Semiempirical scaling laws for electron capture at low energies

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We present semiempirical scaling laws for different electronic rearrangement features in slow collisions with ions of high charge q. The absolute cross section for removing exactly r electrons from the target is found to be well described by the scaling (sc) relation ${}^{sc}\sigma_q^r=(2.7\times10^{-13})qr/[I_1^2I_r^2\Sigma_{j=1}^{j=N}(j/I_j^2)]$ cm^2, where I_j is the jth target ionization potential in eV, and N is the number of outer-shell electrons. This expression and the related total recoil-ion charge-state fractions, ${}^{sc}f_q^r={}^{sc}\sigma_q^r/\Sigma_r{}^{sc}\sigma_q^r$, compare favorably with recent experimental results (${}^{ex}\sigma_{q,q-p}^r$) for slow Xe^{q+} ions ($15\leqslant q\leqslant43$) colliding with He, Ar, and Xe. We discuss the possibility of establishing scaling laws for phenomenological cross sections $\sigma_{q,q-p}$, where only the number of electrons retained by the projectile p is specified. [S1050-2947(96)10211-0]

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I. INTRODUCTION

In this paper we present semiempirical scaling rules for various features of charge transfer in slow collisions ($v\sim0.2$ a.u.) between highly charged ions and atoms. The results are based on our recent data on slow collisions between Xe^{q+} ions and three different targets, He, Ar, and Xe [1]. In Ref. [1], we presented absolute experimental cross sections, ${}^{ex}\sigma_{q,q-p}^r$, for processes

$$A^{q+}+B\rightarrow A^{(q-p)+}+B^{r+}+(r-p)e^-, \qquad (1)$$

where r is the number of electrons removed from the target B and p is the number of electrons retained by the projectile A. The experimental results of Selberg *et al.* [1] contain close to 300 absolute cross sections of the kind ${}^{ex}\sigma_{q,q-p}^r$ (i.e., with q, r, and p defined) covering the parameter ranges $15\leqslant q\leqslant43$ and $r\leqslant9$. Further, three different targets were used (He, Ar, and Xe) and this makes it a rather unique set of data. Similar other data ${}^{ex}\sigma_{q,q-p}^r$ with $q>20$ are scarce and except for the Xe^{30+}-Ar cross sections by Raphaelian *et al.* [2], only data with He as a target have been published (Andersson *et al.* [3]). For $q<15$ a large set of data was presented by Groh *et al.* [4] already in 1983. Here we present a scaling rule for the absolute cross sections for removing exactly r electrons from the target,

$${}^{sc}\sigma_q^r=(2.7\times10^{-13})qr\Bigg/\left[I_1^2I_r^2\sum_{j=1}^{N}(j/I_j^2)\right], \qquad (2)$$

where the result is in units of cm^2 if the ionization potentials, I_j, are given in eV. The sum over j runs over the number of outer-shell electrons of the target, i.e., $N=2$ for He and $N=8$ for Ar, and Xe. Equation (2) compares more favorably than the classical over-the-barrier models [5,6] with the experimental results of Selberg *et al.* [1]. The total recoil-ion charge-state fractions, ${}^{sc}f_q^r={}^{sc}\sigma_q^r/\Sigma_r{}^{sc}\sigma_q^r,>$ derived from Eq. (2) are also closer to the experimental results than the corresponding model quantities. In particular, the scaling law (2) and ${}^{sc}f_q^r$ give good results for the He target, whereas the barrier models fail in this case.

In the past, there have been a few attempts to establish scaling laws for phenomenological cross sections $\sigma_{q,q-p}$, i.e., the projectile keeps p electrons but there is no information about the number of electrons removed from the target. The most famous of these results, mostly based on data below $q=10$, was presented by Müller and Salzborn already before 1980 and gave $\sigma_{q,q-1}\sim q^{1.17}/I^{1.96}$ [7,8]. Shortly thereafter it was realized that electron removal cross sections, σ_q^r [9,10], are a more natural basis for understanding electron-transfer mechanisms since such quantities are independent of the outcomes of the relaxation processes following the initial multiple-electron transfer. That is, σ_q^r is the cross section for formation of an r-times excited state on the highly charged projectile ion in the first step of the reaction

$$A^{q+}+B\rightarrow A^{(q-r)+}+B^{r+}\rightarrow A^{(q-p)+}+B^{r+}+(r-p)e^-. \qquad (3)$$

Thus far it has not been possible to formulate a simple model that is able to account also for the second step in Eq. (3) and no corresponding scaling laws for high q have been established. Recently, Kimura *et al.* [11] presented a scaling law for removing *at least* r electrons from the target, which is consistent with our formula (2) for removing *exactly* r electrons from the target. The establishment of scaling rules are important for two reasons. First, it may provide important input to simulations of, e.g., fusion and astrophysical plasmas in the lack of real experimental data. Second, it may serve as a guidence for attempts to understand the underlying physical processes.

In Sec. II we give a short account of the classical over-the-barrier models by Bárány *et al.* [5] and Niehaus [6]. Section III is devoted to an account of the different ways in which the experimental data [1] are reduced for the following comparisons with the present scaling rules and the models [5,6] in Sec. IV. In the latter section we account for the analysis leading to the the various scaling rules for total-electron capture, recoil-ion charge-state fractions, and elec-

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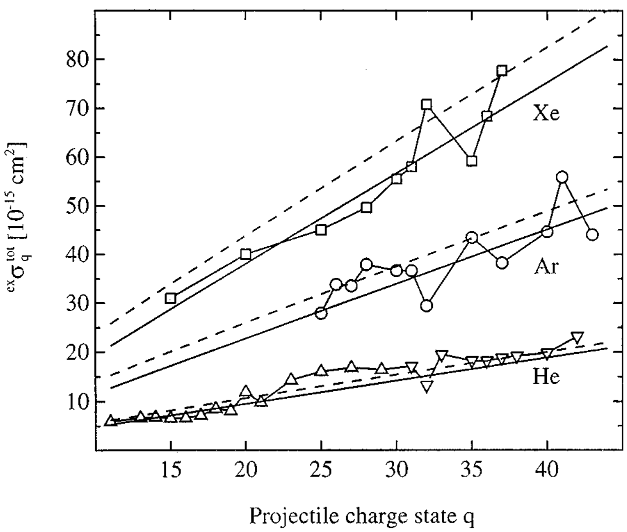

FIG. 1. Total experimental electron-capture cross sections [${}^{ex}\sigma_q^{tot}$ according to (6) in the text] for slow Xe^{q+} ions colliding with He, Ar, and Xe. The full lines show a common fit to the experimental results. The expression Cq^{α}/I_1^{β} was used and the three sets of data were fitted together, yielding the results $C=2.7\times10^{-13}$ cm^2, $\alpha=0.98\pm0.06$, and $\beta=1.96\pm0.04$ with I_1 in units of eV. The dashed lines are the predictions from the extended classical over-the-barrier model ${}^{m}\sigma_q^{tot}=\pi R_1^2$ (cf. text).

tron removal cross sections. Finally, we discuss relaxation processes (of intermediate multiply excited states) and the possibility of formulating scaling rules for phenomenological cross sections ($\sigma_{q,q-p}$) in Sec. V.

II. MODELS

The classical over-the-barrier models by Bárány *et al.* [5] and by Niehaus [6] assume that multiple-electron transfer proceeds sequentially, i.e., the electrons are transferred at different internuclear separations. An electron may leave the target when the top of the internuclear potential barrier is at the same energy as the most loosely bound of the target electrons. In their model, Bárány *et al.* assume that the number of electrons transferred is determined only by the impact parameter in such a way that if it lies between the critical radii for removal of r and $r+1$ electrons (R_r and R_{r+1}), r electrons will be removed from the target with a probability of 1. Niehaus [6] uses a more refined treatment where the capture probabilities are given by the ratios of phase space available on the projectile and target at the moment (on the way out from the collision) when the internuclear barrier rises above the energy of a quasimolecular electron. Further, the two models use slightly different assumptions concerning the screening of the incoming charge q by $(r-1)$ inner electrons as seen by the rth electron when it is about to be transferred to the projectile. Bárány *et al.* [5] use full screening, whereas Niehaus [6] neglects the screening. However, the differences between the predictions of the two models are minor for the cases of interest here, where q is large and r is comparatively small. In the following we will mostly relate to the model by Bárány *et al.*, which assumes a quasicontinuum of capture states. Electron removal cross sections are then given by areas between concentric rings

$$ {}^{m}\sigma_q^r=\pi(R_r^2-R_{r+1}^2), \tag{4}$$

where the critical distance for removing r electrons is given by

$$ R_r=[2\sqrt{r(q-r+1)}+r]/I_r. \tag{5}$$

III. EXPERIMENTAL DATA

The analysis presented in this paper is based on the partial cross sections ${}^{ex}\sigma_{q,q-p}^r$ which we recently reported in another publication [1]. We have summed these partial cross sections in four different ways [(i)–(iv)] in order to establish the scaling rules and to make comparisons with the classical over-the-barrier models.

(i) For a given q, target species, and number of electrons retained by the projectile (p), the cross sections for removal of different numbers of target electrons (r) are summed over r. These results for the different values of p are then

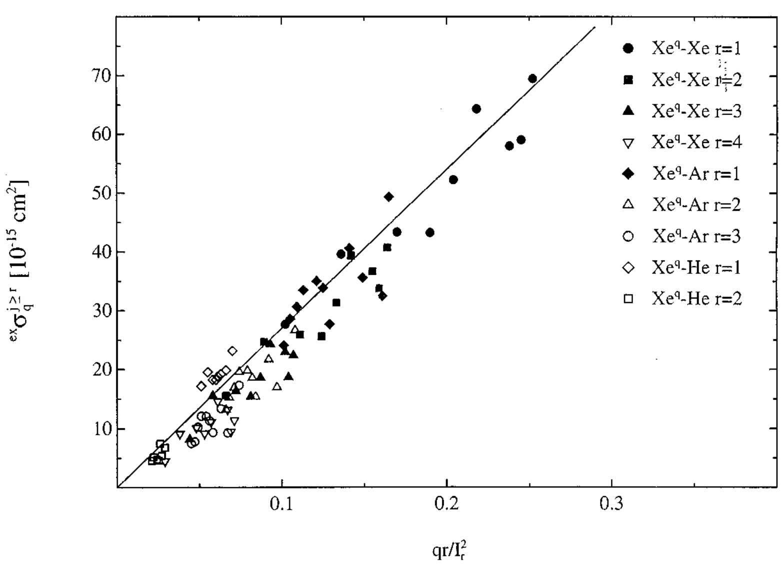

FIG. 2. Absolute experimental cross sections for removing at least r electrons from the target [${}^{ex}\sigma_q^{j\geqslant r}$ according to Eq. (7) in the text] as functions of qr/I_r^2. The results shown are for Xe^{q+} colliding with Xe, Ar, and He. The results for He with $q<30$ are from Andersson *et al.* [3]. The slope of the line has the coefficient 2.7×10^{-13} cm^2 [cf. Eq. (11) in the text].

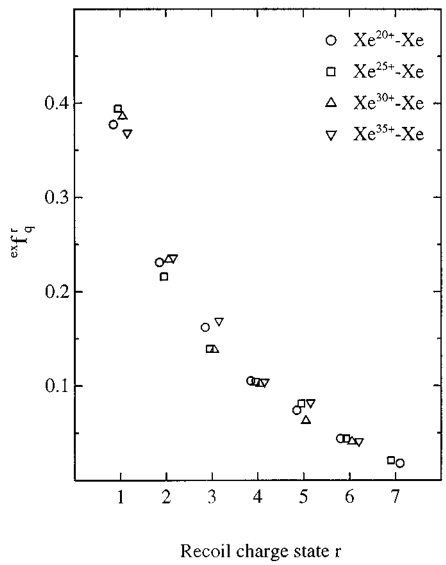

FIG. 3. Total experimental recoil-ion charge-state fractions ${}^{ex}f_q^r$ [cf. Eq. (12) in the text with $pmax=3$] as functions of the number of removed target electrons, r, for Xe^{q+}-Xe collisions with $q=20$, 25, 30, and 35. The experimental cross sections used to deduce these quantities thus include up to three retained electrons on the projectile and recoil-ion charge states up to $r=8$.

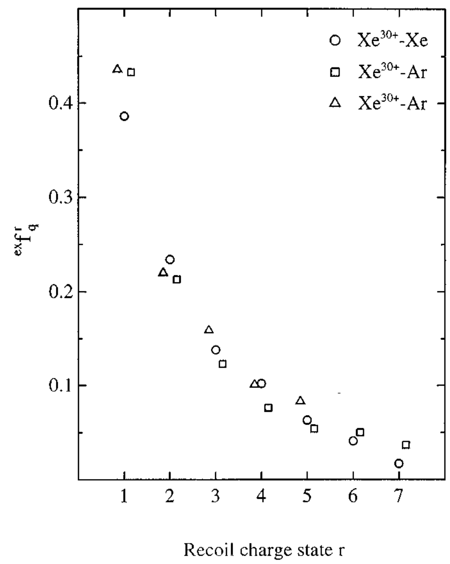

FIG. 4. Total experimental recoil-ion charge-state fractions ${}^{ex}f_q^r$ for Xe^{30+}-Xe (○) and Xe^{30+}-Ar (△) collisions at the velocity $v=0.2$ a.u. as functions of the number of removed target electrons r [cf. Eq. (12) in the text]. The results by Raphaelian *et al.* [2] (□) at $v=0.3$ a.u. are shown for a comparison. The corresponding present results for Ar are somewhat higher since the contributions from $r=6$ and $r=7$ are lacking in the experimental data.

summed over p, which yields the total-electron-capture cross section

$$
{}^{ex}\sigma_q^{tot}=\sum_p \left(\sum_{r\geqslant p}^{N} ({}^{ex}\sigma_{q,q-p}^r)\right)\approx\sum_{p=1}^{pmax}\left(\sum_{r\geqslant p}^{N} ({}^{ex}\sigma_{q,q-p}^r)\right). \tag{6}
$$

Here $pmax$ is limited to 2 or 3 by the content of the experimental data of Selberg *et al.* [1]. This, however, is not believed to be a serious limitation since already the $p=3$ term in the sum above is expected to be very small. This was also observed by Raphaelian *et al.* [2] for the Xe^{30+}-Ar system and they stated that the $p=3$ contribution was only on the order of a few percent of ${}^{ex}\sigma_q^{tot}$. The total charge-exchange cross sections could also be determined without using the coincidence information (i.e., the partial cross sections ${}^{ex}\sigma_{q,q-p}^r$). This was done by summing the phenomenological cross sections, ${}^{ex}\sigma_{q,q-p}$, over p. This method yielded results consistent with the ones obtained through Eq. (6).

(ii) The same procedure as in (i), but here we exclude various numbers of the leading terms in the sums over r in order to obtain the total cross section for removing *at least* r electrons from the target

$$
{}^{ex}\sigma_q^{j\geqslant r}=\sum_p \left(\sum_{j\geqslant r}^{N} ({}^{ex}\sigma_{q,q-p}^j)\right)\approx\sum_{p=1}^{pmax}\left(\sum_{j\geqslant r}^{N} ({}^{ex}\sigma_{q,q-p}^j)\right). \tag{7}
$$

This expression reduces to Eq. (6) when $j\geqslant r=1$.

(iii) Summations over p for specified values of r that give the total cross sections for removal of *exactly* r electrons from the target

$$
{}^{ex}\sigma_q^r=\sum_p ({}^{ex}\sigma_{q,q-p}^r)\approx\sum_{p=1}^{pmax} ({}^{ex}\sigma_{q,q-p}^r). \tag{8}
$$

In some cases, the limitations $p\leqslant 2$ or $p\leqslant 3$ becomes more serious for evaluations according to Eqs. (7) and (8) especially if r is large. We have excluded such cases in the analysis below.

(iv) Finally, we sum over r for specified values of p, which gives the total cross sections for retaining p electrons on the projectile

$$
{}^{ex}\sigma_{q,q-p}=\sum_{r=p}^{N} ({}^{ex}\sigma_{q,q-p}^r). \tag{9}
$$

These cross sections can also be obtained through direct measurements of the charge-exchange yields for the projectiles (singles measurements).

IV. SCALING LAWS

A. Total-electron-capture cross sections

In Fig. 1, we show the total-electron-capture cross sections for Xe^{q+}-projectiles colliding with He, Ar, and Xe from Ref. [1] [summed according to Eq. (6)]. The dashed lines give the values from the extended classical over-the-barrier model by Bárány *et al.* [5] ${}^{m}\sigma_q^{tot}=\pi R_1^2$ [with R_1 given by Eq. (5)], while the full lines are fits to the experimental data. In these fits we used an expression of the form ${}^{sc}\sigma_q^{tot}=C\times q^{\alpha}/I_1^{\beta}$ and arrived at $C=(2.7\pm 0.1)\times 10^{-13}$ cm^2, $\alpha=0.98\pm 0.06$, and $\beta=1.96\pm 0.04$ when the first ionization potential of the target, I_1, is given in units of eV. These parameter values are close to the ones expected from

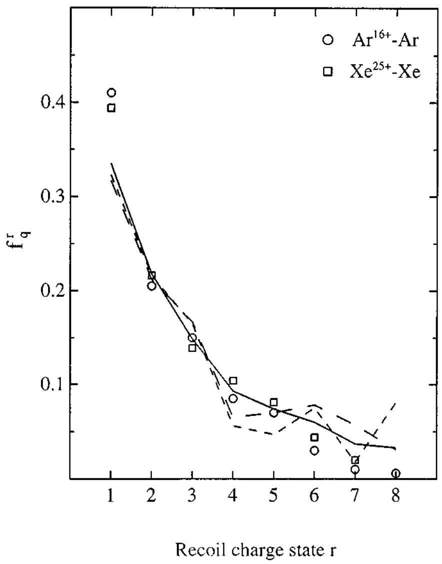

FIG. 5. Total experimental recoil-ion charge-state fractions ${}^{ex}f_q^r$ for Xe^{25+}-Xe [cf. Eq. (12) in the text] and Ar^{16+}-Ar (from Vancura *et al.* [17]) at $v=0.2$ as functions of r. The results of the semiempirical expression ${}^{sc}f_q^r$ (15) are shown as a solid line, whereas the model values ${}^{m}f_q^r$ from Refs. [5] and [6] are indicated by short and long dashed lines, respectively.

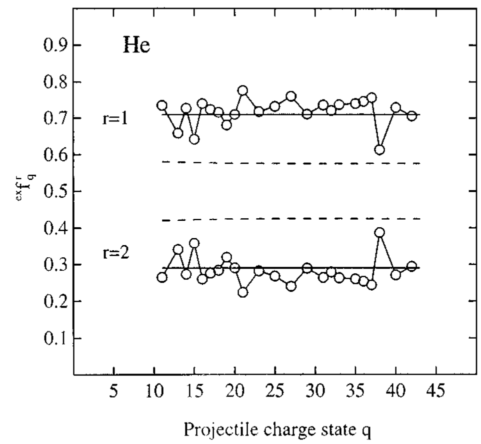

FIG. 6. Total experimental recoil-ion charge-state fractions for He as functions of q [cf. text: Eq. (8) divided by Eq. (6) summing over $p=1$ and 2 in Eq. (8)]. The predictions from the over-the-barrier model (Ref. [5]) are shown as dashed lines. The recoil-ion charge-state fractions, ${}^{sc}f_q^1$ and ${}^{sc}f_q^2$, according to Eq. (15) are shown as full lines.

the over-the-barrier model [5] which yields ${}^{m}\sigma_q^{tot}=(2.6\times10^{-13})(q+\sqrt{q})/I_1^2$ at large q [c.f. Eq. (5) with $r=1$]. We have not included the $\sqrt{q}$ term in the fit and our coefficient C (2.7×10^{-13}) is thus higher than the model coefficient, although the dashed lines (model) in Fig. 1 lie above the corresponding full lines (fits). It is interesting to note that the experimental data on the average (as described by the fits) are slightly lower than the model values, which can be regarded as upper limits for the total cross sections. This might be explained in terms of the deviations from true quasicontinua of projectile capture states, which would make the real capture distance smaller than R_1. The dominant impression from Fig. 1, however, is that the agreement between the model and the experimental result is very good — better than $\pm20\%$. This, in turn, supports the model viewpoint stating that electron capture (single or multiple) will happen with 100% probability for impact parameters that are smaller than the outermost critical radius R_1. In the following we will set $\alpha=1$ and $\beta=2$, since these values are consistent both with our experimental results (the fit) and the model. We thus arrive at

$$ {}^{sc}\sigma_q^{tot}=(2.7\times10^{-13})q/I_1^2. \tag{10}$$

where the result is in cm^2 when I_1 is given in eV.

In Fig. 2 we show a universal scaling of the cross sections for removing *at least* r electrons from the target. The summed experimental cross sections according to Eq. (7) are plotted as a function of qr/I_r^2. This parameter is close to R_r^2, the square of the critical distance for removing the rth electron from the target, when r is considerably smaller than

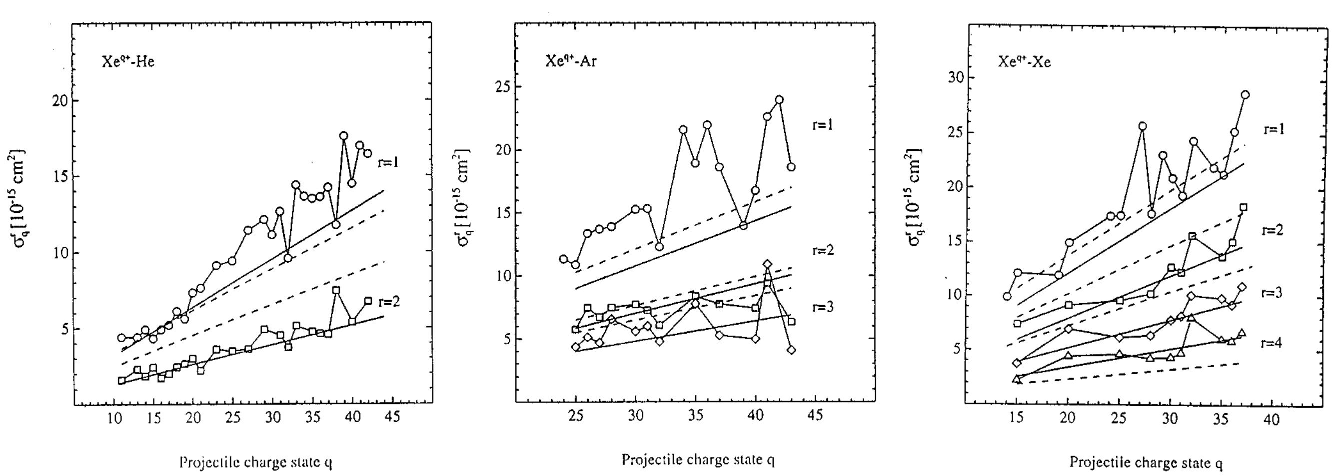

FIG. 7. Absolute experimental cross sections for removing exactly r electrons from the target, ${}^{ex}\sigma_q^r$, are compared with the over-the-barrier model, Eq. (4), and with the scaling law, Eq. (16). The former is represented by dashed lines, while the latter is represented by full lines.

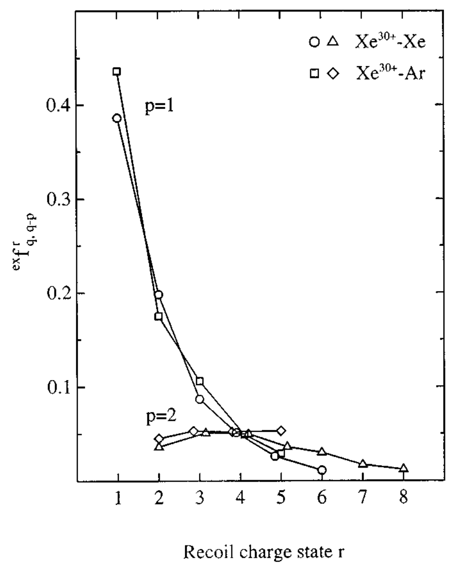

FIG. 8. Partial recoil-ion charge-state fractions ${}^{ex}f^{r}_{q,q-1}$ and ${}^{ex}f^{r}_{q,q-2}$ for projectile charge-state changes $q\rightarrow(q-1)$ and $q\rightarrow(q-2)$, respectively. Results are shown for Xe^{30+}-Xe and Xe^{30+}-Ar collisions.

q [cf. Eq. (5)]. In Fig. 2 we give results for $r=4$ (Xe), $r=3$ (Xe, Ar), and $r=2$, $r=1$ (Xe, Ar, He). The full line in Fig. 2 is given by

$$ {}^{sc}\sigma^{j\geqslant r}_{q}=(2.7\times10^{-13})qr/I_r^2. \tag{11}$$

The fair agreement between the experimental results and Eq. (11) lends some support to the model assumption, saying that at least r electrons will be transferred to the projectile with 100% probability if the impact parameter is smaller than R_r [5].

B. Charge-state fractions for recoil ions

The experimental charge-state fractions ${}^{ex}f^{r}_{q}$ are given by

$$ {}^{ex}f^{r}_{q}=\sum_{p=1}^{pmax}({}^{ex}\sigma^{r}_{q,q-p})/({}^{ex}\sigma^{tot}_{q}) \tag{12}$$

and we find that they are nearly independent of projectile charge state (under the condition that $q>10$). This feature is displayed in Fig. 3, where we show the measured fractions for Xe projectiles of charge states $q=20$, 25, 30, and 35. The four sets of data are very close (within a few percent) between $r=1$ and $r=7$. Further, the recoil-ion charge-state distributions are virtually the same for the two heavy noble gas targets (Ar and Xe), as we show in Fig. 4 with a comparison between Xe^{30+}-Xe and Xe^{30+}-Ar results at $v\sim0.2$ a.u. In this figure, we have also included the Xe^{30+}-Ar results from Raphaelian *et al.* [2] at the slightly higher velocity of 0.3 a.u. This phenomena was also documented by Schuch *et al.* [12] for slow Ne^{10+} ions colliding with the noble gases. Finally, it is evident from Fig. 5 that the recoil-ion fractions are also independent of the projectile species from a comparison between Ar^{16+}-Ar (from Ali *et al.* [13]) and present Xe^{25+}-Xe results.

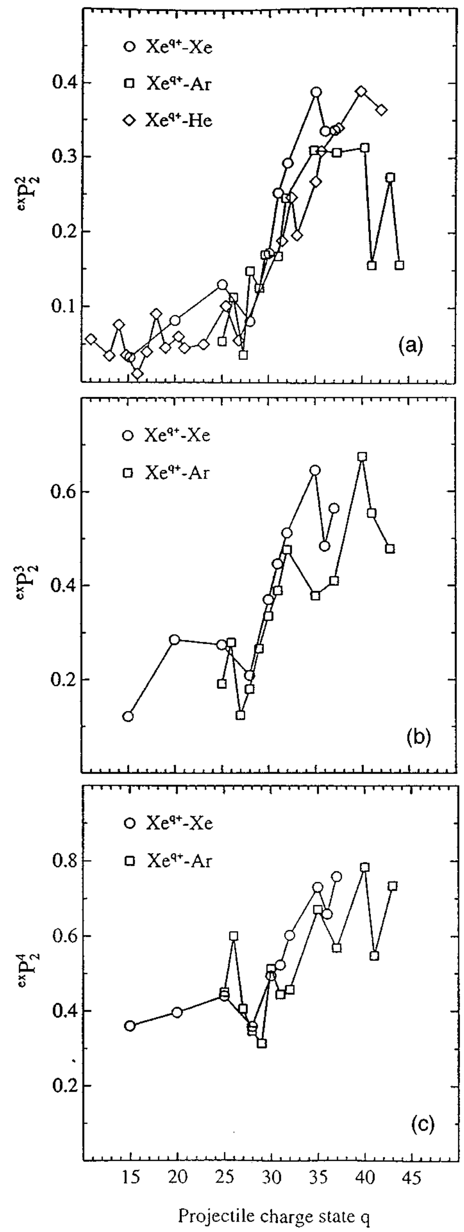

FIG. 9. Experimental probabilities for retaining two electrons on the projectile (${}^{ex}P^{r}_{2}={}^{ex}\sigma^{r}_{q,q-2}/({}^{ex}\sigma^{r}_{q,q-2}+{}^{ex}\sigma^{r}_{q,q-1})$ for the cases $r=2$ (a), $r=3$ (b), and $r=4$ (c). In (a) we show results for He, Ar, and Xe, while (b) and (c) contain results for Ar and Xe. The omission of the cross sections ${}^{ex}\sigma^{r}_{q,q-3}$ and ${}^{ex}\sigma^{r}_{q,q-4}$ for $r=3$ and $r=4$ in the denominator in ${}^{ex}P^{r}_{2}$ are of minor importance since they are much smaller than the ones which are included.

From the success of the scaling laws (10) and (11) for the total-electron-capture cross section and for the cross section for removing at least r electrons from the target one would expect the relative charge-state fractions to be given by

$$ f^{r}=[r(I_1/I_r)^2-(r+1)(I_1/I_{r+1})^2]. \tag{13}$$

This would also be well in line with the classical over-the-barrier model, which predicts ${}^{m}f^{r}_{q}=(R_r^2-R_{r+1}^2)/R_1^2$. Note that Eq. (13) can be derived from the model expression ${}^{m}f^{r}_{q}$ for large values of q and reasonably small values of r. A comparison between model values [5] and the measured frac-

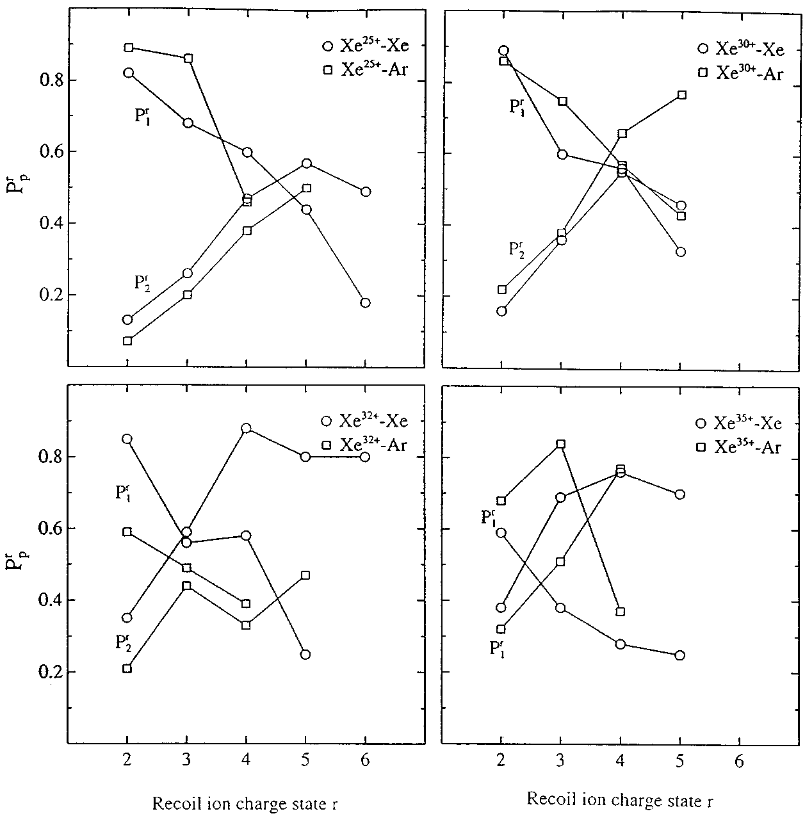

FIG. 10. Semiempirical probabilities for retaining one (P_1^r) and two (P_2^r) electrons on the projectile following the initial transfer of r electrons from the target. These quantities are obtained by dividing the experimental partial cross sections ${}^{ex}\sigma^r_{q,q-p}$ with the scaling law (16). Results for Xe^{q+} ions (q=25, 30, 32, and 35) colliding with Xe and Ar are shown.

tions are shown in Fig. 5. Although the model gives fair results in relation to the measured values for $r=1$, 2, and 3, it is also clear that some of the rather prominent features predicted by the model are missing in the experimental data. According to the model, the variation of f_q^r as a function of r should not be monotonic. Instead it should exhibit peaks in the recoil-ion fractions which are associated with the removal of the last electron in an atomic (sub)shell. Further, it is clear that the experimental values for $r=4$ and $r=5$ are considerably higher than the model values. The peaks in the model distributions at $r=6$ and $r=8$ are due to the large increases in the target ionization potential (giving comparatively large differences between R_r and R_{r+1}). The experimental recoil-ion fractions shown in this work clearly decrease monotonically with r. In Fig. 6, we show recoil-ion fractions ${}^{ex}f_q^1$ and ${}^{ex}f_q^2$ for Xe^{q+}-He collisions in the charge-state range $q=11$ to $q=42$ [3,14]. The values from the classical over-the-barrier model [5], which are indicated by dashed lines in the figure, fail to reproduce the experimental results.

Although the model is reasonably successful in reproducing absolute capture cross sections for removal of a specified *minimum* number of electrons (cf. Figs. 1 and 2) it is much less accurate when it comes to the partition in cross sections for *specified and exact* numbers of removed target electrons, even on a relative scale. Raphaelian *et al.* [2] evaluated the model by Niehaus [6] for Xe^{30+}-Ar and since we have found that the recoil-ion charge-state fractions are independent of projectile charge, species, and whether the target is Ar or Xe, these results may be compared directly with the experimental results for Xe^{25+}-Xe and Ar^{16+}-Ar (cf. Fig. 5). The results of the Niehaus model are very close to the ones of the model by Bárány *et al.* [5] both for multiple-electron targets and for He. Thus both models have limited success in reproducing the experimental relative recoil-ion charge-state distributions.

If we, however, assume that the cross section for removing exactly r target electrons is a constant, $c_x(r)$, times $(2.7\times10^{-13})qr/I_r^2$ we get the following expression for the total-capture cross section

$$ {}^{sc}\sigma_q^{tot}=(2.7\times10^{-13})\sum_{j=1}^{N}[c_x(j)jq/I_j^2]. \tag{14}$$

If we further assume that the constant c_x is independent of r, the recoil-ion charge-state fractions, ${}^{sc}f_q^r$, become

$$ {}^{sc}f_q^r=r(I_1/I_r)^2 \Big/ \sum_{j=1}^{N}[j(I_1/I_j)^2]. \tag{15}$$

This formula has no q dependence, it contains no information about the number of core electrons on the projectile, but it is target dependent through its sensitivity to the target ionization potentials. The sequences of the ratios $(I_1/I_r)^2$ for different *heavy* inert gases are very close and the sequences of ${}^{sc}f_q^r$ according to Eq. (15) are virtually the same for Xe and Ar. This is in agreement with the experimental results for the heavy targets. Further, in contrast to Eq. (13), Eq. (15) gives results for the He target that are very close to the experimental values, as shown by the full lines in Fig. 6.

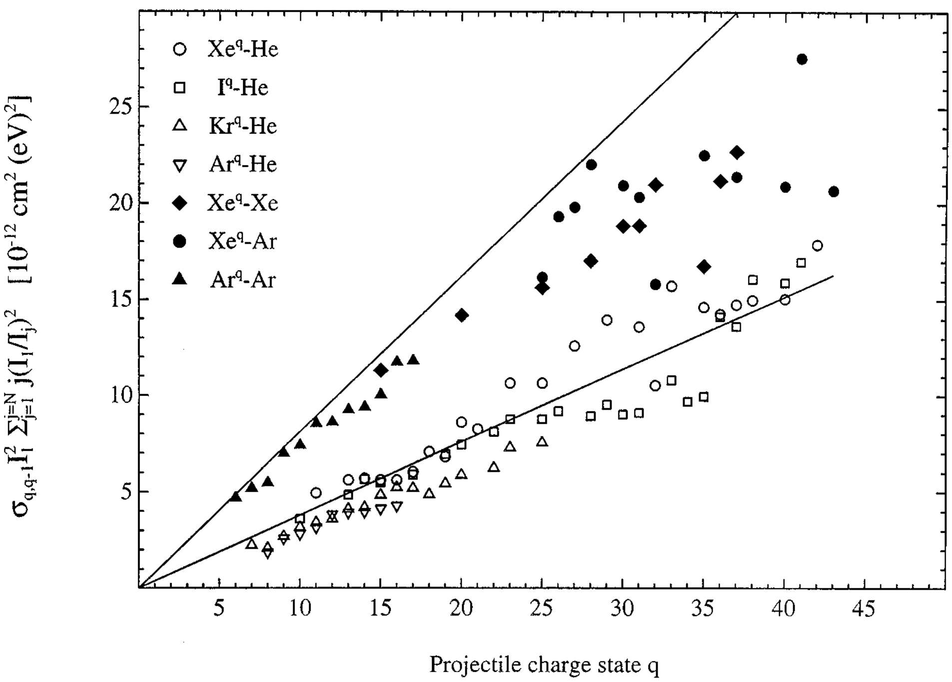

FIG. 11. Experimental phenomenological cross sections $\sigma_{q,q-1}$ multiplied by I_1^2 and the sum $\Sigma_j^N[j(I_1/I_j)^2]$ as functions of the projectile charge q. The results fall in two groups. The upper set is for the two heavy targets, whereas the lower set is for He. The two lines indicate the slopes expected for moderate q in the two cases [cf. Eq. (19)]. Data for projectiles other than Xe are from Refs. [13, 15, 17, 18].

C. Electron-removal cross sections

The absolute cross sections for removal of exactly r electrons from the target can be expressed as

$$^{sc}\sigma_q^r = (2.7\times 10^{-13})qr \Bigg/ \left(I_r^2 \sum_{j=1}^{N} [j(I_1/I_j)^2] \right). \quad (16)$$

This is simply the product between the total-electron-capture cross section $^{sc}\sigma_q^{tot}$ (10) and the recoil-ion charge-state fractions $^{sc}f_q^r$ (15), where both were found to be able to account for the experimental data. A comparison between the scaling law, the extended classical over-the-barrier model [5], and the experimental results are shown for Xe^{q+} colliding with Xe, Ar, and He in Figs. 7(a)–7(c), respectively. From this figure it is clear that the scaling law $^{sc}\sigma_q^r$ (16) is much closer to the experimental results than the model cross sections $^{m}\sigma_q^r$ (4) for $r \geqslant 2$. The agreement is slightly better for Eq. (16) than for Eq. (4) for $r=1$ and the He target, while the opposite relation prevails for $r=1$ and the heavier targets.

From the discussion in Sec. IV B it appears as if the cross section for removing exactly r electrons from the target is a coefficient c_B (target dependent) ($B=$He, Ar, and Xe) times πR_r^2 rather than $\pi(R_r^2 - R_{r+1}^2)$. According to the assumption leading to Eqs. (14) and (15) $^{sc}\sigma_q^r = c_B \times 2.7\times 10^{-13} qr/I_r^2$ cm^2. Comparing this to Eq. (16) we see that

$$c_B = 1 \Bigg/ \left(\sum_{j=1}^{N} j(I_1/I_j)^2 \right), \quad (17)$$

which is 0.71 for He and 0.33 for Ar and Xe. As an alternative to Eq. (16) σ_q^r can be expressed as

$$\sigma_q^r = \pi R_r^2 \Bigg/ \left(\sum_{j=1}^{N} [j(I_1/I_j)^2] \right), \quad (18)$$

using R_r from the model (5).

Note that formula (18) gives different results than the classical over-the-barrier model $^{m}\sigma_q^r = \pi(R_r^2 - R_{r+1}^2)$. The model assumes that the impact parameter of the collision determines the number of removed target electrons, whereas Eq. (18) is consistent with a viewpoint saying that r *or fewer than r electrons* will be removed if the impact parameter lies between R_r and R_{r+1}. That is, different electron-removal processes will compete at a given impact parameter b, except

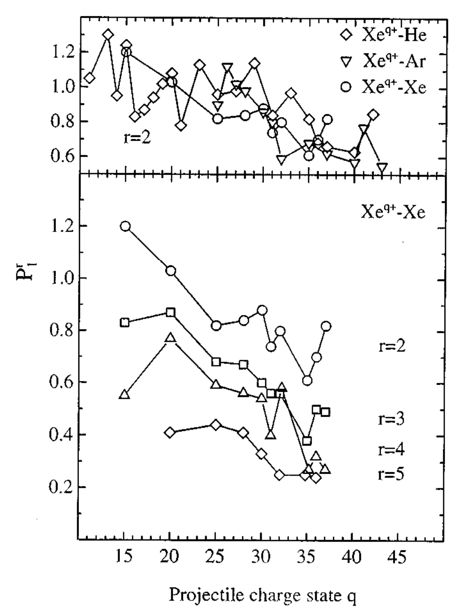

FIG. 12. Semiempirical probabilities (P_1^r) for retaining one electron on the projectile after initial transfer of r electrons from the target. The results are shown as functions of the projectile charge q with $r=2$, 3, 4, and 5 for Xe^{q+}-Xe collisions in the lower figure. The upper figure shows a comparison for $r=2$ for the three targets He, Ar, and Xe (cf. text).

for $b>R_2$, where only single-electron capture is possible.

V. RELAXATION AND PHENOMENOLOGICAL CROSS SECTIONS

A. Decay of multiply excited states

The partial recoil-ion charge-state fraction ${}^{ex}f^r_{q,q-1}$ and ${}^{ex}f^r_{q,q-2}$ for the projectile charge changes $q\rightarrow(q-1)$ and $q\rightarrow(q-2)$ are shown for Xe^{30+}-Xe and Xe^{30+}-Ar in Fig. 8. As can be seen from this figure, the results are very similar for the two targets. This indicates that the outcomes of the relaxation processes for the multiply (r times) excited projectile ion $Xe^{(q-r)+}$ are rather insensitive to whether the target is Xe or Ar.

Figure 9 displays experimental relative probabilities, ${}^{ex}P^r_2$ for retaining two electrons on the projectile if $r=2$ (a), $r=3$ (b), and $r=4$ (c) electrons were transferred initially. For the first case, $r=2$, we show results obtained with He, Ar, and Xe. There are no significant differences between these three sets of results [16]. For $r=3$ and $r=4$, we display results for the two heavy targets (Ar and Xe). Although there are some small differences in a few cases, the general trends are the same for Ar and Xe. We note the rather sharp increase in the region between $r=28$ and $r=36$, which is present in all three Figs. [9(a)–9(c)]. We attribute these phenomena to increased tendencies for radiative decay rather than autoionization when the number of $3d$ vacancies in the projectile increases [16].

In the preceeding section, we showed that the scaling law (16) is able to account for the absolute cross sections for the removal of specified numbers of target electrons (r). Relying on this law, we deduce semiempirical probabilities for retaining one (P^r_1) and two (P^r_2) electrons on the projectile if r electrons were removed initially. This is done by dividing the respective measured partial cross section ${}^{ex}\sigma^r_{q,q-p}$ for $p=1$ and $p=2$ with the expression (16). The results are shown in Fig. 10 for $q=25$, 30, 32, and 35 and there it can be seen that the sums of the probabilities for one- and two-electron retainment for $r=2$ and $r\geqslant 3$ are close to 1 in most cases. The latter is in agreement with the findings of Raphaelian *et al.* [2], who measured three-electron retainment and found it to be unimportant (a few percent of σ^{tot}_q) for multiple-electron transfer to Xe^{30+} projectiles. We also note the similarities between the variations of the semiempirical probabilities with r for the Ar and Xe targets.

B. Phenomenological cross sections

The first scaling laws for slow collisions between highly charged ions and atomic targets were established for phenomenological cross sections $\sigma_{q,q-p}$. We now know that the number of removed electrons may be much higher than the number of electrons retained by the projectile and that autoionization, or cascades of autoioinzation processes, often are very important [cf. Eq. (3)]. At first glance, this may lead one to believe that attempts to identify simple scaling rules for phenomenological cross sections are deemed to fail. To a certain extent this is true and we have been unable to find a scaling law for phenomenological cross sections, which is valid for light and heavy targets and for moderate and high q. It is, however, possible to find some common features, as we show in Fig. 11. There, we have plotted experimental values of $\sigma_{q,q-1}$ (present and others [13,15,17,18]) multiplied by I_1^2 and $\Sigma_j{}^N j(I_1/I_j)^2$ as functions of q. According to Eq. (16), the products should then fulfill the following relation:

$$\sigma_{q,q-1}I_1^2\sum_{j=1}^{N}\,[j(I_1/I_j)^2]=2.7\times10^{-13}q\sum_{j=1}^{N}\,P_1^j j(I_1/I_j)^2, \tag{19}$$

where P^j_1 are the probabilities for retaining one electron on the projectile when j electrons are transferred initially. The rationale of formula (19) can be understood by noting that the total-scaling-capture cross section, ${}^{sc}\sigma^{tot}_q$, deduced from Eq. (16), can be written

$${}^{sc}\sigma_q^{tot}=(2.7\times10^{-13})q\sum_{j=1}^{N}\,(j/I_j^2)\bigg/\sum_{j=1}^{N}\,[j(I_1/I_j)^2], \tag{20}$$

which equals Eq. (14). The phenomenological cross section for one-electron retainment can then be expressed as

$${}^{sc}\sigma_{q,q-1}=(2.7\times10^{-13})q\sum_{j=1}^{N}\,(P_1^j j/I_j^2)\bigg/\sum_{j=1}^{N}\,[j(I_1/I_j)^2]. \tag{21}$$

Formula (19) is obtained by a slight rearrangement of Eq. (21).

For moderate q and low r, the P^j_1 values are close to 1 (cf. Fig. 12) and the slopes of the two curves in Fig. 11 are given by $2.7\times10^{-13}\Sigma_{j=1}^N j(I_1/I_j)^2$. The coefficients take the values 8.0×10^{-13} for Ar and Xe and 3.8×10^{-13} for He. This explains that there are two branches for the experimental data; one for He and one for the heavy target gases. The latter data deviate from the linear behavior when the charge increases. This is due to the fact that the coefficients P^j_1 in Eq. (19) then no longer are close to 1 as is obvious from the upper part of Fig. 12. The experimental data then fall below the line since radiative relaxation becomes a more important process at higher q. Such a behavior is *not* observed for the He data. This might seem to contrast with the results shown in Fig. 12 (upper part), where it is shown that the importance of stabilization processes are rather insensitive to target species. The explanation is that the sum of all possible multiple-electron processes is much less important in relation to true single-electron capture for He than for Ar and Xe (compare Figs. 3–5 with Fig. 6). It is interesting to note that the two branches of data are expected to come close to each other as q increases as all values of P^j_1 (except $P^1_1=1$) decrease with increasing q (see Fig. 12). In this high-q limit all data will fall on a common curve with the slope 2.7×10^{-13}.

VI. CONCLUSIONS

In this paper we have formulated semiempirical scaling rules for removal of well-defined numbers of target electrons from various target atoms. In contrast to different formula-tions of the classical over-the-barrier model, this scaling law works equally well for He and heavier noble gas targets. Further, the experimental recoil-ion charge-state distributions are found to be much better described by the scaling than the classical models, in particular for the He target. The scaling law correctly accounts for the observations that the recoil-ion fractions are independent of projectile charge, the number of projectile core electrons for a given projectile charge, and whether the target is Ar or Xe. We have finally discussed the possibility of establishing scaling laws for phenomenological cross sections and, in particular, we noted that the cross sections for one-electron retainment by the projectile fall on two different curves, one for He and one for the heavier targets. It would certainly be interesting to check the validity of the scaling by performing experiments with other targets, preferably ones which are not noble gases, in a similar high range of projectile charge states. It would also be very interesting with experimental data in a much higher charge-state regime.

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