Effect of feedback on the decoherence of a Schrödinger-cat state: A quantum trajectory description

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The linear stochastic Schrödinger equation of a cavity mode subject to a homodyne measurement and to a phase-sensitive feedback loop realized with part of the output homodyne photocurrent is derived. We show that quantum feedback has stabilizing effects which manifest themselves in a significant retardation of the decoherence of a Schrödinger-cat state. [S1050-2947(96)04411-3]

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I. INTRODUCTION

The basic aspect of quantum mechanics is linearity which gives, in particular, the possibility of preparing systems in linear superposition states. When this superposition principle is extended to the macroscopic world, conceptual difficulties arise as pointed out, e.g., by the Einstein-Podolsky-Rosen [1] and Schrödinger-cat [2] paradoxes, because one is forced to accept the existence, in principle, of linear superposition of macroscopically distinguishable states. Practically we never see this curious superposition states at the macroscopic level because of quantum decoherence. Decoherence is the rapid destruction of the phase relation between two quantum states of a system caused by the entanglement of these two states with two different states of the environment [3,4]. In the case of macroscopic systems, the interaction with the environment can never be escaped; since the decoherence rate is proportional to the ''macroscopic separation'' between the two states [3,5,6], a linear superposition of macroscopically distinguishable states is immediately changed into the corresponding statistical mixture, with no quantum coherence left. Nonetheless, a full comprehension of the fuzzy boundary between classical and quantum world is far from being reached [7,8], and, therefore, there is great interest in the realization of ''Schrödinger-cat'' states in *mesoscopic* systems where one can hope to control the decoherence and see the emergence of classical properties from the quantum domain. A first important achievement has been recently obtained by Monroe *et al.* [9], who have prepared a trapped $^9Be^+$ ion in a superposition of spatially separated coherent states and detected the quantum coherence between the two localized states.

Another promising field for the generation of mesoscopic Schrödinger-cat states is quantum optics in which a large number of proposals have appeared for the generation of linear superpositions of two coherent states of the electromagnetic field in a cavity with opposite phases, $|\psi\rangle = c_+|\alpha\rangle + c_-|-\alpha\rangle$ [10]. The decoherence time of this optical Schrödinger cat state due to the interaction with the outside vacuum modes is equal to $t_{dec} = 1/2\gamma|\alpha|^2$ (γ is the cavity decay rate) [6], which becomes very small even for a moderate number of photons, unless very high-Q cavities are used. It is, therefore, very important to find a way to control and possibly increase this decoherence time in optical systems. A first suggestion has been given by Kennedy and Walls [11], who showed that if the vacuum bath could be replaced by a squeezed bath [12], the interference fringes indicating the presence of quantum coherence would increase their lifetime. Two of the present authors have followed this suggestion and proved that a squeezed bath can be simulated by an appropriate use of quantum feedback, showing in this way that an appreciable retardation of the decoherence process can be achieved using electro-optical feedback [13]. The possibility of controlling the decoherence of a linear superposition state of a cavity field mode is crucial also in the developing field of quantum computation [14]. The revolutionary aspect of quantum computation relies on the ability of evaluating exponentially many parallel inputs and to obtain a result depending on the interferences among various superposed results. The central obstacle for a quantum computer to work is the fragility of the entangled linear superpositions of N ''quantum bits'' with respect to decoherence. The loss of coherence should then be reduced as much as possible, because the decoherence time should be much larger than the calculation time. Since the first experimental realizations of a ''quantum gate,'' i.e., of the fundamental building block of a quantum computer, have been performed in quantum optics [15], the use of quantum feedback to control the decoherence process of a radiation mode may be of great help also for quantum computation.

In this paper we shall illustrate in a very simple model how the application of a feedback loop may slow down the decoherence of an optical Schrödinger-cat state. To that end we shall study a field mode in a cavity subject to a homodyne measurement, in which part of the output photocurrent is fed back to the cavity, and we analyze the mode dynamics using the technique of quantum trajectories [16]. We shall derive a linear stochastic Schrödinger equation (SSE) for the dynamics of the system in the presence of feedback which upon averaging reproduces the homodyne feedback master equation first given by Wiseman and Milburn [17,18]. The quantum trajectory approach will give an intuitive and appealing picture of the effect of homodyne feedback. In fact, single stochastic trajectories clearly exhibit, as we shall show, the stabilizing influence of feedback on the phase fluctuations of the system at the quantum level.

The paper is organized as follows. In Sec. II we present the model and derive the linear and nonlinear homodyne-feedback stochastic Schrödinger equation. In Sec. III we derive and discuss the feedback master equation of Wiseman and Milburn [17,18]. In Sec. IV we analytically solve the master equation and analyze in Sec. V the decoherence process by numerically simulating the nonlinear SSE. Section VI is for concluding remarks.

II. MODEL

Let us consider an electromagnetic mode with bosonic annihilation operator a inside a cavity with decay rate γ_a. In the Markov approximation the interaction of the cavity mode with the vacuum fluctuations of the outside electromagnetic field may be described by the interaction-picture Hamiltonian

$$H_{int}(t)dt = H_0 dt + i\hbar\sqrt{\gamma_a}[a dB_{in}^{\dagger}(t) - dB_{in}(t)a^{\dagger}] \tag{2.1}$$

with the mode Hamiltonian H_0 in the cavity; the white-noise operator $dB_{in}(t)$ and its adjoint satisfy the Ito rules [12]

$$dB_{in}(t)^2 = dB_{in}^{\dagger}(t)^2 = 0, \quad dB_{in}(t)dB_{in}^{\dagger}(t) = dt, \tag{2.2}$$

$$dB_{in}^{\dagger}(t)dB_{in}(t) = 0,$$

$$[dB_{in}(t), a(t')] = [dB_{in}^{\dagger}(t), a(t')] = 0 \quad \text{for} \quad t \geqslant t'. \tag{2.3}$$

The interaction of the cavity mode with the input field generates an output field $dB_{out}(t)$ given by the input-output relation [12]

$$dB_{out}(t) = \sqrt{\gamma_a}a(t)dt + dB_{in}(t). \tag{2.4}$$

Let us now imagine to perform a homodyne measurement of the quadrature component

$$X_{\varphi} = \frac{1}{2}(a e^{-i\varphi} + a^{\dagger}e^{i\varphi}) \tag{2.5}$$

of the cavity field; the corresponding output signal can be easily expressed in terms of the output field and the phase of the local oscillator φ as

$$\begin{aligned} d\Theta(t) &= dB_{out}(t)e^{-i\varphi} + dB_{out}^{\dagger}(t)e^{i\varphi} \\ &= 2\sqrt{\gamma_a}X_{\varphi}(t)dt + d\Xi(t), \end{aligned} \tag{2.6}$$

where we have introduced the quadrature component

$$d\Xi(t) = dB_{in}(t)e^{-i\varphi} + dB_{in}^{\dagger}(t)e^{i\varphi} \tag{2.7}$$

of the input field. In [19] a linear SSE for the cavity mode wave function conditioned on the homodyne measurement of X_{φ} was derived as

$$d|\psi_{\xi}(t)\rangle = \left[-\frac{i}{\hbar}H_0 dt - \frac{\gamma_a}{2}a^{\dagger}a dt + \sqrt{\gamma_a}a e^{-i\varphi}d\xi(t)\right]|\psi_{\xi}(t)\rangle, \tag{2.8}$$

where the real-valued Wiener increment $d\xi(t)$ satisfies $\overline{d\xi(t)} = 0$, $(d\xi(t))^2 = dt$; the index ξ of the wave function $|\psi_{\xi}(t)\rangle$ points to the dependence on the previous history of the noise $\xi(s)$ with $0 \leqslant s < t$.

We now consider the case of feeding back part of the homodyne photocurrent into the cavity. It is reasonable to assume that the effect of feedback is linear in the output signal $d\Theta(t)$ [17] and may, therefore, be described by the Hamiltonian

$$H_{fb}(t)dt = \hbar\sqrt{\gamma_a}d\Theta(t-\tau)A(t), \tag{2.9}$$

where $A(t)$ is a generic system observable and τ is the delay time introduced by the feedback loop. The presence of a nonzero delay makes the feedback-modified dynamics of the cavity mode non-Markovian; therefore, we consider the limit of a vanishing delay time, $\tau \rightarrow 0$, which can be physically justified if the delay time is smaller than the typical time scale of the cavity mode dynamics, which in the present case is the decoherence time $t_{dec} = 1/2\gamma|\alpha|^2$. With realistic loop delays of order 10^{-8} sec, this implies that the Markovian limit is appropriate only for good cavities and not too large mean photon numbers.

The limit of a zero delay time is quite delicate. First of all, ambiguities concerning operator ordering can arise. Indeed, as an output field the increment $d\Theta(t-\tau)$ commutes with all system operators $A(t)$ for $\tau > 0$, but not for $\tau = 0$; however, that commutation relation is preserved also in the limit of a vanishing delay if the condition

$$\lim_{\tau \rightarrow 0} d\Theta(t-\tau)A(t) = d\Xi(t)A(t) = A(t)d\Xi(t) \tag{2.10}$$

is satisfied, i.e., if the photocurrent fed back into the cavity equals the input field in the zero-delay limit. Second, one has to take into account the fact that the feedback always acts *after* the cavity mode has interacted with the vacuum input field, even in the limit $\tau \rightarrow 0$ [18]. The total wave function of the composite system of cavity mode and bath in the presence of feedback may therefore be written as

$$|\psi_{tot}(t+dt)\rangle = \exp\left\{-\frac{i}{\hbar}H_{fb}(t)dt\right\} \times \exp\left\{-\frac{i}{\hbar}H_{int}(t)dt\right\}|\psi_{tot}(t)\rangle. \tag{2.11}$$

Inserting the Hamiltonian $H_{int}(t)$ and $H_{fb}(t)$ of (2.1) and (2.9), respectively, and using the Ito rules (2.3) we obtain

$$
\begin{aligned}
|\psi_{tot}(t+dt)\rangle = &\exp\left\{-\frac{i}{\hbar}H_{fb}(t)dt\right\} \\
&\times\left\{1+\sqrt{\gamma_a}[a dB_{in}^{\dagger}(t)-dB_{in}(t)a^{\dagger}]\right. \\
&\left.-\frac{i}{\hbar}H_0 dt-\frac{\gamma_a}{2}a^{\dagger}a dt\right\}|\psi_{tot}(t)\rangle \\
=&\left\{1+\sqrt{\gamma_a}[(a-iAe^{i\varphi})dB_{in}^{\dagger}(t)\right. \\
&-dB_{in}(t)(a^{\dagger}+iAe^{-i\varphi})]-\frac{i}{\hbar}H_0 dt \\
&-\frac{\gamma_a}{2}a^{\dagger}a dt-\frac{\gamma_a}{2}A^2 dt \\
&\left.-i\gamma_a A a e^{-i\varphi}dt\right\}|\psi_{tot}(t)\rangle. \qquad (2.12)
\end{aligned}
$$

The terms appearing on the right hand side of the foregoing expression may be further rearranged and we arrive at

$$
\begin{aligned}
|\psi_{tot}(t+dt)\rangle = &\left\{1+\sqrt{\gamma_a}[(a-iAe^{i\varphi})dB_{in}^{\dagger}(t)\right. \\
&-dB_{in}(t)(a^{\dagger}+iAe^{-i\varphi})] \\
&-\frac{i}{\hbar}\left(H_0+\frac{\hbar\gamma_a}{2}(Aae^{-i\varphi}+a^{\dagger}Ae^{i\varphi})\right)dt \\
&\left.-\frac{\gamma_a}{2}(a^{\dagger}+iAe^{-i\varphi})(a-iAe^{i\varphi})dt\right\}|\psi_{tot}(t)\rangle, \qquad (2.13)
\end{aligned}
$$

which reads as the total wave function $|\psi_{tot}\rangle$ of (2.12) with $H_{fb}\equiv 0$ except for the replacements

$$a\to a-iAe^{i\varphi}, \qquad (2.14)$$

$$H_0\to H_0+\frac{\hbar\gamma_a}{2}(Aae^{-i\varphi}+a^{\dagger}Ae^{i\varphi}). \qquad (2.15)$$

The derivation of the linear SSE (2.8) presented in [19] may now be taken over and we obtain the following linear SSE for the cavity mode wave function under the influence of homodyne feedback,

$$
\begin{aligned}
d|\psi_{\xi}(t)\rangle = &\left[-\frac{i}{\hbar}H_0 dt-\frac{\gamma_a}{2}a^{\dagger}a dt-\frac{\gamma_a}{2}A^2 dt-i\gamma_a A a e^{-i\varphi}dt\right. \\
&\left.+\sqrt{\gamma_a}(ae^{-i\varphi}-iA)d\xi\right]|\psi_{\xi}(t)\rangle, \qquad (2.16)
\end{aligned}
$$

which is equivalent to a nonlinear homodyne-feedback SSE given by Wiseman and Milburn [17]. In fact, renaming ξ in (2.16) as θ and interpreting the increment $d\theta(t)$ as output noise driven by the input-noise increment $d\xi(t)$ according to (2.4) we arrive at the nonlinear SSE of [17] for a normalized wave function $|\phi_{\xi}\rangle$ [19],

$$
\begin{aligned}
d|\phi_{\xi}(t)\rangle = &\left[-\frac{i}{\hbar}H_0 dt-\frac{\gamma_a}{2}a^{\dagger}a dt-\frac{\gamma_a}{2}A^2 dt-i\gamma_a A a e^{-i\varphi}dt\right. \\
&+\gamma_a\langle X_{\varphi}\rangle(ae^{-i\phi}-iA)dt-\frac{\gamma_a}{2}\langle X_{\varphi}\rangle^2 dt \\
&\left.+\sqrt{\gamma_a}(ae^{-i\varphi}-iA-\langle X_{\varphi}\rangle)d\xi\right]|\phi_{\xi}(t)\rangle \qquad (2.17)
\end{aligned}
$$

with the mean $\langle X_{\varphi}\rangle=\langle\phi_{\xi}|X_{\varphi}|\phi_{\xi}\rangle$. The equivalence between linear and nonlinear SSE's such as (2.16) and (2.17) has also been shown in [20,21].

III. HOMODYNE-FEEDBACK MASTER EQUATION

The master equation for the reduced density operator of the cavity mode may be derived from the linear SSE (2.16) by taking the differential of the projector $|\psi_{\xi}(t)\rangle\langle\psi_{\xi}(t)|$ according to

$$
\begin{aligned}
d|\psi_{\xi}(t)\rangle\langle\psi_{\xi}(t)| = &|d\psi_{\xi}(t)\rangle\langle\psi_{\xi}(t)|+|\psi_{\xi}(t)\rangle\langle d\psi_{\xi}(t)| \\
&+|d\psi_{\xi}(t)\rangle\langle d\psi_{\xi}(t)|; \qquad (3.1)
\end{aligned}
$$

inserting (2.16) and averaging over the Wiener noise leads to the homodyne-feedback master equation [17,18]

$$
\begin{aligned}
\dot{\rho} = &-\frac{i}{\hbar}[H_0,\rho]+\frac{\gamma_a}{2}(2a\rho a^{\dagger}-a^{\dagger}a\rho-\rho a^{\dagger}a) \\
&-i\gamma_a[A,ae^{-i\varphi}\rho+\rho a^{\dagger}e^{i\varphi}]-\frac{\gamma_a}{2}[A,[A,\rho]]. \qquad (3.2)
\end{aligned}
$$

In the absence of feedback that master equation reduces to the well-known master equation for an oscillator coupled to a zero temperature heat bath,

$$\dot{\rho}=-\frac{i}{\hbar}[H_0,\rho]+\frac{\gamma_a}{2}(2a\rho a^{\dagger}-a^{\dagger}a\rho-\rho a^{\dagger}a). \qquad (3.3)$$

As explained in [17] (see also [13]), the third term on the right hand side of (3.2) is the feedback term itself, while the fourth one is a diffusionlike term inevitably induced by the noise introduced in the measurement step of the feedback loop.

Let us now consider a specific example: We assume that the cavity mode has no internal dynamics (in the interaction picture), i.e., $H_0\equiv 0$, and take the feedback operator A as

$$A=gX_{\theta}=\frac{g}{2}(ae^{-i\theta}+a^{\dagger}e^{i\theta}); \qquad (3.4)$$

the constant g represents the gain of the feedback process, while the experimentally controllable phase θ is not specified further. The particular choice (3.4) of A means that the feedback loop adds a driving term to the mode dynamics, which could be achieved, e.g., by using an electro-optic device with variable transmittivity driven by the homodyne photocurrent. Using (3.4) and redefining the phase of a so that $\varphi=0$, the master equation (3.2) takes the form

$$
\begin{aligned}
\dot{\rho}= & \frac{\gamma}{2}(N+1)(2a\rho a^{\dagger}-a^{\dagger}a\rho-\rho a^{\dagger}a) \\
& +\frac{\gamma}{2}N(2a^{\dagger}\rho a-aa^{\dagger}\rho-\rho aa^{\dagger}) \\
& -\frac{\gamma}{2}M(2a^{\dagger}\rho a^{\dagger}-a^{\dagger}a^{\dagger}\rho-\rho a^{\dagger}a^{\dagger}) \\
& -\frac{\gamma}{2}M^{*}(2a\rho a-aa\rho-\rho aa) \\
& -i[(\delta a^{\dagger}a+\tilde{g}^{*}a^{\dagger}a^{\dagger}+\tilde{g}aa),\rho],
\end{aligned}
\tag{3.5}
$$

which may be read as the master equation of an oscillator coupled to a squeezed bath at finite temperature with damping constant $\gamma=\gamma_a(1-g\ \sin\theta)$, thermal photon number $N=\gamma_a g^2/(4\gamma)$, and squeezing parameter $M=-\gamma_a g e^{i\theta}(ge^{i\theta}/2-i)/(2\gamma)$. Furthermore, the feedback process introduces a Hamiltonian term in the last line of the right hand side of (3.5) which amounts to a frequency shift $\delta=(\gamma_a g\cos\theta)/2$ and a second-order self interaction of the cavity mode with coupling coefficient $\tilde{g}=\gamma_a g e^{-i\theta}/4$.

IV. TIME EVOLUTION OF A SCHRÖDINGER-CAT STATE

Let us now consider the effect of homodyne feedback on the time evolution of a Schrödinger-cat state. In [22] a rigorous solution of the homodyne SSE (2.8) for the wave function $|\psi_\theta(t)\rangle$ contingent on the output noise starting from an initial Schrödinger-cat state

$$
|\psi(0)\rangle=\frac{1}{\sqrt{2}}(|\alpha_0\rangle+|-\alpha_0\rangle) \tag{4.1}
$$

with large amplitude α_0 of the two coherent states $|\pm\alpha_0\rangle$ has been presented, giving an appealing picture of how the wave vector chooses between the two possible constituents of (4.1) in each stochastic realization: In each single run the mean displacement $\langle X_0(t)\rangle_\theta$ chooses between the two possible values $\pm\alpha_0 e^{-\gamma_a t/2}$ on a time scale of the order of the decoherence time $t_{dec}=(2\gamma_a|\alpha_0|^2)^{-1}$.

Since there is an enormous interest in the possibility of generating and detecting linear superpositions of classically distinguishable states [3,5,6,10] it is certainly interesting to analyze the effects of feedback processes on the decoherence of the cat state. In fact, an analysis performed on a different but related feedback model based on a quantum nondemolition measurement of a quadrature component [13] has shown that under appropriate conditions the destruction of the interference fringes indicating the presence of quantum coherences between the two coherent states can be retarded. These fringes are extremely sensitive to decoherence effects induced, e.g., by the coupling to a dissipative environment, and any increase in their lifetime would greatly facilitate the detection of Schrödinger-cat states [11,13].

We begin by solving explicitly the feedback master equation (3.2) for initial conditions of the general form $\rho(0)=\Sigma_{\alpha,\beta}N_{\alpha,\beta}|\alpha\rangle\langle\beta|$, where $|\alpha\rangle$ and $|\beta\rangle$ are coherent states. The exact time evolution of the density operator from this initial state may be obtained with the help of the normally ordered characteristic function $\chi(\lambda,\lambda^*;t)=\mathrm{Tr}\{\rho(t)\exp(\lambda a^{\dagger})\exp(-\lambda^* a)\}$, which reads [13]

$$
\chi(\lambda,\lambda^*;t)=\sum_{\alpha,\beta}N_{\alpha,\beta}\langle\beta|\alpha\rangle\exp\Big\{B^*(t)\lambda-A(t)\lambda^* -\nu(t)|\lambda|^2+\frac{\mu(t)}{2}\lambda^{*2}+\frac{\mu(t)^*}{2}\lambda^2\Big\} \tag{4.2}
$$

with the time-dependent coefficients

$$
A(t)=\frac{i}{2\sin\theta}(\alpha e^{-i\theta}+\beta^* e^{i\theta})e^{-\gamma_a t/2} -\frac{ie^{i\theta}}{2\sin\theta}(\alpha+\beta^*)e^{-(1-2g\sin\theta)\gamma_a t/2}, \tag{4.3}
$$

$$
B^*(t)=-\frac{i}{2\sin\theta}(\alpha e^{-i\theta}+\beta^* e^{i\theta})e^{-\gamma_a t/2} +\frac{ie^{-i\theta}}{2\sin\theta}(\alpha+\beta^*)e^{-(1-2g\sin\theta)\gamma_a t/2}, \tag{4.4}
$$

$$
\nu(t)=\frac{g^2}{4}\left[\frac{1-e^{-(1-2g\sin\theta)\gamma_a t}}{1-2g\sin\theta}\right], \tag{4.5}
$$

$$
\mu(t)=-e^{2i\theta}\nu(t). \tag{4.6}
$$

Let us now focus on the time evolution of the interference terms of the initial density operator $\rho(0)$. To that end we employ the marginal probability distribution of the quadrature component X_φ, $P(x_\varphi)=\langle x_\varphi|\rho(t)|x_\varphi\rangle$, where $|x_\varphi\rangle$ is the eigenstate of X_φ with eigenvalue x_φ; the probability distribution P may be reconstructed directly from the outcome of the homodyne measurement. The analytical expression of that distribution may be obtained from the characteristic function (4.2) as [11,13] (again, we choose the phases so that $\varphi=0$)

$$
P(x,t)=\sum_{\alpha,\beta}N_{\alpha,\beta}\frac{\langle\beta|\alpha\rangle}{\sqrt{\pi\sigma_x^2(t)}}\exp\left\{-\frac{(x-\delta_{\alpha,\beta}(t))^2}{\sigma_x^2(t)}\right\}, \tag{4.7}
$$

with $x_{\varphi=0}\equiv x$ and

$$
\sigma_x^2(t)=\frac{1}{2}+\nu(t)+\mathrm{Re}\{\mu(t)\} \tag{4.8}
$$

$$
=\frac{1}{2}\left\{1+g^2\sin^2\theta\left[\frac{1-e^{-(1-2g\sin\theta)\gamma_a t}}{1-2g\sin\theta}\right]\right\} \tag{4.9}
$$

$$
\delta_{\alpha,\beta}(t)=\frac{A(t)+B^*(t)}{2}. \tag{4.10}
$$

For the initial density operator

$$
\rho(0)=\frac{1}{2(1+e^{-2|\alpha_0|^2})}(|\alpha_0\rangle+|-\alpha_0\rangle)(\langle\alpha_0|+\langle-\alpha_0|) \tag{4.11}
$$

pertaining to the cat state (4.1) the distribution (4.7) simplifies as

$$P(x,t)=\frac{1}{2(1+e^{-2|\alpha_0|^2})}\{p_+^2(x,t)+p_-^2(x,t)$$
$$+2p_+(x,t)p_-(x,t)\cos[\Omega(x,t)]|\langle\alpha_0|-\alpha_0\rangle|^{\eta(t)}\}. \tag{4.12}$$

While the Gaussian distributions

$$p_\pm^2(x,t)=\frac{1}{\sqrt{\pi\sigma_x^2(t)}}\exp\left\{-\frac{(x\mp\mathrm{Re}\{\alpha_0\}e^{-(1-2g\sin\theta)\gamma_a t/2})^2}{\sigma_x^2(t)}\right\} \tag{4.13}$$

pertain to the initial coherent state $|\pm\alpha_0\rangle$, the third term in (4.12) describes the quantum interference between the two coherent states. The function

$$\Omega(x,t)=\frac{2x\ \mathrm{Im}\{\alpha_0\}e^{-(1-2g\sin\theta)\gamma_a t/2}}{\sigma_x^2(t)} \tag{4.14}$$

represents probability oscillations associated with the interference fringes, while the factor $|\langle\alpha_0|-\alpha_0\rangle|^{\eta(t)}=\exp\{-2|\alpha_0|^2\eta(t)\}$ describes the suppression of quantum coherence due to dissipation. That suppression is practically immediate for macroscopically distinguishable states (i.e., large amplitude $|\alpha_0|$), unless the decoherence function

$$\eta(t)=1-\frac{e^{-(1-2g\sin\theta)\gamma_a t}}{2\sigma_x^2(t)} \tag{4.15}$$

is nearly zero. To determine the conditions under which the detection of the quantum coherence is facilitated we compare $\eta(t)$ with the corresponding decoherence function without feedback given by [11]

$$\eta_{vac}(t)=1-e^{-\gamma_a t}. \tag{4.16}$$

Therefore, for times $\gamma_a t\ll 1$ the interference term in (4.12) decays as $\exp\{-2|\alpha_0|^2\gamma_a t\}$, leading to the decoherence time $t_{dec}^{vac}=1/2\gamma_a|\alpha_0|^2$. On the contrary, in the presence of feedback this decoherence time can be significantly increased. In fact, at small times (4.15) simplifies as

$$\eta(t)\simeq(1-g\sin\theta)^2\gamma_a t, \tag{4.17}$$

and we may infer the feedback-modified decoherence time

$$t_{dec}\simeq\frac{1}{2\gamma_a|\alpha_0|^2(1-g\sin\theta)^2}, \tag{4.18}$$

implying that the destruction of quantum coherence is retarded by the feedback loop provided the feedback parameters satisfy the condition $0<g\sin\theta<2$.

As can be seen from Eq. (4.18), the case $g\sin\theta=1$ is particularly interesting. Indeed, Eqs. (4.15) and (4.8) yield $\eta(t)\equiv 0$, suggesting that decoherence is completely suppressed and the interference pattern is perfectly preserved. Actually, this is not the whole truth because for $g\sin\theta=1$ the evolution equations are unstable and the probability distribution tends to broaden and flatten. To be more specific, from

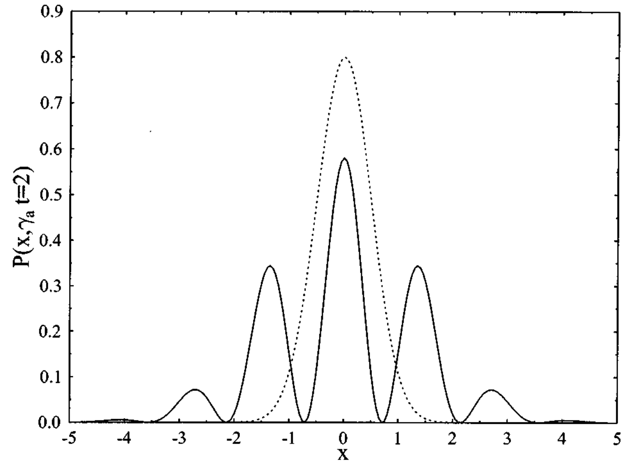

FIG. 1. Marginal density of the eigenvalue x of the quadrature component X_0 for $\gamma_a t=2$, $g=0$ (without feedback, dotted line) and $g\sin\theta=1$ (full line).

(4.3), (4.4), and (4.5) one can immediately see that the dynamics is unstable for $g\sin\theta\geqslant 1/2$; in the particular case $g\sin\theta=1$ the probability distribution $P(x,t)$ decreases exponentially according to

$$P(x,t)=e^{-\gamma_a t/2}P(xe^{-\gamma_a t/2},0) \tag{4.19}$$

for all initial states. Therefore, the probability oscillations eventually vanish at large times but, as can be seen from Fig. 1, the interference pattern remains visible much longer compared to the case without feedback.

A feedback-induced preservation of the probability distribution of the homodyne-measured quadrature X has been obtained recently also by Wiseman [23], using a similar model. He proposed to use feedback to eliminate the measurement backaction, thereby turning the homodyne measurement into a quantum nondemolition measurement of X. Actually, the feedback loop proposed in [23] coincides with the one adopted here in the special case $g\sin\theta=1$. The difference with the present model is that in [23] a parametric amplifier is also introduced to stabilize the motion of the X quadrature without adding any extra noise. In this way the exponential broadening of Eq. (4.19) is eliminated and $P(x)$ is perfectly preserved. Our scheme has the advantage of being simpler and it works well for not too large times.

V. QUANTUM TRAJECTORY DESCRIPTION OF DECOHERENCE

The analysis of the preceding section clearly reveals that homodyne feedback can, under appropriate conditions, increase the decoherence time of a Schrödinger cat state according to (4.18). Let us now consider the behavior of the mean quadrature component $\langle X(t)\rangle_\xi$ for given realizations of the noise ξ and vanishing phase φ of the local oscillator. In the case of a zero temperature heat bath without feedback [22] the mean ''displacement'' $\langle X(t)\rangle_\xi$ was shown to settle, in each run, at either $+\alpha_0$ or $-\alpha_0$ within a ''decision time'' on the order of the vacuum decoherence time, while the subsequent damping towards $\langle X(\infty)\rangle_\xi$ follows on the much

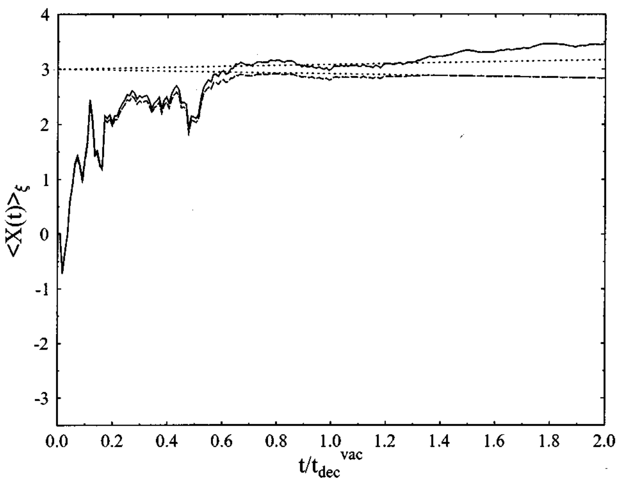

FIG. 2. The mean $\langle X(t)\rangle_\xi$ for a single run of the nonlinear stochastic Schrödinger equation with initial amplitude $\alpha_0=3$ for $g=0$ (without feedback, dashed line) and $g=1$, $\theta=\pi/2$ (full line). The dotted lines show the time evolution of the amplitude of the coherent state $|\alpha(t)\rangle$ according to $\alpha_0 e^{-\gamma_a t/2}$ (without feedback) and $\alpha_0 e^{+\gamma_a t/2}$ (with feedback).

larger time scale γ^{-1}. It is therefore quite natural to ask if feedback is able to delay that decision, i.e., to increase the decision time. However, the numerical study of single realizations of $\langle X(t)\rangle_\xi$ according to the nonlinear SSE (2.17) reveals that this is not the case. In Fig. 2 we compare a typical run of $\langle X(t)\rangle_\xi$ (for a single realization of the vacuum noise ξ) without feedback (dashed line) with the feedback-modified one (full line) for the parameter values $\alpha_0=3$, $g=1$, and $\theta=\pi/2$. The two curves almost coincide except for times after the decision has taken place where the mean evolves as $\pm\alpha_0 e^{-\gamma_a t/2}$ and $\pm\alpha_0 e^{-(1-2g\sin\theta)\gamma_a t/2}$ in the case without and with feedback, respectively [see Eq. (4.13)]. In particular, in both cases the ''decision'' takes place at essentially the same time, in contrast to the decoherence process discussed above [this is quite similar to the behavior of $\langle X(t)\rangle_\xi$ at finite temperatures studied in [24]: while the decoherence is thermally accelerated, the decision process is not influenced by thermal fluctuations]. The failure of the feedback loop to delay the decision can be easily understood by taking a look at the marginal probability distribution $P(x,t)$ of (4.12). For a real-valued amplitude α_0 of the initial cat state the interference term $\Omega(x,t)$ of $P(x,t)$ is identically zero and, therefore, the only effect of feedback is to change the mechanical relaxation of the center of the two Gaussian peaks $p_\pm(x,t)$.

From the above analysis of the behavior of the mean $\langle X(t)\rangle_\xi$ we are, therefore, led to look for a different quantity which is more appropriate to reveal the effect of homodyne feedback on the fate of the cat state in a single run of the homodyne experiment. We shall consider the ''coherence'' function

$$C_\xi(t)=\langle -\alpha_0|\phi_\xi(t)\rangle\langle\phi_\xi(t)|\alpha_0\rangle, \tag{5.1}$$

where the normalized wave function $|\phi_\xi(t)\rangle$ is the solution of the nonlinear SSE (2.17). The ensemble average of $C_\xi(t)$ gives the off-diagonal element

$$\overline{C_\xi(t)}=\langle -\alpha_0|\rho(t)|\alpha_0\rangle \tag{5.2}$$

of the reduced density operator of the cavity mode. In the case of a zero temperature heat bath without feedback that off-diagonal element decays as (in the limit of a ''macroscopic'' initial separation, i.e., $e^{-2|\alpha_0|^2}\simeq 0$)

$$\overline{C_\xi(t)}\simeq\frac{1}{2}\exp\{|\alpha_0|^2(e^{-\gamma_a t}+2e^{-\gamma_a t/2}-3)\}, \tag{5.3}$$

which for small times $\gamma_a t\ll 1$ gives the usual exponential decoherence as

$$\overline{C_\xi(t)}\simeq\frac{1}{2}\exp\{-2|\alpha_0|^2\gamma_a t\}. \tag{5.4}$$

The corresponding expression in the presence of feedback may be obtained with the help of the normally ordered characteristic function according to [25]

$$\langle -\alpha_0|\rho(t)|\alpha_0\rangle=\frac{e^{-|\alpha_0|^2}}{\pi}\int d^2\lambda\, e^{-|\lambda-\alpha_0|^2}\chi(\lambda,t); \tag{5.5}$$

using expression (4.2) for $\chi(\lambda,t)$ the averaged coherence function (5.2), again for $e^{-2|\alpha_0|^2}\simeq 0$, takes the form

$$\begin{aligned}\overline{C_\xi(t)}=&\frac{e^{-4|\alpha_0|^2}}{2\sqrt{2\nu(t)+1}}\exp\bigg\{\frac{1}{2\nu(t)+1}(2\nu(t)(1+e^{-\gamma_a t/2})^2\\&\times(\mathrm{Im}\{\alpha_0\}\cos\theta-\mathrm{Re}\{\alpha_0\}\sin\theta)^2\\&+(\mathrm{Im}\{\alpha_0\})^2(1+e^{-(1-2g\sin\theta)\gamma_a t/2})^2\\&+[\mathrm{Im}\{\alpha_0\}\cot\theta(e^{-(1-2g\sin\theta)\gamma_a t/2}-e^{-\gamma_a t/2})\\&+\mathrm{Re}\{\alpha_0\}(1+e^{-\gamma_a t/2})]^2)\bigg\}.\end{aligned} \tag{5.6}$$

For times t small compared to the mechanical relaxation time, $\gamma_a t\ll 1$, the foregoing expression simplifies as

$$\begin{aligned}\overline{C_\xi(t)}\simeq&\frac{1}{2}\exp[-2\gamma_a t((\mathrm{Re}\{\alpha_0\})^2+[\mathrm{Re}\{\alpha_0\}g\cos\theta-\mathrm{Im}\{\alpha_0\}\\&\times(1-g\sin\theta)]^2)];\end{aligned} \tag{5.7}$$

note that the previously derived expression (4.18) for the decoherence time follows from (5.7) by setting $\mathrm{Re}\{\alpha_0\}=0$. In general, the explicit dependence of (5.6) and (5.7) on both the real and the imaginary part of the amplitude of the initial coherent state $|\alpha_0\rangle$ reveals the ''anisotropic'' aspect of homodyne feedback. In fact, in the absence of feedback the dynamics is simply given by the phase invariant vacuum master equation (3.3). On the contrary, in the presence of feedback the homodyne measured quadrature $X_{\varphi=0}\equiv X$ becomes privileged and one expects that the dynamical effects of quantum feedback mainly manifest along this quadrature. This can be seen by taking the initial Schrödinger-cat state along the measured quadrature, i.e., choosing $\mathrm{Im}\{\alpha_0\}=0$ in (5.7). We thereby obtain

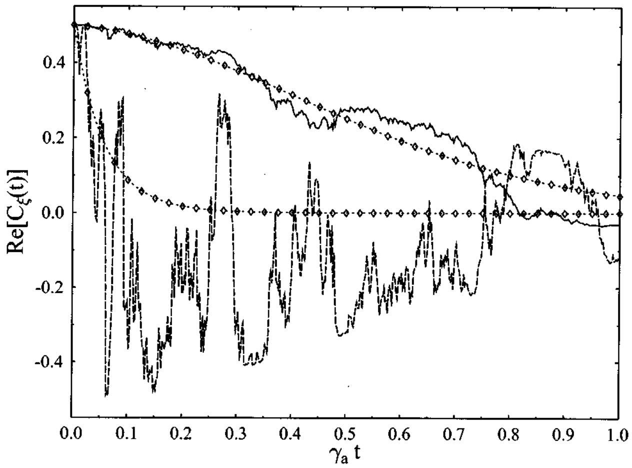

FIG. 3. Real part of the coherence function $C_\xi(t)$ for the same parameter values as in Fig. 2. The dotted lines and diamonds show the analytical and numerical results for the corresponding ensemble averaged coherence function $\overline{C_\xi(t)}$, respectively.

$$\overline{C_\xi(t)} \simeq \frac{1}{2}\exp[-2\gamma_a t(\mathrm{Re}\{\alpha_0\})^2(1+g^2\cos^2\theta)], \quad (5.8)$$

i.e., the coherence function in the presence of feedback always decays faster than in the vacuum bath case. This is due to the fact that for $\mathrm{Im}\{\alpha_0\}=0$ the interference fringes associated to quantum coherence (which always oscillate along the direction orthogonal to the phase of the Schrödinger cat) are along the y direction, i.e., they are $\pi/2$ out of phase with respect to the measured quadrature X, and the feedback mechanism has no relevant influence on them (see also Fig. 2). Instead, the retardation induced by the feedback loop can be observed if we take the interference fringes along the direction of the measured quadrature, i.e., $\mathrm{Re}\{\alpha_0\}=0$. In this case the fringes are stabilized by feedback and Eq. (5.7) simplifies to expression (4.18) for the retarded decoherence time. In particular, again choosing $g\sin\theta=1$, one has to approximate the argument of the exponential in (5.6) up to second order in $\gamma_a t$, which leads to a Gaussian decay for $\overline{C_\xi(t)}$,

$$\overline{C_\xi(t)} \simeq \frac{1}{2}\exp\left\{-\left(\frac{\mathrm{Im}\{\alpha_0\}\gamma_a t}{2}\right)^2\right\}, \quad (5.9)$$

instead of the much faster exponential decay. This fact again indicates the significant retardation of decoherence for the particular choice $g\sin\theta=1$.

We now propose a closer inspection of single realizations of the coherence function as predicted by the nonlinear SSE (2.17) in the ''optimal'' case for decoherence retardation, i.e., $\mathrm{Re}\{\alpha_0\}=0$. In Fig. 3 we compare, for a single run of the noise ξ, the behavior of the real part of $C_\xi(t)$ without feedback (dashed line) and in the presence of feedback (full line) for parameter values $g=1$ and $\theta=\pi/2$ (the corresponding curves for the imaginary part of $C_\xi(t)$ are qualitatively similar). For comparison, the exact analytical results (5.3) and (5.9) for the ensemble averaged quantity $\overline{C_\xi(t)}$ are displayed by the two dotted lines; the diamonds show the corresponding numerical results obtained by taking the average over 1000 single trajectories and are in excellent agreement with the analytical curves. Figure 3 clearly reveals the rather dramatic difference between the two stochastic trajectories; especially, the fluctuations of the coherence function around its average value are significantly suppressed by the feedback loop, suggesting that the feedback process induces a strong stabilization against fluctuations, a well known effect of classical feedback.

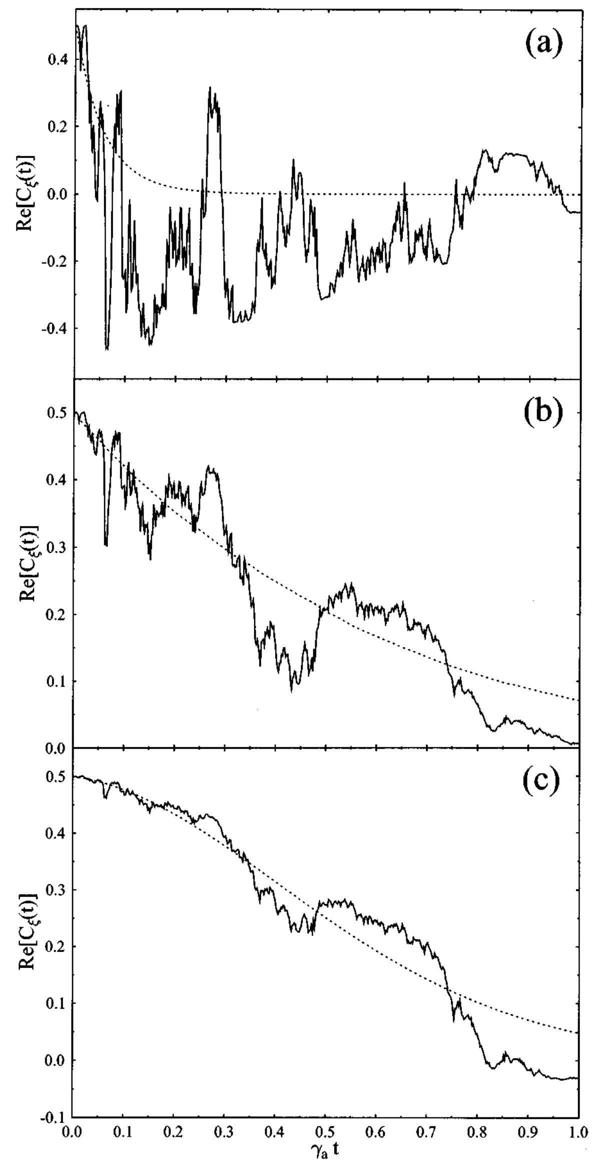

FIG. 4. The same plot as in Fig. 3 for $g=1$ and $\theta=0$ (a), $\theta=\pi/4$ (b), and $\theta=\pi/2$ (c).

A particular character of the present homodyne feedback scheme is its phase sensitivity, i.e., it depends on the value of the experimentally adjustable phase θ. It is now possible to see that quantum fluctuations manifest themselves through phase fluctuations of the wave function of the cavity mode and that, due to the phase sensitivity, feedback is able to stabilize them provided the phase θ is appropriately chosen. This is well shown by Figs. 4(a)–4(c), where single runs of the real part of the coherence function $C_\xi(t)$ and the corresponding ensemble averages for $g=1$ and three different values of the phase θ [$\theta=0$ in Fig. 4(a), $\theta=\pi/4$ in Fig. 4(b),

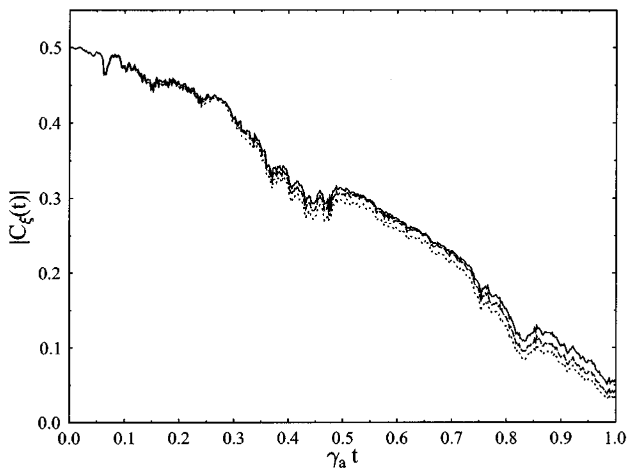

FIG. 5. The modulus $|C_\xi(t)|$ for $g=1$ and $\theta=0$ (dotted line), $\theta=\pi/4$ (dashed line), and $\theta=\pi/2$ (full line).

and $\theta=\pi/2$ in Fig. 4(c)] are displayed. The stochastic trajectories become more and more regular as θ approaches the ''optimal'' value $\pi/2$. The suppression of the phase fluctuations is also revealed in Fig. 5, where the *modulus* of $C_\xi(t)$ instead of its real part for $\theta=\pi/2$ (full line), $\theta=\pi/4$ (dashed line), and $\theta=0$ (dotted line) are plotted; the three trajectories almost coincide and are very similar to the trajectory of $\mathrm{Re}\{C_\xi(t)\}$ in the most stable case $\theta=\pi/2$ [Fig. 4(c)]. We may conclude that the large coherence fluctuations in the cases without feedback (Fig. 3) and with feedback, but phase $\theta\neq\pi/2$, are essentially phase fluctuations, which can be eliminated almost completely by an appropriate choice of the phase θ of the feedback loop.

In our quantum trajectory description of homodyne feedback we have necessarily assumed unit efficiency in the detection of the outgoing light. In fact, the description of a continuously measured system in terms of a stochastic wave function requires complete knowledge of its time evolution. In the more realistic case of an efficiency $\eta<1$, information about the state of the mode is irretrievably lost and it is only possible to work with a stochastic master equation for the conditional density matrix of the system. That stochastic master equation has been derived in [17,18] and, when averaged over the noise, gives the following non-unit efficiency generalization of the homodyne feedback master equation (3.2),

$$\dot{\rho}=-\frac{i}{\hbar}[H_0,\rho]+\frac{\gamma_a}{2}(2a\rho a^\dagger-a^\dagger a\rho-\rho a^\dagger a)$$
$$-i\gamma_a[A,ae^{-i\varphi}\rho+\rho a^\dagger e^{i\varphi}]-\frac{\gamma_a}{2\eta}[A,[A,\rho]]. \tag{5.10}$$

The effect of an inefficient detector is to increase the diffusion term induced by the noise during the measurement step. As can be expected, this increased diffusion limits the capability of our feedback scheme of retarding the quantum decoherence of the Schrödinger-cat state. This may be easily seen from the analytic solution of master equation (5.10), which involves only slight modifications of the solution derived in Sec. IV for the case $\eta=1$. In fact, the coefficient $\nu(t)$ of Eq. (4.5) is modified according to $\nu(t)\rightarrow\nu(t)/\eta$ and, as a consequence, Eq. (4.8) reads

$$\sigma_x^2(t)=\frac{1}{2}\left\{1+\frac{g^2}{\eta}\sin^2\theta\left[\frac{1-e^{-(1-2g\sin\theta)\gamma_a t}}{1-2g\sin\theta}\right]\right\}. \tag{5.11}$$

According to (4.15), an increase of $\sigma_x^2(t)$ implies a worse coherence preservation of our homodyne feedback scheme. The decoherence function $\eta(t)$, Eq. (4.17), generalizes to

$$\eta(t)\simeq\left(1-2g\sin\theta+\frac{g^2}{\eta}\sin^2\theta\right)\gamma_a t \tag{5.12}$$

and leads to the general feedback-modified decoherence time

$$t_{dec}\simeq\frac{1}{2\gamma_a|\alpha_0|^2\left(1-2g\sin\theta+\frac{g^2}{\eta}\sin^2\theta\right)}. \tag{5.13}$$

Therefore, the decoherence of the cat state is retarded if the condition $0<g\sin\theta<2\eta$ is satisfied. The results for the averaged coherence function $\overline{C_\xi(t)}$ in Sec. V are also modified only slightly in the case of inefficient detectors. The short-time expression (5.7) now reads

$$\overline{C_\xi(t)}\simeq\frac{1}{2}\exp\Bigg\{-2\gamma_a t\Bigg[(\mathrm{Re}\{\alpha_0\})^2-2\mathrm{Re}\{\alpha_0\}\mathrm{Im}\{\alpha_0\}g\cos\theta$$
$$+(\mathrm{Im}\{\alpha_0\})^2(1-2g\sin\theta)$$
$$+\frac{g^2}{\eta}(\mathrm{Re}\{\alpha_0\}\cos\theta+\mathrm{Im}\{\alpha_0\}\sin\theta)^2\Bigg]\Bigg\}. \tag{5.14}$$

The above expressions clearly reveal that the case $g\sin\theta=1$ is no more peculiar in the sense that, if $\eta<1$, it does no more show any perfect fringe preservation but only a slowing down of decoherence. In fact, in the case of nonunit efficiency, the exponential broadening (4.19) is no more valid. Moreover, in the case $g\sin\theta=1$ and $\mathrm{Re}\{\alpha_0\}=0$ the Gaussian decay of Eq. (5.9) is replaced by a short-time exponential decay according to

$$\overline{C_\xi(t)}\simeq\frac{1}{2}\exp\left\{-2\gamma_a t(\mathrm{Im}\{\alpha_0\})^2\left(\frac{1}{\eta}-1\right)\right\}. \tag{5.15}$$

VI. CONCLUSION

In this paper we have studied the dynamics of a cavity mode subject to a continuous homodyne measurement and a feedback loop in which part of the output homodyne photocurrent is fed back to the cavity. Using a nonlinear SSE we have analyzed numerically the effect of feedback in the Markovian limit of vanishing feedback delay time and found that the homodyne-feedback mechanism is able to stabilize the dynamics at the quantum level by strongly reducing phase fluctuations of the cavity mode caused by the interaction with the external vacuum modes, which manifests itself in a significant retardation of the decoherence of a Schrödinger-cat state [see Eqs. (4.18) and (5.9)]. The capability of homodyne feedback of controlling the decoherence process may be of great importance both for improving some proposed experiments of optical Schrödinger cat generation as well as quantum computation.

The decoherence control offered by the present homodyne feedback scheme still suffers some important limitations. In fact, as we have shown, it is ''anisotropic,'' i.e., it is mostly efficient along the direction of the homodyne measured quadrature X, while it is almost completely inefficient in the direction orthogonal to it. This means that quantum coherence can be preserved by the present scheme only if one has at least some phase information on the linear superposition state to be preserved. This is not the case for, e.g., quantum computers where the linear superposition state can be very general and one has no information about it.

The best coherence preservation is obtained if the feedback parameters are chosen according to $g\sin\theta=1$ and unit detection efficiency. In this case the interference pattern is perfectly preserved since the probability distribution $P(x)$ exponentially expands according to Eq. (4.19). This is also shown by the behavior of the coherence function (5.2) which, in this case, is characterized by a very slow Gaussian decay [see Eq. (5.9)]. As expected, the use of inefficient detectors limits the coherence preservation properties of the present feedback scheme. Nonetheless, the slowing down of the cat decoherence remains well visible also for $\eta<1$ and could be experimentally observed using currently available high-efficiency detectors. The crossover from the exponential decay of Eq. (5.15) to the Gaussian decay of (5.9) could be perceived, at least indirectly, using various detectors of increasing efficiency.

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