Quasiclassical dynamics in a closed quantum system

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We consider Gell-Mann and Hartle's consistent histories formulation of quantum cosmology in the interpretation in which one history, chosen randomly according to the decoherence functional probabilities, is realized from each consistent set. We show that in this interpretation, if one assumes that an observed quasiclassical structure will continue to be quasiclassical, one cannot infer that it will obey the predictions of classical or Copenhagen quantum mechanics. [S1050-2947(96)03309-4]

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I. INTRODUCTION

Modern cosmological theory strongly suggests that large-scale classical structure now dominating the universe evolved from a highly homogenous quantum state lacking any such structure. Since this process cannot be described in the Copenhagen interpretation of quantum theory, other interpretational ideas, requiring, or claiming to require, no pre-existing classical realm, have attracted increasing attention over the past 40 years. There has been particular interest lately in the consistent histories approach to quantum theory developed by Griffiths [1,2], Omnès [3,4], and Gell-Mann and Hartle [5,6]. Unlike earlier suggestive but imprecise proposals in the literature, this formulation of the quantum theory of a closed system admits a well-defined interpretation that defines an interesting scientific theory, albeit a rather weak one. The purpose of this paper is to explain precisely how weak this theory is when it comes to predicting the formation and evolution of classical structure.

It was recently shown [7], *inter alia*, that one cannot use any version of the consistent histories formalism to predict that the largely classical structure we observe will persist or appear to persist. It is shown here that if we try to evade this difficulty by simply assuming that we will continue to observe a largely classical universe, we cannot use the consistent histories formalism to predict that the classical equations of motion will hold, even approximately, or that the results of quantum experiments will agree with the predictions of the Copenhagen interpretation. The formalism predicts infinitely many different possible outcomes for a typical classical or quantum observation or experiment. The conditional probabilities for these outcomes, given the event that a classical structure persists for any fixed time interval, are not defined.

If the aim is to derive a theory of the formation of the observed large-scale structure, or its present dynamics, from a consistent histories formulation of quantum cosmology, these are clearly rather discouraging facts. Their implications are discussed in Sec. IV.

II. CONSISTENT HISTORIES

We simplify the discussion by using a version of the consistent histories formalism in which the initial conditions are defined by a pure state rather than a density matrix, the basic objects of the formalism are branch-dependent sets of projections, and consistency is defined by Gell-Mann and Hartle's decoherence criterion. Arguments similar to those below apply in the general case.[1] The notation is Gell-Mann and Hartle's [8].

Let ψ be the initial state of the universe. *A branch-dependent set of histories* is a set of products of projection operators chosen from projective decompositions and with a time label. The set is indexed by the variable $\alpha=\{\alpha_n,\alpha_{n-1},\ldots,\alpha_1\}$, where the ranges of the α_k and the projections they define depend on the values of $\alpha_{k-1},\ldots,\alpha_1$ and the histories take the form

$$C_\alpha=P^n_{\alpha_n}(t_n;\alpha_{n-1},\ldots,\alpha_1)\times P^{n-1}_{\alpha_{n-1}}(t_{n-1};\alpha_{n-2};\ldots,\alpha_1)\cdots P^1_{\alpha_1}(t_1). \qquad (2.1)$$

Here, for fixed values of $\alpha_{k-1},\ldots,\alpha_1$, the $P^k_{\alpha_k}(t_k;\alpha_{k-1},\ldots,\alpha_1)$ define a projective decomposition indexed by α_k, so that $\Sigma_{\alpha_k}P^k_{\alpha_k}(t_k;\alpha_{k-1},\ldots,\alpha_1)=1$ and

$$P^k_{\alpha_k}(t_k;\alpha_{k-1},\ldots,\alpha_1)P^k_{\alpha'_k}(t_k;\alpha_{k-1},\ldots,\alpha_1)=\delta_{\alpha_k\alpha'_k}P^k_{\alpha_k}(t_k;\alpha_{k-1},\ldots,\alpha_1). \qquad (2.2)$$

The set of histories is *consistent* if and only if

$$(C_\beta\psi,C_\alpha\psi)=\delta_{\alpha\beta}p(\alpha), \qquad (2.3)$$

in which case $p(\alpha)$ is interpreted as the probability of the history C_α.[2] The histories of nonzero probability in a con-

[1] In particular, if there is an impure initial density matrix, then the discussion of Ref. [7], Sec. 3, can be used to show that a generic quasiclassical history belongs to an infinite family of inequivalent consistent sets. The use of branch-dependent sets, though convenient for discussing structure formation, is also inessential.

[2] Note that, when we use the compact notation C_α to refer to a history, we intend the individual projection operators, not just their product, to be part of the definition of the history.

sistent set thus correspond precisely to the nonzero vectors $C_\alpha\psi$. Only consistent sets are of physical relevance. Although the dynamics are defined purely by the Hamiltonian, with no collapse postulate, each projection in the history can be thought of as corresponding to a historical event, taking place at the relevant time. If a given history is realized, its events correspond to extra physical information, neither deducible from the state vector nor influencing it.

Most projection operators involve rather obscure physical quantities, so that it is hard to interpret a general history in familiar language. Given some sensible model, with Hamiltonian and canonical variables specified, one can construct sets of histories that describe familiar physics and check that they are indeed consistent. For example, a useful set of histories for describing the solar system could be defined by projection operators whose nonzero eigenspace contains states in which a given planet's center of mass is located in a suitably chosen small volumes of space at the relevant times, and one would expect a sensible model to show that this is a consistent set and that the histories of significant probability are those agreeing with the trajectories predicted by general relativity. More generally, Gell-Mann and Hartle [5] introduce the notion of a *quasiclassical domain*: a consistent set that is complete, in the sense that it cannot be consistently extended by more projective decompositions, and is defined by projection operators that involve similar variables at different times and satisfy classical equations of motion, to a very good approximation, most of the time. The notion of a quasiclassical domain seems natural, though presently imprecisely defined. Its heuristic definition is motivated by the familiar example of the hydrodynamic variables—densities of chemical species in small volumes of space, and similar quantities—which characterize our own quasiclassical domain. Here the branch dependence of the formalism plays an important role, since the precise choice of variables (most obviously, the sizes of the small volumes) we use depends on earlier historical events. The formation of our galaxy and solar system influences all subsequent local physics; even present-day quantum experiments have the potential to do so significantly, if we arrange for large macroscopic events to depend on their results. It should be stressed at this point that, according to all the developers of the consistent histories approach, quasiclassicality and related properties are interesting notions to study within, not defining features of, the formalism. All consistent sets of histories have the same physical status.

By an interpretation of the consistent histories formalism we mean a description of physics that uses only basic mathematical quantities defined in the formalism, such as sets and histories, and that respects the democracy among consistent sets. The literature contains a variety of such interpretations, but essentially these are different ways of saying the same thing. One can, with Griffiths, say that precisely one history from each consistent set is realized, these histories being chosen according to the probability distribution defined on their set. One can, more economically, say that, in fact, only one consistent set is physically relevant, but that we have no theoretical rule that identifies this set or its properties [7]. Or one can say, as Gell-Mann and Hartle do, that the predictions one makes depend on the set one uses, though here it must be understood that for almost all sets these predictions will not correspond to the physics one actually observes. In each case, though, it is to be understood that we can only observe events from one history and that the formalism supplies no theoretical criterion characterizing the consistent set from which that history is drawn. These forms of words are scientifically equivalent. When we come to predicting the future from historical data, our predictions all take the form "if S turns out to be the relevant consistent set, then event E will take place with probability p." No event can be predicted independent of the as yet unknown set S, and, in fact, any prediction made in a generic consistent set S will be incompatible with the predictions made in some other consistent set S'.[3] We will use the many-histories language here. Nature consists of a list of histories $H(S)$ drawn from each consistent set S. No observer can observe events from more than one such history. The formalism predicts neither the history $H(S)$ in which we find ourselves nor the set S to which it belongs. It supplies only the probabilities for the possible $H(S)$ given the unknown set S.

I should like to add here the cautionary remark that there is another interpretation of the consistent histories formalism that is *not* equivalent to the many-histories interpretation considered here.[4] This interpretation, which as I understand it is not advocated by Griffiths, Omnès, or Gell-Mann and Hartle (but see the work of Saunders [11]), can be summarized by saying that *every* consistent history is realized in a continuum of copies whose measure is given by the history's probability weight.[5]

III. QUASICLASSICAL HISTORIES IN QUANTUM COSMOLOGY

In the absence of a quantum theory of gravity, we work in some fixed background space-time with preferred timelike directions and suppose that the gravitational interactions of matter can be modeled by a noncovariant quantum potential. This is obviously incorrect, but at least shares qualitative features with the type of description that it is hoped might emerge from a fundamental theory.

A semiclassical treatment, in which the background manifold depends on the large-scale structures in the matter distribution described by the different branches, would presumably give a better description of our quasiclassical domain. But it is hard to see any useful role for a consistency criterion in a semiclassical theory, and in any case no adequate semiclassical theory is available. Gell-Mann and Hartle's definition of a quasiclassical domain has to be understood as a definition that applies to real world cosmology only in the context of a theory of gravity yet to be developed.[6]

[3]These points are discussed in detail in Ref. [7]. The reader might also find the shorter summaries in Refs. [9, 10] useful.

[4]I am very grateful to Todd Brun for pointing this out in the course of an illuminating correspondence.

[5]Though the separate discussion necessary for this interpretation is beyond the scope of this paper, it seems to me that the interpretation suffers from related and equally severe problems.

[6]A discussion of possible generalizations of the formalism to quantum gravity can be found in Ref. [12].

What can be said about cosmology in our model? No consistent set can give *the* correct account of the evolution of large-scale structure, since there is no definitive account of the unobserved past in the consistent histories formalism. However, cosmologically minded consistent historians envisage that the set defining our quasiclassical domain can be extended to a set—there may well be many such sets, but let us fix on one and call it S_0—which gives a particularly interesting account, running very roughly as follows.

Some projective decomposition $P^1_{\alpha_1}$ at an early time t_1 characterizes inhomogeneities that mark the beginning of the formation of structure. Further decompositions $P^2_{\alpha_2}, \ldots, P^k_{\alpha_k}$, which depend on the inhomogeneities already realized, describe the development of greater and finer-grained inhomogeneity. By some later time, say t_{k+1}, almost all of the projections in these decompositions become, to a very good approximation, projections onto ranges of eigenvalues for hydrodynamic variables. At this point the histories of nonzero probability define many distinct branches of the quasiclassical domain, each of which corresponds to the formation of significantly different large-scale structures. This branching process continues through to the present, through processes such as the quantum spreading of macroscopic bodies and those of their interactions with microscopic particles or subsystems that are subsequently macroscopically amplified. Each probabilistic quantum experiment that we perform, for example, defines a new branching. There are thus, by now, a very large number of nonzero history vectors $C_\alpha \psi$ corresponding to distinct quasiclassical branches. The quasiclassical domain is filled out, between all these branchings, by many projective decompositions describing events that are very nearly predictable from the earlier history.

The branching process must stop at some point if the Hilbert space is finite dimensional, since all the nonzero history vectors are orthogonal and any new branching adds to their number [7,9]. The familiar description of quantum experiments cannot be reproduced beyond this point, since all subsequent events are predictable. Indeed, it is not clear that the quasiclassical domain can continue at all. Consistent historians thus generally tacitly assume that the Hilbert space is infinite dimensional, or at least that the present number of branches is very much smaller than its dimension. In order to simplify the discussion we will do so too.

Although Gell-Mann and Hartle generally refer to quasiclassicality as a property of domains, it is obviously sensible and useful to refer to individual histories as being quasiclassical if they are built from projectors defined by hydrodynamic variables and if the conditional probabilities of most of these projectors, given the earlier history, is very close to one. The picture S_0 gives, then, is of a large number of histories C_α, including our own history C_{α_0}, defined up to the present time t_0, almost all of which are quasiclassical in their later stages. Any quantum experiments we now undertake can be described by a consistent set S_0' that extends S_0 by projections, defined for the branch of S_0 corresponding to our own history, which (very nearly) describe future hydrodynamic variables—the local densities around the possible paths of a pointer, say— that record the results. The possible outcomes of these experiments are described by a series of nonzero history vectors $P_{\beta_1} C_{\alpha_0} \psi, \ldots, P_{\beta_k} C_{\alpha_0} \psi$. Each of these outcomes corresponds to a quasiclassical history, which we take to be complete up to time $t > t_0$. Let us suppose we are just about to undertake such an experiment, and for simplicity suppose that the number of outcomes k is 3 or larger.

Now consider a similar branching, corresponding to another quantum process with several macroscopically distinct outcomes, described by vectors $P_{\gamma_1} C_\alpha \psi, \ldots, P_{\gamma_l} C_\alpha \psi$ in a history $C_\alpha \psi$ other than our own. These histories can be described in the equivalent consistent set in which the projective decomposition defined by the P_{γ_i} is replaced by that defined by the one-dimensional projectors P'_{γ_i} onto the states $P_{\gamma_i} C_\alpha \psi$, together with their complement $(1 - \Sigma_i P'_{\gamma_i})$ which defines the zero probability history $(1 - \Sigma_i P'_{\gamma_i}) C_\alpha \psi$. Although there are $(l+1)$ projectors in this decomposition, there are still only l physical branches, since zero probability histories are physically irrelevant in the formalism. We can define other consistent sets, which are inequivalent to S_0 and involve nonquasiclassical histories in the branches extending the history $C_\alpha \psi$, by replacing the projectors P'_{γ_i} in this last decomposition by projectors onto any l states forming an orthogonal basis for the subspace spanned by the vectors $P_{\gamma_i} C_\alpha \psi$.

By making similar substitutions of the projective decompositions on all branches other than our own, we can construct an infinite number of consistent sets S whose only quasiclassical history is our own C_{α_0}. After the branching defined, in any of these sets, by the experiment we are about to undertake, the only quasiclassical histories will be the $P_{\beta_i} C_{\alpha_0}$, corresponding to the k possible experimental results. Finally, we can pick one result i_0 and again define new consistent sets by replacing the projectors P_{β_i} for $i \neq i_0$ by projectors onto another orthonormal basis of the subspace spanned by $\{P_{\beta_i} C_{\alpha_0} : i \neq i_0\}$. In this way we can construct an infinite number of consistent sets S that include precisely one of the histories $P_{\beta_{i_0}} C_{\alpha_0}$, which extend the history C_{α_0} nonquasiclassically between times t_0 and t in all the other $(k-1)$ branches and have no other quasiclassical histories. These sets do not correspond to quasiclassical domains, but contain one quasiclassical branch, which describes our history to date together with one of the possible outcomes of the experiments we are about to undertake.[7]

The probabilities of the histories $P_{\beta_i} C_{\alpha_0}$ are nonzero. Now, according to the many-histories interpretation, one history is realized from each of the sets S and the probability of being realized from any given set is simply the standard history probability. Since we have seen that there are an infinite number of consistent sets containing any of these histories, it follows with probability one that each of these k histories is realized infinitely many times from sets S of the form described above. That is, each of the quasiclassical histories

[7]Though sets of histories of this type do not seem to have been explicitly considered in the literature, most consistent historians would, I believe, take their existence for granted in any sensible cosmological model.

defined by all of our observed data to date, together with one of the possible experimental results, is realized infinitely many times. The consistent histories formalism, in the many-histories interpretation, realizes an infinite number of copies of each possible quasiclassical outcome, and these copies of course include descriptions of ourselves observing our history and the outcome of the experiment.

This is the problem. The formalism supplies us neither with any way of identifying the correct set from which to draw our history nor with any probability measure on the sets. Thus, though we can identify the history describing the data we observe, and when given a particular consistent set, we can calculate the probabilities of its histories, we have no way to compute theoretically, or from empirical data, the probability of belonging to a history realized from any given set or class of sets. If we merely adopt the assumption that our realized history up to time t will be quasiclassical, we can make no probabilistic predictions. In order to do so, we need to make a stronger assumption, for example, that our history will be one realized from a particular set. To make such an assumption is to go beyond the formalism.

The discussion applies *a fortiori* to the predictions of classical mechanics since, of course, these predictions are never made with complete certainty. The argument just outlined holds so long as the probabilities are nonzero, and there is always some tiny probability that the position of a macroscopic object will undergo a significant quantum fluctuation without violating the quasiclassicality of its history. While the classical equations of motion are supposed to hold to a very good approximation, nearly all of the time, in a quasiclassical domain, a macroscopic tunneling event need not violate these criteria. For example, if we study a ball thrown against a wall, a very nearly consistent[8] set can be defined by projection operators whose eigenspaces correspond to states in which the ball's center of mass lies within small volumes of space on either side of the wall and the history in which the ball's center of mass trajectory goes towards the wall and then continues on the far side is a quasiclassical history whose probability, though tiny, is nonzero.

It might possibly be argued against this last point that it is sensible to ignore very small probability histories in the formalism. There are several problems with this line of defense, however. It is true that, as Gell-Mann and Hartle point out [5], it is often sensible and convenient to ignore small probability histories. If, for example, we have found a good theoretical reason to fix a particular set of histories for making predictions and find that within that set the probability of the sun failing to rise tomorrow is $10^{-10^{40}}$, we can, for all practical purposes, take it to be zero. But this does not imply (and Gell-Mann and Hartle do not argue) that small probability histories are meaningless or always theoretically negligible. In particular, small probability histories can still give rise to large conditional probabilities. The construction above produces infinitely many consistent sets in which the only quasiclassical history is one in which the sun fails to rise and, of course, in those sets, the probability of no sunrise conditioned on persisting quasiclassicality is, tautologically, one.

It is true that we could simply declare by fiat that all histories with probability smaller than some parameter ϵ are to be neglected. Some care would be required here, since the probability of our own realized history ϵ_0 is by now extremely small and ϵ/ϵ_0 would also have to be very small if we are to continue observing random events for very long. But in any case this strategy would mean that ϵ/ϵ_0 becomes a key parameter in determining the outcome of experiments. No outcome i whose probability conditioned on the past history $p(i|H)$ is smaller than ϵ/ϵ_0 could arise. However, we would still have no way of deriving from the many-histories interpretation the correct probabilities, conditioned on future quasiclassicality, for outcomes for which $p(i|H) > \epsilon/\epsilon_0$. Many predictions of classical mechanics might, for some finite time interval, be recovered by this strategy, at the price of introducing a new experimentally determinable parameter, but the predictions of Copenhagen quantum mechanics cannot be.

Finally, it should be stressed that the problem identified here is quite different from the generally recognized problems of precisely defining quasiclassicality [5] and of understanding the error limits within which classical physics can be recovered once a set involving classical variables has been specified [3,4]. The arguments here require only a heuristic definition of quasiclassicality [5], but, of course, would remain valid if a precise definition were supplied.

IV. CONCLUSION

The argument we have given is very simple. Predictions within the formalism depend on one's choice of set. If we choose one of the infinitely many sets whose only quasiclassical history describes a series of N measurements of σ_x on spin-1 particles prepared in the eigenstate $\sigma_y = 1$, in each of which $\sigma_x = 1$ is observed, then our prediction is that either quasiclassicality will fail to persist or that $\sigma_x = 1$ will repeatedly be observed. If we condition on the persistence of quasiclassicality, then in this set the latter prediction is made with probability one. Indeed, this sequence of results is realized in an infinite number of sets, as are all other sequences. Without a measure on the space of sets, we cannot assign any *a priori* probability distribution to the choice of set that should be used for prediction and hence, if we assume the persistence of quasiclassicality, cannot assign any probabilities to our quasiclassical predictions.

Is the conclusion interesting? Why should anyone have hoped to calculate conditional probabilities of the type considered? Might the conclusion perhaps rely on a perverse reading of the consistent histories formalism? Can we not easily find another interpretation in which no similar difficulty arises? Is there perhaps a natural measure on the consistent sets that produces the correct probabilistic predictions? If not, is there a simple amendment to the formalism that does the job? We take these points in turn.

At issue here is the relation between the consistent histories formulation of quantum cosmology, classical mechanics, and Copenhagen quantum mechanics. Nothing that the consistent histories formalism says is inconsistent with either of

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[8]Gell-Mann and Hartle require only approximately consistent sets [5]. However, it is conjectured [7] that this set can be approximated by an exactly consistent set that describes essentially the same physics. The conjecture is investigated further in Ref. [13].

these last two theories. It does not contradict their predictions. However, it does not allow us to derive them. Given any quasiclassical history, such as the one we find ourselves in, the formalism makes no probabilistic or deterministic predictions of future events. As we have seen, this still holds true if we assume that the history will continue to be quasiclassical. The predictions of Copenhagen quantum mechanics do not follow even from the consistent histories account of quantum cosmology combined with the assumption of a quasiclassical history obeying standard classical mechanics to a good approximation. All three theories are independent.

Gell-Mann and Hartle argue [5] that, although all consistent sets are equivalent in the formalism, we find ourselves perceiving a quasiclassical history because we have evolved so as to become sensitive to quasiclassical variables and adapted to make use of them. There are implicit assumptions in this argument that need not concern us here [7]. Let us accept that it might be so and suppose that some theory of perception tells us that for the purpose of predicting our own future perceptions we can ignore the possibility that we might find ourselves in a nonquasiclassical history. The preceding discussion still tells us that there are infinitely many observers sharing our evolutionary history, continuing to observe a quasiclassical world in the future, who find their subsequent observations disagreeing with classical mechanics and Copenhagen quantum mechanics. This in itself need not be an insuperable problem; however, the formalism does not define any probability measure that allows us to tell which type of realized quasiclassical history is more probable. Thus, accepting Gell-Mann and Hartle's argument, we find ourselves unable to use the consistent histories formalism to make the predictions of classical mechanics and Copenhagen quantum mechanics.

We need not interpret the formalism in many-histories language. The other interpretations of the formalism in the literature, though, have the same implication when it comes to making predictions, and it is easy to see why without rehearsing the full argument [7] for their equivalence. To predict anything, in any interpretation, we require data, in the form of the observed history H, and a consistent set S that includes the projections defining that history. Neither the inclusion of the projections defining H nor the assumption that S contains quasiclassical histories extending H is a very strong constraint on S. Without assuming some sort of probability measure on the space of sets we cannot characterize the likely properties of S. In particular, we cannot say what type of histories it is likely to contain or which quasiclassical histories it is likely to contain.

There is indeed a natural measure on the consistent sets of histories, defined at least if the Hilbert space is finite dimensional and inherited from the geometric description of consistent sets as algebraic curves [7]. Unfortunately, though unsurprisingly, when restricted to the class of sets containing extensions of a given quasiclassical history, it assigns measure zero to the subclass containing quasiclassical extensions of that history. In other words, if the formalism is amended so that the physically relevant set is chosen according to the natural measure, it predicts with probability one that the quasiclassicality we observe will cease immediately.

The obvious amendment to the formalism is to abandon democracy among consistent sets. If we hypothesize that a set defining our quasiclassical domain is the physically relevant set or, more generally, that among the physically relevant sets it is the only one including some of the projections that characterize the observed data, then we can certainly predict the persistence of quasiclassicality and derive the predictions of classical and Copenhagen quantum mechanics.[9] In practice this is almost precisely what we do when we make experimentally testable predictions: we do not typically use all the projections defining our quasiclassical domain, but the variables we do consider are always quasiclassical projections or operators almost perfectly correlated with those projections. To suppose that a particular set or type of set is fundamentally preferred, of course, is to go beyond orthodox quantum theory, by insisting that particular variables are distinguished. However, it appears to be necessary in order to derive our most successful physical theories from the consistent histories formulation of quantum cosmology.

There are at least two genuine, and genuinely different, interpretations of quantum theory that follow the line of thought that begins with the arguments of Everett *et al.* [14] that quantum theory admits a "many-worlds interpretation." One of these, due to Bell [15,16], abandons the notion of a coherent historical description of physics entirely: the events occurring at any time are uncorrelated with those at earlier or later times. This proposal is logically consistent and, given the correct dynamics and boundary conditions, experimentally unfalsifiable, but is not thought by most physicists (and was not thought by Bell) to be a useful scientific theory, since it makes cosmology redundant, memory fictitious, and useful prediction impossible. The other is the interpretation based on the consistent histories formalism considered here. Neither allows the derivation of a theory of quasiclassical physics.

This is not to say that either the formalism itself or the current ideas about structure formation are misguided. The former suggests at least one possible way of going beyond orthodox quantum theory. The latter implicitly rely on intuitions, which may well be sound, about the variables that might be distinguished. It would seem, though, that if we want a genuine derivation of a theory of the formation and dynamics of the quasiclassical structure in the universe from quantum cosmology, in which we can make the usual quasiclassical predictions, then we have to go beyond orthodox quantum theory as it is presently understood by identifying preferred variables in some way. This need not necessarily involve any change in the dynamics.

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[9] Another possible strategy, though one fraught with problems, is the selection of sets that characterize observers rather than domains [7].

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