

Coherence-induced effects in pulse-pair propagation through absorbing media

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The propagation of a pair of electromagnetic pulses through a coherently prepared atomic medium in the Λ configuration is studied. We find that even in an absorbing medium, it is possible to produce pulses that propagate shape invariant. The results reveal the spatiotemporal dynamics associated with field propagation, and highlight the role of atom preparation, pulse intensities, and medium length, on the final output pulses that one obtains. We also explore the prospects for pulse shaping and control by using dissimilar shaped pulses at the input to the medium and following their evolution. [S1050-2947(96)06109-4]

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I. INTRODUCTION

The pioneering work of McCall and Hahn on self-induced transparency [1] was followed by a number of theoretical and experimental works on pulse propagation through two-level atoms. It is known that an electromagnetic pulse of area between π and 2π , resonant with a two-level transition, will evolve into a characteristic shape that is described by a secant hyperbolic function, which then propagates shape invariant through the medium. In the absence of damping, one can analytically derive this pulse profile. However, if the relaxation of the upper state is accounted for, even this simple problem of a two-level atom is not amenable to an analytic solution, and one resorts to numerical methods.

Lately there has been tremendous interest in quantum coherence effects, such as inversionless lasing and electromagnetically induced transparency, that arise in strongly driven atoms. In this context, Scully and co-workers have discussed a medium in which the atomic population is coherently distributed between two energy levels of a three-level atom [2]. This medium can exhibit an enhanced refractive index with vanishing absorption, even on a transition to which a resonant field is applied. Fleischhauer *et al.* have detailed various techniques for obtaining a coherently prepared medium, and studied the effect of field incoherence and Doppler broadening on the preparation of such a medium [3].

Most studies on pulse propagation through three-level atoms assume the standard initial condition of the population being in the ground state [4(a),4(b),5,6]. These include reports on population transfer to high-lying states, [7] and the recent work of Carboneschi and Arimondo on pulse-pair propagation through a double- Λ system [8]. While all previous works on coherently prepared media have been in the context of continuous-wave fields, application of pulsed fields leads to a number of interesting features. In this paper we study the dynamics associated with pulse-pair propagation through a medium in which the atomic population is coherently distributed between the ground state and the metastable state of a three-level Λ system. Our theoretical

model, outlined in Sec. II, consists of the coupled Schrödinger-Maxwell equations, which describe the temporal and spatial evolution of the atomic states and the two electromagnetic fields that are initially applied to the medium. We will show that the evolution of the pulses in a coherently prepared medium exhibits key differences from the problem where the atoms are prepared in the standard way. Specifically, we find that it is possible to produce twin pulses, which propagate invariant even in an absorbing medium. Our calculations reveal the spatial dynamics associated with pulse propagation through a coherently prepared medium, and illustrate the dependence of these dynamics on the intensity of the input pulses. An important experimental issue that is addressed by our calculations is the role of the medium length in determining some crucial aspects of the pulse-pair behavior. The medium length is not an issue if one invokes the adiabatic approximation [4,5]. However, if one eschews this approximation, the medium length becomes a critical parameter. Lastly, we find that a coherently prepared medium permits one to control the phase and amplitude of the final output pulses. We note here that starting from a different set of initial conditions on the atoms, viz., a V system with inverted population on one transition, Mazets and Matisov recently reported a study on pulse propagation [9].

II. BASIC EQUATIONS

Here we outline the essential features of our theoretical formalism. The atomic scheme we consider is the Λ system shown in Fig. 1, with excited states $|1\rangle$ and $|2\rangle$, and ground state $|3\rangle$, in which $|1\rangle$ decays at a rate γ to states other than $|2\rangle$ and $|3\rangle$. A control (secondary) pulse, with Rabi frequency Ω_p (Ω_s), acts resonantly on the $|1\rangle \leftrightarrow |3\rangle$ ($|1\rangle \leftrightarrow |2\rangle$) transition. The Rabi frequencies are taken to be strong. In the absence of any coherence between $|2\rangle$ and $|3\rangle$ one will get the conventional Λ system, and pulse propagation in such a system has been recently reported [4,5]. From Schrödinger's and Maxwell's equations, the coupled temporal and spatial evolution of atoms and pulses in the slowly varying envelope approximation can be written as

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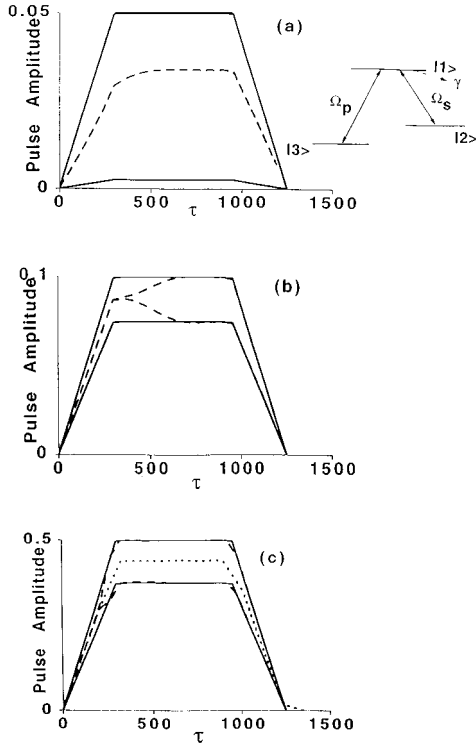


FIG. 1. Control and secondary pulse profiles as functions of τ for $\theta = 3\pi/4$, $\gamma = 0.04$, and (a) $\Omega_p^0 = 0.05$, $\Omega_s^0 = 0.05\Omega_p^0$, $\zeta = 0$ (solid), and 60 (dashed); (b) $\Omega_p^0 = 0.1$, $\Omega_s^0 = 0.75\Omega_p^0$, $\zeta = 0$ (solid), and 60 (dashed); (c) $\Omega_p^0 = 0.5$, $\Omega_s^0 = 0.75\Omega_p^0$, $\zeta = 0$ (solid), 210 (dashed), and 5000 (dotted); also plotted is the adiabatic result (indistinguishable from $\zeta = 5000$), where precise matching occurs within a very short propagation length. All quantities are in dimensionless units, as defined in the text.

$$\frac{\partial ic_1}{\partial t} = -i\gamma c_1 - \frac{1}{2}\Omega_p c_3 - \frac{1}{2}\Omega_s c_2, \quad (1a)$$

$$\frac{\partial c_2}{\partial t} = \frac{1}{2}\Omega_s^*(ic_1), \quad (1b)$$

$$\frac{\partial c_3}{\partial t} = \frac{1}{2}\Omega_p^*(ic_1), \quad (1c)$$

$$\left(\frac{\partial}{\partial z} + \frac{\partial}{\partial ct}\right)\Omega_p = 2i\mu_p c_3^* c_1, \quad (1d)$$

$$\left(\frac{\partial}{\partial z} + \frac{\partial}{\partial ct}\right)\Omega_s = 2i\mu_s c_2^* c_1, \quad (1e)$$

where c_1 , c_2 , and c_3 are the probability amplitudes of the atomic levels, $\mu_p (\alpha\gamma/2)$ and μ_s (assumed equal to μ_p) are the propagation constants (which depend on the dipole moments and atomic number density) for the two pulses, [10] and α is the absorption coefficient of the medium. The two lower states of our medium are initially coherently prepared [2,3] as

$$|\Psi\rangle = \cos\theta|3\rangle + \sin\theta|2\rangle, \quad (2)$$

where θ is the coherence angle. Equation (1) can be numerically solved by specifying the initial conditions at $t=0$ for all z , viz., $c_3 = \cos\theta$, $c_2 = \sin\theta$, and $c_1 = 0$, and the boundary conditions at $z=0$ for all t , which we take as

$$0 < \tau < 300, \quad \Omega_p = \frac{\Omega_p^0 \tau}{300}, \quad (3a)$$

$$300 < \tau < 950, \quad \Omega_p = \Omega_p^0, \quad (3b)$$

$$950 < \tau < 1250, \quad \frac{\Omega_p^0(1250 - \tau)}{300}, \quad (3c)$$

where Ω_p^0 is the peak Rabi frequency of the control pulse. We adopt the usual convention of utilizing pulse-local variables, $\tau = t - z/c$ and $\zeta = z$, which are in units of $1/\alpha c$ and $1/\alpha$, respectively. The secondary pulse has the same form as the control pulse, with Ω_p^0 replaced by Ω_s^0 the peak Rabi frequency of the secondary pulse. Though most of the results presented in this paper utilize two pulses with identical envelopes, we emphasize that the principal results are quite general and can be applied to pulses with different envelopes also. We will demonstrate this explicitly in Sec. IV.

III. ANALYTIC RESULTS

Equation (1) is not, in general, amenable to an analytic solution for obtaining the characteristics of the atoms and fields. Under some limits, and assumptions, it is, however, possible to derive certain analytical results. In this section, we describe two distinct types of results, one that describes the *conservation of coherence* within our system, and another that enables us to predict the ratio of intensities of the steady-state pulses after propagating through the coherent ensemble of atoms, as well as the relative phases of the two pulses.

Law of conservation of coherence: Starting from the basic formalism in Eq. (1), one can derive a general conservation law that relates the atomic state amplitudes c_2 and c_3 to the fields Ω_p and Ω_s , as

$$4\mu \frac{\partial}{\partial \tau}(c_2 c_3) = \frac{\partial}{\partial \zeta}(\Omega_p \Omega_s), \quad (4a)$$

which can be written as

$$4\mu \frac{\partial}{\partial \tau} \int d\zeta (c_2 c_3) = (\Omega_p \Omega_s)_{z=L} - (\Omega_p \Omega_s)_{z=0}. \quad (4b)$$

This expression, true for all t , can be interpreted as the rate of change of *atomic coherence* being equal to the flow of the pulses' coherence across the boundary of the medium. This is analogous to the Poynting theorem, which relates the change of energy to flux across a boundary. Note that this law is similar to the law of conservation of charge in electrodynamics, and to the equation of continuity in hydrodynamics. It is interesting that such a law can be derived even for a dissipative system, such as our three-level atoms interacting with two pulses.

Twin pulses in the long length limit: If we focus on the special case of $\theta = 3\pi/4$, since for $\Omega_p^0 = \Omega_s^0$ one would obtain

a trapped state [11] as the initial condition, identical input pulses would propagate through the atoms unchanged (results not shown). However, it can be demonstrated analytically that even if one starts with unequal pulse amplitudes, the two pulses become identical during propagation, provided there is no limitation on the medium length. Assuming that eventually Ω_p and Ω_s become independent of ζ (i.e., $(\partial/\partial\zeta)\Omega_{p,s}=0$), Eq. (1) suggests that

$$c_1(\tau, \zeta \rightarrow \infty) = 0. \quad (5a)$$

Thus, from Eqs. (1b) and (1c),

$$\frac{\partial}{\partial\tau} c_{2,3}(\tau, \zeta \rightarrow \infty) = 0, \quad (5b)$$

which implies that

$$c_{2,3}(\tau, \zeta \rightarrow \infty) = c_{2,3}(\tau=0, \zeta \rightarrow \infty). \quad (5c)$$

Then, Eq. (1a) can be written as

$$\Omega_p(\tau, \zeta \rightarrow \infty) c_3(\tau, \zeta \rightarrow \infty) + \Omega_s(\tau, \zeta \rightarrow \infty) c_2(\tau, \zeta \rightarrow \infty) = 0. \quad (5d)$$

Clearly, the decay rate γ now becomes irrelevant. Substituting from Eq. (5c) into (5d), the final expression is

$$\begin{aligned} &\Omega_p(\tau, \zeta \rightarrow \infty) c_3(\tau=0, \zeta \rightarrow \infty) + \Omega_s(\tau, \zeta \rightarrow \infty) c_2(\tau=0, \zeta \rightarrow \infty) \\ &= 0. \end{aligned} \quad (5e)$$

For a coherently prepared medium with $\theta=3\pi/4$, the initial condition is $c_3(\tau=0, \zeta \rightarrow \infty) = -c_2(\tau=0, \zeta \rightarrow \infty)$, which then leads to the result

$$\Omega_p(\tau, \zeta \rightarrow \infty) = \Omega_s(\tau, \zeta \rightarrow \infty), \quad (5f)$$

which indicates that the two pulses will be matched. In contrast to the coherently prepared medium, if atoms are prepared in the standard way with the population in the ground state, then the initial condition is $c_2=0$, and one finds from Eq. 5(e) that $\Omega_p(\tau, \zeta \rightarrow \infty)=0$, which means the control pulse would simply decay to zero. Thus, the analytical argument leads us to conclude that even if we do not invoke the approximation that the evolution of $|1\rangle$ adiabatically follows the evolution of states $|2\rangle$ and $|3\rangle$, one can still obtain matched pulses after sufficient propagation distance. Note that even though the above analysis predicts pulse matching in coherently prepared media for all atom and field parameters, it provides no information about the transient spatiotemporal dynamics, nor about the final steady-state pulse shapes. This information is, however, available from the numerical calculations, which are reported next. We emphasize here that analytic solutions, especially for arbitrary values of γ , θ , etc. are very difficult, due to the complexity of the problem. As we have mentioned at the outset, even the seemingly simple problem of solitons in two-level atoms becomes analytically intractable when relaxation effects are incorporated into the formalism.

IV. NUMERICAL RESULTS

This section describes our numerical results. We begin by describing the dynamical behavior of the system for $\theta=3\pi/4$, and input pulses with identical envelopes. Later, we also analyze the role of different initial conditions, i.e., coherence angles other than $3\pi/4$, and input pulses with different envelopes.

The standard case of $\theta=0$ has been studied by Harris, who obtained a set of normal modes of the atom and field system, when matched pulses were applied at the input [5]. Agarwal has derived the quantum theory of this process [12]. Eberly *et al.* provided the spatial details associated with transparency in such a medium [4(a)]. In fact, our analytic result that Ω_p decays to zero is evident in the numerical results of Ref. [4(a)]. We now present numerical results for the case when $\theta=3\pi/4$. In Fig. 1(a), where $\Omega_p^0=0.05$ and Ω_s^0 is 5% of Ω_p^0 it is observed that Ω_s matches the amplitude and shape of Ω_p at $\zeta=60$. In general, if $\Omega_p^0 > \Omega_s^0$ the medium amplifies the secondary pulse, at the expense of the control pulse, until the two match each other identically, whereafter they propagate through the medium unchanged. This is a consequence of the fact that once the Rabi frequencies of the two pulses become identical, the atom and field system evolves into a trapped state, whereafter there is no further change in the dynamics of the pulse propagation. This is in agreement with Ref. [5].

If the control pulse is made stronger, while maintaining all other parameters constant, one can illustrate the role of the medium length. In fact, the ability of the medium to produce matched pulses is crucially dependent on the medium length, as shown in Fig. 1(b), where we change Ω_p^0 to 0.1 and Ω_s^0 to 75% of Ω_p^0 . The inability of the pulses to completely match at $\zeta=60$ is in contrast to the matching that does take place for smaller values of Ω_p^0 . In Fig. 1(c) are results for an even higher Ω_p^0 ($=0.5$), and there is almost no change in the pulses as they propagate through the medium, up to distances as large as $\zeta=210$. Thus, if one has a medium that corresponds to a length, say, of $\zeta=60$, then for large values of Ω_p^0 the medium is transparent to the fields and the atoms are unable to match the two pulses. One can now raise the following question: for a given set of atom and field parameters, should one expect a critical value of the control pulse Rabi frequency, such that for Ω_p^0 less than this value, one will get matched pulses, and for Ω_p^0 greater than this value one would not? The answer is in Fig. 1(c), where pulse profiles are shown at $\zeta=5000$, where they match. Clearly, if there is no limit to the length of the available medium, one will eventually produce twin pulses after sufficient propagation. This is supported by the analysis in Eq. (5). However, for a fixed medium length (as is typical in experiments), there will be a critical value of Ω_p^0 such that only for strengths lower than this critical value does one obtain matched pulses. This aspect of pulse matching cannot be seen if the adiabatic approximation is invoked [4(a),4(c),5]. For comparison, we also show in Fig. 1(c) the result of the adiabatic approximation (obtained by setting $\dot{c}_1=0$), which, as expected, restores pulse matching within a very short propagation length. Note that these profiles are indistinguishable from the profiles at $\zeta=5000$. In a recent

paper, using the standard initial condition, Harris and Luo also examined the impact of relaxing the adiabatic approximation when matched pulses are applied to a medium of three-level atoms, and discussed the requirements on the pulse energy for producing transparency [6]. They found that transparency is achieved when the number of photons in the control laser becomes equal to the number of atoms in the path of the pulses (if oscillator strengths of the two transitions are the same). Their results also indicate that as γ is reduced, the time required to induced transparency becomes longer.

Even though one obtains matched pulses for all Ω_p^0 , the spatial evolution exhibits two distinct types of behavior, depending on the strength of the control pulse. In Fig. 2(a) are snapshots of the pulses for various propagation distances within the medium, when $\Omega_p^0 < \gamma$. The control pulse loses energy uniformly across its temporal profile, and the secondary pulse gains energy uniformly across its profile, until the two pulses are identical, whereafter they propagate invariant. There is no significant distortion of the pulse shapes during the evolution. In Fig. 2(b) are the snapshots when $\Omega_p^0 > \gamma$, and the profiles are quite different. Note that there is a strong distortion of both pulses as they propagate into the medium. Surprisingly, when the two pulses reach steady state, they have almost the same shape as at the input [input pulses are identical to those in Fig. 1(a)]. Also evident from this figure is that for strong control fields, the matching takes place across the temporal profile of the pulses, such that the leading edges match first, and then the trailing parts are progressively matched as the pulses propagate further.

The differences in response, between strong control pulse ($\Omega_p^0 > \gamma$) and weak control pulse ($\Omega_p^0 < \gamma$), also become apparent in the propagation distance at which stable pulses are formed. For example, in Fig. 3 we show how the pulse-pair propagation in a coherently prepared medium is modified if we reduce the decay rate of the excited state. For $\gamma=0.004$, $\Omega_p^0=0.05$, and $\Omega_s^0=0.05\Omega_p^0$ twin pulses are now obtained at $\zeta=390$. Due to the complexity of the equations, and the resulting behavior of the atoms and fields, it is not possible to provide analytical estimates of how the propagation distance, at which pulse matching occurs, varies with γ . Our numerical experiments, however, can answer this question. In Table I we indicate the values of ζ at which twin pulses are first attained, for different Ω_p^0 and γ . We have obtained similar data for a number of different parameter values, and find two distinct types of behavior. For strong control pulse and a constant Ω_p/γ , the distance at which stable pulses are formed varies significantly with γ , i.e., an order of magnitude change in γ leads to approximately an order of magnitude change in ζ . Thus, even if the ratio Ω_p^0/γ is the same, the pulses may have to propagate different distances before attaining steady state. On the other hand, the table also shows that for a weak control pulse, and a given ratio of Ω_p to γ , the value of ζ at which twin pulses are formed is nearly independent of γ , i.e., an order of magnitude change in γ produces a very small change in ζ . Finally, note that for a given ratio of Ω_p^0/Ω_s^0 (all entries in Table I), the ratio of steady state Ω_p^0 to the input Ω_p^0 is, to a good approximation, independent of γ . This, in fact, is also the reason why the adiabatic result and the profiles at $\zeta=5000$

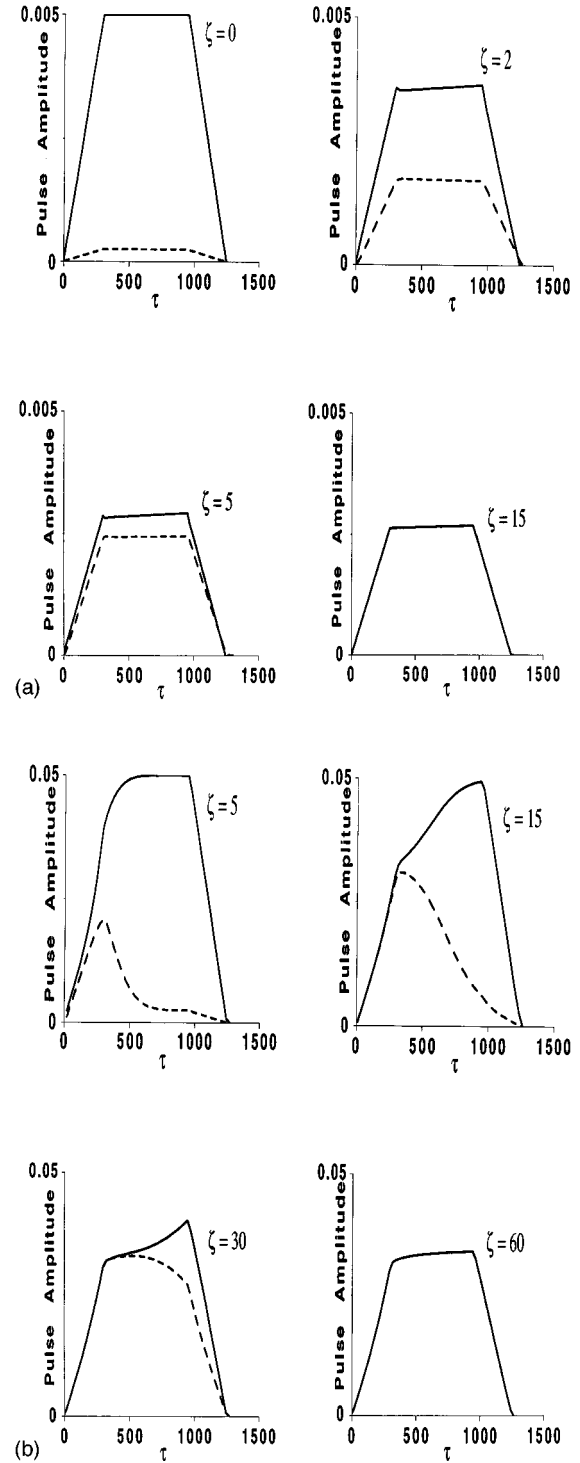


FIG. 2. Snapshots of control (solid) and secondary (dashed) pulse profiles as functions of τ at various propagation distances (ζ) within the medium for $\theta=3\pi/4$, $\gamma=0.04$, and (a) $\Omega_p^0=0.005$, $\Omega_s^0=0.05\Omega_p^0$; (b) $\Omega_p^0=0.05$, $\Omega_s^0=0.05\Omega_p^0$. All quantities are in dimensionless units, as defined in the text.

are indistinguishable in Fig. 1(c).

If instead of a coherence angle of $3\pi/4$, we choose $\pi/4$, the secondary pulse is out of phase (by π radians) with the control pulse (result not shown). From Eq. (2) we note that $\theta=\pi/4$ implies that the atomic system is prepared as $|\Psi\rangle=(|3\rangle+|2\rangle)/\sqrt{2}$. Thus, only if Ω_p and Ω_s have opposite

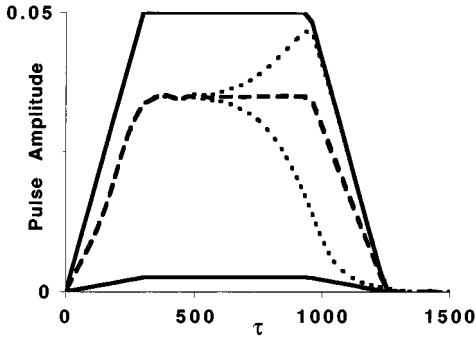


FIG. 3. Control and secondary pulse profiles as functions of τ for $\theta = 3\pi/4$, $\gamma = 0.004$, $\Omega_p^0 = 0.05$, $\Omega_s^0 = 0.05\Omega_p^0$, $\zeta = 0$ (solid), 210 (dotted), and 390 (dashed). All quantities are in dimensionless units, as defined in the text.

signs is it possible to form a trapped state, which would then lead to the generation of pulses that are out of phase. It must be borne in mind that we are considering the cases when $\theta = \pi/4$ or $3\pi/4$, for which, not only are the pulses matched in envelope but also in amplitude.

Dissimilar pulse envelopes, and other coherence angles:

So far we have concentrated on the control pulse and the secondary pulse having identical envelopes, and $\theta = 3\pi/4$. The question then arises: Are the results presented specific to the choices of the initial conditions we made, or can they be generalized? To answer this question, we next explore several different sets of initial conditions. Both from a fundamental viewpoint as well as an applied perspective, it is interesting to examine the consequences of utilizing input pulses with different shapes. In Fig. 4(a), we show what happens when we use a soliton for Ω_p , and maintain the same shape for Ω_s as previously. Thus, the control pulse is now defined by

$$\Omega_p = \frac{2}{\sigma} \operatorname{sech}\left(\frac{\tau - \tau_0}{\sigma}\right). \quad (6)$$

In Fig. 4(a), $\sigma = 40$ (this choice makes $\Omega_p^0 = 0.05$), and τ_0 , the peak of the soliton, is at 600. Note that in addition to the

TABLE I. How the propagation distance ζ , at which pulse matching takes place, varies with γ , for different values of Ω_p^0 . Other parameters are $\Omega_s^0 = 0.05\Omega_p^0$ and $\theta = 3\pi/4$. The last column indicates the steady-state amplitude of the control pulse. One can note that, e.g., when $\Omega_p^0 = 0.05$, the steady-state value is nearly the same for γ , varying over three orders of magnitude. All quantities are in dimensionless units, as defined in the text.

Ω_p^0	γ	ζ	Ω_p^0 (steady state)
0.005	0.4	10	0.003
0.005	0.04	15	0.003
0.05	0.4	16	0.03
0.05	0.04	60	0.03
0.05	0.004	390	0.035
0.05	0.0004	$\sim 10,000$	0.036
0.5	0.04	5,000	0.36
0.1	0.04	195	0.07

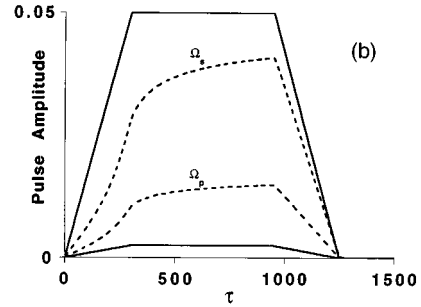
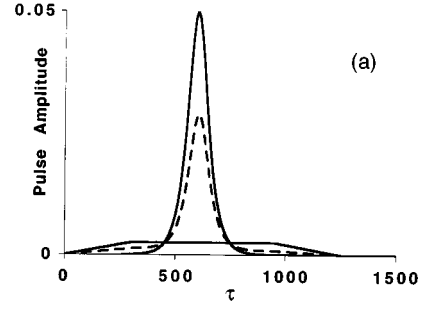


FIG. 4. (a) Control and secondary pulse profiles as functions of τ , when the input pulses have different envelopes. Ω_p is a soliton (definition in text), and Ω_s is identical to those in other figures. $\Omega_p^0 = 0.05$, $\Omega_s^0 = 0.05\Omega_p^0$, $\gamma = 0.04$, and $\zeta = 0$ (solid) and 60 (dashed); (b) control and secondary pulse profiles as functions of τ for $\gamma = 0.04$, $\Omega_p^0 = 0.05$, $\Omega_s^0 = 0.05\Omega_p^0$, $\theta = 8\pi/9$, and $\zeta = 0$ (solid) and 60 (dashed). All quantities are in dimensionless units, as defined in the text.

envelopes, now the widths of the two pulses are also different at the input to the atomic medium. At a propagation distance of approximately $\zeta = 60$, it is clear that one obtains twin pulses of identical shapes and amplitudes. We have experimented with a number of different pulse envelopes, and found that the production of twin pulses by a coherently prepared medium is a very general and robust phenomenon. We note that no matter what the shape, width, and amplitude of the input pulses, they attain nearly the same shape as the stronger control pulse. For example, in Fig. 4(a), if the steady state twin pulses are fit to a sech profile, there is a mismatch only in the tails (the extent of this mismatch decreases as the ratio of Ω_s^0/Ω_p^0 decreases). These results suggest a powerful technique for pulse control and shaping. To produce a pulse of desired shape at the $|1\rangle \leftrightarrow |2\rangle$ transition frequency, one needs to apply a pulse of that shape at the $|1\rangle \leftrightarrow |3\rangle$ transition frequency. This scheme can be especially useful for producing tailored pulses at frequencies where conventional pulsed lasers are not easily available.

Next we examine whether it is the specific choice of $\theta = 3\pi/4$ that produces stable, shape-invariant pulses, or whether other coherence angles can provide similar results. In Fig. 4(b) are the pulse profiles for $\theta = 8\pi/9$. Once again we find that at certain distances within the medium, one obtains stable, shape-invariant pulses, which then propagate through the medium. While a coherence angle of $3\pi/4$ can provide amplitude and envelope matched pulses, we now find that the final steady-state intensity of the secondary pulse can be either larger or smaller than the steady-state

intensity of the control pulse. It is, in fact, possible to predict from the analysis of Eq. (5) whether, for a given coherence angle, the steady-state intensity of the control pulse would be smaller or larger than that of the secondary pulse, and also what their relative phases would be. One can rewrite Eq. (5e) as

$$\Omega_p(\tau, \zeta \rightarrow \infty) \cos \theta + \Omega_s(\tau, \zeta \rightarrow \infty) \sin \theta = 0. \quad (7)$$

This implies that the ratio Ω_p/Ω_s will be equal to $-\sin \theta/\cos \theta$ [13] and explains why the final pulses are out of phase when $\theta = \pi/4$, and also how the ratio of the final pulse intensities depends on the coherence angle. It is therefore apparent that depending on the value of the coherence angle, one can arrange for the final secondary pulse to be either more or less intense than the final control pulse.

V. CONCLUSIONS

In summary, we have reported on a study of pulse-pair propagation through a coherently prepared system. It is shown that one can produce twin pulses of arbitrary shapes that propagate unaltered through an absorbing medium. For certain coherence angles one can generate twin pulses, which can be important in studies related to the production of non-classical states of light, and investigating ultrafast chemical and biological processes via transient coherent phenomena, and we hope to examine the consequences of our results on correlated, twin pulses [14]. A conservation law for coherence within the system has been derived. An analytic argument is presented, which indicates that two pulses in a coherently prepared medium will become identical to each other in the steady state, whereas if all atoms are initially in the ground state, the control pulse would simply decay. Thus, medium preparation is the key to having shape-invariant pulses propagate through this ensemble of atoms. The analytic and numerical work indicates that one can tailor the amplitude and phase of a pulse through suitable medium preparation, and suggests a method of pulse control that is entirely different from conventional techniques that rely on the characteristics of the control field [15]. Furthermore, it is

shown that a coherently prepared medium acts as a frequency converter and amplifier, where a weak field at one frequency is amplified by utilizing a control field at a different frequency.

The numerical results provide insight into the spatial evolution of the pulses, which is a sensitive function of the control pulse intensity. For a weak control pulse, there is no distortion of the pulse shapes as they propagate through the medium, and eventually become identical. However, when the control pulse is strong, there is a significant reshaping of the pulse profiles, and it is interesting that in spite of this distortion during the transient stages, the final steady-state pulses have nearly the same shape as the input pulses. This implies that the steady-state output pulses are independent of γ , which can provide significant flexibility in experiments. Also, for high intensities of the control pulse, the exact results from numerically solving Eq. (1) indicate a much longer propagation distance to attain steady state than do the results of the adiabatic approximation. This distinction becomes significant for designing experiments with realistic propagation lengths.

Finally, we have explored the consequences of using dissimilar shaped pulses at the input to the medium, and find that for $\theta = 3\pi/4$, one always obtains twin pulses. For a strong control pulse, and a weaker secondary pulse, with different envelopes, both fields evolve into the shape of the control pulse. As mentioned previously, this provides a technique for pulse control and shaping at frequencies not conventionally possible. The results also indicate that for coherence angles other than $3\pi/4$, one still obtains stable, shape-invariant pulses, though of unequal amplitudes. In the context of pulse control, it is clear from our work that through a suitable choice of a single parameter, the coherence angle, one can control the final amplitude and phase of the steady-state pulses.

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