Broadband phase-noise squeezing of traveling waves in electromagnetically induced transparency

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We show that driven Λ-type atoms in a cell under conditions of electromagnetically induced transparency squeeze the phase noise of a traveling-wave input field in a broad spectral region. The maximum squeezing is about 41% below the shot-noise level. [S1050-2947(96)03910-8]

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In the course of the search for sources of bright phase-squeezed light we recently suggested the utilization of electromagnetically induced transparency [1]. The proposed setup was based on an atomic Λ scheme driven on one transition as in the experiments of Harris and co-workers [2–5] (see Fig. 1). A medium consisting of such atoms placed inside a cavity was shown to reduce the phase noise of an injected field up to 50% (in the output of the cavity) under optimum conditions.

The use of cavity setups has, however, technical drawbacks (necessity of correct frequency locking, etc.) and leads to a noise reduction only in a relatively narrow band determined by the cavity width. In the present report we show that squeezing is also possible in a traveling-wave configuration. If we shine traveling-wave laser light onto a cell containing the transparent medium, the input light is shown to be phase squeezed up to about 41% at the output under optimum conditions. The bandwidth of noise reduction is thereby determined by the width of the Λ resonance which, in the case of strong driving, is the Rabi frequency of the driving field.

The method we employ to obtain a propagation equation for the mean value and the fluctuations of the probe field closely follows the formulation of Ref. [8] based on the original approach of Ref. [9]. In exactly the same way as described in Ref. [8], we define a slowly varying space- and time-dependent field annihilation operator $a(z,t)$ that obeys the propagation equation

$$\left(\frac{\partial}{\partial t}+c\,\frac{\partial}{\partial z}\right)a(z,t)=igN\sigma_0(z,t), \tag{1}$$

where N is the total number of atoms interacting with the field. Here we have assumed that the carrier frequency ν is resonant with the atomic transition $|a\rangle\rightarrow|b\rangle$. $g=\wp\sqrt{\nu/2\hbar\epsilon_0 AL}$ is the corresponding coupling strength, with $\wp$ representing the atomic dipole moment and AL the quantization volume (which we here identify with the cell volume for simplicity). $\sigma_0(z,t)$ is a space- and time-dependent collective variable describing the atomic dipole. The collective atomic variables are related to the single atom operators

$$\sigma_0^i=|b\rangle\langle a|_i, \tag{2}$$

$$\sigma_1^i=|b\rangle\langle c|_i, \tag{3}$$

$$\sigma_2^i=|c\rangle\langle a|_i, \tag{4}$$

$$\sigma_x^i=|x\rangle\langle x|_i, \tag{5}$$

by means of the following definition. The interaction volume of total length L is divided into $2M+1$ layers, each of thickness $L/(2M+1)$ with the center at $z_l=lL/(2M+1)$, $(l=-M,\ldots,M)$. Then the space- and time-dependent variables are

$$\sigma_\mu(z,t)=\frac{1}{N}\lim_{M\rightarrow\infty}(2M+1)\sum_j \left.\sigma_\mu^j(t)\right|_{z_l\rightarrow z}, \tag{6}$$

where the sum over j is to be taken over all atoms in the layer l around z. These collective atomic operators inherit the Heisenberg equations of motion from their one-atom counterparts [Eqs. (3a)–(3e) of Ref. [1]]

$$\begin{aligned}\dot{\sigma}_a=&-(\gamma+\gamma')\sigma_a-i(\Omega'^*\sigma_2-\text{H.a.})\\&-ig(a^\dagger\sigma_0-\text{H.a.})+F_a,\end{aligned} \tag{7}$$

$$\dot{\sigma}_b=\gamma\sigma_a+\gamma_c\sigma_c+ig(a^\dagger\sigma_0-\text{H.a.})+F_b, \tag{8}$$

$$\dot{\sigma}_0=-\tfrac{1}{2}(\gamma+\gamma')\sigma_0+iga(\sigma_b-\sigma_a)+i\Omega'\sigma_1+F_{\sigma_0}, \tag{9}$$

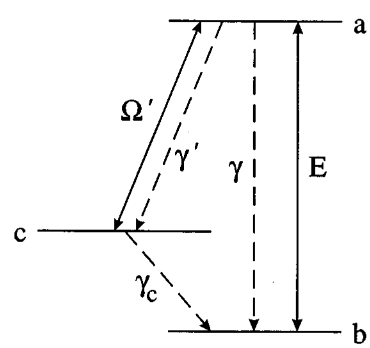

FIG. 1. Atomic level scheme. A quantized probe field E couples to the transition $|a\rangle-|b\rangle$, whose upper state is coupled to level $|c\rangle$ by a classical driving field of Rabi frequency Ω'. γ, γ', and γ_c are longitudinal relaxation rates.

$$\dot{\sigma}_1 = -\tfrac{1}{2}\gamma_c\sigma_1 - iga\sigma_2^\dagger + i\Omega'^*\sigma_0 + F_{\sigma_1}, \tag{10}$$

$$\dot{\sigma}_2 = -\tfrac{1}{2}(\gamma+\gamma'+\gamma_c)\sigma_2 + i\Omega'(\sigma_c-\sigma_a) + iga\sigma_1^\dagger + F_{\sigma_2}. \tag{11}$$

Ω' is the Rabi frequency of the driving-field resonant with the $|a\rangle \rightarrow |c\rangle$ transition. In the present paper, the driving field is treated as a classical or coherent field with undepleted amplitude. This approximation is justified if we assume that the coupling on the $|a\rangle \rightarrow |c\rangle$ transition is much weaker than the coupling on the $|a\rangle \rightarrow |b\rangle$ transition, which implies $\gamma' \ll \gamma$.

Following Ref. [1], we now resort to a c-number formulation of the problem. To this end we introduce a generalized P distribution [10] by choosing a normal operator ordering

$$a^\dagger, \sigma_2^\dagger, \sigma_1^\dagger, \sigma_0^\dagger, \sigma_a, \sigma_b, \sigma_c, \sigma_0, \sigma_1, \sigma_2, a. \tag{12}$$

Using standard scaling arguments [10], we obtain a Fokker-Planck-type equation of motion for P, which is equivalent to a set of c-number stochastic differential equations formally identical to the operator equations. Then the field variable $\alpha(z,t)$ obeys the equation

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right)\alpha(z,t) = -g^2N\Sigma_0(z,t), \tag{13}$$

where we used the variable $\Sigma_0(z,t)$ defined through $ig\alpha(z,t)\Sigma_0(z,t) = \sigma_0(z,t)$ in analogy to Ref. [1]. In order to simplify the problem, we restrict our analysis to field fluctuations slow compared to the time scale of the atomic evolution. In this limit we may eliminate the atomic degrees of freedom adiabatically and find in the limit of strong driving ($|\Omega'|^2 \gg \gamma\gamma_c, \gamma'\gamma_c$)

$$\Sigma_0(z,t) = \overline{\Sigma_0}(z,t) + F_\Sigma(z,t), \tag{14}$$

where $F_\Sigma(z,t)$ is the effective Langevin force corresponding to $F_\Sigma(t)$ in Eq. (14) of Ref. [1] and

$$\overline{\Sigma_0}(z,t) = \frac{\gamma_c}{2}\frac{[|\Omega(z,t)|^2 + |\Omega'|^2]\gamma + |\Omega(z,t)|^2\gamma'}{[\gamma'|\Omega(z,t)|^2 + \gamma|\Omega'|^2][|\Omega(z,t)|^2 + |\Omega'|^2]}. \tag{15}$$

Here we have used $\Omega(z,t) = g\alpha(z,t)$. In the limit $\gamma' \ll \gamma$ and for $|\Omega(z,t)| \sim |\Omega'|$, we have

$$\overline{\Sigma_0}(z,t) = \frac{\gamma_c}{2|\Omega'|^2}. \tag{16}$$

We proceed by assuming stationary conditions and small fluctuations of the field around the semiclassical steady-state value $\alpha_0(z)$, $\alpha(z,t) = \alpha_0(z) + \delta\alpha(z,t)$. With Eqs. (13) and (16) we find for the semiclassical amplitude the equation of motion

$$c\frac{d}{dz}\alpha_0(z) = -\frac{g^2N\gamma_c}{2|\Omega'|^2}\alpha_0(z), \tag{17}$$

which corresponds to linear absorption with rate $\gamma_1 = g^2N\gamma_c/(2|\Omega'|^2)$, i.e.,

$$\alpha_0(z) = \alpha_0(0)e^{-\gamma_1 z/c}. \tag{18}$$

On the other hand, the propagation equation for the Fourier transform of the field fluctuations is

$$c\frac{d}{dz}\delta\tilde{\alpha}(z,\omega) = -(\gamma_1 - i\omega)\delta\tilde{\alpha}(z,\omega) - g^2\sqrt{N}\alpha_0(z)\tilde{F}_\Sigma(z,\omega). \tag{19}$$

We now define the Fourier transform of the phase fluctuation

$$\delta\tilde{\phi}(z,\omega) = \frac{1}{2i}\left(\frac{\delta\tilde{\alpha}(z,\omega)}{\alpha_0(z)} - \frac{\delta\tilde{\alpha}^*(z,-\omega)}{\alpha_0^*(z)}\right), \tag{20}$$

and the corresponding noise operator

$$\tilde{F}_\phi(z,\omega) = g^2\sqrt{N}\frac{\tilde{F}_\Sigma(z,\omega) - \tilde{F}_\Sigma^*(z,-\omega)}{2i}. \tag{21}$$

$\tilde{F}_\phi(z,\omega)$ has the correlation function [1,8]

$$\langle\tilde{F}_\phi(z,\omega)\tilde{F}_\phi(z',\omega')\rangle = \langle F_\phi F_\phi\rangle L\,\delta(z-z')2\pi\delta(\omega+\omega'), \tag{22}$$

where the diffusion coefficient $\langle F_\phi F_\phi\rangle$ is given in the limit $|\Omega(z,t)| \ll \gamma$ by [1]

$$\langle F_\phi F_\phi\rangle = -\frac{g^2\gamma_1}{|\Omega'|^2[1+\kappa^2(z)]^2}. \tag{23}$$

$\kappa(z)$ is the ratio of the probe Rabi frequency to the driving-field Rabi frequency $\kappa(z) = |\Omega(z)/\Omega'|$.

From Eq. (19) we see that $\delta\tilde{\phi}(z,\omega)$ is governed by

$$c\frac{d}{dz}\delta\tilde{\phi}(z,\omega) = i\omega\delta\tilde{\phi}(z,\omega) - \tilde{F}_\phi(z,\omega). \tag{24}$$

Formally integrating this differential equation, we find for the correlation function of the phase fluctuations

$$\begin{aligned}\langle\delta\tilde{\phi}(z,\omega)\delta\tilde{\phi}(z,\omega')\rangle &= \langle\delta\tilde{\phi}(0,\omega)\delta\tilde{\phi}(0,\omega')\rangle \\ &\quad\times e^{i(\omega+\omega')z/c} + \frac{1}{c^2}\int_0^z dz'\int_0^z dz'' \\ &\quad\times\langle\tilde{F}_\phi(z',\omega)\tilde{F}_\phi(z'',\omega')\rangle \\ &\quad\times e^{i\omega(z-z')/c + i\omega'(z-z'')/c} \\ &= -\frac{2\pi\delta(\omega+\omega')Lg^2\gamma_1}{c^2|\Omega'|^2} \\ &\quad\times\int_0^z dz'\frac{1}{[1+\kappa^2(z')]^2}.\end{aligned} \tag{25}$$

In the second equation the term containing $\langle\delta\tilde{\phi}(0,\omega)\delta\tilde{\phi}(0,\omega')\rangle$ has been dropped, because it leads to

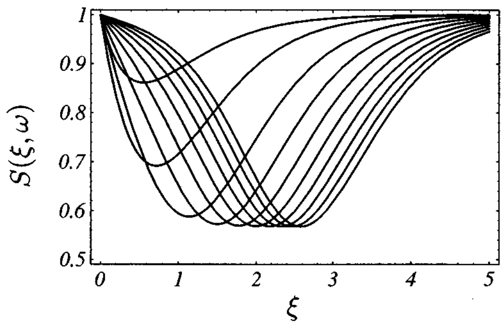

FIG. 2. Squeezing spectrum over normalized propagation depth for different values of $\kappa(0)$. In the order of stronger squeezing $\kappa(0)=1/2,1,2,...,9$.

no contribution for coherent input light. The squeezing spectrum $S(z,\omega)$, normalized such that $S(z,\omega)=1$ corresponds to no squeezing at all and $S(z,\omega)=0$ corresponds to perfect squeezing, is given in terms of our quantities by

$$S(z,\omega)=1+4\alpha_0^*(z)\alpha_0(z)\,\frac{c}{L}\times\int\frac{d\omega'}{2\pi}\,\langle\delta\widetilde{\phi}(z,\omega)\,\delta\widetilde{\phi}(z,\omega')\rangle. \tag{26}$$

Inserting Eq. (25) into Eq. (26) and subsequent integration yields

$$S(z,\omega)=1+2\kappa^2(0)e^{-2\zeta}\left\{\frac{1}{1+\kappa^2(0)e^{-2\zeta}}-\frac{1}{1+\kappa^2(0)}+\ln\left[\frac{[1+\kappa^2(0)]e^{-2\zeta}}{1+\kappa^2(0)e^{-2\zeta}}\right]\right\}, \tag{27}$$

where we introduced the dimensionless propagation length $\zeta=\gamma_1 z/c$. This is the main result of the present paper. We see that the squeezing spectrum depends only on the propagation length and the ratio of the probe input intensity to the driving-field intensity at the entrance plane of the medium. The spectrum does not depend on ω as a consequence of the adiabatic limit and is valid for Fourier frequencies smaller than the reciprocal values of typical atomic time scales. Depending on the atomic parameters, the corresponding spectral width may be of the order of the drive-field Rabi frequency and thus substantially broader than in our previously analyzed cavity-based setup, where it is determined by the cavity-decay rate [1]. In Fig. 2, we plot the behavior of $S(z,\omega)$ for several different values of the ratio $\kappa(0)$ as a function of the normalized propagation length.

At the point ζ_0 of maximum squeezing, the spectrum can be shown to take the form

$$S(\zeta_0,\omega)=1-\frac{2\kappa^2(0)e^{-2\zeta_0}}{[1+\kappa^2(0)e^{-2\zeta_0}]^2}, \tag{28}$$

where ζ_0 is the solution of

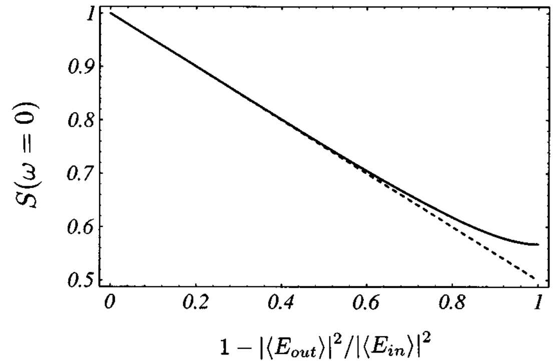

FIG. 3. Maximum achievable squeezing at $\omega=0$ vs loss in coherent amplitude for traveling-wave configuration (full line) and cavity setup (dashed line).

$$\frac{2+\kappa^2(0)e^{-2\zeta}}{(1+\kappa^2(0)e^{-2\zeta})^2}-\frac{1}{1+\kappa^2(0)}+\ln\left[\frac{[1+\kappa^2(0)]e^{-2\zeta}}{1+\kappa^2(0)e^{-2\zeta}}\right]=0. \tag{29}$$

For example, for $\kappa(0)=2$, we find $\zeta_0\simeq1.14$ and $S(\zeta_0,\omega)\simeq0.59$. In this case, the coherent amplitude of the input is attenuated to about $e^{-1.14}\simeq1/3$. It can be seen that $S(\zeta,\omega)$ does not reach considerably below 41% squeezing. Thus we have found a scheme in which for reasonably strong input intensities, an attenuated but possibly still bright transmitted signal displays squeezed phase noise of at best 41% below the shot-noise level.

A seeming drawback of the present scheme is the relatively large attenuation of the input field proportional to $e^{-2\zeta}$. However, as pointed out in a recent work by Gheri, Walls, and Marte [6], the optimum conditions under which squeezing occurs in the cavity setup are such that destructive interference of the circulating and the reflected component of the field lead to a small coherent amplitude at the output [7]. In fact one finds, that under otherwise optimum conditions squeezing and output-amplitude reduction scale in the same way with the cavity and medium loss rate:

$$S(\omega=0)=\frac{|\langle E_{\rm out}\rangle|^2}{|\langle E_{\rm in}\rangle|^2}=\frac{(\gamma_0-\gamma_1)^2}{(\gamma_0+\gamma_1)^2}, \tag{30}$$

where γ_0 represents the cavity-loss rate and γ_1 the effective-loss rate due to the Λ medium. In Fig. 3 we have plotted the maximum achievable squeezing at $\omega=0$ as a function of the loss in coherent amplitude for both configurations, the cavity setup (dashed line) and the traveling-wave configuration (full line). Except for the region of very large losses both curves are identical, showing that both schemes are equally efficient in generating bright squeezed output. However, the bandwidth of noise reduction in the traveling-wave setup can be made much larger as compared to the cavity scheme.

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