Quantum jumps in hydrogenlike systems

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In this paper it is shown that the Lyman-α transition of a single hydrogenlike system driven by a laser exhibits macroscopic dark periods, provided there exists an additional constant electric field. We describe the photon-counting process under the condition that the polarization of the laser coincides with the direction of the constant electric field. The theoretical results are given for the example of ^{4}He$^+$. We show that the emission behavior depends sensitively on the Lamb shift [W. E. Lamb and R. C. Retherford, Phys. Rev. **72**, 241 (1947)] between the $2s_{1/2}$ and $2p_{1/2}$ energy levels. A possibly realizable measurement of the mean duration of the dark periods should give quantitative information about the above energy difference by using the proposed photon-counting process. [S1050-2947(96)08611-8]

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I. INTRODUCTION

For the first time coherence effects in hydrogenlike systems were found by the observation of quantum beats (Stark beats) in the Lyman-α transition [1]. In these experiments only the metastable $2s$ state of atomic hydrogen is initially populated. Switching on a constant electric field leads to a build-up of a coherent superposition of the upper $2p$ and $2s$ levels, and the radiative decay shows an interference pattern known as quantum beats.

This uncommon behavior of hydrogenlike systems suggests that interesting effects may occur if the Lyman-α transition is driven by monochromatic laser light and if an additional constant electric field leads to a coherence between the upper levels $2p$ and $2s$. We show that the resulting photon-counting process is similar to the one predicted by Dehmelt [2] for a different system with two excited states, one rapidly decaying and the other metastable, driven by two lasers. Semiclassically one expects for the Dehmelt system periods of constant fluorescence intensity due to the strong transition (light period), interrupted by periods of zero intensity, while the atomic electron is shelved in the metastable state (dark period). These photon statistics have been observed experimentally [3], and the above semiclassical idea has been analyzed quantum mechanically [4–6]. In an alternative experiment Hulet and Wineland proved the existence of macroscopic dark periods in the fluorescence intensity of a single ion influenced by a magnetic field, when a single laser is tuned near one of the principal transition resonances [7]. Hegerfeldt and Plenio [8] proposed another mechanism for dark periods that is not based on the existence of a metastable state. They studied a three-level atom with two strong electric-dipole transitions to one common ground state, driven by a single laser. The existence of macroscopic dark periods in the fluorescence light is due to a quantum coherence effect. The premise is a very small energy separation of the upper levels in conjunction with parallel transition dipole moments. Because of this an experimental realization of the latter physical system seems to be difficult [9].

In this paper we discuss the photon-counting process of a single hydrogenlike system, driven by a single linearly polarized laser and additionally influenced by a constant electric field. As for the Dehmelt system, there exists a semiclassical explanation for the occurrence of macroscopic dark periods in the emission process of the hydrogenlike system as follows. The strong Lyman-α transition ($2p\rightarrow1s$) is driven by the laser light. Because of this we expect a constant fluorescence intensity. The constant electric field leads to the possibility that the atomic electron makes a transition from the $2p$ to the $2s$ energy level. In this case we have zero intensity (dark period), because a dipole transition to the $1s$ ground state is impossible. However, due to the constant electric field, there exists the possibility that the atomic electron gets out of the $2s$ back into the $2p$ energy level, and the emission process starts again. By the quantum-mechanical treatment of the problem we show that under the assumption that the polarization of the laser coincides with the direction of the constant electric field the above semiclassical explanation describes the photon-emission process qualitatively [10].

As will be seen later, one can regulate the mean duration of the dark and light periods almost independently by varying the intensity of the laser beam and the strength of the constant electric field. The mean duration of the dark periods depends sensitively on the Lamb shift [11] between the $2s_{1/2}$ and $2p_{1/2}$ energy level. A possibly realizable measurement of this mean duration, as it was done in the experiments [3,7] in the case of other systems, should give quantitative information about the above energy difference. We show that there exists a correspondence between this system and the above mentioned mechanism of macroscopic dark periods without a metastable state [8].

II. QUANTUM-MECHANICAL DESCRIPTION OF THE PHOTON-COUNTING PROCESS

We consider a single hydrogenlike system without hyperfine structure [12] driven by a single linearly polarized laser with electric field $\mathbf{F}_L$ and additionally influenced by a weak constant electric field $\mathbf{F}$. The laser is supposed to be tuned near the $2p_{1/2}\rightarrow1s_{1/2}$ transition resonance. The Hamiltonian in dipole form for the atom interacting with the quantized radiation field is given by
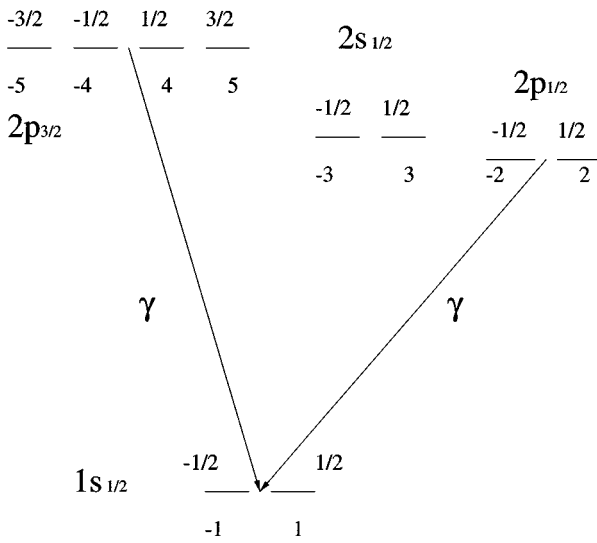
FIG. 1. Relevant energy levels of ^{4}He$^+$. The fine structure frequency between the $2p_{3/2}$ and the $2p_{1/2}$ energy level is given by 1.75×10^{11} Hz, while the Lamb shift frequency between the $2s_{1/2}$ and the $2p_{1/2}$ energy levels is 1.4×10^{10} Hz. In addition $\gamma=10^{10}$ s^{-1} is the Einstein coefficient of the Lyman-α transition. Note that the above Lamb shift splitting is appreciably larger than γ.

$$H=H_A+\sum_{\mathbf{k}\lambda}\hbar\omega_k a_{\mathbf{k}\lambda}^{\dagger}a_{\mathbf{k}\lambda}+\sum_{\mathbf{k}\lambda}i\sqrt{\frac{\hbar\omega_k}{2\varepsilon_0 V}}(a_{\mathbf{k}\lambda}-a_{\mathbf{k}\lambda}^{\dagger})$$

$$\times e\mathbf{D}\cdot\boldsymbol{\varepsilon}_{\mathbf{k}\lambda}+e\mathbf{D}\cdot[\mathbf{F}_L(t)+\mathbf{F}]. \tag{1}$$

Here H_A is the atomic fine-structure Hamiltonian [13]. We assume the Lamb shift to be incorporated in H_A [14]. As in Fig. 1 the relevant atomic eigenstates with positive magnetic quantum number are labeled from $|1\rangle$ to $|5\rangle$. For every $|i\rangle$ $(i=1,\ldots,5)$ with positive magnetic quantum number m_i, there exists a corresponding atomic eigenstate with the same principal quantum number, the same total angular-momentum quantum number, the same parity and the magnetic quantum number $-m_i$, which we denote by $|-i\rangle$. Then the atomic Hamiltonian is given by

$$H_A=\sum_{|i|=1}^{5}\hbar\omega_{i1}|i\rangle\langle i|, \tag{2}$$

where ω_{ij} is the transition frequency between the states $|i\rangle$ and $|j\rangle$. Note that $|i|>|j|$ implies $\omega_{i1}\geq\omega_{j1}$.

To describe the photon-counting process one needs the probability density $w(t_1,\ldots,t_N;[t_0,t])$ for finding exactly N photons at times $t_1<\cdots<t_N$ in the interval $[t_0,t]$. We assume that the initial state of the complete system is $|\Omega\rangle\langle\Omega|\otimes\rho(t_0)$, where $|\Omega\rangle$ is the vacuum of the quantized radiation field, and $\rho(t_0)$ is the initial atomic density operator. The probability density w is of the form [15–17]

$$w(t_1,\ldots,t_N;[t_0,t])=\mathrm{Tr}[\hat{S}(t,t_N)\hat{J}\hat{S}(t_N,t_{N-1})$$

$$\times\cdots\hat{J}\hat{S}(t_1,t_0)\rho(t_0)], \tag{3}$$

where $\hat{S}(t,t')$ and $\hat{J}$ are atomic superoperators. Using the quantum jump approach [5] (which is essentially equivalent to the Monte Carlo wave-function approach [18] and to the use of quantum trajectories [19]) Hegerfeldt [17] has determined the superoperators $\hat{J}$ and $\hat{S}(t,t')$ for an arbitrary atom. Considering Refs. [5,17] one finds that $|\Omega\rangle\langle\Omega|\otimes[\hat{J}\rho(t)/\mathrm{Tr}(\cdot)]$ is the state right after the detection of a photon, provided we have a measurement by absorption, which we assume from now on. There exists a nonunitary atomic operator $U_c(t,t_0)$, the so called conditional (reduced) evolution operator, describing the time development of an atom under the condition that no photon is observed in the time interval $[t_0,t]$, and $\hat{S}(t,t_0)$ is given by $\hat{S}(t,t_0)\rho(t_0)=U_c(t,t_0)\rho(t_0)U_c(t,t_0)^{\dagger}$. The conditional evolution operator is generated by the conditional Hamiltonian H_c and with $\mathbf{D}_{ij}\equiv\langle i|\mathbf{D}|j\rangle$, the generalized damping terms

$$\Gamma_{ijkl}\equiv\frac{e^2}{6\pi\varepsilon_0\hbar c^3}\mathbf{D}_{ij}\cdot\mathbf{D}_{kl}|\omega_{kl}|^3 \tag{4}$$

and the decay matrix

$$\Gamma\equiv\sum_{\substack{|i|,|j|,|k|=1\\|i|,|k|>|j|}}^{5}\Gamma_{ijjk}|i\rangle\langle k| \tag{5}$$

one finds following Refs. [5,17] that H_c is given by

$$H_c=H_A+e\mathbf{D}\cdot[\mathbf{F}_L(t)+\mathbf{F}]-i\hbar\Gamma. \tag{6}$$

For the reset operator $\hat{J}$ one obtains [17]

$$\hat{J}\rho=\sum_{\substack{ijkl\\|i|>|j|\\|k|>|l|}}(\Gamma_{jikl}+\Gamma_{klji})\langle i|\rho|k\rangle|j\rangle\langle l|. \tag{7}$$

The above operators H_c and $\hat{J}$ are rather complicated in the case of our ten level hydrogenlike system. There are three main difficulties in calculating the probability density Eq. (3) as follows. The superoperator $\hat{J}$ generally carries pure states into statistical mixtures of the two ground states $|1\rangle$, $|-1\rangle$, with in general nonzero off-diagonal matrix elements. The state right after the detection of a photon $|\Omega\rangle\langle\Omega|\otimes[\hat{J}\rho(t)/\mathrm{Tr}(\cdot)]$ generally depends on $\rho(t)$ [20]. The superoperators $\hat{J}$ and $\hat{S}(t,t')$ act on a 100 dimensional vector space. In the next section we show a way out of these difficulties.

III. SYMMETRY CONSIDERATIONS

Here we make the assumption that the laser light is linearly polarized with polarization in the same direction as the constant electric field, $\mathbf{F}=F\mathbf{e}_z$ and $\mathbf{F}_L(t)=F_L\cos(\omega_L t+\varphi_L)\mathbf{e}_z$. Therefore we have an invariance of the Hamiltonian Eq. (1) with respect to the group O(2) of those orthogonal transformations which leave the z-axis invariant. The results of the following symmetry considerations are given at the end of this section.

By T we denote the standard double-valued representation of the group O(2) on the atomic Hilbert space. The invariance of the Hamiltonian Eq. (1) leads to

$$T(g)(\hat{J}\rho)T(g)^{\dagger}=\hat{J}[T(g)\rho T(g)^{\dagger}] \qquad (8)$$

for every group element g and to the analogous equation for $\hat{S}(t,t')$. Special elements of O(2) are given by $\mathcal{D}(\varphi)$, the rotation through the angle φ about the z-axis, and by $\mathcal{S}$, the reflection in the y-z plain. One can show that the projector $\hat{P}^{(0)}$ onto the subspace of the scalar operators with respect to the group O(2) is given by [21]

$$\hat{P}^{(0)}\rho=\frac{1}{4\pi}\int_{0}^{2\pi}\mathrm{d}\varphi[T(\mathcal{D}(\varphi))\rho T(\mathcal{D}(\varphi))^{\dagger}$$

$$+T(\mathcal{D}(\varphi))T(\mathcal{S})\rho T(\mathcal{S})^{\dagger}T(\mathcal{D}(\varphi))^{\dagger}]$$

$$=\frac{1}{2}\sum_{\substack{i,j>0\\m_i=m_j}}(\langle i|\rho|j\rangle+\varepsilon_i^{*}\varepsilon_j\langle -i|\rho|-j\rangle)$$

$$\times(|i\rangle\langle j|+\varepsilon_i\varepsilon_j^{*}|-i\rangle\langle -j|), \qquad (9)$$

where ρ is an arbitrary atomic operator and ε_i is a phase factor with the property $T(\mathcal{S})|i\rangle=\varepsilon_i|-i\rangle$. Using the cyclic invariance of the trace and Eq. (3), Eq. (8) we obtain

$$w(t_1,\ldots,t_N;[t_0,t])=\mathrm{Tr}[\hat{S}(t,t_N)\hat{J}\hat{S}(t_N,t_{N-1})$$

$$\times\cdots\hat{J}\hat{S}(t_1,t_0)\hat{P}^{(0)}\rho(t_0)]. \qquad (10)$$

Since the atomic state right after the detection of a photon is a statistical mixture of the ground states $|1\rangle$, $|-1\rangle$ one finds

$$\hat{J}\hat{P}^{(0)}\rho=\mathrm{Tr}(\hat{J}\rho)\rho_r \qquad (11)$$

with the scalar operator

$$\rho_r=\frac{1}{2}(|1\rangle\langle 1|+|-1\rangle\langle -1|). \qquad (12)$$

From Eq. (8) we also know that $\hat{J}$ and $\hat{S}(t,t')$ leave the subspace of the scalar operators invariant, and using additionally Eq. (11) we come to the conclusion that the probability density Eq. (3) factorizes into single-photon probabilities

$$w(t_1,\ldots,t_N;[t_0,t])$$

$$=\mathrm{Tr}[\hat{S}(t,t_N)\rho_r]\mathrm{Tr}[\hat{J}\hat{S}(t_N,t_{N-1})\rho_r]\cdots\mathrm{Tr}[\hat{J}\hat{S}(t_2,t_1)\rho_r]$$

$$\times\mathrm{Tr}[\hat{J}\hat{S}(t_1,t_0)\hat{P}^{(0)}\rho(t_0)]. \qquad (13)$$

From this we see that the photon-counting process is governed by $P_0(t)$, the probability density of counting no photons until t starting with the ground state, since $-\dot{P}_0(t-t')=\mathrm{Tr}[\hat{J}\hat{S}(t,t')\rho_r]$ [17]. The conditional evolution operator $U_c(t,t')$ is a scalar operator and from Eq. (9) one obtains that $U_c(t,t')$ leaves the atomic subspace generated by the states with positive magnetic quantum number invariant. Using the definition of ρ_r, the relation $T(\mathcal{S})|1\rangle\langle 1|T(\mathcal{S})^{\dagger}=|-1\rangle\langle -1|$, the fact that $U_c(t,t')$ commutes with $T(\mathcal{S})$ and the cyclic invariance of the trace we obtain

$$P_0(t)=\mathrm{Tr}[\hat{S}(t,0)\rho_r]=\mathrm{Tr}[U_c(t,0)\rho_r U_c(t,0)^{\dagger}]$$

$$=\|U_c(t,0)|1\rangle\|^2. \qquad (14)$$

Therefore we only have to consider the atomic states with positive magnetic quantum number. By a similar reasoning it is easy to check that the latter is also true in the more general case of the first single-photon probability $\mathrm{Tr}[\hat{J}\hat{S}(t_1,t_0)\hat{P}^{(0)}\rho(t_0)]$ in Eq. (13).

Because of our symmetry assumption the original ten-level atom behaves like a five-level system with a single ground state [22]. In the special case of the spontaneous Lyman-α transition influenced by the Stark effect this agrees with the previous results in literature (see, for example, [23]). Starting with the state right after the detection of a photon even the state $|5\rangle$ drops out, because it is coupled neither by the laser nor by the constant electric field. Going over to an interaction picture the explicit time dependence of H_c in Eq. (6) vanishes. We introduce the operator

$$M\equiv\frac{i}{\hbar}H_c=\begin{pmatrix} 0 & \dfrac{i\Omega_L}{2} & 0 & -\dfrac{i\Omega_L}{\sqrt{2}} \\[2mm] i\dfrac{\Omega_L}{2} & \dfrac{\gamma}{2}-i\Delta_2 & i\Omega & 0 \\[2mm] 0 & i\Omega & -i\Delta_3 & -i\sqrt{2}\Omega \\[2mm] -i\dfrac{\Omega_L}{\sqrt{2}} & 0 & -i\sqrt{2}\Omega & \dfrac{\gamma}{2}-i\Delta_4 \end{pmatrix}$$

$$(15)$$

in matrix form with respect to the atomic basis $|1\rangle$, $|2\rangle$, $|3\rangle$, $|4\rangle$, where γ is the Einstein coefficient of the Lyman-α transition, $\Omega_L\equiv(e/\hbar)F_L\langle 2|z|1\rangle$ is the real Rabi frequency of the laser with respect to the $2p_{1/2}\rightarrow 1s_{1/2}$ transition, $\Omega\equiv(e/\hbar)F\langle 2|z|3\rangle$ is the analogous real constant of the constant electric field and $\Delta_i\equiv\omega_L-\omega_{i1}$ is the detuning of the laser with respect to the state $|i\rangle$. Then we finally have $P_0(t)=\|\mathrm{e}^{-Mt}|1\rangle\|^2$ [24].

IV. EMISSION BEHAVIOR

We assume the laser to be tuned near the $2p_{1/2}\rightarrow 1s_{1/2}$ transition resonance, such that $|\Delta_2|\ll\gamma$. In this case $\hbar|\Delta_3|$ is essentially given by the Lamb shift between the $2s_{1/2}$ and $2p_{1/2}$ energy level, and $|\Delta_4|$ is a fine structure frequency, which leads to $|\Delta_3|\ll|\Delta_4|$. We also assume the electric fields to be chosen to satisfy the relation $|\Omega|\ll|\Omega_L|<|\Delta_3|$. We approximate the function $P_0(t)$ by means of a perturbative approach based on the book of Kato [25]. Since additionally the $2p_{3/2}$ level couples weakly to the laser, we have two perturbation parameters Ω/Δ_3 and Ω_L/Δ_4. Putting $\Omega=0$ in Eq. (15) we denote the resulting operator by $M^{(0)}$, and we define $M^{(1)}\equiv M-M^{(0)}$. There exists a decomposition of the form

$$M^{(0)}=\Lambda_1^{(0)}P_1^{(0)}+\lambda_2^{(0)}P_2^{(0)}+\lambda_3^{(0)}P_3^{(0)}, \qquad (16)$$

where $\lambda_2^{(0)}\equiv-i\Delta_3$ is one of the eigenvalues, another one is given by

$$\lambda_3^{(0)} \approx \frac{\gamma}{2} - i\Delta_4 \qquad (17)$$

in first order in Ω_L/Δ_4 and $P_i^{(0)}$, $i=2,3$ are the respective eigenprojectors, and we define $P_1^{(0)} \equiv 1 - P_2^{(0)} - P_3^{(0)}$. The operator $\Lambda_1^{(0)}$ is chosen to commute with $P_1^{(0)}$. We obtain for $\Lambda_1^{(0)}$ in first order in Ω_L/Δ_4 the result

$$-i\hbar\Lambda_1^{(0)} \approx \hbar\frac{\Omega_L}{2}(|2\rangle\langle 1| + |1\rangle\langle 2|) - i\hbar\left(\frac{\gamma}{2} - i\Delta_2\right)|2\rangle\langle 2|. \qquad (18)$$

Note that the right-hand side of Eq. (18) is the conditional Hamiltonian of a two-level atom [5,17]. In an analogous manner we can decompose the operator M in the form

$$M = \Lambda_1 P_1 + \lambda_2 P_2 + \lambda_3 P_3 \qquad (19)$$

with each P_i corresponding to $P_i^{(0)}$, and we have

$$e^{-Mt}|1\rangle = e^{-\Lambda_1 t}P_1|1\rangle + e^{-\lambda_2 t}P_2|1\rangle + e^{-\lambda_3 t}P_3|1\rangle. \qquad (20)$$

The main idea of our perturbative approach is to approximate Λ_1, λ_2, λ_3 and the respective projectors separately with the aid of Ref. [25]. First of all we are interested in the behavior of $P_0(t)$ assuming $t \gg \gamma^{-1}$. By using Eqs. (17) and (18) one can verify that the first and the third term in Eq. (20) decay exponentially on the time scale γ^{-1} while in first order in Ω/Δ_3 the real part of λ_2 vanishes. Because of this we only have to approximate $\mathrm{Re}(\lambda_2)$ and $\|P_2|1\rangle\|^2$. In first order in Ω/Δ_3 we have

$$P_2|1\rangle \approx -P_2^{(0)}M^{(1)}[M^{(0)} - \lambda_2^{(0)}(1 - P_2^{(0)})]^{-1}|1\rangle. \qquad (21)$$

With the definition of the complex number

$$\alpha \equiv 1 - \frac{\Delta_3}{\Delta_4} - \frac{\Omega_L^2}{4\Delta_3^2} - \frac{\Delta_2}{\Delta_3} + \frac{3\Omega_L^2}{4\Delta_3\Delta_4} + \frac{\gamma^2}{4\Delta_3\Delta_4} + \frac{\Delta_2}{\Delta_4} - \frac{\Omega_L^2\Delta_2}{2\Delta_3^2\Delta_4}$$
$$-i\left(\frac{\gamma}{2\Delta_3} - \frac{\gamma}{\Delta_4} + \frac{3\Omega_L^2\gamma}{8\Delta_3^2\Delta_4} + \frac{\gamma\Delta_2}{2\Delta_3\Delta_4}\right) \qquad (22)$$

we obtain

$$P_0(t) \approx e^{-2\,\mathrm{Re}(\lambda_2)t}\frac{\Omega^2\Omega_L^2}{4\Delta_3^4}\frac{\left(1 - 3\frac{\Delta_3}{\Delta_4} + 2\frac{\Delta_2}{\Delta_4}\right)^2 + \frac{9\gamma^2}{4\Delta_4^2}}{|\alpha|^2} \qquad (23)$$

assuming $t \gg \gamma^{-1}$, up to small relative deviations of the order Ω/Ω_L. In second order in Ω/Δ_3 one finds

$$\lambda_2 \approx \lambda_2^{(0)} - \langle 3|M^{(1)}[M^{(0)} - \lambda_2^{(0)}(1 - P_2^{(0)})]^{-1}M^{(1)}|3\rangle \qquad (24)$$

and this leads to

$$\mathrm{Re}\lambda_2 \approx \frac{\Omega^2\gamma}{2\Delta_3^2}\frac{1 - 2\frac{\Delta_3}{\Delta_4} + 3\frac{\Delta_3^2}{\Delta_4^2} - 4\frac{\Delta_2\Delta_3}{\Delta_4^2} + \frac{3\gamma^2}{4\Delta_4^2} + 2\frac{\Delta_2^2}{\Delta_4^2}}{|\alpha|^2}. \qquad (25)$$

It is comparatively easy to describe the behavior of $P_0(t)$ on the other time scale $t \gg \gamma^{-1}$. In zeroth order in Ω_L/Δ_4, Ω/Δ_3 we have

$$P_0(t) \approx \|e^{-\Lambda_1^{(0)}t}|1\rangle\|^2, \qquad (26)$$

where $\Lambda_1^{(0)}$ can be approximated by Eq. (18).

We introduce a time T_0 such that $\gamma^{-1} \ll T_0 \ll (2\,\mathrm{Re}\lambda_2)^{-1}$. Then for $t \ll T_0$ the function $P_0(t)$ is governed by the behavior of a two-level atom with a strong transition, while in a large time interval around T_0 it is very small, though not vanishingly small, and slowly varying. An interruption of the atomic fluorescence longer than T_0 is called a dark period. The above results concerning $P_0(t)$ guarantee the occurrence of light and dark periods in the resonance fluorescence of the atom (see, for example, [6]). Following Refs. [5,6] we can calculate the mean durations T_L, T_D of the light and dark periods and the probability p for the occurrence of a dark period. One finds $p = P_0(T_0)$ and $T_D = (2\,\mathrm{Re}\lambda_2)^{-1}$, which is given by Eqs. (23) and Eq. (25), respectively. The value of T_L can be obtained from $T_L = \tau_L/p$, where τ_L is the mean time between two photons in a light period. This is intuitively obvious, since p^{-1} is the mean number of photons in a light period. We have

$$\tau_L = -\int_0^{T_0} t\frac{\dot{P}_0(t)}{1-p}\,dt \approx \frac{1}{\gamma}\frac{\gamma^2 + 2\Omega_L^2 + 4\Delta_2^2}{\Omega_L^2} \qquad (27)$$

by using Eq. (26). Thus one finds

$$T_D = \frac{\Delta_3^2|\alpha|^2}{\Omega^2\gamma\left(1 - 2\frac{\Delta_3}{\Delta_4} + 3\frac{\Delta_3^2}{\Delta_4^2} - 4\frac{\Delta_2\Delta_3}{\Delta_4^2} + \frac{3\gamma^2}{4\Delta_4^2} + 2\frac{\Delta_2^2}{\Delta_4^2}\right)} \qquad (28)$$

and

$$T_L = \frac{4\Delta_3^4|\alpha|^2(\gamma^2 + 2\Omega_L^2 + 4\Delta_2^2)}{\gamma\Omega_L^4\Omega^2\left[\left(1 - 3\frac{\Delta_3}{\Delta_4} + 2\frac{\Delta_2}{\Delta_4}\right)^2 + \frac{9\gamma^2}{4\Delta_4^2}\right]}. \qquad (29)$$

All the above mean values can be obtained from a single trajectory of the photon-counting process.

As a typical example of the occurrence of macroscopic dark periods in the emission process of $^4\mathrm{He}^+$ we discuss $\Delta_2 = 0$, $F = 3.6\times10^3$ V/m, $F_L = 2.9\times10^6$ V/m, which means $\Omega = 0.025\gamma$, $\Omega_L = 5\gamma$. One finds

$$T_D = 1.1\times10^{-5} \text{ s}, \quad T_L = 4\times10^{-4} \text{ s}, \qquad (30)$$

and for the mean number of photons in a light period we obtain $p^{-1} = 2\times10^6$. Since the Lyman-α transition is remarkably strong with a lifetime of about 0.1 ns, one has a high fluorescence intensity in a light period and a different

time scale in comparison with the Dehmelt systems in Ref. [3].

Under consideration of our premises with respect to the electric fields we know that T_L/T_D is almost independent of Ω. However, T_D only depends weakly on Ω_L, which is intuitively obvious. As a conclusion, one can regulate the emission process with the aid of the electric fields.

V. DISCUSSION

From the calculation above we have seen that macroscopic dark periods occur in hydrogenlike systems like $^4\text{He}^+$ provided the external electric fields are suitably chosen. One might wonder, however, in which way one can reach a dark period. As the quantum-mechanical calculation shows and the intuitive explanation in the introduction suggests, in order to reach a macroscopic dark period the system must be mostly in the $2s$ state.

One might be tempted to argue in a simplified way as follows. One could assume that the coherent evolution of the atom is started by the absorption of a $1s \rightarrow 2p$ photon, and terminated by spontaneous emission into this channel. In order to evolve to an extended dark period spontaneous emission must not occur for many lifetimes. If we take the effective Rabi frequency $\Omega = 1/40\gamma$ of the Stark field one would estimate that the probability to obtain an even mixing of $2p$ and $2s$ is smaller than $\exp(-20)$, or 2×10^{-9}, and at this point the atom is not dark at all. From the 0.1 ns lifetime of the $2p$ state of $^4\text{He}^+$ one estimates a lower limit of $T_L > 0.1$s for the mean time of a light period in the emission process. This result much exceeds the previously calculated value of $T_L = 4 \times 10^{-4}$ s from the quantum mechanical description, and macroscopic dark periods should be very seldom.

At this point we have to remember that it is the relative weight of the $2s$ state in the emission-free subensemble that counts rather than the absolute population. There are *two* mechanisms that make the $2s$ state become rapidly predominant in the emission-free subensemble as follows. The relative weight of the $2p$ and the $1s$ state in the emission-free subensemble decreases rapidly on the time scale γ^{-1}, because those atoms with a spontaneous emission from the strong Lyman-α transition leave the emission-free subensemble and do not contribute. On the other hand the $2s$ state is metastable and weakly coupled to the $2p$ state. Therefore if the atom is once in the $2s$ state it stays with a high probability, and it remains in the emission-free subensemble for a long time. As a conclusion we obtain the possibly astonishing result that the $2s$ state becomes predominant fairly quickly in the emission-free subensemble although the absolute population of this metastable state is very small. An estimation of the population dynamics in the emission-free subensemble from a very simple rate equation model is given in the appendix.

For the mechanism of quantum jumps in hydrogenlike systems there exists a close relation to the proposal of macroscopic dark periods without a metastable state by Hegerfeldt and Plenio [8] as follows. If we neglect the existence of the weakly coupled $2p_{3/2}$ level, and if we consider the conditional Hamiltonian, which is given in matrix form with respect to the atomic orthonormal basis $|1\rangle$,

$1/\sqrt{2}(|2\rangle + |3\rangle)$, $1/\sqrt{2}(|2\rangle - |3\rangle)$ by

$$H_c = \hbar \begin{pmatrix} 0 & \dfrac{\Omega_L}{2\sqrt{2}} & \dfrac{\Omega_L}{2\sqrt{2}} \\[2ex] \dfrac{\Omega_L}{2\sqrt{2}} & \Omega - i\dfrac{\gamma}{4} - \dfrac{\Delta_2+\Delta_3}{2} & -i\dfrac{\gamma}{4} + \dfrac{\Delta_3-\Delta_2}{2} \\[2ex] \dfrac{\Omega_L}{2\sqrt{2}} & -i\dfrac{\gamma}{4} + \dfrac{\Delta_3-\Delta_2}{2} & -\Omega - i\dfrac{\gamma}{4} - \dfrac{\Delta_2+\Delta_3}{2} \end{pmatrix}, \tag{31}$$

then Eq. (31) corresponds directly to Eq. (5) of [8], except that the off-diagonal frequency shift terms $(\Delta_3 - \Delta_2)/2$ (half of the negative Lamb shift frequency) are absent [26]. As a conclusion, the physical system of Hegerfeldt and Plenio behaves like a hydrogenlike system without quantum electrodynamical corrections of the atomic spectrum.

In the case of our realistic hydrogenlike system the photon-counting process is governed by the detuning Δ_3, where $\hbar|\Delta_3|$ is essentially given by the Lamb shift between the $2s_{1/2}$ and the $2p_{1/2}$ energy level. We assume a possibly realizable measurement of the mean duration of the dark periods, as it was done in the experiments [3,7] for Dehmelt systems. In this case we can calculate the detuning Δ_3 with the aid of the approximation Eq. (28) to high accuracy by solving Eq. (28) for Δ_3, which leads to a polynomial equation of sixth degree. If the other parameters in Eq. (28) are known, this provides a detection of the Lamb shift by using the proposed photon-counting process.

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APPENDIX

In this appendix we show that, in the emission-free subensemble, the relative weight of the $2s$ state becomes rapidly predominant on the time scale of the inverse Lyman-α Einstein coefficient γ^{-1}. This can be seen by a simple rate equation model for the emission-free subensemble as follows. For convenience we neglect the existence of the weakly coupled $2p_{3/2}$ level. By $P_i(t)$ ($i = 1,2,3$) we denote the probability that no photon has been detected until t *and* that the atom is in the state $|i\rangle$ at time t. We note that $t > 0$ implies $\Sigma_{i=1}^{3} P_i(t) < 1$. By R_B, R_R we denote the transition rates due to stimulated emission of the blue transition $2p \rightarrow 1s$ and the red transition $2s \rightarrow 2p$ respectively. For this subensemble one has the rate equations

$$\dot{P}_1 = -R_B P_1 + R_B P_2, \tag{A1}$$

$$\dot{P}_2 = R_B P_1 - (\gamma + R_B + R_R) P_2 + R_R P_3, \tag{A2}$$

$$\dot{P}_3 = R_R P_2 - R_R P_3. \tag{A3}$$

The only difference to the usual rate equations [14] is that those atoms with a spontaneous emission from the blue tran-

sition leave the emission-free subensemble. Therefore the term γP_2 in Eq. (A1) is absent. This leads to a decay of $\Sigma_i P_i(t)$ in time. Assuming $R_R \ll R_B, \gamma$ and starting with the ground state one easily obtains the perturbative expressions

$$P_1(t) = e^{-\mu_2 t} \frac{\mu_1 - R_B}{\mu_1 - \mu_2} - e^{-\mu_1 t} \frac{\mu_2 - R_B}{\mu_1 - \mu_2},$$

$$P_2(t) = \frac{R_B}{\mu_1 - \mu_2}(e^{-\mu_2 t} - e^{-\mu_1 t}),$$

$$P_3(t) = \frac{R_R R_B}{\mu_1 - \mu_2}\left(\frac{e^{-\mu_1 t}}{\mu_1 - \mu_3} - \frac{e^{-\mu_2 t}}{\mu_2 - \mu_3} \right)$$
$$+ e^{-\mu_3 t}\left(\frac{R_R R_B}{(\mu_2 - \mu_3)(\mu_1 - \mu_2)} \right.$$
$$\left. - \frac{R_R R_B}{(\mu_1 - \mu_3)(\mu_1 - \mu_2)} \right), \qquad (A4)$$

where the real numbers μ_i are defined by

$$\mu_{1/2} \equiv \frac{1}{2}[(\gamma + R_R + 2R_B) \pm \sqrt{(\gamma + R_R)^2 + 4R_B^2}],$$

$$\mu_3 \equiv R_R. \qquad (A5)$$

We note that $\mu_1 > \mu_2 \gg \mu_3$, and we see that the population $P_3(t)$ of the $2s$ state $|3\rangle$ in the emission-free subensemble increases *rapidly* from $P_3(0) = 0$ on the time scale $\mu_{1/2}^{-1} \sim R_B^{-1} \sim \gamma^{-1}$ and then remains on a low level for a long
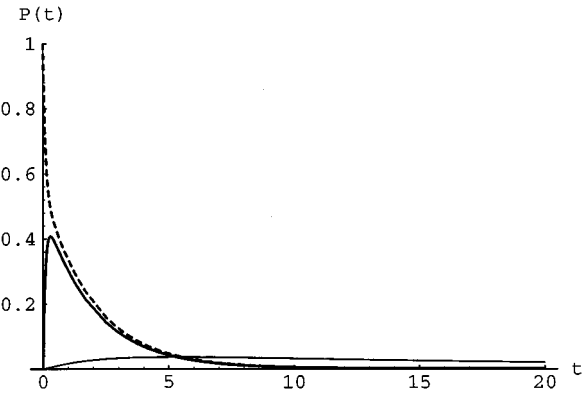

FIG. 2. Estimation of the expected population dynamics by means of the simplified rate equation model in the case of the parameters $R_B = 5\gamma$, $R_R = 0.05\gamma$. The dashed line, fat solid line, thin solid line indicate the population $P_1(t)$, $P_2(t)$, $P_3(t)$ respectively. The time axis is given in natural units of the inverse Lyman-α Einstein coefficient.

time of the order $\mu_3^{-1} \sim R_R^{-1}$. On the other hand the population $P_1(t)$, $P_2(t)$ of the $1s$, $2p$ state respectively decreases *rapidly* on the time scale $R_B^{-1} \sim \gamma^{-1}$ so that $P_3(t) > P_2(t), P_3(t)$ is reached fairly quickly. This behavior can also be seen in Fig. 2. Because of the normalization ($\Sigma_i P_i(t)$ = weight of the emission-free subensemble) the conditional probabilities are $P_j(t)/\Sigma_i P_i(t)$ ($j=1,2,3$). A similar behavior is obtained in the quantum-mechanical calculation of the paper, but a quantitative agreement is not easily available from this simple rate equation model.

[1] H. J. Andrä, Phys. Rev. A **2**, 2200 (1971); H. J. Andrä, *Fast-Beam (Beam-Foil) Spectroscopy* in *Progress in Atomic Spectroscopy*, Part B, edited by W. Hanle and H. Kleinpoppen (Plenum, New York, 1979), p. 893.

[2] H. G. Dehmelt, Bull. Am. Phys. Soc. **20**, 60 (1975).

[3] W. Nagourney, J. Sandberg, and H. Dehmelt, Phys. Rev. Lett. **56**, 2797 (1986); J. C. Bergquist, R. G. Hulet, W. M. Itano, and D. J. Wineland, *ibid.* **57**, 1699 (1986); Th. Sauter, W. Neuhauser, R. Blatt, and P. E. Toschek, *ibid.* **57**, 1696 (1986).

[4] A. Schenzle, R. G. DeVoe, and R. G. Brewer, Phys. Rev. A **33**, 2127 (1986); J. Javanainen, *ibid.* **33**, 2121 (1986); D. T. Pegg, R. Loudon, and P. L. Knight, *ibid.* **33**, 4085 (1986); G. Nienhuis, *ibid.* **35**, 4639 (1987); P. Zoller, M. Marte, and D. J. Walls, *ibid.* **35**, 198 (1987); M. Porrati and S. Putterman, *ibid.* **36**, 925 (1987); S. Renaud, J. Dalibard, and C. Cohen-Tannoudji, IEEE J. Quantum Electron. **24**, 1395 (1988); D. T. Pegg and P. L. Knight, Phys. Rev. A **37**, 4303 (1988).

[5] T. S. Wilser, Ph.D. dissertation, Göttingen, 1991 (unpublished); G. C. Hegerfeldt and T. S. Wilser, *Proceedings of the Second International Wigner Symposium, Goslar (1991)*, edited by H. D. Doebner, W. Scherer, and F. Schroeck (World Scientific, Singapore, 1992).

[6] C. Cohen-Tannoudji and J. Dalibard, Europhys. Lett. **1**, 441 (1986).

[7] R. G. Hulet and D. J. Wineland, Phys. Rev. A **36**, 2758 (1987).

[8] G. C. Hegerfeldt and M. B. Plenio, Phys. Rev. A **46**, 373 (1992).

[9] G. C. Hegerfeldt and M. B. Plenio, Z. Phys. B **96**, 533 (1995).

[10] H. J. Kimble, R. J. Cook, and A. L. Wells, Phys. Rev. A **34**, 3190 (1986) have used a Bloch equation approach to the Dehmelt system which might be carried over to the present case. However, as the authors point out their approximations become problematic for a large detuning of the weak transition.

[11] W. E. Lamb and R. C. Retherford, Phys. Rev. **72**, 241 (1947).

[12] In the case of $^4He^+$ there is no hyperfine structure since the nucleus consists of an even number of both neutrons and protons.

[13] H. A. Bethe and E. E. Salpeter, in *Encyclopedia of Physics*, Atoms I, edited by S. Flügge (Springer, Berlin, 1957), Vol. 35.

[14] R. Loudon, *The Quantum Theory of Light* (Clarendon, Oxford, 1983).

[15] M. D. Srinivas and E. B. Davies, Opt. Acta **28**, 981 (1981).

[16] C. W. Gardiner, *Quantum Noise* (Springer, Berlin, 1991).

[17] G. C. Hegerfeldt, Phys. Rev. A **47**, 449 (1993).

[18] J. Dalibard, Y. Castin, and K. Mølmer, Phys. Rev. Lett. **68**, 580 (1992); K. Mølmer, Y. Castin, and J. Dalibard, J. Opt. Soc. Am. B **10**, 524 (1993).

[19] H. J. Carmichael, *An Open System Approach to Quantum Optics*, edited by W. Beiglböck, Lecture Notes in Physics, Vol. 18 (Springer, Berlin, 1993).

[20] If we assume a single ground state this would also be the state right after the detection of a photon, and in this case it clearly does not depend on the state before.

[21] Here we use the invariant measure [see, for example, M. A. Naimark and A. I. Štern, *Theory of Group Representations* (Springer, Berlin, 1982)] of the compact group O(2). It is also possible to find the same results by a purely algebraic reasoning. In this paper we choose the simplest calculation.

[22] By a similar reasoning one can generalize the approach of

dimensional reduction to the case of a unitary representation of an arbitrary symmetry group.

[23] G. Lüders, Z. Naturforsch. **5a**, 608-611 (1950).

[24] For convenience we neglect the spontaneous decay of the $2s$ state. For the example of $^4\text{He}^+$ we have a lifetime of about $2\,\text{ms}$.

[25] T. Kato, *Perturbation Theory for Linear Operators* (Springer, Berlin, 1966), p. 62.

[26] Compare also T. P. Altenmüller, Z. Phys. D **34**, 157 (1995).