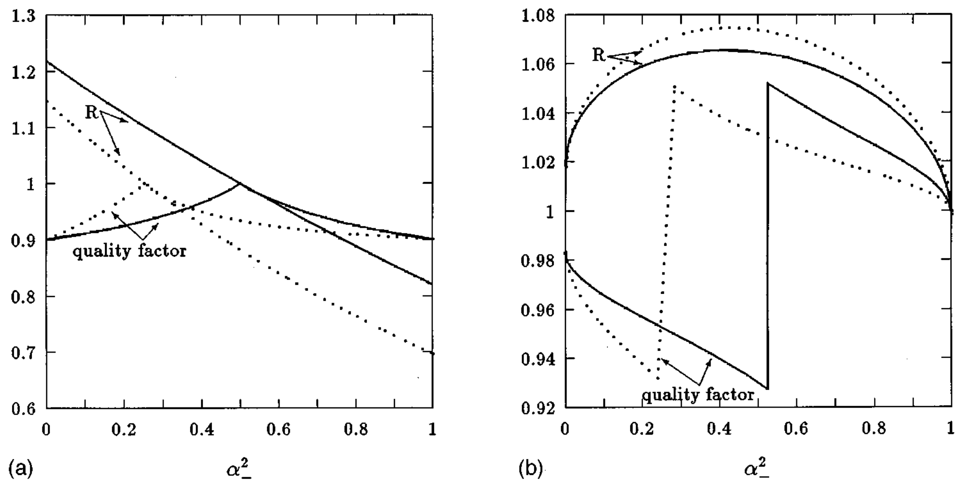

FIG. 4. The retention R and the quality factor (28) as functions of α_-^2, for $|\beta_1\rangle$ (solid lines) and $|\beta_2\rangle$ (dotted lines).

Another measure of the efficiency of the process can be given by the retention of the ratio between the two components $b_\pm^\dagger$ in Eq. (17). This starts from $|\beta_+/\beta_-|^2$ and if retained the parameter

$$R=\left(\frac{|C_{++}|^2+|C_{-+}|^2}{|C_{+-}|^2+|C_{--}|^2}\right)\left(\frac{\beta_-}{\beta_+}\right)^2 \tag{29}$$

should be close to unity. The retention parameter R for case I and the inputs $|\beta_1\rangle$ and $|\beta_2\rangle$ are shown in Fig. 4(a) together with the corresponding quality factor in Eq. (28). In Fig. 4(b) the same parameters are shown for the asymmetric case II. As we can see, the retention parameter R is at its worst about 70%; in case II it is better than 90%. In case I, the quality factor (28) is good to within 90% and in the asymmetric case II to better than 95%.

Finally we want to return to the question of the influence of the initial phases. These do affect the outcome, but their influence seems to be smaller than the influence of the magnitudes. We consider the achieved phase shifts as functions of the superposition coefficients α and β. In Fig. 5 we plot the phase shifts $\Delta\varphi_\pm$ against α_-^2 in the asymmetric case II shown for $|\beta_1\rangle$ and $|\beta_2\rangle$. For $|\beta_1\rangle$, we also consider the case when the initial phase φ_+^a is set to the value $\varphi_+^a=\pi/4$. The behavior is close to ideal; in the range $\alpha_-^2\in(0.1,0.9)$, nearly ideal behavior is observed, $\Delta\varphi_+\simeq 9.5°$ and $|\Delta\varphi_-|<0.4°$. The effect of the initial phase is small. In the symmetric case I, the behavior was found to be less optimal: we saw only a small difference for the two β states, but for α_-^2 in the range $(0.1,0.9)$ the phase shift changed from 30° to 10°. Thus in case I, the magnitude of the angle remains considerable but it does depend on the value of α. We have not carried out a systematic investigation of the influence of the phase factors; the results reported here indicate that they cause no drastic changes. If needed, their effects can easily be evaluated using the method presented here.

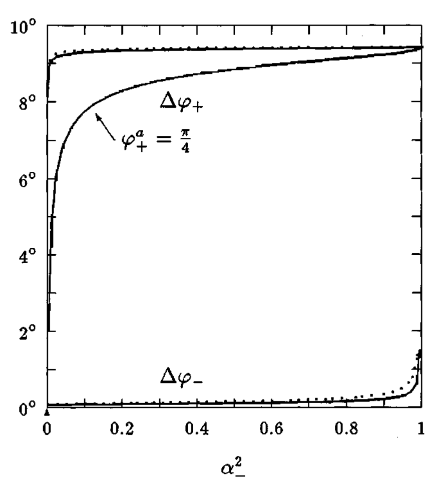

FIG. 5. The phase shifts $\Delta\varphi_\pm$ as functions of α_-^2, for $|\beta_1\rangle$ (solid lines) and $|\beta_2\rangle$ (dotted lines). The shift $\Delta\varphi_+$ is shown also for the case of a nonzero initial phase φ_+^a (case II).

IV. CONCLUSION

As a conclusion, we discuss how well a quantum gate can be realized in our model. We choose to look at a gate which changes the value of the target bit whenever the control bit has the value one. Based on the considerations above, we conclude that the asymmetric case II is better suited to work as a gate. Its performance can easily be improved from the results presented above by increasing $\Delta_2^{\rm II}$, $\Delta_1^{\rm II}$, and λ_2 in a suitable way. Here we use the parameters $\Delta_2^{\rm II}=70$, $\Delta_1^{\rm II}=65$, $\lambda_2=6.85$, $\lambda_1=2$, and $t=\pi$: this enables us to approximate the transformation $U_{\rm II}$ to the accuracy 10^{-3} with a phase shift of 60°. This has to be applied three times in sequence in order to get a phase shift of 180° in the circular polarization basis, which is needed to flip the value of the target bit in the linear polarization basis. After performing suitable transformations between the circular and linear bases (see [21]), we obtain as the final result the transformation C:

$$C=\begin{bmatrix} 0.995e^{-i33°} & O(10^{-3}) & O(10^{-2}) & O(10^{-2}) \\ O(10^{-3}) & 0.995e^{-i33°} & O(10^{-2}) & O(10^{-2}) \\ O(10^{-2}) & O(10^{-2}) & O(10^{-3}) & -0.997 \\ O(10^{-2}) & O(10^{-2}) & -0.997 & O(10^{-3}) \end{bmatrix}.$$

We see that the gate can be realized in this case to the accuracy 10^{-2}. The relative phase difference between $e^{-i33°}$ and

-1 could be cancelled by applying another gate transformation. This transformation together with the transformation C would then produce the standard controlled-NOT gate. The performance of a quantum gate is often characterized by the so-called fidelity parameter [22], which is the modulus squared of the overlap between the desired and actual output states of the gate. For the transformation C the order of magnitude estimate for the fidelity is $|0.997+10^{-2}E+10^{-3}E|^2$, where E is an error amplitude which depends on the input state, and is always greater than or equal to -4. Thus even in the worst possible case this gives 0.91; an average over all possible input states would give a bigger number.

The present scheme has been found to perform reasonably well as a computing device. It is naturally not good enough to be an element of a computer network of realistic size, but none of the suggestions in the literature satisfies this criterion yet, although many of them are very promising and have been demonstrated to work at the one-gate level. The controlled-NOT quantum logic gate has been realized experimentally using an ion trap system [16]. The achieved accuracy in the performance of the gate in this experiment was about 80%. Conditional phase shifts of the order 10°–20° were observed in the experiments reported in Ref. [17]. The basic idea of how to realize conditional logic in this experiment, viz., to let two light fields interact via an atom, is similar to ours, but, for example, the atomic transitions utilized are of a different type.

The performance of our scheme can be improved by sequential application of the $b^\dagger$ and $a^\dagger$ photons, with final restoration of the $b^\dagger$ state by a third pulse. Such a scheme seems to require perfectly controlled pulses, which we regard as even more unrealistic than the model we have investigated. To implement our method in a multistep computation we assume all initial information to be coded in a set of field modes residing uncoupled in the same cavity. During their coherence time, we shoot through the cavity volume a sequence of suitably chosen atoms which couple the photon pairs, i.e., perform the two-qubit operations. To affect all possible unitary transformations, the cavity has to be rather complicated, containing a suitable arrangement of λ plates to give access to all desired polarization states. Also the atoms have to be able to couple just the desired modes at each stage of the calculation. This and the restrictions imposed by loss rates and decoherence times pose extremely strict limitations on the computations possible. If several cavities are necessary, the dissipative effects on photons transferred between them raise further problems. However, such difficulties seem to afflict other schemes suggested too. Which one can be optimized the most remains an experimental challenge.

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