Two-mode quantum-optical state measurement: Sampling the joint density matrix

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We propose schemes for measuring the joint quantum state of a two-mode optical field using balanced homodyne detection, thereby generalizing earlier single-mode schemes. We distinguish between two cases—the case where the two modes are spatially separable and the case where they are nonseparable. For each case we present a generalization of the direct sampling method of state reconstruction and point out that its advantages over inverse Radon methods are even greater here than in the single-mode case. In the case of two modes that cannot be separated we propose a scheme in which a single balanced homodyne detector is used for data collection. [S1050-2947(96)00309-5]

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One of the essential concepts in physics is that of the state of a system, whether particle or field. Recently quantum states, describable as arbitrary superpositions of eigenstates, have become accessible to measurement in simple systems—the angular state of an electron in an hydrogen atom; [1] the vibrational state of a molecular wave packet; [2] or the state of a single mode of an optical field [3]. In the optical and molecular cases, which correspond to continuous degrees of freedom (DOF), only a single DOF was measured. A set of variables relating to the DOF was identified, having the property that the set of their separately measured probability densities provides sufficient information to enable the full reconstruction of the quantum state for this DOF. Because the algorithm for reconstruction is analogous to classical tomography (inverse Radon transform) [3–5], we refer to such a set of variables as ''tomographically complete'' [6].

One of the important applications of state measurement of an optical field is the determination of photon-number statistics. In this case balanced homodyne detection (BHD) is used to measure a set of tomographically complete variables. Because the photodetectors used in this measurement technique can have high quantum efficiency ($>99\%$ [7]), the photon statistics can be measured without the degradation that occurs when using standard photomultiplers and conventional photon counting. Recently the oscillations of photon number probability $p(n)$ in a squeezed vacuum state have been measured using optical homodyne tomography, a result which could not be obtained by conventional means [8]. Another powerful feature of homodyne detection is that it is mode selective. Only the signal light that is in the space-time mode defined by the LO field contributes to the measurement [9]. Using this concept, Munroe *et al.* have time resolved photon statistics with subpicosecond sampling time, two orders of magnitude better than possible with conventional methods [10].

In this paper we propose schemes for measuring the joint quantum state of a pair of optical modes, thereby generalizing earlier single-mode schemes. The modes are defined by the measuring apparatus (balanced homodyne detector) and are (nonmonochromatic) spatial-temporal modes, defined with a particular spatial, temporal, and spectral form. The reconstruction schemes distinguish between two possible cases—the case where the two modes are spatially separable [11] and the case where they are nonseparable. An important application of two-mode state reconstruction using balanced homodyne detection is the determination of two-time photon-number correlations on ultrafast time scales.

The case in which the two modes of interest are separable is straightforward to handle—simply send the two separated modes into two separate BHDs and measure the joint statistics of their outputs. In a BHD setup, the signal field and a reference (LO) field are mixed coherently on a 50/50 beam splitter and detected by high-quantum-efficiency photodiodes, the outputs of which are time integrated and subtracted. The two BHDs give tomographically complete measurements for determining the Wigner function for the two-mode field, as first discussed in Ref. [5] and generalized by Kuhn, Welsch, and Vogel [12]. The signal field is represented by a photon-flux amplitude $\hat{\Phi}_S^{(+)}(\mathbf{r},t)$ which, for a quasimonochromatic field, equals the electric-field operator $\hat{E}_S^{(+)}(\mathbf{r},t)$ scaled by $(c/2\pi\hbar\overline{\omega})^{-1/2}$, where $\overline{\omega}$ is the center frequency of the field. For each BHD we use an independent LO beam, each an intense coherent-state field with amplitude at the surface of the respective detectors given by $\mathbf{\Phi}_{Li}^{(+)}(\mathbf{x},t)=i\alpha_{Li}\nu_i(\mathbf{x},t)$, for $i=1,2$, where α_{Li} is the coherent-state amplitude for the normalized spatial-temporal mode $\nu_i(\mathbf{x},t)$ of the LO beam used in the ith BHD and $\mathbf{x}$ is the two-dimensional transverse spatial variable. The difference signal from each BHD can be normalized to yield the measured values of the two (generalized) quadrature-amplitude operators of the signal $\hat{q}_{1\theta}=(\hat{a}_1e^{-i\theta}+\hat{a}_1^\dagger e^{i\theta})/2^{1/2}$ and $\hat{q}_{2\beta}=(\hat{a}_2e^{-i\beta}+\hat{a}_2^\dagger e^{i\beta})/2^{1/2}$, where $\hat{a}_i$ is the photon annihilation operator for the portion of the signal field projected onto the ith LO mode

$$\hat{a}_i=-i\int_0^T dt\int_{\text{Det}} d^2x\,\nu_i^*(\mathbf{x},t)\hat{\Phi}_s^{(+)}(\mathbf{x},t)\quad (i=1,2). \quad (1)$$

The detection integration time T is assumed to be long compared to the pulse durations, and the spatial integrals are over

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the detectors' surfaces. The phase-shifts θ,β are determined by the phases of the two independent LO beams, whose amplitudes can be written as $\alpha_{L1}=|\alpha_{L1}|\exp(i\theta)$ and $\alpha_{L2}=|\alpha_{L2}|\exp(i\beta)$. The signal-field annihilation operators obey the Bose commutation relation $[\hat{a}_i,\hat{a}_j^\dagger]=\delta_{ij}$ (in the paraxial-beam approximation). They can be represented in terms of two Hermitean quadrature operators, $\hat{q}_i=(\hat{a}_i+\hat{a}_i^\dagger)/2^{1/2}$ and $\hat{p}_i=(\hat{a}_i-\hat{a}_i^\dagger)/i2^{1/2}$, which obey the commutator for a pair of conjugate variables, $[\hat{q}_i,\hat{p}_j]=i\delta_{ij}$. The measured variables $\hat{q}_{1\theta},\hat{q}_{2\beta}$ can be thought of as resulting from a phase-space rotation of the quadrature variables

$$\begin{pmatrix}\hat{q}_{i\theta}\\ \hat{p}_{i\theta}\end{pmatrix}=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\hat{q}_i\\ \hat{p}_i\end{pmatrix}, \tag{2}$$

where the conjugate variables $\hat{p}_{1\theta},\hat{p}_{2\beta}$ are similarly defined.

Let us briefly consider the output of only one of the BHDs which gives a measurement of $\hat{q}_{1\theta}$. The now-standard way to reconstruct the state of the single-mode field entering this detector is to measure the value of $q_{1\theta}$ many times and build up a histogram (probability density) $P_\theta(q_{1\theta})$. This is repeated for many discrete values of the phase-shift θ. Then apply the inverse Radon transform to the set of probabilities to obtain the Wigner function $W(x_1,p_1)$ corresponding to the state of the measured mode [3–5]. The physical basis for this reconstruction theorem is that the measured distributions $P_\theta(q_{1\theta})$ are related to the Wigner function by line-projection integrals in which $q_{1\theta}$ is held constant. It should be noted that the kernel for the inverse Radon transform is unbounded and requires appropriate regularization for numerical implementation. The commonly used filtered back-projection algorithm accomplishes this by imposing a frequency cutoff which acts as a smoothing filter in the reconstruction process [13].

An alternative which is in some cases superior to the standard Radon method is the direct sampling method of state reconstruction, introduced by D'Ariano, Machiavello, and Paris [14], and developed significantly since [15]. This more powerful approach is to reconstruct the density matrix in the photon-number basis $\rho_{nm}=\langle n|\rho|m\rangle$ by direct averaging: $\rho_{nm}=\langle F_{nm}(q_{1\theta},\theta)\rangle_{q_{1\theta},\theta}$, where $\langle\ \rangle_{q_{1\theta,\theta}}$ represents an ensemble average over the experimentally measured variables $q_{1\theta},\theta$. The sampling functions are given by $F_{nm}(q_{1\theta},\theta)=f_{nm}(q_{1\theta})\exp[i(n-m)\theta]$ in which the so-called pattern functions are [15] $f_{nm}(q)=\partial_q\psi_n(q)\varphi_m(q)$, for $n\leqslant m$, and $f_{nm}(q)=f_{mn}(q)$ otherwise. Here $\psi_n(q)$ and $\varphi_m(q)$ are, respectively, the (real) wave functions of the regular and irregular eigenstates of the number operator (harmonic-oscillator energy operator) and ∂_q represents the derivative with respect to q. No smoothing of phase-space densities is required here because the sampling function in the number basis is bounded. In this rigorous sense it is preferred to measure the density matrix ρ_{nm} directly rather than going through the Wigner function. In a practical sense it is also preferred because it is no longer necessary to store large amounts of data—the reconstruction is done in *real time*.

Now for the two-mode case consider the output of both BHDs with which we measure $q_{1\theta}$ and $q_{2\beta}$ many times to build up the joint probability density $P_{\theta\beta}(q_{1\theta},q_{2\beta})$. This is repeated varying the two phase-shifts θ,β discretely and independently. $P_{\theta\beta}(q_{1\theta},q_{2\beta})$ is formally the projection of the four-dimensional Wigner function for the two-mode state $W(q_1,p_1,q_2,p_2)$ onto the plane $(q_{1\theta},q_{2\beta})$ [5]. We can think of this as a first projection in the (q_1,p_1) plane to give $P_\theta(q_{1\theta};q_2,p_2)$ followed by a second projection in the (q_2,p_2) plane to obtain $P_{\theta\beta}(q_{1\theta},q_{2\beta})$. These projections are identical to the Radon transform for which the inversion is known. Thus we can tomographically reconstruct the Wigner function using the inverse Radon transform by first inverting $P_{\theta\beta}(q_{1\theta},q_{2\beta})$ once for each combination of θ, $q_{1\theta}$—to yield $P_\theta(q_{1\theta};q_2,p_2)$. Then the resulting distributions are inverted, once for each combination of q_2,p_2 values to finally obtain $W(q_1,p_1,q_2,p_2)$. While a single inversion is needed for the single-mode case, two-mode reconstruction requires on the order of 10 000 such inversions and therefore much greater computer memory size [5].

This added complexity motivates the generalization of the direct sampling method of state reconstruction to the case of two modes. A straightforward generalization of the sampling method for reconstructing $\rho_{nmjk}={}_1\langle n|_2\langle j|\hat{\rho}|k\rangle_2|m\rangle_1$ is given by

$$\begin{aligned}\rho_{nmjk}&=\langle S_{nm}^{jk}(q_{1\theta},\theta,q_{2\beta},\beta)\rangle_{q_{1\theta},\theta,q_{2\beta},\beta}\\ &=\int_{-\infty}^{\infty}\int_0^{\pi}\int_{-\infty}^{\infty}\int_0^{\pi}P_{\theta\beta}(q_{1\theta},q_{2\beta})S_{nm}^{jk}(q_{1\theta},\theta,q_{2\beta},\beta)\\ &\quad\times dq_{1\theta}d\theta\,dq_{2\beta}d\beta,\end{aligned} \tag{3}$$

i.e., the density matrix in the photon-number basis is obtained by averaging a sampling function $S_{nm}^{jk}(q_{1\theta},\theta,q_{2\beta},\beta)$ for which a valid choice is easily shown to be a simple product of single-mode sampling functions, $S_{nm}^{jk}(q_{1\theta},\theta,q_{2\beta},\beta)=F_{nm}(q_{1\theta},\theta)F_{jk}(q_{2\beta},\beta)$. This is not surprising given that the two modes are independently measurable DOFs of the optical field. To evaluate Eq. (3) from experiment we measure the variables $q_{1\theta}$ and $q_{2\beta}$ many times, while the phases θ,β are independently and uniformly varied over a π interval.

In the case of two modes that cannot be separated it is not possible to measure the quadratures independently of each other, as was necessary in the reconstruction method presented above. If a simple 50-50 beam splitter were used to produce two beams for measurement by a pair of BHDs, then the extra noise added by the beam splitter would degrade the precision of the state measurement [16]. Here we show that it is possible to reconstruct the joint state of a nonseparable two-mode field without the use of a beam splitter. For this we use a single four-port BHD in which the LO field is in a linear superposition of the two modes of interest

$$\Phi_L^{(+)}(\mathbf{x},t)=i|\alpha_L|\exp(i\theta)[\nu_1(\mathbf{x},t)\cos\alpha+\nu_2(\mathbf{x},t)e^{i\gamma}\sin\alpha], \tag{4}$$

where α and γ are parameters that can be varied and the mode functions $\nu_1(\mathbf{x},t),\nu_2(\mathbf{x},t)$ are orthonormal in $(\mathbf{x},t)$. The quadrature measured from the output of the BHD in this case is $\hat{Q}=(\hat{a}e^{-i\theta}+\hat{a}^\dagger e^{i\theta})/2^{1/2}$ where $\hat{a}$ is the annihilation operator for the portion of the signal field projected onto the LO mode,

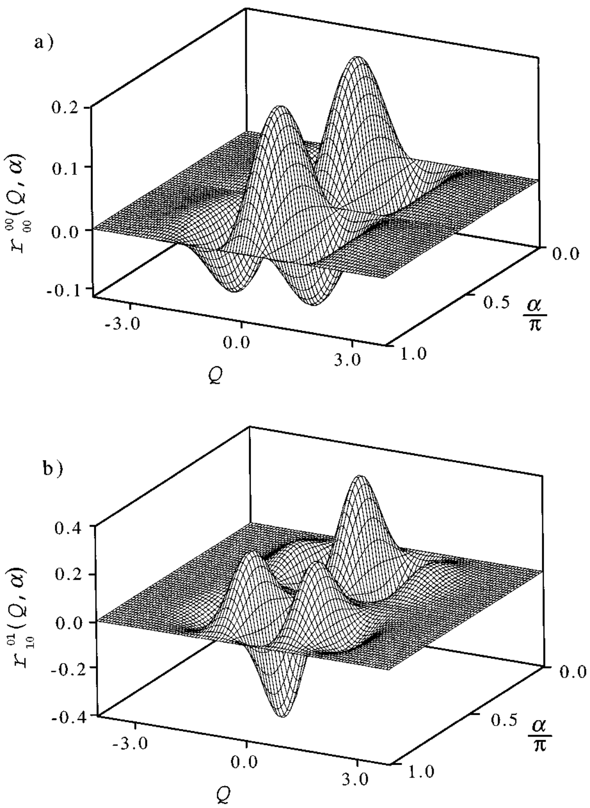

FIG. 1. Two-mode pattern functions.

$$\hat{a} = -i \int_0^T dt \int_{\rm Det} d^2x \hat{\Phi}_S^{(+)}(\mathbf{x},t)[\nu_1^*(\mathbf{x},t)\cos\alpha + \nu_2^*(\mathbf{x},t)e^{-i\gamma}\sin\alpha]. \tag{5}$$

Using $\hat{a}_i = (\hat{q}_i + i\hat{p}_i)/2^{1/2}$ we can write the measured quadrature as

$$\hat{Q} = \cos(\alpha)[\hat{q}_1 \cos\theta + \hat{p}_1 \sin\theta] + \sin(\alpha)[\hat{q}_2 \cos(\theta-\gamma) + \hat{p}_2 \sin(\theta-\gamma)]. \tag{6}$$

If we define $\theta-\gamma=\beta$ then the bracketed terms are recognized as the generalized quadratures in Eq. (2) so that we can write $\hat{Q} = \hat{q}_{1\theta}\cos\alpha + \hat{q}_{2\beta}\sin\alpha$. The measured quadrature is equivalent to a linear combination of the generalized quadratures associated with the individual space-time modes that make up the LO.

From the joint probability density for the two quadratures $P_{\theta\beta}(q_{1\theta},q_{2\beta})$ the probability density for the linear combination Q is

$$P^{\alpha}_{\theta\beta}(Q) = \int\int \delta(Q - q_{1\theta}\cos\alpha - q_{2\beta}\sin\alpha) \times P_{\theta\beta}(q_{1\theta},q_{2\beta})dq_{1\theta}\,dq_{2\beta}, \tag{7}$$

which is parametrized by three adjustable quantities—two optical phases θ and β, and α which determines the relative amplitudes of the two modes that make up the LO. Equation (7) is a projection integral for which we know the inversion (the inverse Radon transform). Using the BHD to measure $P^{\alpha}_{\theta\beta}(Q)$ for many values of α, the joint distribution $P_{\theta\beta}(q_{1\theta},q_{2\beta})$ can be tomographically reconstructed via the inverse Radon transform [17]. For convenience we write this here as

$$P_{\theta\beta}(q_{1\theta},q_{2\beta}) = \frac{1}{\pi^2}\int_{-\infty}^{\infty} dQ \int_0^{\pi} d\alpha\; P^{\alpha}_{\theta\beta}(Q) \times K(Q - q_{1\theta}\cos\alpha - q_{2\beta}\sin\alpha), \tag{8}$$

where $K(x)$ is the unbounded kernel defined in Eqs. (1) and (2) in Ref. [18]. This inversion must be performed once for each combination of θ,β values. From $P_{\theta\beta}(q_{1\theta},q_{2\beta})$ we can determine the two-mode Wigner function, $W(q_1,p_1,q_2,p_2)$, by performing a set of successive inversions. This third level of tomographic inversion further complicates the data storage and processing requirements, as discussed earlier.

Is it possible to extend the direct sampling method so that ρ_{nmjk} is obtained by directly averaging some new sampling function over the set Q,α,θ,β? To see that this can be done we use Eqs. (3) and (8) to write

$$\rho_{nmjk} = \langle R^{jk}_{nm}(Q,\alpha,\theta,\beta)\rangle_{Q,\alpha,\theta,\beta} = \int_{-\infty}^{\infty}\int_0^{\pi}\int_0^{\pi}\int_0^{\pi} P^{\alpha}_{\theta\beta}(Q) R^{jk}_{nm}(Q,\alpha,\theta,\beta) \times dQ\,d\alpha\,d\theta\,d\beta, \tag{9}$$

where the sampling function is $R^{jk}_{nm}(Q,\alpha,\theta,\beta) = r^{jk}_{nm}(Q,\alpha)\exp[i(n-m)\theta + i(j-k)\beta]$, with

$$r^{jk}_{nm}(Q,\alpha) = -\frac{1}{2\pi^2}\int\int {\rm P}\,\frac{1}{(Q - q_{1\theta}\cos\alpha - q_{2\beta}\sin\alpha)^2} \times f_{nm}(q_{1\theta})f_{jk}(q_{2\beta})dq_{1\theta}dq_{2\beta}. \tag{10}$$

We have made use of $F_{\mu\nu}(q,\phi) = f_{\mu\nu}(q)\exp[i(\mu-\nu)\phi]$ and have also chosen a convenient regularization of the kernel $K(x) = -(1/2){\rm P}x^{-2}$ [18], where P indicates Cauchy's principle value. Using $f_{nm}(q_{1\theta}) = \partial_{q_{1\theta}}\psi_n(q_{1\theta})\varphi_m(q_{1\theta})$ (assuming $n \leqslant m$) and performing the $q_{1\theta}$ integral by parts Eq. (10) can be written as

$$r^{jk}_{nm}(Q,\alpha) = -\frac{1}{2\pi^2}\int f_{jk}(y)\frac{\partial^2}{\partial Q^2} \times \int {\rm P}\,\frac{\psi_n(x)\varphi_m(x)}{(Q - y\sin\alpha)/\cos\alpha - x}\,dx\,dy. \tag{11}$$

We have also replaced $q_{1\theta}$ and $q_{2\beta}$ with x and y, respectively. Using the Hilbert transform and the theory of analytic signals one can prove [16] that for $n \leqslant m$

$$-\frac{1}{\pi}\int \mathrm{P}\,\frac{\psi_n(x)\varphi_m(x)}{\xi-x}\,dx=\psi_n(\xi)\psi_m(\xi), \qquad (12)$$

while for $n>m$ Eq. (12) is valid under the exchange $n\leftrightarrow m$. Using this in Eq. (11) gives the final result

$$r_{nm}^{jk}(Q,\alpha)=\frac{1}{2\pi}\int f_{jk}(y)\,\frac{\partial^2}{\partial Q^2}\,\psi_n\left(\frac{Q-y\cos\alpha}{\sin\alpha}\right)\times\psi_m\left(\frac{Q-y\cos\alpha}{\sin\alpha}\right)dy, \qquad (13)$$

for $n\leqslant m$, while for $n>m$ we can use $r_{nm}^{jk}(Q,\alpha)=r_{mn}^{jk}(Q,\alpha)$. Note that all evidence of the regularization has vanished. Thus the evaluation of Eq. (9) is not subject to smoothing and is completely analogous to the sampling methods developed previously. This result significantly simplifies the state reconstruction of a two-mode field over the multiple inversions required in the Radon method. In Fig. 1 we show some examples of $r_{nm}^{jk}(Q,\alpha)$ which we refer to as two-mode pattern functions. Note that in Fig. 1(a) the full sampling function is also shown since $R_{nn}^{jj}(Q,\alpha,\theta,\beta)=r_{nn}^{jj}(Q,\alpha)$. It is easily shown from Eq. (13) that $r_{nm}^{jk}(Q,\alpha)\rightarrow 0$ as $\cos\alpha\rightarrow 0$ or $\sin\alpha\rightarrow 0$.

In order to test this reconstruction method we have performed numerical simulations of Eq. (9) using analytically calculated histograms treated as (noiseless) experimental data. We simulated several Fock states of the form $|\psi\rangle=|J\rangle|N\rangle$ and in each case obtained the density matrix $\rho_{nmjk}=\delta_{nN}\delta_{mN}\delta_{jJ}\delta_{kJ}$, accurate to within 10^{-5}.

In the case that only the two-mode joint photon-number statistics are desired, a random-phase technique [10] can be used—randomize both θ and β and replace Eq. (9) by an integral over Q and α only, with $P_{\theta\beta}^{\alpha}(Q)$ replaced by its phase-random version $P^{\alpha}(Q)$, and the sampling function R_{nm}^{jk} replaced by $r_{nn}^{jj}(Q,\alpha)$.

In conclusion we have proposed two schemes for measuring the joint quantum state of a two-mode optical field. This generalizes earlier single-mode schemes and allows one to measure quantum correlations, such as photon-number correlations, between two space-time modes of the optical field. One important application is when the spatial modes are identical and the two temporal modes are chosen as localized pulses separated by a variable (nonzero) delay. In this way the two-time correlations of a field, or of two fields, can be measured on ultrafast time scales. The first scheme makes use of a pair of BHDs and is appropriate when the two modes of interest can be spatially separated such as by propagation direction, polarization, wavelength, or by very large spatial or temporal separation. The second reconstruction scheme applies when the two modes cannot be spatially separated and uses a single BHD for data collection. In both cases we showed that with proper data analysis involving the inverse Radon transform one can tomographically reconstruct the Wigner function describing the two-mode state. Furthermore, we derived alternative direct sampling methods in which the joint density matrix can be obtained in the Fock basis by directly averaging over the experimentally determined quadrature histograms.

Note added in proof. Our attention has recently been drawn to work by Opatrny, Welsch, and Vogel [19], who have proposed much the same scheme as ours in the special case of two temporal modes (having the same spatial properties). Their data-analysis method yields the two-mode density matrix represented in the quadrature basis. For this basis there are no unbounded sampling functions, in contrast to the case of the Fock basis, which we discuss here [18].

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[1] J. R. Ashburn *et al.*, Phys. Rev. A **41**, 2407 (1990).

[2] T. J. Dunn, I. A. Walmsley, and S. Mukamel, Phys. Rev. Lett. **74**, 884 (1995).

[3] D. T. Smithey, M. Beck, M. G. Raymer, and A. Faridani, Phys. Rev. Lett. **70**, 1244 (1993); D. T. Smithey, M. Beck, J. Cooper, M. G. Raymer, and A. Faridani, Phys. Scr. **T48**, 35 (1993).

[4] J. Bertrand and P. Bertrand, Found. Phys. **17**, 397 (1987); K. Vogel and H. Risken, Phys. Rev. A **40**, 2847 (1989); for a review, see U. Leonhardt and H. Paul, Prog. Quantum Electron. **19**, 89 (1995).

[5] M. G. Raymer, D. T. Smithey, M. Beck, M. E. Anderson, and D. F. McAlister, in Proceedings of the Third International Wigner Symposium, 1993 [Int. J. Mod. Phys. B (to be published)].

[6] These variables can be used to reconstruct the Wigner function, which is in one-to-one correspondence with the quantum state. Note that other variables may be used to construct other phase-space densities (e.g., Q function) from which constructing the state requires deconvolution of a smoothing function, a procedure which is unstable in many experimental situations due to statistical and systematic errors in the data. So they are not tomographically complete in a practical sense.

[7] E. S. Polzik, J. Carry, and H. J. Kimble, Phys. Rev. Lett. **68**, 3020 (1992).

[8] S. Schiller, G. Breitenbach, S. F. Pereira, T. Müller, and J. Mylnek (unpublished).

[9] H. P. Yuen and J. H. Shapiro, IEEE Trans. Inform. Theor. **IT-26**, 78 (1980); M. G. Raymer, J. Cooper, H. J. Carmichael, M. Beck, and D. T. Smithey, J. Opt. Soc. Am. B **12**, 1801 (1995).

[10] M. Munroe, D. Boggavarapu, M. E. Anderson, and M. G. Raymer, Phys. Rev. A **52**, R924 (1995).

[11] By spatially separable we mean that the two modes can be separated, without suffering losses, by any means (e.g., using spatial filters, polarizers, prisms, time gates, etc.) whereby they can be subsequently separated in space.

[12] H. Kuhn, D.-G. Welsch, and W. Vogel, Phys. Rev. A **51**, 4240 (1995).

[13] F. Natterer, *The Mathematics of Computerized Tomography* (Wiley, New York, 1986).

[14] G. M. D'Ariano, C. Macchiavello, and M. B. A. Paris, Phys. Rev. A **50**, 4298 (1994).

[15] U. Leonhardt, M. Munroe, T. Kiss, Th. Richter, and M. G. Raymer, Opt. Commun. **127**, 144 (1996), and references therein.

[16] This method would give, at best, a measurement of the Q function describing the two-mode field; see U. Leonhardt and H. Paul, Phys. Rev. A **48**, 4598 (1993).

[17] In the present case we have considered an LO comprised of two space-time modes with varying relative amplitudes, including the case where most of the LO photons are in one mode and only a small number are in the other mode. It is easily shown that the desired distribution for the sum quadrature $P^{\alpha}_{\theta\beta}(Q)$ is correctly obtained from the BHD given the constraint that the total number of photons in the LO field is large.

[18] G. M. D'Ariano, U. Leonhardt, and H. Paul, Phys. Rev. A **52**, R1801 (1995).

[19] T. Opatrny, D.-G. Welsch, and W. Vogel, Opt. Commun. (to be published).