Maxwell-Bloch equations for spatially inhomogeneous semiconductor lasers. I. Theoretical formulation

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Space-dependent Maxwell-Bloch equations are derived for the description of spatiotemporal dynamics of spatially inhomogeneous semiconductor lasers. The dynamics of the charge carriers is described in a density-matrix approach using a Wigner function representation. On this basis, the coupled set of equations of motion for the active medium and the space-dependent light field is derived. Based on typical length and time scales, approximations are performed to obtain a numerically tractable problem. The many-body interactions give rise to space-dependent energy renormalizations, Coulomb enhancement, and scattering processes. The latter ones are considered in the form of momentum- and density-dependent microscopic relaxation rates due to carrier-carrier and carrier-phonon interaction for the carrier distribution functions and the polarization. For the spatial transport or the carriers an ambipolar diffusion model is derived. [S1050-2947(96)01409-6]

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I. INTRODUCTION

In early theoretical studies of the semiconductor laser the interplay between the light field and the laser medium has mainly been analyzed in terms of rate equations for the density of photons and charge carriers [1–3]. Disregarding the spatial dependence, these models generally describe the dynamic behavior of a single-stripe semiconductor laser rather well. Various methods from nonlinear dynamics were applied for the description of the dynamics of the semiconductor laser within the rate-equations approximation, e.g., to characterize the relaxation oscillations [2,3] and the behavior upon variation of the applied injection current. With delayed optical feedback and in linear and *T*-shaped external resonators dynamic instabilities and chaotic behavior have experimentally been observed [4].

In spite of their popularity, general limitations in the theoretical description of semiconductor lasers in terms of phenomenological models (involving, e.g., the linewidth enhancement factor) have been pointed out previously. Generally, the interaction of the optical light field and the active semiconductor medium is strongly dependent on spectral properties of the semiconductor and on the local electron and hole occupation in the conduction and valence band. The active medium, represented by its microscopic polarization and carrier distributions, in turn, acts as the source for the optical field generated by the annihilation of an electron-hole pair. Typically, however, the polarization is adiabatically eliminated and, disregarding the wave-number dependence of the gain function, effectively the band structure of a direct-gap semiconductor is reduced to a two-level system without spectral broadening. The linear dependence of the gain on the density N of charge carriers assumed in many phenomenological approximations does not include nonlinear effects such as the saturation of the gain medium at very high pumping rates. Recently, attempts have been made to include the dynamics of the polarization variable on various levels. Assuming spatial homogeneity, i.e., disregarding any transport processes, Maxwell-Bloch formulations have been proposed on the basis of phenomenological two-level (instead of more realistic two-band) approximations for the active laser medium [5,6]. Using homogeneous semiconductor Bloch equations the ultrashort time nonequilibrium carrier and intensity dynamics of microcavity lasers have recently been investigated [7]. Describing the dynamics of broad-area lasers on the basis of a spatially dependent Maxwell-Bloch, first results on the spatiotemporal dynamics and propagation of optical filaments have been presented [8]. Those results have given the strong indication of the necessity to include in the theoretical description the spatial degrees of freedom together with the band structure of the semiconductor medium.

In this paper we will derive a hierarchy of space-dependent Maxwell-Bloch equations for spatially inhomogeneous semiconductor lasers. The basic equations are valid for arbitrarily strong inhomogeneities. Based on typical length scales in a semiconductor laser and on the separation of the time scales between the dynamics in momentum and in real space, transport processes are hierarchically included. In a successive paper, from now on termed II, we show results of extensive numerical simulations on the spatiotemporal dynamics of broad-area semiconductor lasers. While always considering the spatial dependence of the microscopic variables and of the optical field in the calculations, approximations in the description of the active semiconductor laser medium are made regarding the level of complexity in the explicit consideration of many-body effects in the model. In particular, the formation and propagation of optical filaments

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in broad-area semiconductor lasers is studied in which, as we will show in II, the microscopic properties strongly influence the macroscopic spatiotemporal dynamics of the whole device.

II. THE ACTIVE SEMICONDUCTOR

The active medium in a semiconductor laser is the source of the optical radiation which is produced and amplified in the laser. In the spirit of the semiclassic laser theory only the classical character of the optical field is considered while one accounts for the quantum-mechanical properties of the semiconductor medium. In the derivation outlined in the following, the concept of the density-matrix theory [9–13] is followed [14]. A characteristic feature of a semiconductor as the active medium of a laser is the fact that, besides the interaction with the light field, other types of interactions play an important role. These are, in particular, the Coulomb interaction among the carriers giving rise to many-body renormalizations, screening, and a thermalization of the nonequilibrium carrier distribution, as well as the interaction with phonons leading to an energy exchange between the carriers and the crystal lattice.

The single-particle Hamiltonian and the mean-field part of the many-body Hamiltonian will lead us to the semiconductor Bloch equations, which are generalized to include arbitrary spatial inhomogeneities. They describe the coherent interaction between the carriers and the laser light field as well as transport due to spatial gradients and electrostatic forces. The remaining part of the Hamiltonian describing carrier-carrier and carrier-phonon correlations gives rise to a screening of the bare Coulomb interaction and to scattering processes leading to a dephasing of the polarization and a relaxation of the carrier distribution functions towards a local quasiequilibrium. Since we are here mainly interested in the laser dynamics, we will not discuss in detail the derivation of the latter contributions. Instead, we will treat them in a simplified way in terms of a static screening of the Coulomb potential and a relaxation-time approximation for the scattering dynamics. The scattering rates, however, are obtained from the microscopic transition rates and, therefore, they become functions of wave vector, space and time.

A. Microscopic variables and Hamiltonian

The semiconductor is described in terms of an isotropic two-band model where, for the case of the holes, a suitably averaged effective mass is taken. A generalization to more bands is straightforward. The basic variables describing the carrier dynamics are given by the single-particle density matrices. Due to the formal analogy with semiclassical transport theory, it is convenient to use a Wigner representation for a spatially inhomogeneous system. In the following we will generalize the concept of the Wigner function [15] to the case of a two-band model. Starting from the off-diagonal density-matrices $f^{e}_{\mathbf{k},\mathbf{k}'}=\langle c^{\dagger}_{\mathbf{k}}c_{\mathbf{k}'}\rangle$, $f^{h}_{\mathbf{k},\mathbf{k}'}=\langle d^{\dagger}_{\mathbf{k}}d_{\mathbf{k}'}\rangle$, and $p_{\mathbf{k},\mathbf{k}'}=\langle d_{-\mathbf{k}}c_{\mathbf{k}'}\rangle$ space-dependent distribution functions $f^{e,h}$ and interband polarization p can be introduced by performing a Fourier transformation with respect to the relative momentum according to

$$f^{e,h}(\mathbf{k},\mathbf{r})=\sum_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}} f^{e,h}_{\mathbf{k}-\frac{1}{2}\mathbf{q},\mathbf{k}+\frac{1}{2}\mathbf{q}}, \tag{1a}$$

$$p(\mathbf{k},\mathbf{r})=\sum_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}} p_{\mathbf{k}-\frac{1}{2}\mathbf{q},\mathbf{k}+\frac{1}{2}\mathbf{q}}. \tag{1b}$$

Here, $c^{\dagger}_{\mathbf{k}}$ and $d^{\dagger}_{\mathbf{k}}$ ($c_{\mathbf{k}}$ and $d_{\mathbf{k}}$) denote creation (annihilation) operators for electrons and holes with wave-vector $\mathbf{k}$, respectively.

The free carriers in the semiconductor are described by the Hamiltonian

$$H_0=\sum_{\mathbf{k}} \epsilon^{e}_{\mathbf{k}} c^{\dagger}_{\mathbf{k}}c_{\mathbf{k}}+\sum_{\mathbf{k}} \epsilon^{h}_{\mathbf{k}} d^{\dagger}_{\mathbf{k}}d_{\mathbf{k}}, \tag{2}$$

where $\epsilon^{e}_{\mathbf{k}}=\mathcal{E}_g+\hbar^2k^2/2m_e$ and $\epsilon^{h}_{\mathbf{k}}=\hbar^2k^2/2m_h$ are the dispersion relations of electrons and holes, respectively, m_e and m_h being their effective masses, and $\mathcal{E}_g$ is the band gap.

The light field couples electron and hole states. In the semiclassic laser theory the optical field is described by a classical electric field $\mathbf{E}(\mathbf{r},t)$ with positive ($\mathbf{E}^{(+)}$) and negative ($\mathbf{E}^{(-)}=\mathbf{E}^{(+)*}$) frequency components. Expanding the light field in a Fourier series according to

$$\mathbf{E}^{(+)}(\mathbf{r},t)=\sum_{\mathbf{q}} \mathbf{E}^{0}_{\mathbf{q}}(t)e^{i(\mathbf{q}\mathbf{r}-\omega t)}=\sum_{\mathbf{q}} \mathbf{E}^{(+)}_{\mathbf{q}}(t)e^{i\mathbf{q}\mathbf{r}}, \tag{3}$$

where ω denotes the central frequency, the carrier-light interaction Hamiltonian is given by

$$H^{cL}=-\sum_{\mathbf{k},\mathbf{q}} [\mathbf{d}_{cv}(\mathbf{k})\cdot\mathbf{E}^{(+)}_{\mathbf{q}}(t)c^{\dagger}_{\mathbf{k}+\frac{1}{2}\mathbf{q}}d^{\dagger}_{-\mathbf{k}+\frac{1}{2}\mathbf{q}} + \mathbf{d}^{*}_{cv}(\mathbf{k})\cdot\mathbf{E}^{(-)}_{\mathbf{q}}(t)d_{-\mathbf{k}+\frac{1}{2}\mathbf{q}}c_{\mathbf{k}+\frac{1}{2}\mathbf{q}}]. \tag{4}$$

Here, the coupling is treated in rotating wave approximation and $\mathbf{d}_{cv}(\mathbf{k})$ denotes the dipole matrix element between valence band and conduction-band states.

Inserting the Wigner representation of the interband polarization, the expectation value of the interaction Hamiltonian can be written as

$$\langle H^{cL}\rangle=-\int d^3r[\mathbf{E}^{(+)}(\mathbf{r},t)\cdot\mathbf{P}^{(-)}(\mathbf{r},t) + \mathbf{E}^{(-)}(\mathbf{r},t)\cdot\mathbf{P}^{(+)}(\mathbf{r},t)] \tag{5}$$

which allows us to identify the space-dependent macroscopic polarizations

$$\mathbf{P}^{(+)}(\mathbf{r},t)=\frac{1}{\mathcal{V}}\sum_{\mathbf{k}} \mathbf{d}^{*}_{cv}(\mathbf{k})p(\mathbf{k},\mathbf{r},t) \tag{6}$$

and $\mathbf{P}^{(-)}=\mathbf{P}^{(+)*}$ entering Maxwell's equations. Here $\mathcal{V}$ denotes a normalization volume of the crystal.

Electrons and holes as charged particles interact via the Coulomb potential $V_{\mathbf{q}}$. In a two-band model the interaction Hamiltonian is given by

$$H^{cc}=\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} V_{\mathbf{q}}\left[\frac{1}{2}c^{\dagger}_{\mathbf{k}}c^{\dagger}_{\mathbf{k}'+\mathbf{q}}c_{\mathbf{k}'}c_{\mathbf{k}-\mathbf{q}}+\frac{1}{2}d^{\dagger}_{\mathbf{k}}d^{\dagger}_{\mathbf{k}'+\mathbf{q}}d_{\mathbf{k}'}d_{\mathbf{k}-\mathbf{q}}-c^{\dagger}_{\mathbf{k}}d^{\dagger}_{\mathbf{k}'+\mathbf{q}}c_{\mathbf{k}'}d_{\mathbf{k}-\mathbf{q}}\right] \tag{7}$$

with

$$V_{\mathbf{q}}=\frac{4\pi e^2}{\mathcal{V}\varepsilon_0 q^2}, \tag{8}$$

ε_0 being the static background dielectric constant. The three parts refer to electron-electron, hole-hole, and electron-hole interaction, respectively. We have neglected terms which lead to transitions between valence and conduction bands, i.e., impact ionization and Auger recombination [16]. Furthermore, the interband exchange energy has been neglected which is justified if, as in the present case, the exciton binding energy is much smaller than the band gap [17].

The carrier-carrier Hamiltonian can be separated into a mean-field (Hartree-Fock) part H^{cc}_{HF} and a remaining part H^{cc}_{corr} depending on two-particle correlations. For an inhomogeneous system the Hartree-Fock contribution is given by

$$\begin{aligned}H^{cc}_{HF}=\sum_{\mathbf{k},\mathbf{k}',\mathbf{q}}\{&[V_{\mathbf{k}-\mathbf{k}'}(f^{e}_{\mathbf{q},\mathbf{q}-\mathbf{k}+\mathbf{k}'}-f^{h}_{\mathbf{q},\mathbf{q}-\mathbf{k}+\mathbf{k}'})\\&-V_{\mathbf{q}}f^{e}_{\mathbf{k}+\mathbf{q},\mathbf{k}'+\mathbf{q}}]c^{\dagger}_{\mathbf{k}'}c_{\mathbf{k}}+[V_{\mathbf{k}-\mathbf{k}'}(f^{h}_{\mathbf{q},\mathbf{q}-\mathbf{k}+\mathbf{k}'}-f^{e}_{\mathbf{q},\mathbf{q}-\mathbf{k}+\mathbf{k}'})\\&-V_{\mathbf{q}}f^{h}_{\mathbf{k}+\mathbf{q},\mathbf{k}'+\mathbf{q}}]d^{\dagger}_{\mathbf{k}'}d_{\mathbf{k}}-V_{\mathbf{q}}p_{\mathbf{k}+\mathbf{q},\mathbf{k}'+\mathbf{q}}c^{\dagger}_{\mathbf{k}'}d^{\dagger}_{-\mathbf{k}}\\&-V_{\mathbf{q}}p^{*}_{\mathbf{k}+\mathbf{q},\mathbf{k}'+\mathbf{q}}d_{-\mathbf{k}}c_{\mathbf{k}'}\}.\end{aligned} \tag{9}$$

The carriers also interact with static and dynamic perturbations of the ideal lattice periodicity. Of particular importance in a polar semiconductor is the interaction with optical phonons via the Fröhlich interaction. The corresponding Hamiltonian is given by

$$H^{cp}=\sum_{\mathbf{q},\mathbf{k}}\gamma_{\mathbf{q}}[c^{\dagger}_{\mathbf{k}+\mathbf{q}}b_{\mathbf{q}}c_{\mathbf{k}}-c^{\dagger}_{\mathbf{k}}b^{\dagger}_{\mathbf{q}}c_{\mathbf{k}+\mathbf{q}}-d^{\dagger}_{\mathbf{k}+\mathbf{q}}b_{\mathbf{q}}d_{\mathbf{k}}+d^{\dagger}_{\mathbf{k}}b^{\dagger}_{\mathbf{q}}d_{\mathbf{k}+\mathbf{q}}], \tag{10}$$

where $b^{\dagger}_{\mathbf{q}}$ ($b_{\mathbf{q}}$) denote creation (annihilation) operators of a phonon with wave-vector $\mathbf{q}$ and the Fröhlich coupling matrix element is given by

$$\gamma_{\mathbf{q}}=-i\left[\frac{2\pi e^2\hbar\omega_{op}}{\mathcal{V}}\left(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_0}\right)\right]^{1/2}\frac{1}{q} \tag{11}$$

with the energy of the optical phonon $\hbar\omega_{op}$ and the optical dielectric constant ε_∞. Carrier-phonon interaction has a mean-field contribution only in the presence of coherent phonons which require strong spatial inhomogeneities [18–20]. In a semiconductor laser which is operated at room temperature the phonon system typically can be assumed to remain close to thermal equilibrium, thus there are no carrier-phonon mean-field contributions.

Then, the effective single-particle Hamiltonian consists of Eqs. (2), (4), and (9) and can be written as

$$\begin{aligned}H_{\mathrm{eff}}&=H_0+H^{cL}+H^{cc}_{HF}\\&=\sum_{\mathbf{k},\mathbf{k}'}[\mathcal{E}^{e}_{\mathbf{k}\mathbf{k}'}c^{\dagger}_{\mathbf{k}}c_{\mathbf{k}'}+\mathcal{E}^{h}_{\mathbf{k}\mathbf{k}'}d^{\dagger}_{\mathbf{k}}d_{\mathbf{k}'}+\mathcal{U}_{\mathbf{k}\mathbf{k}'}c^{\dagger}_{\mathbf{k}}d^{\dagger}_{-\mathbf{k}'}+\mathcal{U}^{*}_{\mathbf{k}\mathbf{k}'}d_{-\mathbf{k}'}c_{\mathbf{k}}]\end{aligned} \tag{12}$$

with the matrices $\mathcal{E}^{e,h}_{\mathbf{k}\mathbf{k}'}$ of the renormalized energies,

$$\begin{aligned}\mathcal{E}^{e,h}_{\mathbf{k}\mathbf{k}'}=\epsilon^{e,h}_{\mathbf{k}}\delta_{\mathbf{k}\mathbf{k}'}-\sum_{\mathbf{q}}[&V_{\mathbf{q}}f^{e,h}_{\mathbf{k}'+\mathbf{q},\mathbf{k}+\mathbf{q}}\mp V_{\mathbf{k}-\mathbf{k}'}(f^{e}_{\mathbf{q},\mathbf{q}-\mathbf{k}'+\mathbf{k}}\\&-f^{h}_{\mathbf{q},\mathbf{q}-\mathbf{k}'+\mathbf{k}})],\end{aligned} \tag{13}$$

where the upper (lower) sign refers to electrons (holes), and the matrix $\mathcal{U}_{\mathbf{k}\mathbf{k}'}$ of the effective field,

$$\mathcal{U}_{\mathbf{k}\mathbf{k}'}=-\mathbf{d}_{cv}\left(\frac{1}{2}(\mathbf{k}+\mathbf{k}')\right)\cdot\mathbf{E}^{(+)}_{\mathbf{k}-\mathbf{k}'}(t)-\sum_{\mathbf{q}}V_{\mathbf{q}}p_{\mathbf{k}'+\mathbf{q},\mathbf{k}+\mathbf{q}}. \tag{14}$$

B. Equations of motion

By using Heisenberg's equations of motion, the equations of motion for the single-particle density matrices in the Wigner representation can be derived. The effective single-particle Hamiltonian leads to a closed set of equations of motion for distribution functions and interband polarization. The result, e.g., for the electron distribution function is given by

$$\begin{aligned}i\hbar\frac{\partial}{\partial t}f^{e}(\mathbf{k},\mathbf{r})=\frac{1}{\mathcal{V}}\int d^3r'\sum_{\mathbf{k}',\mathbf{q}'}e^{i\mathbf{q}'\mathbf{r}'}\{&[e^{i\mathbf{k}'\mathbf{r}}\mathcal{E}^{e}_{\mathbf{k}+(1/2)\mathbf{q}'+(1/2)\mathbf{k}',\mathbf{k}+(1/2)\mathbf{q}'-(1/2)\mathbf{k}'}-e^{-i\mathbf{k}'\mathbf{r}}\mathcal{E}^{e}_{\mathbf{k}-(1/2)\mathbf{q}'-(1/2)\mathbf{k}',\mathbf{k}-(1/2)\mathbf{q}'+(1/2)\mathbf{k}'}]\\&\times f^{e}(\mathbf{k}-(1/2)\mathbf{k}',\mathbf{r}-\mathbf{r}')+e^{i\mathbf{k}'\mathbf{r}}\mathcal{U}_{\mathbf{k}+(1/2)\mathbf{q}'+(1/2)\mathbf{k}',\mathbf{k}+(1/2)\mathbf{q}'-(1/2)\mathbf{k}'}p^{*}(\mathbf{k}-\tfrac{1}{2}\mathbf{k}',\mathbf{r}-\mathbf{r}')\\&-e^{-i\mathbf{k}'\mathbf{r}}\mathcal{U}^{*}_{\mathbf{k}-(1/2)\mathbf{q}'+(1/2)\mathbf{k}',\mathbf{k}-(1/2)\mathbf{q}'-(1/2)\mathbf{k}'}p(\mathbf{k}-\tfrac{1}{2}\mathbf{k}',\mathbf{r}-\mathbf{r}')\}.\end{aligned} \tag{15}$$

Performing a Taylor expansion, this integro-differential equation can be formally written in terms of a differential equation of infinite order. By using

$$f^e\left(\mathbf{k}-\frac{1}{2}\mathbf{k}',\mathbf{r}-\mathbf{r}'\right)=\sum_{n,m=0}^{\infty}\frac{1}{2^m n!m!}\left(-\mathbf{r}'\cdot\frac{\partial}{\partial\mathbf{r}}\right)^n\times\left(-\mathbf{k}'\cdot\frac{\partial}{\partial\mathbf{k}}\right)^m f^e(\mathbf{k},\mathbf{r}) \tag{16}$$

and after a partial integration the equation of motion, e.g., for the electron distribution function is obtained as

$$\begin{aligned}\frac{\partial}{\partial t}f^e(\mathbf{k},\mathbf{r})=\frac{1}{i\hbar}\sum_{n,m=0}^{\infty}\frac{i^{n+m}}{2^{n+m}n!m!}\Bigg\{&[(-1)^n-(-1)^m]\\&\times\left(\frac{\partial}{\partial\mathbf{k}'}\cdot\frac{\partial}{\partial\mathbf{r}}\right)^n\left(\frac{\partial}{\partial\mathbf{r}'}\cdot\frac{\partial}{\partial\mathbf{k}}\right)^m\mathcal{E}^e(\mathbf{k}',\mathbf{r}')f^e(\mathbf{k},\mathbf{r})\\&+(-1)^n\left(\frac{\partial}{\partial\mathbf{k}'}\cdot\frac{\partial}{\partial\mathbf{r}}\right)^n\left(\frac{\partial}{\partial\mathbf{r}'}\cdot\frac{\partial}{\partial\mathbf{k}}\right)^m\mathcal{U}(\mathbf{k}',\mathbf{r}')\\&\times p^*(\mathbf{k},\mathbf{r})-(-1)^m\left(\frac{\partial}{\partial\mathbf{k}'}\cdot\frac{\partial}{\partial\mathbf{r}}\right)^n\\&\times\left(\frac{\partial}{\partial\mathbf{r}'}\cdot\frac{\partial}{\partial\mathbf{k}}\right)^m\mathcal{U}^*(\mathbf{k}',\mathbf{r}')p(\mathbf{k},\mathbf{r})\Bigg\}\Bigg|_{\substack{\mathbf{k}'=\mathbf{k}\\\mathbf{r}'=\mathbf{r}}}.\end{aligned} \tag{17}$$

The equations for the hole distribution function and the polarization are given in Appendix A. Here, we have introduced the space-dependent renormalized energies and fields

$$\mathcal{E}^{e,h}(\mathbf{k},\mathbf{r})=\sum_{\mathbf{q}}e^{i\mathbf{q}\mathbf{r}}\mathcal{E}^{e,h}_{\mathbf{k}+\frac{1}{2}\mathbf{q},\mathbf{k}-\frac{1}{2}\mathbf{q}}=\epsilon^{e,h}_{\mathbf{k}}+\Delta\mathcal{E}^{e,h}(\mathbf{k},\mathbf{r})\mp e\Phi(\mathbf{r}), \tag{18a}$$

$$\begin{aligned}\mathcal{U}(\mathbf{k},\mathbf{r})&=\sum_{\mathbf{q}}e^{i\mathbf{q}\mathbf{r}}\mathcal{U}_{\mathbf{k}+\frac{1}{2}\mathbf{q},\mathbf{k}-\frac{1}{2}\mathbf{q}}\\&=-\mathbf{d}_{cv}(\mathbf{k})\cdot\mathbf{E}^{(+)}(\mathbf{r},t)+\Delta\mathcal{U}(\mathbf{k},\mathbf{r}),\end{aligned} \tag{18b}$$

where

$$\Delta\mathcal{E}^{e,h}(\mathbf{k},\mathbf{r})=-\sum_{\mathbf{k}'}V_{\mathbf{k}-\mathbf{k}'}f^{e,h}(\mathbf{k}',\mathbf{r}) \tag{19}$$

denotes the energy renormalization due to the exchange interaction and

$$\Delta\mathcal{U}(\mathbf{k},\mathbf{r})=-\sum_{\mathbf{k}'}V_{\mathbf{k}-\mathbf{k}'}p(\mathbf{k}',\mathbf{r}) \tag{20}$$

is the internal field being responsible for excitonic effects and Coulomb enhancement. The electrostatic potential $\Phi(\mathbf{r})$ due to the Hartree terms in the mean-field Hamiltonian satisfies the Poisson equation

$$\nabla^2\Phi(\mathbf{r})=-\frac{4\pi}{\varepsilon_0}\rho(\mathbf{r})=\frac{4\pi e}{\varepsilon_0\mathcal{V}}\sum_{\mathbf{k}}[f^e(\mathbf{k},\mathbf{r})-f^h(\mathbf{k},\mathbf{r})], \tag{21}$$

$\rho(\mathbf{r})$ being the electric charge density.

We notice that in each order (n,m) spatial derivatives of the order $n+m$ appear. If the length scales of spatial inhomogeneities are sufficiently large, one may assume that terms with increasing order will be of decreasing importance. Let us consider the lowest orders: In zeroth order $(n=m=0)$ we obtain the well-known semiconductor Bloch equations

$$\frac{\partial}{\partial t}f^e(\mathbf{k},\mathbf{r},t)=\frac{\partial}{\partial t}f^h(-\mathbf{k},\mathbf{r},t)=g(\mathbf{k},\mathbf{r},t), \tag{22a}$$

$$\begin{aligned}\frac{\partial}{\partial t}p(\mathbf{k},\mathbf{r},t)=&\frac{1}{i\hbar}[\mathcal{E}^e(\mathbf{k},\mathbf{r},t)+\mathcal{E}^h(-\mathbf{k},\mathbf{r},t)]p(\mathbf{k},\mathbf{r},t)\\&+\frac{1}{i\hbar}\mathcal{U}(\mathbf{k},\mathbf{r},t)[1-f^e(\mathbf{k},\mathbf{r},t)-f^h(-\mathbf{k},\mathbf{r},t)],\end{aligned} \tag{22b}$$

with the generation rate

$$g(\mathbf{k},\mathbf{r},t)=\frac{1}{i\hbar}[\mathcal{U}(\mathbf{k},\mathbf{r},t)p^*(\mathbf{k},\mathbf{r},t)-\mathcal{U}^*(\mathbf{k},\mathbf{r},t)p(\mathbf{k},\mathbf{r},t)]. \tag{23}$$

The space coordinate $\mathbf{r}$ appears only as a parameter, i.e., locally the dynamics is the same as in the homogeneous case and there are no effects of spatial transport. Typically, four-wave-mixing experiments are interpreted on the basis of these equations [21,22]. In the present case of a broad-area laser, however, carrier diffusion plays a crucial role and transport terms have to be included.

In first order ($n=1,m=0$ and $n=0,m=1$) we obtain, e.g., for the electron distribution function

$$\begin{aligned}\frac{\partial}{\partial t}f^{e(1)}(\mathbf{k},\mathbf{r})=&\frac{1}{\hbar}\left\{-\frac{\partial\mathcal{E}^e(\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}\cdot\frac{\partial f^e(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}+\frac{\partial\mathcal{E}^e(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}\cdot\frac{\partial f^e(\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}\right\}\\&+\frac{1}{2\hbar}\left\{-\frac{\partial\mathcal{U}(\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}\cdot\frac{\partial p^*(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}+\frac{\partial\mathcal{U}(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}\cdot\frac{\partial p^*(\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}-\frac{\partial\mathcal{U}^*(\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}\cdot\frac{\partial p(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}+\frac{\partial\mathcal{U}^*(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}\cdot\frac{\partial p(\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}\right\}.\end{aligned} \tag{24}$$

The corresponding equations for the hole distribution function and the polarization are given in Appendix B. In the first row of (24) we recover the two contributions of the drift term in the Boltzmann transport equation exactly as in the case of a one-band model. The second and third row can be interpreted as a local generation rate due to an in or out flow of

polarization in the corresponding volume element of the phase space. Thus we obtain the generalization of the Boltzmann equation to the case of a two-band model including the coherent interband transport contributions. Higher derivatives, which become important if the variables exhibit spatial variations on very short length scales, are responsible for typically quantum-mechanical transport features like tunneling as, e.g., in resonant tunneling diodes [23]. However, in the present laser structures studied here they can be safely neglected.

C. Screening and scattering processes

The correlation part of the carrier-carrier interaction Hamiltonian and the carrier-phonon interaction Hamiltonian give rise to two phenomena: screening of the bare interaction and scattering processes among the carriers. Here, we will not discuss this part explicitly, but model the effects in terms of a static screening model and a relaxation-time approximation for the scattering processes [17,21,24,25].

The bare Coulomb potential is replaced by the screened potential

$$V_{\mathbf{q}}^{s}=\frac{4\pi e^{2}}{\mathcal{V}\varepsilon_{0}\kappa^{2}}\,\frac{1+\alpha x^{2}}{1+x^{2}+\alpha x^{4}}, \tag{25}$$

with $x=q/\kappa$, $\alpha=(\varepsilon_{0}\hbar^{2}\kappa^{4})/(16\pi m_{r}Ne^{2})$, and the space- and time-dependent screening wave-vector

$$\kappa^{2}=-\frac{4\pi e^{2}}{\varepsilon_{0}\mathcal{V}}\sum_{\nu=e,h}\sum_{\mathbf{k}}\left(\frac{\partial\epsilon_{\mathbf{k}}^{\nu}}{\partial k}\right)^{-1}\left(\frac{\partial f^{\nu}(\mathbf{k},\mathbf{r})}{\partial k}\right), \tag{26}$$

$N(\mathbf{r})=N^{e}(\mathbf{r})=N^{h}(\mathbf{r})$ being the carrier density, $m_{r}=(m_{e}^{-1}+m_{h}^{-1})^{-1}$ being the reduced mass, and an isotropic distribution function has been assumed. For sufficiently weak deviations of the carrier distributions from local quasiequilibrium, the energy renormalization can be modeled by a rigid shift of the bands according to

$$\Delta\mathcal{E}^{e,h}(\mathbf{r})=-\frac{a}{2}\mathcal{E}_{0}\left(1+b^{2}\frac{(k_{B}T)^{2}}{Na_{0}^{3}\mathcal{E}_{0}^{2}}\right)^{-1/4}, \tag{27}$$

with the exciton binding energy $\mathcal{E}_{0}=m_{r}e^{4}/(2\varepsilon_{0}^{2}\hbar^{2})$, the exciton Bohr radius $a_{0}=\hbar^{2}\varepsilon_{0}/(e^{2}m_{r})$, the temperature T, Boltzmann's constant k_{B}, and the numerical factors are given by $a=4.64$ and $b=0.107$ [26]. Furthermore, under the condition of a pronounced gain, as is the case in an active semiconductor laser, a high-density approximation for the internal field describing the Coulomb enhancement can be used. It is obtained by replacing the wave-vector $\mathbf{k}$ in Eq. (20) by the Fermi wave-vector $\mathbf{k}_{F}$ [27] resulting in

$$\Delta\mathcal{U}(\mathbf{r})=-\sum_{\mathbf{k}'}V^{s}_{\mathbf{k}_{F}-\mathbf{k}'}p(\mathbf{k}',\mathbf{r}). \tag{28}$$

Within a semiclassical picture, i.e., when describing scattering processes in terms of a scattering rate, the scattering contributions to the equations of motion have the structure of the Boltzmann collision terms [13,28]

$$\left.\frac{\partial}{\partial t}f^{e,h}(\mathbf{k},\mathbf{r})\right|_{\mathrm{col}}=-\sum_{\mathbf{k}'}\{W^{e,h}_{\mathbf{k}',\mathbf{k}}f^{e,h}(\mathbf{k},\mathbf{r})[1-f^{e,h}(\mathbf{k}',\mathbf{r})]-W^{e,h}_{\mathbf{k},\mathbf{k}'}f^{e,h}(\mathbf{k}',\mathbf{r})[1-f^{e,h}(\mathbf{k},\mathbf{r})]\}, \tag{29a}$$

$$\left.\frac{\partial}{\partial t}p(\mathbf{k},\mathbf{r})\right|_{\mathrm{col}}=-\sum_{\mathbf{k}'}[W^{p}_{\mathbf{k}',\mathbf{k}}p(\mathbf{k},\mathbf{r})-W^{p}_{\mathbf{k},\mathbf{k}'}p(\mathbf{k}',\mathbf{r})]. \tag{29b}$$

Here, terms which are quadratic or of higher order in the polarization (polarization scattering) have been neglected. The Boltzmann scattering matrices $W^{e,h}_{\mathbf{k}',\mathbf{k}}$ for electrons and holes are real quantities, the scattering matrices $W^{p}_{\mathbf{k}',\mathbf{k}}$ in the equation of motion for the polarization in general are complex, the real part being related to $W^{e,h}_{\mathbf{k}',\mathbf{k}}$ according to

$$\begin{aligned}\mathrm{Re}(W^{p}_{\mathbf{k}',\mathbf{k}})=\tfrac{1}{2}\{&W^{e}_{\mathbf{k}',\mathbf{k}}[1-f^{e}(\mathbf{k}',\mathbf{r})]+W^{e}_{\mathbf{k},\mathbf{k}'}f^{e}(\mathbf{k}',\mathbf{r})\\&+W^{h}_{-\mathbf{k}',-\mathbf{k}}[1-f^{h}(-\mathbf{k}',\mathbf{r})]\\&+W^{h}_{-\mathbf{k},-\mathbf{k}'}f^{h}(-\mathbf{k}',\mathbf{r})\}.\end{aligned} \tag{30}$$

The imaginary part describes second-order corrections to the band-gap renormalization. Typically, these corrections can be neglected compared to the first-order (Hartree-Fock) renormalization terms.

The scattering rates are obtained from Fermi's golden rule and the matrices are given by

$$W^{e,h(cc)}_{\mathbf{k}-\mathbf{q},\mathbf{k}}=\frac{2\pi}{\hbar}|V^{s}_{\mathbf{q}}|^{2}\sum_{\nu=e,h}\sum_{\mathbf{k}'}\delta(\epsilon^{e,h}_{\mathbf{k}-\mathbf{q}}+\epsilon^{\nu}_{\mathbf{k}'+\mathbf{q}}-\epsilon^{\nu}_{\mathbf{k}'}-\epsilon^{e,h}_{\mathbf{k}})\times\{f^{\nu}(\mathbf{k}',\mathbf{r})[1-f^{\nu}(\mathbf{k}'+\mathbf{q},\mathbf{r})]\} \tag{31}$$

for carrier-carrier interaction and

$$W^{e,h(cp)}_{\mathbf{k}-\mathbf{q},\mathbf{k}}=\frac{2\pi}{\hbar}\sum_{\pm}|\gamma^{e,h}_{\mathbf{q}}|^{2}\delta(\epsilon^{e,h}_{\mathbf{k}-\mathbf{q}}-\epsilon^{e,h}_{\mathbf{k}}\pm\hbar\omega_{op})\left(n_{\mathbf{q}}+\frac{1}{2}\pm\frac{1}{2}\right) \tag{32}$$

for carrier-phonon interaction.

Carrier-carrier scattering processes, due to the conservation of carrier density, total momentum, and energy, tend to establish a local heated displaced Fermi-Dirac distribution, while carrier-phonon scattering processes, which conserve only the carrier density, lead to a relaxation towards a Fermi-Dirac distribution at lattice temperature. Both kinds of processes, by destroying phase coherence, lead to a dephasing of the interband polarization.

A full kinetic treatment of the scattering processes can be performed by using various techniques, e.g., Monte Carlo simulations [12,29,30], or a direct integration [25,31] which, already for the case of spatially homogeneous system, involve extensive numerical calculations. In the present case, however, due to the high densities and, consequently, the high scattering rates, the distribution functions of electrons and holes will not exhibit strong deviations from local quasiequilibrium distributions. Therefore, with

$$f^{e,h}(\mathbf{k},\mathbf{r}) = f^{e,h}_{eq}(\mathbf{k},\mathbf{r}) + \delta f^{e,h}(\mathbf{k},\mathbf{r}), \tag{33}$$

$$f^{e,h}_{eq}(\mathbf{k},\mathbf{r}) = (\exp[\beta(\epsilon^{e,h}_{\mathbf{k}} - \mu^{e,h}(\mathbf{r}))] + 1)^{-1}, \tag{34}$$

$\beta = (k_B T)^{-1}$, the Boltzmann collision terms can be linearized with respect to $\delta f^{e,h}$ and p.

Due to the conservation of carrier density, the deviations satisfy the sum rule

$$\sum_{\mathbf{k}} \delta f^{e,h}(\mathbf{k},\mathbf{r}) = 0 \tag{35}$$

which determines the chemical potentials $\mu^{e,h}(\mathbf{r})$. Their dependence on carrier density can be approximated by a *2/1-Padé* approximation [32] and as a function of the normalized density $\nu^{e,h} = N^{e,h}/N^{e,h}_0$ they are given by

$$\beta\mu^{e,h} \approx \ln\nu^{e,h} - K_1 \ln(K_2 \nu^{e,h} + 1) + K_3 \nu^{e,h}, \tag{36}$$

where

$$N^{e,h}_0 = \frac{1}{4}\left(\frac{2m_{e,h}}{\hbar^2 \pi \beta}\right)^{3/2} \tag{37}$$

and the coefficients are [32] $K_1 = 4.896\,685\,1$, $K_2 = 0.044\,964\,57$, and $K_3 = 0.133\,376\,0$.

Linearizing the collision term for the polarization [Eq. (29b)] simply consists in calculating the scattering matrices with the quasiequilibrium distribution functions. Since the polarization as a complex quantity is typically an oscillating function of $\mathbf{k}$, we may assume that the second term in Eq. (29b) is negligible due to random phases in the summation. As a result, the dephasing is described in terms of a dephasing rate [33]. Linearizing the collision term for the distribution functions results in a similar structure with a term which has the structure of a relaxation rate and other terms involving a summation over different $\mathbf{k}$ components of δf. Due to the sum rule [Eq. (35)] the latter terms usually are much smaller than the former one and may be neglected [34]. Thus the collision terms can be expressed in terms of a relaxation-time approximation

$$\left.\frac{\partial}{\partial t} f^{e,h}(\mathbf{k},\mathbf{r},t)\right|_{\text{col}} = -\tau^{-1}_{e,h}(\mathbf{k})[f^{e,h}(\mathbf{k},\mathbf{r},t) - f^{e,h}_{eq}(\mathbf{k},\mathbf{r},t)], \tag{38a}$$

$$\left.\frac{\partial}{\partial t} p(\mathbf{k},\mathbf{r},t)\right|_{\text{col}} = -\tau^{-1}_{p}(\mathbf{k}) p(\mathbf{k},\mathbf{r},t) \tag{38b}$$

with the relaxation rates

$$\tau^{-1}_{e,h}(\mathbf{k}) = \sum_{\mathbf{k}'} \{W^{e,h}_{\mathbf{k}',\mathbf{k}}[1 - f^{e,h}_{eq}(\mathbf{k}',\mathbf{r})] + W^{e,h}_{\mathbf{k},\mathbf{k}'} f^{e,h}_{eq}(\mathbf{k}',\mathbf{r})\}, \tag{39a}$$

$$\tau^{-1}_{p}(\mathbf{k}) = \tfrac{1}{2}[\tau^{-1}_{e}(\mathbf{k}) + \tau^{-1}_{h}(\mathbf{k})]. \tag{39b}$$

The scattering matrices are calculated with the quasiequilibrium distributions [35].

In Fig. 1 the relaxation rates for carrier-carrier [Figs. 1(a)–1(c)] and carrier-phonon scattering [Figs. 1(d)–1(f)] are plotted as functions of k and the carrier density. (Note the different orientations of the left and right column.) Both rates have been calculated in the static screening model. Above a density of the order of 10^{18} cm^{-3}, the Fermi edge leads to a pronounced minimum in the relaxation rates due to phase-space filling. This dip is more pronounced for electrons due to their smaller density of states resulting in a higher degree of degeneracy. In the case of carrier-phonon interaction we also see the strong increase in the relaxation rate if, with increasing k, the threshold for the onset of phonon emission is reached.

D. Injection and spontaneous recombination

Modeling the behavior of a semiconductor laser requires additional processes to be taken into account. In a laser diode the carriers are electrically injected from the contacts into the active region. This injection is modeled by an injection rate

$$\left.\frac{\partial}{\partial t} f^{e,h}\right|_{\text{inj}} = \Lambda^{e,h}(\mathbf{k},\mathbf{r},t) = \frac{\eta_i j_p(\mathbf{r})}{ed} \frac{\hat{f}^{e,h}(\mathbf{k},\mathbf{r},t)[1 - f^{e,h}(\mathbf{k},\mathbf{r},t)]}{\mathcal{V}^{-1}\sum_{\mathbf{k}} \hat{f}^{e,h}(\mathbf{k},\mathbf{r},t)}, \tag{40}$$

where $\hat{f}^{e,h}(\mathbf{k},\mathbf{r},t)$ denotes the pump generated Fermi-Dirac distribution just outside the active region, j_p is the electric pump current density, d is the vertical thickness of the active region, and η_i is the injection efficiency, i.e., the proportion of carriers which actually reaches the active region.

Since in semiclassic laser theory the light field is treated in terms of a classical electric field, only absorption and stimulated emission are obtained. Thus, in the absence of a light field, no processes occur and the laser cannot turn on. Spontaneous emission, being related to the quantum nature of the light field, has to be added to the equations in order to start the lasing process. This leads to a recombination term in the equations of motion for the distribution functions

$$\left.\frac{\partial}{\partial t} f^{e,h}\right|_{\text{rec}} = -\Gamma_{sp}(\mathbf{k}) f^{e}(\pm\mathbf{k},\mathbf{r},t) f^{h}(\mp\mathbf{k},\mathbf{r},t), \tag{41}$$

the upper sign refering to electrons and the lower sign refering to holes, with the spontaneous recombination coefficient

$$\Gamma_{sp}(\mathbf{k}) = \frac{4n_l^3}{\hbar^4 \pi c^3} |d_{cv}(\mathbf{k})|^2 \left(\mathcal{E}_g + \frac{\hbar^2 k^2}{2m_r}\right)^3, \tag{42}$$

and n_l being the refractive index of the active layer. Typically, in a semiconductor additional nonradiative recombination mechanisms are present reducing the efficiency of the carrier-light coupling. These are modeled by a constant rate γ_{nr} according to

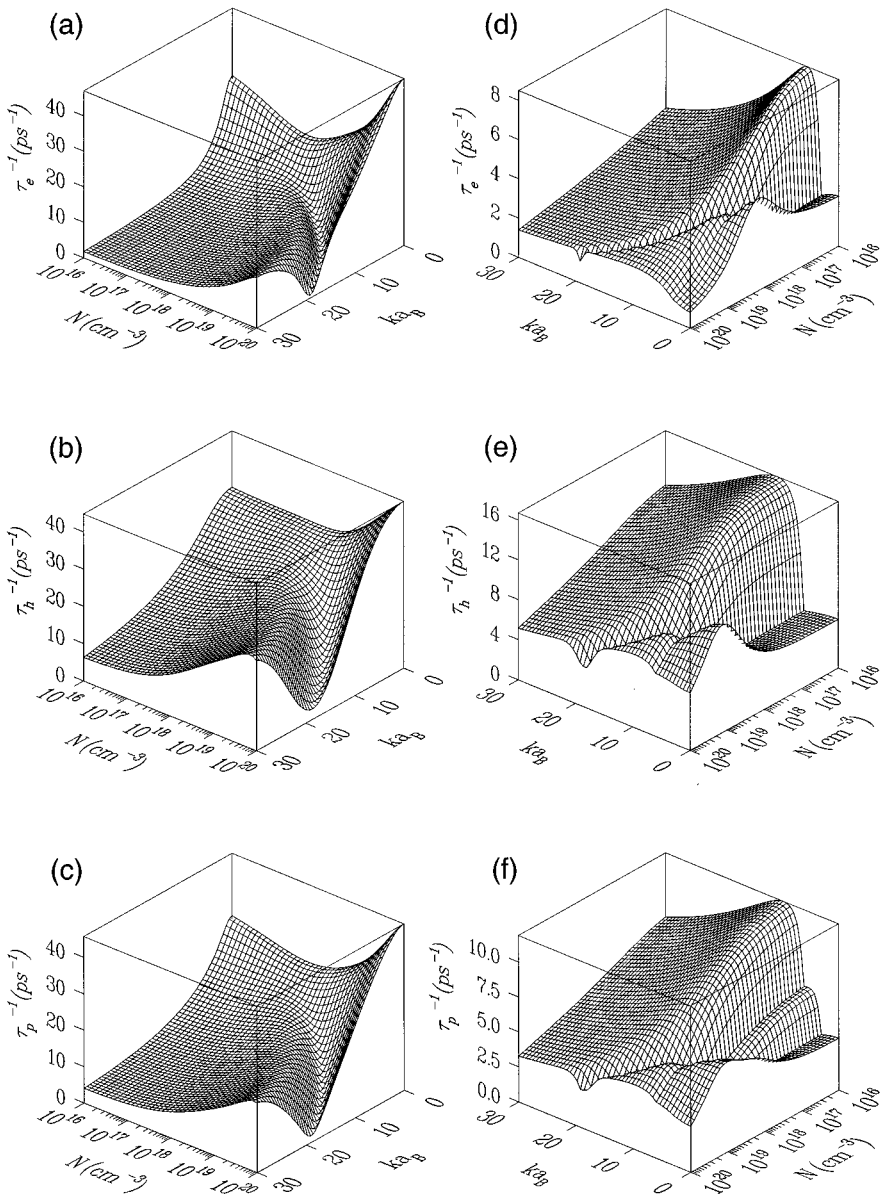

FIG. 1. Relaxation rates for carrier-carrier [(a)–(c)] and carrier-phonon scattering [(d)–(f)], plotted as functions of k and the carrier density N. Note the different orientations of the left and right column.

$$\left.\frac{\partial}{\partial t} f^{e,h}\right|_{\mathrm{nr}} = -\gamma_{nr} f^{e,h}(\mathbf{k},\mathbf{r},t). \tag{43}$$

E. Plasma diffusion and ambipolar approximation

The microscopic semiconductor dynamics is described by distribution functions of electrons and holes and by the interband polarization. All these quantities are functions of space and momentum. Typically, however, there is a strong separation of time scales between the **k**-space and the **r**-space dynamics. Scattering and dephasing processes lead to a fast relaxation of the microscopic variables towards their local quasiequilibrium values on a femtosecond time scale. The spatial transport occurs on a much slower time scale. For this reason, the influence of spatial gradients on the **k**-space dynamics is often negligible. However, the scattering terms in the equations of motion for the distribution functions conserve the density of carriers. Therefore, the density is not influenced by the fast relaxation processes and its spatial transport cannot be neglected. In the equation of motion for the polarization, on the other hand, no conserved quantities exist and thus spatial transport of polarization is usually not important.

Keeping the first-order spatial derivatives of the distribution functions and neglecting any spatial transport of polarization, the equations of motion for electron and hole distribution functions are given by the Boltzmann equations

$$\begin{aligned}
&\frac{\partial}{\partial t} f^{e,h}(\mathbf{k},\mathbf{r},t) + \frac{\hbar\mathbf{k}}{m_{e,h}} \cdot \frac{\partial f^{e,h}(\mathbf{k},\mathbf{r})}{\partial \mathbf{r}} \\
&\quad - \frac{1}{\hbar}\left(\frac{\partial \Delta\mathcal{E}(\mathbf{r})}{\partial \mathbf{r}} \mp e\frac{\partial \Phi(\mathbf{r})}{\partial \mathbf{r}}\right) \cdot \frac{\partial f^{e,h}(\mathbf{k},\mathbf{r})}{\partial \mathbf{k}} \\
&\quad = -\frac{1}{\tau_{e,h}(\mathbf{k})}[f^{e,h}(\mathbf{k},\mathbf{r},t) - f^{e,h}_{eq}(\mathbf{k},\mathbf{r},t)] \\
&\qquad + g(\pm\mathbf{k},\mathbf{r},t) + \Lambda^{e,h}(\mathbf{k},\mathbf{r},t) \\
&\qquad - \Gamma_{sp}(\mathbf{k}) f^{e}(\pm\mathbf{k},\mathbf{r},t) f^{h}(\mp\mathbf{k},\mathbf{r},t) - \gamma_{nr} f^{e,h}(\mathbf{k},\mathbf{r},t),
\end{aligned} \tag{44}$$

where, again, the upper sign refers to electrons and the lower sign to holes. Integrating these Boltzmann equations over $\mathbf{k}$ results in the continuity equations for the carrier densities

$$\frac{\partial}{\partial t}N^{e,h}+\mathrm{div}\mathbf{j}^{e,h}=\Lambda^{e,h}(\mathbf{r},t)+G(\mathbf{r},t)-\gamma_{nr}N^{e,h}(\mathbf{r},t)-W(\mathbf{r},t) \tag{45}$$

with the densities $N^{e,h}(\mathbf{r},t)=\mathcal{V}^{-1}\Sigma_{\mathbf{k}}f^{e,h}(\mathbf{k},\mathbf{r},t)$, the current densities $\mathbf{j}^{e,h}(\mathbf{r},t)=\mathcal{V}^{-1}\Sigma_{\mathbf{k}}(\hbar\mathbf{k}/m_{e,h})f^{e,h}(\mathbf{k},\mathbf{r},t)$, the total generation rate $G(\mathbf{r},t)=\mathcal{V}^{-1}\Sigma_{\mathbf{k}}g(\mathbf{k},\mathbf{r},t)$, the total injection rate $\Lambda^{e,h}(\mathbf{r},t)$, and the total spontaneous emission rate $W(\mathbf{r},t)$.

If the scattering processes are sufficiently fast, the scattering term in the Boltzmann equation is the dominant contribution and the carrier distribution deviates only weakly from the local quasiequilibrium distribution $f_{eq}^{e,h}$ In this case, in all other terms except for the scattering term the distribution function may be approximately replaced by $f_{eq}^{e,h}$ resulting in the approximate solution

$$\begin{aligned}\delta f^{e,h}(\mathbf{k},\mathbf{r},t)=-\tau_{e,h}(\mathbf{k})\Bigg[&\frac{\partial}{\partial t}f_{eq}^{e,h}(\mathbf{k},\mathbf{r},t)+\frac{\hbar\mathbf{k}}{m_{e,h}}\cdot\frac{\partial f_{eq}^{e,h}(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}\\&-\frac{1}{\hbar}\left(\frac{\partial\Delta\mathcal{E}(\mathbf{r})}{\partial\mathbf{r}}\mp e\frac{\partial\Phi(\mathbf{r})}{\partial\mathbf{r}}\right)\cdot\frac{\partial f_{eq}^{e,h}(\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}\\&-g(\mathbf{k},\mathbf{r},t)-\Lambda^{e,h}(\mathbf{k},\mathbf{r},t)+\Gamma_{sp}(\mathbf{k})f_{eq}^{e}\\&\times(\pm\mathbf{k},\mathbf{r},t)f_{eq}^{h}(\mp\mathbf{k},\mathbf{r},t)+\gamma_{nr}f_{eq}^{e,h}(\mathbf{k},\mathbf{r},t)\Bigg].\end{aligned} \tag{46}$$

Thus due to the isotropy of f_{eq} in the $\mathbf{k}$ space, the current density is given by

$$\begin{aligned}\mathbf{j}^{e,h}(\mathbf{r},t)&=\mathcal{V}^{-1}\sum_{\mathbf{k}}\frac{\hbar\mathbf{k}}{m_{e,h}}\delta f^{e,h}(\mathbf{k},\mathbf{r},t)\\&=-D^{e,h}\mathrm{grad}N^{e,h}\pm\sigma^{e,h}\frac{1}{e}\mathrm{grad}\Phi,\end{aligned} \tag{47}$$

with the diffusivities

$$D^{e,h}=-\frac{1}{3\mathcal{V}}\sum_{\mathbf{k}}\tau_{e,h}(\mathbf{k})\frac{\hbar^2k^2\partial f_{eq}^{e,h}(\mathbf{k},\mathbf{r})}{m_{e,h}^2}\frac{\partial(\mu^{e,h}+\Delta\mathcal{E}^{e,h})}{\partial\epsilon_{\mathbf{k}}^{e,h}}\frac{\partial(\mu^{e,h}+\Delta\mathcal{E}^{e,h})}{\partial N^{e,h}} \tag{48}$$

and the conductivities

$$\sigma^{e,h}=N^{e,h}e\tilde{\mu}^{e,h}=-\frac{e^2}{3\mathcal{V}}\sum_{\mathbf{k}}\tau_{e,h}(\mathbf{k})\frac{\hbar^2k^2\partial f_{eq}^{e,h}(\mathbf{k},\mathbf{r})}{m_{e,h}^2}\frac{\partial f_{eq}^{e,h}(\mathbf{k},\mathbf{r})}{\partial\epsilon_{\mathbf{k}}^{e,h}}. \tag{49}$$

Here, $\tilde{\mu}^{e,h}$ denote electron and hole mobilities. In Fig. 2 the mobilities and diffusivities obtained from the microscopic scattering rates are plotted as functions of the carrier density. At low densities we find approximately constant values. When the carriers become strongly degenerate, which occurs for electrons at about 10^{18} cm^{-3} and for holes at about 10^{19} cm^{-3}, both transport parameters exhibit a strong increase related to the reduction of the scattering rate close to the Fermi energy due to phase-space filling effects.

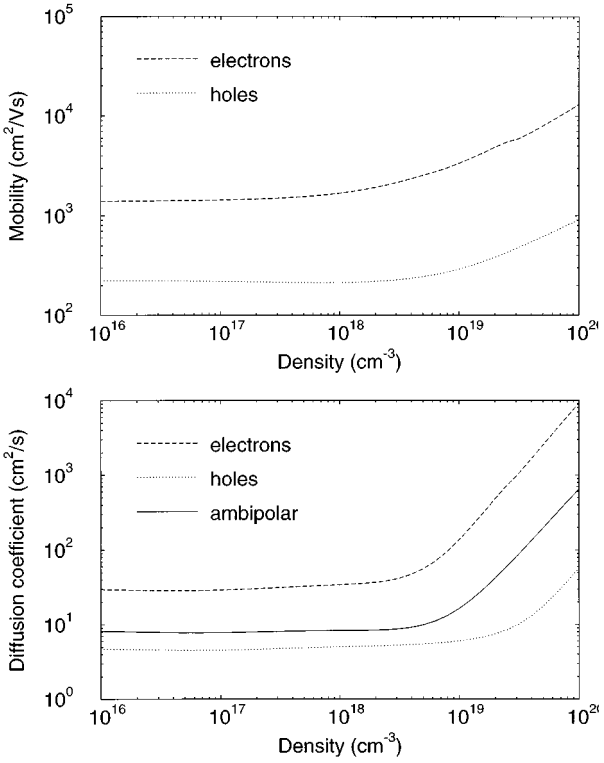

FIG. 2. Electron and hole mobility (a) and electron, hole and ambipolar diffusion coefficient (b) plotted as functions of the carrier density N.

The spatial transport of electrons and holes is coupled by electrostatic forces expressed by the electrostatic potential Φ which, in turn, is obtained from the carrier densities by solving Poisson's equation (21). This coupling strongly increases with increasing densities resulting eventually in an effective one-component behavior, the ambipolar transport of electrons and holes. In this case, no space charges exist and the densities of electrons and holes are equal $N^e=N^h=N$. In order to keep this neutrality, also the current densities are equal $\mathbf{j}^e=\mathbf{j}^h=\mathbf{j}$. Eliminating the drift term in Eq. (47) results in the ambipolar diffusion current density

$$\mathbf{j}=-D_f\mathrm{grad}N \tag{50}$$

with the ambipolar diffusion coefficient

$$D_f=\frac{\sigma^hD^e+\sigma^eD^h}{\sigma^e+\sigma^h}. \tag{51}$$

Under nondegenerate conditions, where due to the Einstein relation $\sigma^e/\sigma^h=D^e/D^h$ holds, Eq. (51) reduces to the well-known form $D_f^{-1}=1/2(D^{e-1}+D^{h-1})$ [36–38]. Combining Eqs. (45) and (50), the macroscopic transport equation for the carrier density is given by

$$\begin{aligned}\frac{\partial}{\partial t}N=&\mathrm{div}(D_f\mathrm{grad}N)+\Lambda(\mathbf{r},t)+G(\mathbf{r},t)-\gamma_{nr}N(\mathbf{r},t)\\&-W(\mathbf{r},t).\end{aligned} \tag{52}$$

The solid line in Fig. 2 shows the ambipolar diffusion coefficient as a function of the density. The curve is flat up to several 10^{18} cm^{-3} and then exhibits the same strong increase as discussed above for the electron and hole diffusion coefficients.

F. Linear and nonlinear polarization

For a numerical modeling of a semiconductor laser it is convenient to separate the polarization in a ''linear'' and a ''nonlinear'' part $p(\mathbf{k},\mathbf{r},t)=p_l(\mathbf{k},\mathbf{r},t)+p_{nl}(\mathbf{k},\mathbf{r},t)$. The internal field [Eq. (28)] is separated accordingly $\Delta\mathcal{U}(\mathbf{r},t)=\Delta\mathcal{U}_l(\mathbf{r},t)+\Delta\mathcal{U}_{nl}(\mathbf{r},t)$. Starting from Eqs. (22b),(38b), this leads to the equations of motion

$$\frac{\partial}{\partial t}p_l(\mathbf{k},\mathbf{r},t)=\frac{1}{i\hbar}[\mathcal{E}^e(\mathbf{k},\mathbf{r},t)+\mathcal{E}^h(-\mathbf{k},\mathbf{r},t)]p_l(\mathbf{k},\mathbf{r},t)+\frac{1}{i\hbar}[-d_{cv}E^{(+)}(\mathbf{r},t)+\Delta\mathcal{U}_l(\mathbf{r},t)]-\tau_p^{-1}(\mathbf{k})p_l(\mathbf{k},\mathbf{r},t), \tag{53a}$$

$$\frac{\partial}{\partial t}p_{nl}(\mathbf{k},\mathbf{r},t)=\frac{1}{i\hbar}[\mathcal{E}^e(\mathbf{k},\mathbf{r},t)+\mathcal{E}^h(-\mathbf{k},\mathbf{r},t)]p_{nl}(\mathbf{k},\mathbf{r},t)+\frac{1}{i\hbar}\Delta\mathcal{U}_{nl}(\mathbf{r},t)-\frac{1}{i\hbar}\mathcal{U}(\mathbf{k},\mathbf{r},t)[f^e(\mathbf{k},\mathbf{r},t)+f^h(-\mathbf{k},\mathbf{r},t)]-\tau_p^{-1}(\mathbf{k})p_{nl}(\mathbf{k},\mathbf{r},t), \tag{53b}$$

with $d_{cv}=\mathbf{d}_{cv}\cdot\mathbf{e}$, $\mathbf{e}$ being the polarization vector of the light field, and the $\mathbf{k}$ dependence of the dipole matrix element has been neglected. Strictly speaking, p_l is not exactly the linear part since in Eq. (53a) we have included energy renormalizations, Coulomb enhancement, and dephasing rates which depend on the carrier density. However, we can take advantage of the separation of time scales disscussed above: The nonlinear polarization is determined by the full $\mathbf{k}$ dependence of the carrier-distribution functions and thus it is a fast variable. The linear polarization, on the other hand, is governed by the carrier density. Its temporal evolution is much slower and retardation effects can be neglected. Then, p_l can be expressed in terms of a $\mathbf{k}$-dependent susceptibility defined as

$$d_{cv}^{*}p_l(\mathbf{k},\mathbf{r},t)=\tilde{\chi}(\omega;\mathbf{k},\mathbf{r},t)E^{(+)}(\mathbf{r},t). \tag{54}$$

In the high-density approximation for the internal field, the susceptibility is given by

$$\tilde{\chi}(\omega;\mathbf{k},\mathbf{r},t)=\tilde{\chi}'+i\tilde{\chi}''=\frac{\tilde{\chi}^0(\omega;\mathbf{k},\mathbf{r},t)}{1-\frac{1}{|d_{cv}|^2}\sum_{\mathbf{k}'}V^s_{\mathbf{k}_F-\mathbf{k}'}\tilde{\chi}^0(\omega;\mathbf{k}',\mathbf{r},t)}, \tag{55}$$

with

$$\tilde{\chi}^0(\omega;\mathbf{k},\mathbf{r},t)=\frac{|d_{cv}|^2}{\mathcal{E}^e(\mathbf{k},\mathbf{r},t)+\mathcal{E}^h(-\mathbf{k},\mathbf{r},t)-\hbar\omega-i\hbar\tau_p^{-1}(\mathbf{k})}. \tag{56}$$

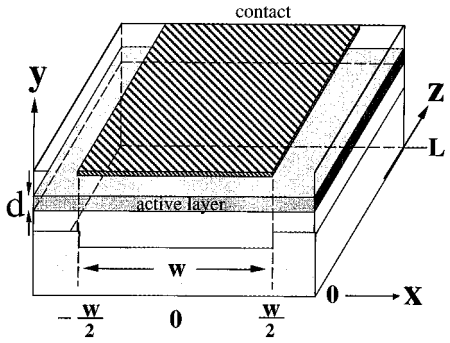

FIG. 3. Schematic of the broad-area semiconductor laser. The active GaAs layer (shaded dark) is sandwiched between two cladding layers of $Al_xGa_{1-x}As$ (white). Charge carriers, injected through the contact stripe(s) at the top of the device (hatched) may recombine in the active zone. Light propagates in longitudinal (z) direction. The transverse ribbed cladding structure at the bottom serves as a passive waveguide.

The macroscopic linear polarization is given by

$$P_l^{(+)}(\mathbf{r},t)=\chi(\omega;\mathbf{r},t)E^{(+)}(\mathbf{r},t), \tag{57}$$

with the susceptibility

$$\chi(\omega;\mathbf{r},t)=\chi'(\omega;\mathbf{r},t)+i\chi''(\omega;\mathbf{r},t)=\frac{1}{\mathcal{V}}\sum_{\mathbf{k}}\tilde{\chi}(\omega;\mathbf{k},\mathbf{r},t). \tag{58}$$

III. MAXWELL-BLOCH EQUATIONS

Based on the microscopic semiconductor model discussed above, in this section a general Maxwell-Bloch formulation for spatially inhomogeneous semiconductor lasers is developed and applied for the description of edge-emitting broad-area semiconductor lasers. Figure 3 shows a schematic plot of a typical broad-area laser structure. The microscopic effects of the active semiconductor medium are considered self-consistently with the macroscopic properties of the laser devices. Macroscopically, Maxwell's wave equation

$$\nabla^2\mathbf{E}+4\pi\nabla(\nabla\cdot\mathbf{P})-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{E}=\frac{4\pi}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{P}, \tag{59}$$

for the optical field $\mathbf{E}$ and the macroscopic polarization $\mathbf{P}$ is the basis for the theoretical description of the spatiotemporal behavior of the broad-area semiconductor lasers. In semiconductor lasers with transversely wide active zones, e.g., the broad-area lasers, a careful consideration of the spatial dependence is vital. Transversely in the x direction the evanescent optical-fields scatter and charge carriers diffuse away from the point where they were generated by the external pump current. Strong waveguiding effects in the vertical y direction which are a consequence of the sequence of semiconductor heterolayers, however, confine the optical field and limit the diffusion of the charge carriers in this direction. This allows the approximate consideration of the optical fields in the vertical y direction in the steady state. In the longitudinal z direction the laser cavity forms a Fabry-Perot

resonator so that the optical fields counterpropagate in a positive and negative z direction. Effectively, the problem is thus reduced to a quasi-two-dimensional one with the transverse direction denoted by x.

Due to the consideration of counterpropagation in a Fabry-Perot resonator, the optical fields are split into longitudinally slowly varying counterpropagating fields $\mathbf{E}^+$ and $\mathbf{E}^-$

$$\mathbf{E}=\tfrac{1}{2}(\mathbf{E}^+e^{iK_zz-i\omega t}+\mathbf{E}^-e^{-iK_zz-i\omega t}+\text{c.c.}) \tag{60}$$

traveling in an opposite positive (''+'') and negative (''−'') z direction, where c.c. indicates complex-conjugate contributions. Thus, the positive frequency components in Eqs. (3) and (6) are given by

$$\mathbf{E}^{(+)}(\mathbf{r},t)=\tfrac{1}{2}[\mathbf{E}^+(\mathbf{r},t)e^{iK_zz-i\omega t}+\mathbf{E}^-(\mathbf{r},t)e^{-iK_zz-i\omega t}]. \tag{61}$$

In the laser medium, the optical field induces the polarization

$$\mathbf{P}=\tfrac{1}{2}(\mathbf{P}^+e^{iK_zz-i\omega t}+\mathbf{P}^-e^{-iK_zz-i\omega t}+\text{c.c.}) \tag{62}$$

which, on the microscopic level, is related to the interband polarization

$$p(\mathbf{k},\mathbf{r},t)=\tfrac{1}{2}[p^+(\mathbf{k},\mathbf{r},t)e^{iK_zz-i\omega t}+p^-(\mathbf{k},\mathbf{r},t)e^{-iK_zz-i\omega t}]. \tag{63}$$

Accordingly, the effective field is decomposed in $\mathcal{U}^\pm$.

The fields $\mathbf{E}^\pm$ and $\mathbf{P}^\pm$ as well as the differential operator ∇ are further separated into transverse and longitudinal parts according to

$$\mathbf{E}^\pm=\mathbf{E}_T^\pm+\mathbf{e}_zE_z^\pm, \tag{64}$$

$$\mathbf{P}^\pm=\mathbf{P}_T^\pm+\mathbf{e}_zP_z^\pm, \tag{65}$$

and

$$\nabla=\nabla_T+\mathbf{e}_z\frac{\partial}{\partial z}. \tag{66}$$

Expansion of the resulting longitudinal and transverse wave equations in terms of the parameter $s=w/\mathfrak{l}=(K_zw)^{-1}$, w being the width and $\mathfrak{l}=K_zw^2$ being the diffraction length of the laser, leads to a hierarchy of wave equations [39]. In the following we will restrict ourselves to the first-order wave equation of the hierarchy which implies that the field variables are purely transverse, while still allowing for their longitudinal dependence.

In first order of s the spatiotemporal dynamics of the counterpropagating optical fields E^+ and E^- is described by the system of partial differential equations

$$\frac{\partial}{\partial z}E^+ + \frac{n_l}{c}\frac{\partial}{\partial t}E^+ = \frac{i}{2}\frac{1}{K_z}\frac{\partial^2}{\partial x^2}E^+ - \frac{\alpha}{2}E^+ + \frac{2\pi i\Gamma}{n_l^2L}P_{nl}^+, \tag{67a}$$

$$-\frac{\partial}{\partial z}E^- + \frac{n_l}{c}\frac{\partial}{\partial t}E^- = \frac{i}{2}\frac{1}{K_z}\frac{\partial^2}{\partial x^2}E^- - \frac{\alpha}{2}E^+ + \frac{2\pi i\Gamma}{n_l^2L}P_{nl}^-, \tag{67b}$$

with the index of refraction n_l, the absorption coefficient $\alpha(\omega)=(4\pi\omega\chi'')/(n_lc)$, the (longitudinal) length of the laser L, the wave number in (longitudinal) propagation direction $K_z=n_lK_0$, K_0 being the unperturbed wave number in the vacuum, and the confinement factor Γ describing the waveguiding properties of the heterostructure.

The fields are coupled to the active semiconductor via the linear polarizations, expressed in terms of n_l and α, and the nonlinear polarizations

$$P_{nl}^\pm(\mathbf{r},t)=\frac{d_{cv}^*}{\mathcal{V}}\sum_{\mathbf{k}}p_{nl}^\pm(\mathbf{k},\mathbf{r},t). \tag{68}$$

Due to the strong carrier-carrier and carrier-phonon scattering, the **k**-space dynamics occurs on a much faster time scale than the **r**-space dynamics. Therefore, in a first step the latter one can be disregarded for the calculation of the microscopic carrier dynamics. In this approximation, the spatial coordinates enter only as parameters and the distribution functions and interband polarizations are isotropic in **k** space, i.e., they depend only $k=|\mathbf{k}|$. Furthermore, the confinement of the carriers in the vertical y direction as a consequence of the semiconductor heterostructure reduces also the semiconductor part to a quasi-two-dimensional problem with the spatial coordinate $\mathbf{x}=(x,z)$.

The microscopic dynamics of the distribution functions and the nonlinear polarizations are then governed by the equations of motion

$$\begin{aligned}\frac{\partial}{\partial t}f^{e,h}(k,\mathbf{x},t)=&\,g(k,\mathbf{x},t)+\Lambda^{e,h}(k,\mathbf{x},t)-\tau_{e,h}^{-1}(k)[f^{e,h}(k,\mathbf{x},t)\\&-f_{eq}^{e,h}(k,\mathbf{x},t)]-\Gamma_{sp}(k)f^e(k,\mathbf{x},t)f^h(k,\mathbf{x},t)\\&-\gamma_{nr}f^{e,h}(k,\mathbf{x},t),\end{aligned} \tag{69a}$$

$$\begin{aligned}\frac{\partial}{\partial t}p_{nl}^\pm(k,\mathbf{x},t)=&-[i\overline{\omega(k)}+\tau_p^{-1}(k)]p_{nl}^\pm(k,\mathbf{x},t)+\frac{1}{i\hbar}\Delta\mathcal{U}_{nl}^\pm(\mathbf{x},t)\\&-\frac{1}{i\hbar}\mathcal{U}^\pm(\mathbf{x},t)[f^e(k,\mathbf{x},t)+f^h(k,\mathbf{x},t)],\end{aligned} \tag{69b}$$

with the generation rate

$$\begin{aligned}g(k,\mathbf{x},t)=&\frac{1}{4\hbar}\text{Im}[\mathcal{U}^+(\mathbf{x},t)p^{+*}(k,\mathbf{x},t)+\mathcal{U}^-(\mathbf{x},t)p^{-*}(k,\mathbf{x},t)\\&+\mathcal{U}^+(\mathbf{x},t)p^{-*}(k,\mathbf{x},t)e^{2iK_zz}\\&+\mathcal{U}^-(\mathbf{x},t)p^{+*}(k,\mathbf{x},t)e^{-2iK_zz}],\end{aligned} \tag{70}$$

Im[z] denoting the imaginary part of z, and the ''detuning''

$$\hbar\overline{\omega(k)}=\mathcal{E}_g+\frac{\hbar^2k^2}{2m_r}+\Delta\mathcal{E}^e(\mathbf{x})+\Delta\mathcal{E}^h(\mathbf{x})-\hbar\omega. \tag{71}$$

The ''grating terms'' which involve high-wave-vector modulations can be disregarded in a first approximation due to the presence of diffusion which has the effect to wash out these amplitude gratings. In this approximation, when separating linear and nonlinear polarizations, one retains

$$g(k,\mathbf{x},t)=\frac{\tilde{\chi}''(\omega;k,\mathbf{x},t)}{2\hbar}[|E^+|^2+|E^-|^2] - \frac{1}{2\hbar}\mathrm{Im}[d_{cv}E^+(\mathbf{x},t)p_{nl}^{+}{}^*(k,\mathbf{x},t) + d_{cv}E^-(\mathbf{x},t)p_{nl}^{-}{}^*(k,\mathbf{x},t)-\Delta\mathcal{U}^+(\mathbf{x},t)p^{+}{}^*(k,\mathbf{x},t) - \Delta\mathcal{U}^-(\mathbf{x},t)p^{-}{}^*(k,\mathbf{x},t)]. \tag{72}$$

The spatial transport of the carriers is described by the ambipolar diffusion equation for the carrier density

$$\frac{\partial N}{\partial t}=\left(\frac{\partial}{\partial x}D_f\frac{\partial}{\partial x}+\frac{\partial}{\partial z}D_f\frac{\partial}{\partial z}\right)N+\Lambda(\mathbf{x},t)+G(\mathbf{x},t)-\gamma_{nr}N - W(\mathbf{x},t) \tag{73}$$

with the total generation rate

$$G=\frac{\chi''(\omega;\mathbf{x},t)}{2\hbar}[|E^+|^2+|E^-|^2]-\frac{1}{2\hbar}\mathrm{Im}[E^+(\mathbf{x},t)P_{nl}^{+}{}^*(\mathbf{x},t) + E^-(\mathbf{x},t)P_{nl}^{-}{}^*(\mathbf{x},t)]+\frac{1}{2\hbar d_{cv}}\mathrm{Im}[\Delta\mathcal{U}^+(\mathbf{x},t)P^{+}{}^*(\mathbf{x},t) + \Delta\mathcal{U}^-(\mathbf{x},t)P^{-}{}^*(\mathbf{x},t)]. \tag{74}$$

The spatial transport of the electron-hole plasma is coupled to the k-space dynamics by the chemical potentials in the quasiequilibrium distribution functions for electrons and holes which are obtained from the carrier density determined by the ambipolar diffusion equation (73).

The partial differential equations for the fields and the carrier density have to be supplemented by boundary conditions. A certain fraction of the charge carriers recombines at the surface of the laser structure with the characteristic surface recombination velocity v_{sr} resulting in

$$D_f\frac{\partial N}{\partial x}=\pm v_{sr}N \quad \text{at} \quad x=\mp\frac{W}{2}, \tag{75a}$$

$$D_f\frac{\partial N}{\partial z}=\pm v_{sr}N \quad \text{at} \quad z=0,L. \tag{75b}$$

The semiconductor laser cavity being a Fabry-Perot resonator, the *longitudinal* boundary conditions

$$E^+(x,z=0,t)=-\sqrt{R_1}E^-(x,z=0,t), \tag{76a}$$

$$E^-(x,z=L,t)=-\sqrt{R_2}E^+(x,z=L,t) \tag{76b}$$

represent reflection of the optical fields at the facet mirrors of the laser structure, R_1 and R_2 being the reflection coefficients. Transversely the optical fields are strongly damped in the cladding layers on both sides of the main laser stripe area. The appropriate *transverse* boundary conditions are

$$\frac{\partial E^+}{\partial x}=\pm\alpha_w E^+ \quad \text{at} \quad x=\mp\frac{W}{2}, \tag{77a}$$

$$\frac{\partial E^-}{\partial x}=\pm\alpha_w E^- \quad \text{at} \quad x=\mp\frac{W}{2}, \tag{77b}$$

where $W=w+w_c$, with w_c being the width of the absorbing cladding wings. In practice, with sufficiently wide absorbing layers (≈ 10 μm), the optical fields and thus the polarizations are damped to insignificantly small values.

IV. CONCLUSIONS

Maxwell-Bloch equations for spatially inhomogeneous semiconductor lasers have been derived which include both space dependence and momentum dependence of the charge-carrier distributions and the polarization. Based on a Wigner function representation of the single-particle density matrices, a system of equations for space-dependent distribution functions and interband polarization is obtained which holds for arbitrary inhomogeneities. The momentum and density dependent relaxation rates for the carrier distributions as well as for the polarization include carrier-carrier scattering and carrier-phonon interactions. From the relaxation rates, the ambipolar diffusion coefficient has been obtained. Thus spatial transport and momentum space dynamics are described in a consistent model. Assuming sufficiently slow variations in space and, consequently, a separation of the characteristic time scales between the fast momentum space and the slow real-space dynamics, the latter has been approximated by an ambipolar diffusion model. This microscopic semiconductor model is coupled to Maxwell's equation via the linear and nonlinear polarization. Taking into account the specific structure of a broad-area semiconductor laser, i.e., a Fabry-Perot resonator in the longitudinal direction and strong waveguiding in the vertical direction, the system of equations (67)–(74) describes the dynamics of the counterpropagating optical fields together with the microscopic properties of the spatially inhomogeneous active laser medium. This model accounts for the mechanisms and consequences of spectral as well as spatial hole burning. A detailed numerical analysis of the spatiotemporal dynamics of the broad-area laser structure shown in Fig. 3 will be given in II.

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APPENDIX A

For completeness, we give here the full effective single-particle parts of the equations of motion for the hole distribution function and the interband polarization

$$\frac{\partial}{\partial t}f^h(-\mathbf{k},\mathbf{r})=\frac{1}{i\hbar}\sum_{n,m=0}^{\infty}\frac{i^{n+m}}{2^{n+m}n!m!}\Bigg\{[(-1)^m-(-1)^n]\times\left(\frac{\partial}{\partial\mathbf{k}'}\cdot\frac{\partial}{\partial\mathbf{r}}\right)^n\left(\frac{\partial}{\partial\mathbf{r}'}\cdot\frac{\partial}{\partial\mathbf{k}}\right)^m\times\mathcal{E}^h(-\mathbf{k}',\mathbf{r}')f^h(-\mathbf{k},\mathbf{r})+(-1)^m\left(\frac{\partial}{\partial\mathbf{k}'}\cdot\frac{\partial}{\partial\mathbf{r}}\right)^n\left(\frac{\partial}{\partial\mathbf{r}'}\cdot\frac{\partial}{\partial\mathbf{k}}\right)^m\times\mathcal{U}(\mathbf{k}',\mathbf{r}')p^*(\mathbf{k},\mathbf{r})-(-1)^n\left(\frac{\partial}{\partial\mathbf{k}'}\cdot\frac{\partial}{\partial\mathbf{r}}\right)^n\times\left(\frac{\partial}{\partial\mathbf{r}'}\cdot\frac{\partial}{\partial\mathbf{k}}\right)^m\mathcal{U}^*(\mathbf{k}',\mathbf{r}')p(\mathbf{k},\mathbf{r})\Bigg\}\Bigg|_{\substack{\mathbf{k}'=\mathbf{k}\\ \mathbf{r}'=\mathbf{r}}}, \tag{A1a}$$

$$\frac{\partial}{\partial t}p(\mathbf{k},\mathbf{r})=\frac{1}{i\hbar}\mathcal{U}(\mathbf{k},\mathbf{r})+\frac{1}{i\hbar}\sum_{n,m=0}^{\infty}\frac{i^{n+m}}{2^{n+m}n!m!}\times\Bigg\{(-1)^n\left(\frac{\partial}{\partial\mathbf{k}'}\cdot\frac{\partial}{\partial\mathbf{r}}\right)^n\left(\frac{\partial}{\partial\mathbf{r}'}\cdot\frac{\partial}{\partial\mathbf{k}}\right)^m\times\mathcal{E}^e(\mathbf{k}',\mathbf{r}')p(\mathbf{k},\mathbf{r})+(-1)^m\left(\frac{\partial}{\partial\mathbf{k}'}\cdot\frac{\partial}{\partial\mathbf{r}}\right)^n\times\left(\frac{\partial}{\partial\mathbf{r}'}\cdot\frac{\partial}{\partial\mathbf{k}}\right)^m\mathcal{E}^h(-\mathbf{k}',\mathbf{r}')p(\mathbf{k},\mathbf{r})-(-1)^n\times\left(\frac{\partial}{\partial\mathbf{k}'}\cdot\frac{\partial}{\partial\mathbf{r}}\right)^n\left(\frac{\partial}{\partial\mathbf{r}'}\cdot\frac{\partial}{\partial\mathbf{k}}\right)^m\mathcal{U}(\mathbf{k}',\mathbf{r}')f^h(-\mathbf{k},\mathbf{r})-(-1)^m\left(\frac{\partial}{\partial\mathbf{k}'}\cdot\frac{\partial}{\partial\mathbf{r}}\right)^n\left(\frac{\partial}{\partial\mathbf{r}'}\cdot\frac{\partial}{\partial\mathbf{k}}\right)^m\times\mathcal{U}(\mathbf{k}',\mathbf{r}')f^e(\mathbf{k},\mathbf{r})\Bigg\}\Bigg|_{\substack{\mathbf{k}'=\mathbf{k}\\ \mathbf{r}'=\mathbf{r}}}. \tag{A1b}$$

APPENDIX B

The first-order ($n=1,m=0$ and $n=0,m=1$) equations of motion for the hole distribution function and the polarization read

$$\frac{\partial}{\partial t}f^{h(1)}(\mathbf{k},\mathbf{r})=\frac{1}{\hbar}\left\{\frac{\partial\mathcal{E}^h(-\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}\cdot\frac{\partial f^h(-\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}-\frac{\partial\mathcal{E}^h(-\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}\cdot\frac{\partial f^h(-\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}\right\}+\frac{1}{2\hbar}\left\{\frac{\partial\mathcal{U}(\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}\cdot\frac{\partial p^*(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}-\frac{\partial\mathcal{U}(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}\cdot\frac{\partial p^*(\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}+\frac{\partial\mathcal{U}^*(\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}\cdot\frac{\partial p(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}-\frac{\partial\mathcal{U}^*(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}\cdot\frac{\partial p(\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}\right\}, \tag{B1a}$$

$$\frac{\partial}{\partial t}p^{(1)}(\mathbf{k},\mathbf{r})=\frac{1}{\hbar}\mathcal{U}(\mathbf{k},\mathbf{r})+\frac{1}{2\hbar}\left\{-\left(\frac{\partial\mathcal{E}^e(\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}-\frac{\partial\mathcal{E}^h(-\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}\right)\cdot\frac{\partial p(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}+\left(\frac{\partial\mathcal{E}^e(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}-\frac{\partial\mathcal{E}^h(-\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}\right)\cdot\frac{\partial p(\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}+\left(\frac{\partial f^e(\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}-\frac{\partial f^h(\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}\right)\cdot\frac{\partial\mathcal{U}(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}-\left(\frac{\partial f^e(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}-\frac{\partial f^h(\mathbf{k},\mathbf{r})}{\partial\mathbf{r}}\right)\cdot\frac{\partial\mathcal{U}(\mathbf{k},\mathbf{r})}{\partial\mathbf{k}}\right\}. \tag{B1b}$$

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