Optimum decision scheme with a unitary control process for binary quantum-state signals

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A scheme for distinguishing between binary signals of nonorthogonal coherent states with the minimum average error is proposed. In contrast to the well-known Dolinar scheme, it does not use a feedback process. Instead, it achieves the same minimum error bound by only unitary transformations and photon number counting. It is shown that the required transformation should produce the appropriate Schrödinger-cat states. An example of the Hamiltonian generating such a process is derived from a multiphoton nonlinear optical process. [S1050-2947(96)03410-5]

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I. INTRODUCTION

How one can distinguish nonorthogonal quantum states is one of the fundamental topics in quantum mechanics. Recently it also has received much attention in application to quantum communication, where different quantum states representing certain messages are transmitted and received. If the transmitted states are kept to be mutually orthogonal, all the messages can be accurately extracted at the receiver with von Neumann's standard measurement [1]. However, for many reasons, including noises in the transmitter, energy loss in the channel, and so on, the delivered states in the quantum communication system are nonorthogonal in almost all cases. This causes errors in the extraction of the information from the quantum states. To distinguish the nonorthogonal quantum states optimally is an important key for the new coming technology of quantum communication [2].

We are interested in how one can achieve a low average error in a decision among quantum-state signals. The decision process in quantum mechanics is expressed by the operator defined on the Hilbert space, which is usually called the detection operator. To derive the detection operator achieving the minimum average error probability, one can rely on the quantum detection theory [3–5], which gives an optimization strategy for certain cost functions. According to it, the detection operator is expressed by the probability operator measure (POM), which is a set of nonnegative Hermitian operators satisfying the resolution of the identity and describing a generalized quantum measurement including the decision. It can also give a lower bound for the error probability, which is often called the Helstrom bound. Hereafter we use the term *optimum* for achieving the Helstrom bound in quantum hypothesis testing.

Only in the case of linearly independent and pure-state signals, is the mathematical framework well established. In this case, the optimum detection operator becomes a projection valued measure (PVM) which is an *orthogonal* resolution of the identity on the Hilbert space spanned by the signal states, as the special case of the POM. It is the standard measurement process, which is considered to be, in principle, physically realizable. This meaning is well understood for common observables in physics, such as position, momentum and spin. That is, measurement results correspond to eigenvalues of these observables and measurement procedures are well known. In contrast, it does not seem easy to imagine physical correspondence to the optimum detection operators as the PVM for most linearly independent and pure-state signals even if one is told it is in principle realizable. It is so even for simple and commonly used optical signals $\{|\alpha\rangle,|-\alpha\rangle\}$, which is binary phase-shift keyed (BPSK). In fact, the realization problem of the optimum decision process has been a question, since the quantum detection theory was founded. Only in a few cases, are realizations of the optimum decision process at a level of physical implementation [6,7].

On the other hand, practical methods for achieving a lower error probability, although still not optimum, have been studied for particular quantum-state signals by heuristic approach. We would like to mention two examples concerned with BPSK signals $\{|\alpha\rangle,|-\alpha\rangle\}$. The conventional method is the homodyne detection, where the quadrature-phase amplitude is measured. The obtained error probability is the minimum value in the classical detection theory. (Such a limit in the classical detection theory is sometimes called the standard quantum limit [8,9]. The Helstrom bound is much lower than this.) The first example is an improved scheme, simply by installing an optical Kerr medium in front of the homodyne detector. Usuda and Hirota found that the error probability can be reduced from the one obtained by the conventional homodyne detection [10]. Its mechanism is explained as a quantum interference due to the nonlinear effect of the Kerr medium [11]. Unfortunately, the amount of the error reduction is not so prominent compared to the Helstrom bound. The second example is more remarkable and is known as the Kennedy receiver [12]. In this receiver, the signal states, $\{|\alpha\rangle,|-\alpha\rangle\}$ are transformed to the states $\{|0\rangle,|-2\alpha\rangle\}$ by combining a local oscillating field via a beam splitter. They are then detected by a photon counter. The obtained error probability only differs from the Helstrom bound by at most a factor of two, which may be considered to be near optimum.

Both examples consist of a state controller (Kerr medium

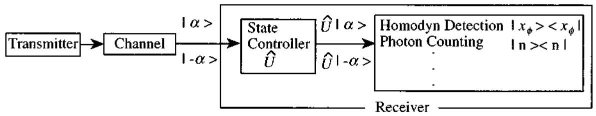

FIG. 1. A scheme of the received quantum-state control. The receiver consists of a state controller described by a unitary operator $\hat{U}$ and a well-known quantum measurement process, such as homodyne detection, photon counting, and so on.

or a beam splitter combining the signal lights with the local oscillating light) and a detector (homodyne detector or photon counter, respectively). This kind of detection scheme is called the received quantum-state control in quantum communication technology [13]. It is based on a practical idea that the performance-improving process is best made at the very last stage, after the main degradation of the signals caused by transmission through long, distant, and noisy channels has occurred. The scheme is summarized in Fig. 1. The state controllers for both cases can be expressed by the unitary operators: $\hat{U}_K = e^{ig\hat{a}^{\dagger 2}\hat{a}^2}$ for the Kerr medium where g is an interaction parameter, $\hat{a}^\dagger$ and $\hat{a}$ are the creation and annihilation operators, respectively, for the signal field, and $\hat{D}(-\alpha) = e^{-\alpha\hat{a}^\dagger + \alpha^*\hat{a}}$ for the Kennedy receiver.

Our question comes out along these studies; *How can one realize the Helstrom bound standing on the above scheme,* i.e., *a unitary process as signal state control plus a well-known physical measurement?*

In recent papers [14,15], the authors gave an answer. That is, for $\{|\alpha\rangle,|-\alpha\rangle\}$, the state control is a unitary transformation expressed by the following operator:

$$\hat{U}(\gamma) = \exp\gamma(|\alpha\rangle\langle-\alpha| - |-\alpha\rangle\langle\alpha|), \tag{1}$$

where γ is a certain interaction parameter. And the photon counting follows this transformation, in which one can distinguish the signals depending on whether the counted photon number is even or odd. The above operator would describe some kind of macroscopic quantum tunneling between $|\alpha\rangle$ and $|-\alpha\rangle$. However, it has been questioned how one can realize this as a practical device. Toward this goal, this paper is intended to study further the structure of the optimum decision process with a unitary transformation as the state control.

A realization method of the optimum decision between $\{|\alpha\rangle,|-\alpha\rangle\}$ was proposed by Dolinar [6]. It is based on a photon counting measurement, including a feedback arrangement where a local oscillating field depending causally on the cumulated results in photon counting is added to the signal field via a beam splitter. Dolinar derived this method as an extension of the Kennedy receiver. Our method is quite a different kind but achieves the same error performance as the Dolinar method.

This paper is organized as follows. In Sec. II, we summarize the theory of the optimum decision for binary signals of linearly independent and pure states. The optimum detection operator is constructed as the PVM. In addition, we briefly mention the studies on the physical realization of the POM, which may be, in general, a nonorthogonal resolution of the identity. And we specify the problem addressed here which focuses exclusively on the realization of the PVM (the orthogonal resolution of the identity), and not on the POM. In Sec. III, confining ourselves to the BPSK signals $\{|\alpha\rangle,|-\alpha\rangle\}$, we will show how to realize the optimum decision process, based on the unitary transformation as the state control and the photon counting. The mathematical structure of the transformation is discussed and a Hamiltonian generating this process is derived based on the nonlinear quantum optical phenomena. In Sec. IV, we derive the optimum detection operators corresponding to the two equivalent realization methods, the Dolinar method and ours. Their mathematical structures will be discussed. In Sec. V, we summarize our results.

II. OPTIMUM DECISION PROCESS

In this section, we describe the optimum decision process for linearly independent binary signals $\{|\rho_1\rangle,|\rho_2\rangle\}$ with the respective prior probabilities ξ_1 and ξ_2 $(=1-\xi_1)$. The overlap $\langle\rho_1|\rho_2\rangle$ $(\equiv\kappa)$ is assumed to be real nonzero value. Let $\mathcal{H}_s$ be a Hilbert space spanned by them. The optimum detection operator is described by a probability operator measure (POM) on $\mathcal{H}_s$, that is, a set of nonnegative definite operators $\hat{\Pi}_i$ summing up to the identity operator $\hat{I}_s$ on $\mathcal{H}_s$,

$$\hat{\Pi}_i \geqslant 0, \quad \hat{\Pi}_1 + \hat{\Pi}_2 = \hat{I}_s, \quad i=1,2. \tag{2}$$

The operator $\hat{\Pi}_i$ represents a quantum-mechanical process by which we decide the received signal is $\hat{\rho}_i(\equiv|\rho_i\rangle\langle\rho_i|)$. When the transmitted signal is actually $\hat{\rho}_i$, we know the probability that the received signal is decided to be $\hat{\rho}_j$ as,

$$P(j|i) = \mathrm{Tr}(\hat{\Pi}_j\hat{\rho}_i). \tag{3}$$

If the operators $\hat{\Pi}_i$ are orthogonal projectors,

$$\hat{\Pi}_i\hat{\Pi}_j = \delta_{ij}\hat{\Pi}_i \tag{4}$$

the POM is called projection valued measure (PVM). Since the signal states here are linearly independent and pure states, the optimum detection operator becomes the PVM (Kennedy's lemma) [16]. That is, the operator $\hat{\Pi}_i$ can be expressed as

$$\hat{\Pi}_i = |\omega_i\rangle\langle\omega_i| \quad (i=1,2) \tag{5}$$

with a complete orthonormal set $\{|\omega_i\rangle\}$ on $\mathcal{H}_s$. This set is determined so as to minimize the decision error probability P_e, which is expressed as,

$$P_e = 1 - \sum_{i=1}^{2} \xi_i \mathrm{Tr}(\hat{\rho}_i\hat{\Pi}_i) \tag{6}$$

$$= 1 - \sum_{i=1}^{2} \xi_i |\langle\omega_i|\rho_i\rangle|^2. \tag{7}$$

The state vectors $|\omega_i\rangle$ are obtained as,

$$|\omega_1\rangle = -[\sqrt{(1+c)/2} + \kappa\sqrt{(1-c)/2(1-\kappa^2)}]|\rho_1\rangle + \sqrt{(1-c)/2(1-\kappa^2)}|\rho_2\rangle, \tag{8}$$

$$|\omega_2\rangle = -\sqrt{(1+c)/2(1-\kappa^2)}|\rho_2\rangle - [\sqrt{(1-c)/2} - \kappa\sqrt{(1+c)/2(1-\kappa^2)}]|\rho_1\rangle, \quad (9)$$

with $c=(1-2\xi_2\kappa^2)/\sqrt{1-4\xi_1\xi_2\kappa^2}$ [17,18]. Consequently, the minimum error probability, that is, the Helstrom bound is

$$P_e(\text{opt}) = \tfrac{1}{2}(1-\sqrt{1-4\xi_1\xi_2\kappa^2}). \quad (10)$$

It is worth mentioning that $|\omega_1\rangle$ and $|\omega_2\rangle$ are eigenvectors of the Hermite operator $\hat{\rho}_2 - \lambda\hat{\rho}_1$ ($\lambda = \xi_1/\xi_2$), with the respective eigenvalues $\omega_1 = 1/2\{1-\lambda-\sqrt{(1+\lambda)^2-4\lambda\kappa^2}\}$ and $\omega_2 = 1/2\{1-\lambda+\sqrt{(1+\lambda)^2-4\lambda\kappa^2}\}$. Actually, since the error probability is written as

$$P_e = \xi_2 - \xi_2 \text{Tr}[(\hat{\rho}_2 - \lambda\hat{\rho}_1)\hat{\Pi}_2], \quad (11)$$

the minimum value of P_e is attained by maximizing the term $\text{Tr}[(\hat{\rho}_2-\lambda\hat{\rho}_1)\hat{\Pi}_2]$. Its maximum is simply the positive eigenvalue ω_2. Therefore $\hat{\Pi}_2$ should be $|\omega_2\rangle\langle\omega_2|$ and $\hat{\Pi}_1 = \hat{I}_s - \hat{\Pi}_2 = |\omega_1\rangle\langle\omega_1|$.

As seen, the measurement with the above $\{\hat{\Pi}_i\}$ is a standard measurement in von Neumann's sense. It is widely accepted that this type of measurement is in principle physically realizable. For example, in the case of binary nonorthogonal signals of the spin-1/2 particle, the Stern-Gerlach type measurement corresponds to $\{\hat{\Pi}_i\}$ [3]. The Hilbert space describing this system is spanned by spin-up and -down states and is two dimensional, which is the same as the signal space. However, in the case of the BPSK signals $\{|\alpha\rangle, |-\alpha\rangle\}$, things are not that simple. The dimension of the physical Hilbert space describing the single mode optical field of which the signal states are made is larger than the number of signals. Then the state evolving from $|\alpha\rangle$ or $|-\alpha\rangle$ may go out of the minimal subspace $\mathcal{H}_s$ spanned by $\{|\alpha\rangle, |-\alpha\rangle\}$ in almost optical phenomena known so far. But the optimum decision process is specified in the two-dimensional space $\mathcal{H}_s$. It brings a difficulty in physical realizations of the optimum decision process. In fact, how to implement the optimum quantum receiver as a practical device has been a question.

In closing this section, we briefly mention the realization problem for nonorthogonal POM. Suppose the spin-1/2 particle, which may take three directions, separated in the angle 120° to each other with the same prior probabilities ([3], Sec. IV). It forms linearly dependent ternary signals $\hat{\rho}_i = 1/2(\hat{I}+\sigma_i)$ with $\sigma_i = \sigma_x\cos\theta_i + \sigma_y\sin\theta_i$ and $\theta_i = (2\pi i)/3$ ($i=1,2,3$), where σ_x and σ_y are the Pauli matrices. The optimum detection operators can be described by the POM $\hat{\Pi}_i = 1/3(\hat{I}+\sigma_i)$, which are nonorthogonal to each other. These POM can be ideally physically realized by constructing the PVM (orthogonal projector) in an extended Hilbert space based on the Naimark extention, where an ancillary system, i.e., independently prepared quantum system, is combined with the original system. A practical recipe was also given by Peres [7]. This kind of realization problem, how to convert a non-orthogonal POM into a PVM physically, is of great interest and only a few simple cases have been studied so far. But it is beyond the scope of this paper. The question addressed here is on the physical realization of the PVM itself, not on the POM. Further we confine ourselves to the case of commonly used binary optical signals.

III. UNITARY CONTROL PROCESS FOR THE OPTIMUM DECISION

In this section, the method proposed in Ref. [14], i.e., the unitary transformation of Eq. (1) plus photon counting, is analyzed in a more general form. Let us assume the prior probabilities ξ_1 and ξ_2 for $|\alpha\rangle$ and $|-\alpha\rangle$, respectively. A convenient algorithm for obtaining the orthonormal set $\{|\omega_i\rangle\}$ constructing the optimum detection operator in the case of M-ary linearly independent signals is proposed by Ban, Osaki, and Hirota [18]. In this method, an orthonormal set $\{|\eta_i\rangle\}$ is initially constructed from the signal states by the Schmidt orthogonalization. Then one can get the set $\{|\omega_i\rangle\}$ by finding an appropriate unitary matrix transforming $\{|\eta_i\rangle\}$ into $\{|\omega_i\rangle\}$. The method in Ref. [14] can be interpreted along this algorithm. (The only exception is that the initial orthonormal set consists of the even and odd coherent states instead of the ones made by the Schmidt orthogonalization.)

Here we shall construct an initial orthonormal set $\{|\eta_i\rangle\}$ in the following way for later convenience. First we transform $\{|\alpha\rangle, |-\alpha\rangle\}$ into $\{|0\rangle, |-2\alpha\rangle\}$ by use of the displacement operator $\hat{D}(-\alpha) = e^{-\alpha\hat{a}^\dagger + \alpha\hat{a}}$, as in the Kennedy receiver (see Introduction). In this section α is assumed to be real. Second we prepare $|\eta_i\rangle$ from $\{|0\rangle, |-2\alpha\rangle\}$ by the Schmidt orthogonalization,

$$|\eta_1\rangle = |0\rangle, \quad (12)$$

$$|\eta_2\rangle = \frac{|-2\alpha\rangle - c_0|0\rangle}{\sqrt{1-c_0^2}}, \quad (13)$$

where c_0 ($=\langle 0|-2\alpha\rangle$) is the real coefficient in the expansion of the state $|-2\alpha\rangle$,

$$|-2\alpha\rangle = \sum_{n=0}^{\infty} c_n |n\rangle, \quad (14)$$

with the photon-number states of the signal optical field $|n\rangle$. Now the Hermite operator in Eq. (11) is expressed in the following form:

$$\begin{aligned}\hat{D}(-\alpha)(\hat{\rho}_2-\lambda\hat{\rho}_1)\hat{D}^\dagger(-\alpha) &= \hat{D}(-\alpha)(\omega_1|\omega_1\rangle\langle\omega_1| + \omega_2|\omega_2\rangle\langle\omega_2|)\hat{D}^\dagger(-\alpha) \\ &= (c_0^2-\lambda)|\eta_1\rangle\langle\eta_1| + (1-c_0^2)|\eta_2\rangle\langle\eta_2| + c_0\sqrt{1-c_0^2} \\ &\quad \times(|\eta_1\rangle\langle\eta_2| + |\eta_2\rangle\langle\eta_1|). \end{aligned} \quad (15)$$

The right-hand side of the last equality can be diagonalized by a simple rotation of the basis $\{|\eta_i\rangle\}$,

$$\hat{D}(-\alpha)(\hat{\rho}_2-\lambda\hat{\rho}_1)\hat{D}^\dagger(-\alpha) = \hat{U}_s^\dagger(\gamma)(\omega_1|\eta_1\rangle\langle\eta_1| + \omega_2|\eta_2\rangle\langle\eta_2|)\hat{U}_s(\gamma), \quad (16)$$

with the unitary operator in the form,

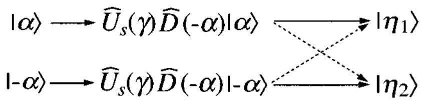

FIG. 2. The channel model for the BPSK signals $\{|\alpha\rangle,|-\alpha\rangle\}$. The signal states are transformed by the unitary process described by $\hat{U}_s(\gamma)\hat{D}(-\alpha)$. They are then detected by the PVM $|\eta_1\rangle\langle\eta_1|$ and $|\eta_2\rangle\langle\eta_2|$. The solid lines represent the correct decision channels, while the dotted lines the error channels.

$$\hat{U}_s(\gamma)=\exp\gamma(|\eta_1\rangle\langle\eta_2|-|\eta_2\rangle\langle\eta_1|), \tag{17}$$

where the parameter γ is chosen as

$$\gamma=-\tan^{-1}\left(\frac{\sqrt{1-4\xi_1\xi_2\kappa^2}-1+2\xi_2\kappa^2}{\sqrt{1-4\xi_1\xi_2\kappa^2}+1-2\xi_2\kappa^2}\right)^{1/2}. \tag{18}$$

By substituting Eq. (16) into Eq. (11), the error probability is written as

$$P_e(\text{opt})=\xi_1|\langle\eta_2|\hat{U}_s(\gamma)\hat{D}(-\alpha)|\alpha\rangle|^2 + \xi_2|\langle\eta_1|\hat{U}_s(\gamma)\hat{D}(-\alpha)|-\alpha\rangle|^2. \tag{19}$$

It corresponds to the channel model shown in Fig. 2. The solid lines represent the correct decision channels while the dotted lines the error channels. The detection process is now decomposed into the unitary transformation by $\hat{U}_s(\gamma)\hat{D}(-\alpha)$ and the standard measurement by the orthogonal projectors $|\eta_1\rangle\langle\eta_1|$ and $|\eta_2\rangle\langle\eta_2|$. The operator $\hat{U}_s(\gamma)$ transforms the signal states, $\hat{D}(-\alpha)|\alpha\rangle$ $(=|0\rangle)$ and $\hat{D}(-\alpha)|\alpha\rangle$ $(=|-2\alpha\rangle)$ into the superposition states between $|\eta_1\rangle$ and $|\eta_2\rangle$ as,

$$\hat{U}_s(\gamma)|0\rangle=\cos\gamma|\eta_1\rangle-\sin\gamma|\eta_2\rangle, \tag{20}$$

$$\hat{U}_s(\gamma)|-2\alpha\rangle=(c_0\cos\gamma+\sqrt{1-c_0^2}\sin\gamma)|\eta_1\rangle + (-c_0\sin\gamma+\sqrt{1-c_0^2}\cos\gamma)|\eta_2\rangle. \tag{21}$$

In the decision of these states, one should note that the state $|\eta_1\rangle$ is the vacuum state while the state $|\eta_2\rangle$ only contains finite number of photons $\{|1\rangle,|2\rangle\cdots\}$. It means that the decision by $\{|\eta_1\rangle\langle\eta_1|,|\eta_2\rangle\langle\eta_2|\}$ can be physically replaced by $\{|0\rangle\langle 0|,\Sigma_{n=1}^{\infty}|n\rangle\langle n|\}$, that is, the photon counting judging the photons are registered or not. This corresponds to the orthogonal resolution of the identity $\hat{I}$ on the whole Hilbert space describing the signal optical field,

$$|0\rangle\langle 0|+\sum_{n=1}^{\infty}|n\rangle\langle n|=\hat{I}. \tag{22}$$

The error probability is rewritten as,

$$P_e(\text{opt})=\xi_1\sum_{n=1}^{\infty}|\langle n|\hat{U}_s(\gamma)\hat{D}(-\alpha)|\alpha\rangle|^2 + \xi_2|\langle 0|\hat{U}_s(\gamma)\hat{D}(-\alpha)|-\alpha\rangle|^2. \tag{23}$$

We would like to call the process described by $\hat{U}_s(\gamma)\hat{D}(-\alpha)$ the unitary control process. By this process, one can appropriately control the input signals for reaching the optimum decision in well-known measurement such as photon counting.

Now the problem is the unitary process described by $\hat{U}_s(\gamma)$. It represents a simple rotation of the state vectors in the plane spanned by $\{|0\rangle,|-2\alpha\rangle\}$. Its generator $|\eta_1\rangle\langle\eta_2|-|\eta_2\rangle\langle\eta_1|$ describes a transition process between the states $|\eta_1\rangle$ and $|\eta_2\rangle$, each of which has macroscopic quantum coherence. Compare them with the nonorthogonal signals of spin-1/2 particles with two orientations of polarization. Then one can take $\{|\eta_1\rangle,|\eta_2\rangle\}$ as the spin-up and -down states $\{|\uparrow\rangle,|\downarrow\rangle\}$. And the rotation in the signal plane can be caused by irradiating the spin particles with certain alternating magnetic fields. Unlike this spin-1/2 case, the transition process between the states $|\eta_1\rangle$ and $|\eta_2\rangle$ of the present case does not seem so trivial.

It produces a so-called Schrödinger-cat state, that is, a superposition state of distinct quantum states with macroscopic coherence $|\eta_1\rangle$ and $|\eta_2\rangle$. In the works so far, the optical Kerr effect is commonly known as a unitary process producing the Schrödinger-cat state [19], whose unitary evolution is described by the operator $\hat{U}_K=e^{ig\hat{a}^{\dagger 2}}\hat{a}^2$, as mentioned in the introduction. If one inputs the coherent state $|\alpha\rangle$ into the Kerr medium with $g=\pi/2$, the output state is in a superposition of $|\alpha\rangle$ and $|-\alpha\rangle$ with the same probability amplitudes. However, one can find no particular merits for quantum communication in this Schrödinger-cat state. (The role of the Schrödinger-cat state in quantum communication is systematically studied in Ref. [20].) Rather for certain regions of small values of g, this effect can cause the reduction in decision error probability, being installed in front of the homodyne detector as explained in the introduction, although the output state from the Kerr medium is not like the Schrödinger-cat state any more.

In each of the Schrödinger-cat states required here, either of two coefficients are very small, $\sin\gamma$ in Eq. (20) and $c_0\cos\gamma+\sqrt{1-c_0^2}\sin\gamma$ in Eq. (21). They are directly related to the minimum error probability in such a way that

$$P_e(\text{opt})=\xi_1\sin^2\gamma+\xi_2(c_0\cos\gamma+\sqrt{1-c_0^2}\sin\gamma)^2. \tag{24}$$

As the generator $|\eta_1\rangle\langle\eta_2|-|\eta_2\rangle\langle\eta_1|$ producing such superposition states, we shall consider here the Hamiltonian representing some nonlinear optical medium. In fact, it can be expanded in a power series in the creation and annihilation operators as

$$|\eta_1\rangle\langle\eta_2|-|\eta_2\rangle\langle\eta_1|=\frac{c_0}{\sqrt{1-c_0^2}}\left(\sum_{l=0}^{\infty}\frac{(-\hat{a}^\dagger)^l\hat{a}^l}{l!} \times\sum_{n=1}^{\infty}\frac{(-2\alpha\hat{a})^n}{n!}-\text{h. c.}\right). \tag{25}$$

Unlike the Hamiltonian for the optical Kerr medium, it includes a power series of nonlinear terms in $\hat{a}$ and $\hat{a}^\dagger$ up to infinite order. But in some cases we do not necessarily require all of these terms. Suppose that the signal power $|\alpha|^2$ is small, where the decision error due to nonorthogonality becomes more serious. In such a weak signal case, the transformed state $|-2\alpha\rangle$ might be replaced by the following finite sum of the Fock states in a good approximation,

$$|\rho_2'\rangle = \sum_{n=0}^{M} c_n'|n\rangle, \quad c_n' = \frac{c_n}{\sqrt{\sum_{n=0}^{M} c_n^2}}, \tag{26}$$

where M is taken to be large enough for the given amplitude $|\alpha|$ to ensure that

$$\sum_{n=M+1}^{\infty} c_n^2 \ll P_e(\mathrm{opt}). \tag{27}$$

The new orthogonal bases are

$$|\eta_1'\rangle = |0\rangle, \tag{28}$$

$$|\eta_2'\rangle = \frac{|\rho_2'\rangle - c_0'|0\rangle}{\sqrt{1-c_0'^2}}. \tag{29}$$

And let us denote the subspace spanned by them as $\mathcal{H}_s'$. The generator becomes

$$|\eta_1'\rangle\langle\eta_2'| - |\eta_2'\rangle\langle\eta_1'| = \sum_{l=0}^{\infty} \frac{(-\hat{a}^\dagger)^l \hat{a}^l}{l!} \sum_{n=1}^{M} d_n \frac{\hat{a}^n}{\sqrt{n!}} - \text{h. c.}, \tag{30}$$

where $d_n = (c_n'/\sqrt{1-c_0'^2})$, This is still a power series of nonlinear terms up to infinite order.

One should note that the optimum decision process needs not to be confined in the two-dimensional space. For example, adding terms like $\Sigma_{n\geqslant 3}\Sigma_{n'\geqslant 1}(c_{nn'}|\omega_n\rangle\langle\omega_{n'}| + c_{nn'}^*|\omega_{n'}\rangle\langle\omega_n|)$ with $\{|\omega_i\rangle | i=3,4,\ldots\}$ being orthogonal to $\{|\omega_1\rangle, |\omega_2\rangle\}$, to the PVM, $\hat{\Pi}_1$ and $\hat{\Pi}_2$ in Eq. (5) never affects the optimum decision, that is, results in the same error probability. The new PVM with these additional terms represent physically different processes from the original ones, although their effects on the signal states are completely equivalent. Thus, in the case that the dimension of the physical Hilbert space of the optical field is larger than the total number of signals, the PVM describing the optimum decision strategy is not unique. Therefore, there are various kind of physically different realization of the optimum decision strategy.

This fact enables us to simplify the generator in Eq. (30), which is derived by the direct translation of the projectors $|0\rangle\langle n|$ and $|0\rangle\langle n|$ into the expression with $\hat{a}$ and $\hat{a}^\dagger$ in the whole Fock space. So the first two summations in the right-hand side of Eq. (30) transform *any* state in the whole Fock space into the vacuum state $|0\rangle$. But we are not concerned with the Fock state $|n\rangle$ with $n > M$, since they are not the constituents of the signal states. In other words, they do not enter the receiver, therefore they are not necessarily transformed into the vacuum state $|0\rangle$. Then the higher-order terms in the right-hand side of Eq. (30) can be neglected, and the generator can be replaced by the following operator $\hat{P}$,

$$\hat{P} = \sum_{l=0}^{M} \frac{(-\hat{a}^\dagger)^l \hat{a}^l}{l!} \sum_{n=1}^{M} d_n \frac{\hat{a}^n}{\sqrt{n!}} - \text{h. c.}, \tag{31}$$

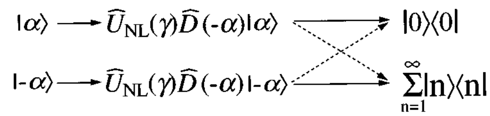

FIG. 3. The channel model for the BPSK signals $\{|\alpha\rangle, |-\alpha\rangle\}$ modified from the one in Fig. 2 by the physical consideration. The unitary control process for the signal states is described by $\hat{U}_{NL}(\gamma)\hat{D}(-\alpha)$. They are then detected by the photon counter. The solid lines represent the correct decision channels, while the dotted lines the error channels.

The new generator $\hat{P}$ is related to the old one $|\eta_1'\rangle\langle\eta_2'| - |\eta_2'\rangle\langle\eta_1'|$ in the following way:

$$\hat{P} = |\eta_1'\rangle\langle\eta_2'| - |\eta_2'\rangle\langle\eta_1'| + \hat{R}, \tag{32}$$

with

$$\hat{R} = \sum_{n=M+1}^{\infty} \sum_{k=1}^{M} c_{n+k,n}(|n\rangle\langle n+k| - |n+k\rangle\langle n|), \tag{33}$$

where

$$c_{n+k,n} = (-1)^M \frac{(n-1)!}{M!(n-1-M)!} \sqrt{[(n+k)!]/(n!k!)}\, d_k. \tag{34}$$

The generator $\hat{P}$ is directly related to the Hamiltonian for a nonlinear optical medium which can cause multiphoton processes up to $3M$th order simultaneously. Then the unitary operator $\hat{U}_s(\gamma)$ can be replaced by $\hat{U}_{NL}(\gamma) = e^{\gamma\hat{P}}$. So one possible way realizing the optimum decision process is the unitary control process by $\hat{U}_{NL}(\gamma)\hat{D}(\alpha)$ for the signal states and the successive photon counting. The corresponding channel model is depicted in Fig. 3. The definition of the lines are the same as in Fig. 2.

Let us estimate the achievable error probabilities after neglecting higher-order terms in our detection scheme. Figure 4 shows the performance of different truncations of M as well as the Helstrom bound and the error probability in the Kennedy receiver. All of them are normalized by the standard quantum limit (SQL) which is reached by the conventional homodyne detection where the quadrature phase amplitude $\hat{X} = 1/2(\hat{a} + \hat{a}^\dagger)$ is measured, and given by $P_e(\mathrm{SQL}) = 1/2\,\mathrm{erfc}(\sqrt{2N_s})$ with $\mathrm{erfc}(x) = (2/(\sqrt{\pi})\int_x^\infty dt\, e^{-t^2}$. It represents the minimum bound in classical detection theory. The horizontal axis is the signal power $N_s = |\alpha|^2$. As seen, the Kennedy receiver which merely shifts the signals $\{|\alpha\rangle, |-\alpha\rangle\}$ to $\{|0\rangle, |-2\alpha\rangle\}$ and then counts the photon number, improves the performance drastically. For $N_s > 1$, it differs from the Helstrom bound by a factor of two. To eliminate this difference, we should install another transformation represented by $\hat{U}_{NL}(\gamma)$ between the above displacement operation and the photon counting in the Kennedy receiver. The effect of neglecting higher-order terms appears in the larger side of the signal power. Nonlinearity M required for the accurate transformation becomes small as the signal power decreases. For $N_s < 0.8$, $M = 4$ seems enough. Figure 5 shows how the error probabilities increase as M

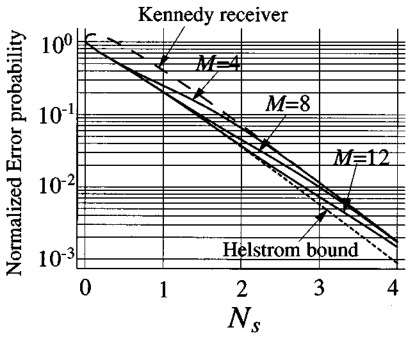

FIG. 4. The achievable error probabilities after neglecting higher-order terms in the unitary transformation. The horizontal axis is the signal power $N_s = |\alpha|^2$ in the BPSK signals $\{|\alpha\rangle, |-\alpha\rangle\}$. The error probabilities corresponding to $M=4$, 8, and 12 are shown by the solid lines. The dotted and dashed lines correspond to the Helstrom bound and the error probability of the Kennedy receiver. All of them are normalized by the SQL which is given in the text. The effect of neglecting higher-order terms than M in the unitary transformation $\hat{U}_{NL}(\gamma)$ is revealed in the larger side of the signal power.

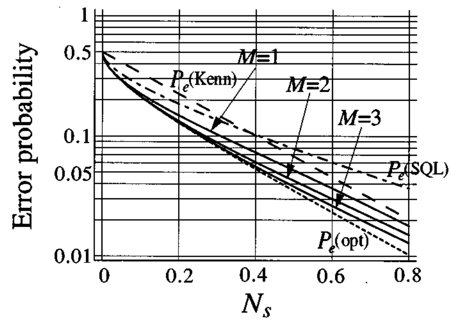

FIG. 5. The comparison of the error performance in a weak signal region. The error probabilities for $M=1$, 2, and 3 are shown by the solid lines. Physical realizations of the generator $\hat{P}$ for $M=1$ and 2 are mentioned in the text. The Helstrom bound $[P_e(\mathrm{opt})]$, the error probability of the Kennedy receiver $[P_e(\mathrm{Kenn})]$ and the SQL $[P_e(\mathrm{SQL})]$ are represented by the dotted, dashed, and one-dotted lines, respectively.

goes down from 3 to 1 (solid lines) in such a weak signal region. The dotted, dashed, and one-dotted lines represent the Helstrom bound $[P_e(\mathrm{opt})]$, the error probability by the Kennedy receiver $[P_e(\mathrm{Kenn})]$ and the SQL $[P_e(\mathrm{SQL})]$, respectively. In the lowest case $M=1$, the deviation from the Helstrom bound is apparent, however, it can still overcome the Kennedy receiver. The corresponding generator $\hat{P}$ can be realized as a passive medium with certain nonlinear susceptibilities up to the third order. Such a kind of Hamiltonian has already been proposed by Kilin and Horoshko [21] in the context of a one-photon Fock state generation from the vacuum state input. In the case of $M=2$, one can get some hints for realizing $\hat{P}$ in the work by Leoński and Tanaś [22], where two-photon Fock state generation by similar nonlinear optical phenomena is proposed. For larger signal power, the cavity QED scheme with the two-channel Raman interaction, proposed recently by Law and Eberly [23], seems useful. It allows the vacuum state to evolve to an arbitrarily prescribed superposition of Fock states. If such a pure-state evolution is truly attainable in the cavity QED scheme, i.e., spontaneous decay of atomic level and cavity damping can be ignored as assumed in Ref. [23], this scheme can be applicable for the unitary control process required in our detection method.

IV. THE OPTIMUM DETECTION OPERATOR FOR BINARY COHERENT-STATE SIGNALS

As mentioned in the introduction, Dolinar proposed the practical receiver, for the first time, which can achieve the Helstrom bound for binary coherent-state signals [6]. Both the Dolinar receiver and our method are not derived by direct use of the quantum detection theory. Actually the resulting detection operators are quite different from the one obtained by the quantum detection theory. It might be meaningful to discuss here the relation among them.

At first, the Dolinar method for $\{|\alpha\rangle, |-\alpha\rangle\}$ is briefly reviewed based on Holevo's formalism [24]. In the receiver, the signal field is combined by a local oscillating field (LOF) through a beam splitter, and is then detected by a photon counter. The LOF is set depending causally on the actual output of the photon counter, and is quickly changed every time after a single photon is registered. Since the probability that a single photon is detected during certain interval depends on the intensity of the field coming into the photon counter, the probability of a certain cumulative outcome can be controlled by adjusting the LOF. It is optimally chosen so as to maximize the probability that even number photons are detected under the signal $|\alpha\rangle$. This kind of receiver may be identified with the set of all possible feedback functions with the LOF. Let us denote the cumulative outcome of n photons as $\vec{t}_n = (t_n, \ldots, t_1)$ where a single photon is registered only at each time of $t_1, \ldots, t_n$ during a period T. The effect of the feedback field during the interval $[t_k, t_{k+1})$ can be described by the following Hamiltonian:

$$\hat{H}_I(t) = i\hbar \xi(t;\vec{t}_k)(-\hat{a}\alpha^* e^{i\omega t} + \hat{a}^\dagger \alpha e^{-i\omega t}), \tag{35}$$

where $\xi(t;\vec{t}_k)$ represents the beating effect by the feedback field, and ω being the angular frequency of the signal field. The Heisenberg evolution due to the Hamiltonian $\hbar\omega\hat{a}^\dagger\hat{a} + \hat{H}_I(t)$ is expressed as follows:

$$\hat{a}(t) = \hat{a}_0(t) + \zeta(t;\vec{t}_k)e^{-i\omega t}, \tag{36}$$

where the first term represents the free evolution $\hat{a}_0(t) = \hat{a}\mathrm{e}^{-i\omega t}$ and

$$\zeta(t;\vec{t}_k) = \alpha \int_{t_k}^{t} d\tau \text{ to } \xi(\tau;\vec{t}_k). \tag{37}$$

By defining the intensity operator as

$$\hat{\Lambda}(t;\vec{t}_k) \equiv \mu[\hat{a}^\dagger + \zeta(t;\vec{t}_k)][\hat{a} + \zeta(t;\vec{t}_k)], \tag{38}$$

where μ is the interaction parameter with the detector, the probability of n photons being counted during the interval T is given by

$$P_i(n;T) = \begin{cases} \langle\rho_i|\hat{\Gamma}_1(T)|\rho_i\rangle, & (n \text{ is even}) \\ \langle\rho_i|\hat{\Gamma}_2(T)|\rho_i\rangle, & (n \text{ is odd}) \end{cases} \tag{39}$$

with

$$\hat{\Gamma}_1(T) = :\exp\left\{ -\int_0^T dt\ \hat{\Lambda}(t;0) \right\} + \sum_{k=1}^{\infty} \int_0^T dt_{2k} \int_0^{t_{2k}} dt_{2k-1} \cdots \int_0^{t_2} dt_1 \times \hat{\Lambda}(t_{2k};\vec{t}_{2k-1}) \cdots \hat{\Lambda}(t_1;0) \cdot \exp\left\{ -\int_{t_{2k}}^T dt\ \hat{\Lambda}(t;\vec{t}_{2k}) - \cdots - \int_{t_1}^{t_2} dt\ \hat{\Lambda}(t;t_1) - \int_0^{t_1} dt\ \hat{\Lambda}(t;0) \right\}:, \tag{40}$$

$$\hat{\Gamma}_2(T) = :\sum_{k=1}^{\infty} \int_0^T dt_{2k-1} \int_0^{t_{2k-1}} dt_{2k-2} \cdots \int_0^{t_2} dt_1 \times \hat{\Lambda}(t_{2k-1};\vec{t}_{2k-2}) \cdots \hat{\Lambda}(t_1;0) \cdot \exp\left\{ -\int_{t_{2k-1}}^T dt \times \hat{\Lambda}(t;\vec{t}_{2k-1}) - \cdots - \int_{t_1}^{t_2} dt\ \hat{\Lambda}(t;t_1) - \int_0^{t_1} dt\ \hat{\Lambda}(t;0) \right\}:, \tag{41}$$

where $i=1$ or 2, $|\rho_1\rangle = |\alpha\rangle$ and $|\rho_2\rangle = |-\alpha\rangle$, and : : indicates the normally ordered expression. These operators satisfy the resolution of the identity $\hat{\Gamma}_1 + \hat{\Gamma}_2 = \hat{I}$, where $\hat{I}$ is the unit operator of the whole Hilbert space. One should note that the above equations become inapplicable for an unbounded time interval. Here we assume a small value of T. For the optimum decision, $\xi(t;\vec{t}_k)$ and equivalently $\zeta(t;\vec{t}_k)$ via Eq. (37) is set in such a way that the phase is changed in π just after t_k, and the absolute intensity decreases monotonically as t increases,

$$\zeta(t;\vec{t}_k) = (-1)^{k+1} \frac{\alpha}{f(t)}, \tag{42}$$

$$f(t) = \sqrt{1 - 4\xi_1\xi_2 \exp(-4\mu|\alpha|^2 t)}. \tag{43}$$

Then the intensity operators are classified in the following two kinds:

$$\hat{\Lambda}_+(t) \equiv \hat{\Lambda}(t;\vec{t}_{2k}) = \mu\left[\hat{a}^\dagger - \frac{\alpha^*}{f(t)}\right]\left[\hat{a} - \frac{\alpha}{f(t)}\right], \tag{44}$$

$$\hat{\Lambda}_-(t) \equiv \hat{\Lambda}(t;\vec{t}_{2k-1}) = \mu\left[\hat{a}^\dagger + \frac{\alpha^*}{f(t)}\right]\left[\hat{a} + \frac{\alpha}{f(t)}\right]. \tag{45}$$

And it is easy to see that the operators $\hat{\Gamma}_1$ and $\hat{\Gamma}_2$ satisfy the following differential equations:

$$\frac{d}{dT}\hat{\Gamma}_1(T) = :-[\hat{\Lambda}_+(T) + \hat{\Lambda}_-(T)]\hat{\Gamma}_1(T) + \hat{\Lambda}_-(T):, \tag{46}$$

$$\frac{d}{dT}\hat{\Gamma}_2(T) = :-[\hat{\Lambda}_+(T) + \hat{\Lambda}_-(T)]\hat{\Gamma}_2(T) + \hat{\Lambda}_+(T):. \tag{47}$$

We will assume the initial condition

$$\hat{\Gamma}_1(0) = \begin{cases} \hat{I}, & \text{for } \xi_1 > \xi_2, \\ \frac{1}{2}\hat{I}, & \text{for } \xi_1 = \xi_2, \\ 0, & \text{for } \xi_1 < \xi_2, \end{cases} \tag{48}$$

and consider the case $\xi_1 > \xi_2$, from which it is easy to solve the problem for other cases. With the initial condition $\hat{\Gamma}_2(0) = 0$, the solution of the equation for $\hat{\Gamma}_2(T)$ is expressed as

$$\hat{\Gamma}_2(T) = :\int_0^T dt\ \hat{\Lambda}_+(t) \exp\left(-\int_t^T d\tau [\hat{\Lambda}_+(\tau) + \hat{\Lambda}_-(\tau)] \right):. \tag{49}$$

It leads to the following results:

$$\hat{\Gamma}_1(T) = :\frac{1}{2}\left(\hat{I} + \frac{\xi_1 - \xi_2}{f(T)} e^{-2(\hat{a}^\dagger\hat{a} + |\alpha|^2)} \right) + \frac{\alpha\hat{a}^\dagger + \alpha^*\hat{a}}{f(T)} \sum_{n=1}^{\infty} \frac{\{-2(\hat{a}^\dagger\hat{a} + |\alpha|^2)\}^{n-1}}{n!}:, \tag{50}$$

$$\hat{\Gamma}_2(T) = :\frac{1}{2}\left(\hat{I} - \frac{\xi_1 - \xi_2}{f(T)} e^{-2(\hat{a}^\dagger\hat{a} + |\alpha|^2)} \right) - \frac{\alpha\hat{a}^\dagger + \alpha^*\hat{a}}{f(T)} \sum_{n=1}^{\infty} \frac{\{-2(\hat{a}^\dagger\hat{a} + |\alpha|^2)\}^{n-1}}{n!}:. \tag{51}$$

In the ideal case of $\mu T = 1$, it can be easily seen that $\hat{\Gamma}_1$ and $\hat{\Gamma}_2$ are equivalent on the signal space $\mathcal{H}_s$, to the detection operators $\hat{\Pi}_1$ and $\hat{\Pi}_2$, respectively, in Eqs. (5), (8), and (9) with $|\rho_1\rangle = |\alpha\rangle$ and $|\rho_2\rangle = |-\alpha\rangle$.

On the other hand, the detection operator in our scheme in Sec. III are obtained as

$$\hat{\Upsilon}_i = \hat{D}^\dagger(-\alpha)\hat{U}_s^\dagger(\gamma)\hat{\Pi}_i'\hat{U}_s(\gamma)\hat{D}(-\alpha) \quad (i=1,2), \tag{52}$$

where $\hat{\Pi}_1' = |0\rangle\langle 0|$ and $\hat{\Pi}_2' = \Sigma_{n=1}^{\infty}|n\rangle\langle n|$. It is easy to get

$$\hat{\Upsilon}_1 = \hat{I}_s \frac{f - 1 + 2\xi_1\kappa^2}{2f} + \frac{1}{2f} :\{(1 + 2\xi_1\kappa^2) e^{-2\alpha(\hat{a}^\dagger - \alpha^*)} \times e^{-(\hat{a}^\dagger - \alpha^*)(\hat{a} - \alpha)} + \text{h.c.}\}:, \tag{53}$$

$$\hat{\Upsilon}_2 = \hat{I} - \hat{\Upsilon}_1, \tag{54}$$

where $f \equiv f(1/\mu)$ [see Eq. (43)]. Here note that $\hat{I}_s$ and $\hat{I}$ are the unit operators of the signal and the whole Hilbert spaces, respectively. Generally, $\hat{\Upsilon}_i$ is different from $\hat{\Gamma}_i$ in matrix components on the space outside $\mathcal{H}_s$. For example, consider the other coherent state $|\alpha'\rangle (\alpha' \neq \pm\alpha)$, and make the orthonormal basis $|\eta_3\rangle$ as,

$$|\eta_3\rangle = \frac{\hat{D}(-\alpha)|\alpha'\rangle - |\eta_1\rangle\langle\eta_1|\eta_3\rangle - |\eta_2\rangle\langle\eta_2|\eta_3\rangle}{\sqrt{1-|\langle\eta_1|\eta_3\rangle|^2-|\langle\eta_2|\eta_3\rangle|^2}}. \tag{55}$$

Then all of the components $\{\langle\eta_i|\hat{\Upsilon}_1|\eta_3\rangle\}$ $(i=1,2,3)$ are zero, while $\{\langle\eta_i|\hat{\Gamma}_1|\eta_3\rangle\}$ have, in general, nonzero value. Thus $\{\hat{\Gamma}_i\}$ and $\{\hat{\Upsilon}_i\}$ are two different kinds of extension into the whole Hilbert space from $\{\hat{\Pi}_i\}$ on the two-dimensional space $\mathcal{H}_s$.

As long as the error performance for the BPSK signals is concerned, the Kennedy receiver seems enough in practical application as shown in the preceding section. The detection operator representing the Kennedy receiver is expressed by the following PVM [25]:

$$\hat{\Pi}_1^K = |\alpha\rangle\langle\alpha|, \tag{56}$$

$$\hat{\Pi}_2^K = \hat{I} - |\alpha\rangle\langle\alpha|. \tag{57}$$

Compared to this detection operator, $\{\hat{\Gamma}_i\}$ and $\{\hat{\Upsilon}_i\}$ seems much more difficult not only mathematically but in physical realization. In lowering the error level of the Kennedy receiver to the Helstrom bound (the gap is only a factor), a significant difficulty seems to be there. This difficulty directly appears in the process represented by $\hat{U}_{NL}(\gamma)$, which is the only difference between our scheme and the Kennedy receiver.

Finally, it could be an interesting question whether the operators $\{\hat{\Gamma}_i\}$ representing the Dolinar receiver belong to the class of operators of the form $\hat{U}^\dagger\hat{\Pi}_i'\hat{U}$ with a unitary operator $\hat{U}$ just like $\hat{\Upsilon}_i$, although they seem physically different. The former is based on the feedback measurement so the detection process is stochastic, while the latter keeps the unitarity till the last step measurement by $\{\hat{\Pi}_i'\}$.

V. SUMMARY

The decision process with a minimum error for BPSK signals is discussed. Since they are linearly independent, the optimum detection operator becomes the PVM on the two-dimensional Hilbert space spanned by the signal states. However, its physical realization is not trivial. In this paper, we demonstrated a physical scheme consisting of the unitary transformations of the signal states and the simple photon counting. The required unitary control process is supposed to generate the appropriate Schrödinger-cat states. The example of the Hamiltonian to generate such a process was derived from a nonlinear optical phenomenon. But how to implement this kind of nonlinear effect in the practical device is the remaining problem in quantum optics.

Our scheme was also compared to the Dolinar method by deriving the detection operator as the PVM defined on the whole Hilbert space of the signal optical field. Although, as is expected, both detection operators coincide on the minimal subspace spanned by the binary signal states, they are not equivalent to each other physically, therefore they lead to different forms on the whole space. The considerations there give rise to a question of whether the PVM for the Dolinar receiver can be expressed in the similar form to our scheme as $\hat{U}^\dagger\hat{\Pi}_i'\hat{U}$ with $\hat{\Pi}_1'=|0\rangle\langle 0|$, $\hat{\Pi}_2'=\Sigma_{n=1}{}^{\infty}|n\rangle\langle n|$ and a certain unitary operator $\hat{U}$.

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