Motion-quantized Jaynes-Cummings models with an arbitrary intensity-dependent medium

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The exact time dependence of the density operator and various physical quantities for the motion-quantized Jaynes-Cummings models (MQJCMs) is given using the operator transformation under which it is shown that the MQJCMs have the same dynamics as the ordinary JCMs in the Kerr medium. The quantum collapses and revivals of the atomic motion due to the quantum properties of light fields and the quantization of atomic position and momentum, are predicted. The effects of the initial atomic momentum distribution and the arbitrary intensity-dependent medium on the collapse and revival phenomena of the population inversion, atomic momentum and radiation are formulated in detail. [S1050-2947(96)00408-8]

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I. INTRODUCTION

The simplest and most important model in quantum optics is the Jaynes-Cummings model (JCM) [1] enabling one to calculate exactly all the quantum-mechanical properties of a system. It predicts many interesting effects such as vacuum-field Rabi oscillations, collapses and revivals of Rabi oscillations due to the quantized aspects of a coherent field, etc., [2–4]. It has now become possible to experimentally test [5–7] many of the predictions of this model. In realistic situations, one does experiments in cavities with the atomic beam at almost fixed velocity and hence, it is desirable to generalize the ordinary JCM to include the quantization of atomic momentum and position. This generalization has been done by some papers [8] although *nobody* has derived the *exact dynamics*. On the other hand, the JCM has been extended and generalized in many directions among which the JCM in a Kerr-like medium has been studied recently [9], and naturally one should ask if the JCM in an *arbitrary* intensity-dependent medium can be solved. In this paper, we will answer these two questions. We adopt the *operator transformation* to discuss the motion-quantized Jaynes-Cummings models (MQJCMs) in an *arbitrary* intensity-dependent medium and obtain the exact results for the time dependence of various physical quantities, such as inversion, *momentum*, *radiation force*, *momentum diffusion*, and field amplitude and fluctuations.

This paper is organized as follows. In Sec. II we will introduce the operator transformation leading to the solvability of the MQJCM considered and the exact evolution of the density operator in this model is given. In Sec. III the expectation values of atomic population, atomic momentum, radiation force, momentum diffusion, and photon number are given. Their quantum-collapse-revival phenomena due to the quantization of atomic motion and the statistical properties of the light field and the destruction of quantum revivals are shown. The generalized MQJCM including the multimode interaction is considered in Sec. IV, followed by the summary in Sec. V.

II. THE OPERATOR TRANSFORMATION AND TIME EVOLUTION OPERATOR IN A SINGLE-MODE CASE

The motion-quantized Jaynes-Cummings model in this paper considers the interaction of a single two-level *moving* atom characterized by the spin operators $\hat{\sigma}_{ij}$ $(i,j=0,1)$, the momentum and position operators $\hat{\vec{p}}$ and $\hat{\vec{R}}$ with a traveling-wave mode of the electromagnetic field characterized by annihilation and creation operators $\hat{a}$ and $\hat{a}^\dagger$, respectively. The total Hamiltonian of the system in RWA can be written as

$$\hat{H}=\frac{\hat{\vec{p}}^2}{2M}+\hbar\left[\omega_0\hat{\sigma}_0+\omega_1\hat{\sigma}_1+\Omega\hat{a}^\dagger\hat{a}+\sum_{j=1}d_j(\hat{a}^\dagger\hat{a})^j\right]+\hbar[g\hat{a}^l f(\hat{a}^\dagger\hat{a})\hat{\sigma}_{10}e^{il\vec{k}\cdot\hat{\vec{R}}}+g^*f(\hat{a}^\dagger\hat{a})(\hat{a}^\dagger)^l\hat{\sigma}_{01}e^{-il\vec{k}\cdot\hat{\vec{R}}}],\tag{1}$$

where g denotes the coupling between the atom and field. In this MQJCM Hamiltonian, there occur the three conservative quantities (atomic probability, excited number, and atomic-field momentum) as follows:

$$\hat{\sigma}_0+\hat{\sigma}_1=\hat{N},\quad \hat{a}^\dagger\hat{a}+l\hat{\sigma}_1=\hat{N}_e,\quad \hat{\vec{p}}+\hbar\vec{k}\hat{a}^\dagger\hat{a}=\hat{\vec{N}}_p.\tag{2}$$

For solving Eq. (1) where there exist three commutation relations $[\hat{a},\hat{a}^\dagger]=1$, $\hat{\sigma}_{ij}\hat{\sigma}_{kl}=\hat{\sigma}_{il}\delta_{jk}$, and $[\hat{x},\hat{p}_x]=i\hbar$, one introduces the *time-independent operator transformation* $\hat{T}=\exp(i\hat{a}^\dagger\hat{a}\vec{k}\cdot\hat{\vec{R}})$ to simplify it. Thus we have

$$\hat{H}'=\hat{T}\hat{H}\hat{T}^{-1}=\frac{(\hat{\vec{p}}-\hat{a}^\dagger\hat{a}\hbar\vec{k})^2}{2M}+\hbar\left[\omega_0\hat{\sigma}_0+\omega_1\hat{\sigma}_1+\Omega\hat{a}^\dagger\hat{a}+\sum_{j=1}d_j(\hat{a}^\dagger\hat{a})^j\right]+\hbar[g\hat{a}^l f(\hat{a}^\dagger\hat{a})\hat{\sigma}_{10}+g^*f(\hat{a}^\dagger\hat{a})\times(\hat{a}^\dagger)^l\hat{\sigma}_{01}].\tag{3}$$

Comparing Eq. (3) with Eq. (2), we know that after the transformation: (1) the atomic momentum is a motion constant in Eq. (3) so that the MQJCM can be processed as the ordinary JCM which includes only two commutation relations $[\hat{a},\hat{a}^\dagger]=1$ and $\hat{\sigma}_{ij}\hat{\sigma}_{kl}=\hat{\sigma}_{il}\delta_{jk}$; 2) the effects of the motion quantization are equivalent to the *Kerr-like intensity-dependent medium* plus the Doppler effect. These results can be explained by using the transformation on the system total momentum operator because $\hat{T}\hat{\vec{N}}_p\hat{T}^{-1}=\hat{\vec{p}}$ means this transformation leads to a momentum conservation. Now we can start from Eq. (3) to solve the MQJCM.

Let the total Hamiltonian $\hat{H}'=\hbar(\hat{H}_0+\hat{V})$ where the ''free part'' being the nonlinear combination of the conservative quantities, different from the linear combination in an ordinary JCM, and the ''interaction part'' are

$$\hat{H}_0=\frac{\hat{\vec{p}}^2}{2\hbar M}+\left(\Omega-\frac{\vec{k}\cdot\hat{\vec{p}}}{M}\right)\hat{N}_e+\varepsilon\hat{N}_e^2\sum_{j=1} d_j\hat{N}_e^j+\left(\omega_0-\frac{\hat{\Delta}}{2}\right)\hat{N}, \tag{4}$$

$$\hat{V}=-\frac{\hat{\Delta}}{2}(\hat{\sigma}_1-\hat{\sigma}_0)+g\hat{a}^l f(\hat{a}^\dagger\hat{a})\hat{\sigma}_{10}+g^*f(\hat{a}^\dagger\hat{a})(\hat{a}^\dagger)^l\hat{\sigma}_{01}, \tag{5}$$

$$\hat{\Delta}=l\left\{\Omega+\frac{1}{l}\sum_{j=1} d_j[\hat{N}_e^j-(\hat{N}_e-l)^j]-\frac{\vec{k}(\hat{\vec{p}}-\hat{N}_e\hbar\vec{k})}{M}\right\} -[(\omega_1-\omega_0)+\varepsilon l^2], \tag{6}$$

where a special detuning operator $\hat{\Delta}$ which also consists of the conservative quantities and includes the contribution of the intensity-dependent medium, is introduced and behaves like a constant. $\varepsilon=(\hbar k^2/2M)$ is the recoil frequency shift per photon. By using the commutation relations, it is simple to show that $\hat{H}_0$ and $\hat{V}$ are also the motion constant. So in the ''interaction picture,'' if we define

$$|1\rangle=\begin{pmatrix}0\\1\end{pmatrix},\quad |0\rangle=\begin{pmatrix}1\\0\end{pmatrix}. \tag{7}$$

The matrix representation of the time evolution operator is obtained from Eq. (5) and Eq. (6) as

$$\hat{U}(t,0)=e^{-i\hat{V}t} =\begin{pmatrix}\cos\hat{\mu}_1 t-i\dfrac{\hat{\Delta}_1\sin_1 t}{2\hat{\mu}_1} & -ig*\dfrac{\sin\hat{\mu}_1 t}{\hat{\mu}_1}f(\hat{a}^\dagger\hat{a})(\hat{a}^\dagger)^l\\ -ig\hat{a}^l f(\hat{a}^\dagger\hat{a})\dfrac{\sin\hat{\mu}_1 t}{\hat{\mu}_1} & \cos\hat{\mu}_2 t+i\dfrac{\hat{\Delta}_2\sin\hat{\mu}_2 t}{2\hat{\mu}_2}\end{pmatrix}, \tag{8}$$

where

$$\hat{\Delta}_1=l\left\{\Omega+\frac{1}{l}\sum_{j=1} d_j[(\hat{a}^\dagger\hat{a})^j-(\hat{a}^\dagger\hat{a}-l)^j]-\frac{\vec{k}(\hat{\vec{p}}-\hat{a}^\dagger\hat{a}\hbar\vec{k})}{M}\right\} -[(\omega_1-\omega_0)+\varepsilon l^2],$$

$$\hat{\Delta}_2=l\left\{\Omega+\frac{1}{l}\sum_{j=1} d_j[(\hat{a}^\dagger\hat{a}+l)^j-(\hat{a}^\dagger\hat{a})^j] -\frac{\vec{k}[\hat{\vec{p}}-(\hat{a}^\dagger\hat{a}+l)\hbar\vec{k}]}{M}\right\}-[(\omega_1-\omega_0)+\varepsilon l^2],$$

$$\hat{\mu}_1^2=\left(\frac{\hat{\Delta}_1}{2}\right)^2+|g|^2f^2(\hat{a}^\dagger\hat{a})(\hat{a}^\dagger)^l\hat{a}^l \quad\text{and}\quad \hat{\mu}_2^2=\left(\frac{\hat{\Delta}_2}{2}\right)^2 +|g|^2f^2(\hat{a}^\dagger\hat{a}+l)\hat{a}^l(\hat{a}^\dagger)^l.$$

Using the inverse operator transformation, the ''true'' time evolution operator is

$$\hat{U}_d(t,0)=\hat{T}^{-1}\hat{U}\hat{T}=\begin{pmatrix}\cos\hat{\mu}_1' t-i\dfrac{\hat{\Delta}_1'\sin\hat{\mu}_1' t}{2\hat{\mu}_1'} & -ig*\dfrac{\sin\hat{\mu}_1' t}{\hat{\mu}_1'}f(\hat{a}^\dagger\hat{a})(\hat{a}^\dagger e^{-i\vec{k}\cdot\hat{\vec{R}}})^l\\ -ig(\hat{a}e^{i\vec{k}\cdot\hat{\vec{R}}})^l f(\hat{a}^\dagger\hat{a})\dfrac{\sin\hat{\mu}_1' t}{\hat{\mu}_1'} & \cos\hat{\mu}_2' t+i\dfrac{\hat{\Delta}_2'\sin\hat{\mu}_2' t}{2\hat{\mu}_2'}\end{pmatrix}, \tag{9}$$

where

$$\hat{\Delta}_1'=l\left\{\Omega+\frac{1}{l}\sum_{j=1} d_j[(\hat{a}^\dagger\hat{a})^j-(\hat{a}^\dagger\hat{a}-l)^j]-\frac{\hat{k}\cdot\hat{\vec{p}}}{M}\right\}-[(\omega_1 -\omega_0)+\varepsilon l^2],$$

$$\hat{\Delta}_2'=l\left\{\Omega+\frac{1}{l}\sum_{j=1} d_j[(\hat{a}^\dagger\hat{a}+l)^j-(\hat{a}^\dagger\hat{a})^j]-\frac{\vec{k}(\hat{\vec{p}}-l\hbar\vec{k})}{M}\right\} -[(\omega_1-\omega_0)+\varepsilon l^2],$$

$$\hat{\mu}_1'^2=\left(\frac{\hat{\Delta}_1'}{2}\right)^2+|g|^2f^2(\hat{a}^\dagger\hat{a})(\hat{a}\dagger)^l\hat{a}^l \quad\text{and}\quad \hat{\mu}_2'^2=\left(\frac{\hat{\Delta}_2'}{2}\right)^2 +|g|^2f^2(\hat{a}^\dagger\hat{a}+l)\hat{a}^l(\hat{a}^\dagger)^l.$$

The expectation value of any operator as a function of time t can be determined by Eq. (9) and the initial combined atom-field state through the formula

$$\langle\hat{o}(t)\rangle=\langle\psi(0)|\hat{U}_d(t,0)\hat{o}(0)\hat{U}_d^\dagger|\psi(0)\rangle, \tag{10}$$

where $\hat{\rho}(0)=|\psi(0)\rangle\langle\psi(0)|$. We shall consider some statistical aspects for the present model.

Note that the methods of the operator transformation, through which the MQJCMs are solvable, are not unique. In Ref. [8], Sleator and Wilkens have given an another operator transformation and used it to discuss the quantum nondemolition measurement of a photon number under some *additional approximations*.

III. THE EXACT DYNAMICS OF A MOTION-QUANTIZED ATOM INTERACTING WITH A SINGLE-MODE FIELD

If we assume that the initial statistics of the atomic momentum eigenstates is Gaussian, the initial atomic internal state is the ground state and the initial field is prepared in a coherent state, the initial density operator is

$$\hat{\rho}(0)=\sum_{n,m=0}^{\infty} e^{-\bar{n}}\frac{\alpha^n\alpha^{*m}}{\sqrt{n!m!}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d^3\vec{p}_0\cdot d^3\vec{p}_0'|\vec{p}_0,0,n\rangle \times\langle\vec{p}_0',0,m|F(\vec{p}_0)F^*(\vec{p}_0'), \tag{11}$$

$$|F(\vec{p}_0)|^2=\frac{\delta(p_{y0}-p_{yc})\,\delta(p_{z0}-p_{zc})}{\chi\sqrt{2\pi}}\,e^{-[(p_{x0}-p_{xc})^2/2\chi^2]}, \tag{12}$$

where $\vec{p}_0=p_{x0}\vec{e}_x+p_{y0}\vec{e}_y+p_{z0}\vec{e}_z$ and χ is the initial momentum diffusion coefficient. Let us consider the wave vector of a light field in the x direction, it is easy to get the expectation values of some physical quantities

$$\langle\hat{\sigma}_{11}\rangle=\sum_{n=0}^{\infty} e^{-\bar{n}}\frac{\bar{n}^{n+l}}{(n+l)!}|g|^2\frac{(n+l)!}{n!}f^2(n+l)\times\int_{-\infty}^{\infty}dp_{x0}\frac{e^{-[(p_{x0}-p_{xc})^2/2\chi^2]}}{\chi\sqrt{2\pi}}\frac{\sin^2\mu_n(p_{x0})t}{[\mu_n(p_{x0})]^2}, \tag{13}$$

$$\langle\Delta\hat{\vec{p}}\rangle=\langle\hat{\vec{p}}\rangle-(p_{xc}\vec{e}_x+p_{yc}\vec{e}_y+p_{zc}\vec{e}_z)=l\hbar\vec{k}\langle\hat{\sigma}_{11}\rangle, \tag{14}$$

$$\langle\Delta\hat{n}\rangle=\langle\hat{a}^\dagger\hat{a}\rangle-\bar{n}=-l\langle\hat{\sigma}_{11}\rangle, \tag{15}$$

$$\langle\hat{\vec{F}}\rangle=\frac{1}{i\hbar}\langle[\hat{\vec{p}},\hat{H}]\rangle=l\hbar\vec{k}\langle[-ig(\hat{a}e^{-i\vec{k}\cdot\hat{\vec{R}}})^l f(\hat{a}^\dagger\hat{a})\hat{\sigma}_{10}+\text{h.c.}]\rangle =l\hbar k\sum_{n=0}^{\infty}e^{-\bar{n}}\frac{\bar{n}^{n+1}}{(n+l)!}|g|^2\frac{(n+l)!}{n!}f^2(n+l)\times\int_{-\infty}^{\infty}dp_{x0}\frac{e^{-[(p_{x0}-p_{xc})^2/2\chi^2]}}{\chi\sqrt{2\pi}}\frac{\sin 2\mu_n t}{\mu_n}, \tag{16}$$

$$\langle(\Delta\hat{\vec{p}})^2\rangle=\chi^2+(l\hbar\vec{k})^2\langle\hat{\sigma}_{11}\rangle(1-\langle\hat{\sigma}_{11}\rangle)+2l\hbar k\sum_{n=0}^{\infty}e^{-\bar{n}}\frac{\bar{n}^{n+l}}{(n+l)!}|g|^2\frac{(n+l)!}{n!}f^2(n+l)\times\int_{-\infty}^{\infty}dp_{x0}(p_{x0}-p_{xc})\frac{e^{-[(p_{x0}-p_{xc})^2/2\chi^2]}}{\chi\sqrt{2\pi}}\frac{\sin^2\mu_n t}{\mu_n^2}, \tag{17}$$

$$\langle(\Delta\hat{n})^2\rangle=\bar{n}+l^2\langle\hat{\sigma}_{11}\rangle(1-\langle\hat{\sigma}_{11}\rangle)-2l\sum_{n=0}^{\infty}e^{-\bar{n}}(n+l-\bar{n})\times\frac{\bar{n}^{n+l}}{(n+l)!}|g|^2\frac{(n+l)!}{n!}f^2(n+l)\times\int_{-\infty}^{\infty}dp_{x0}\frac{e^{-[(p_{x0}-p_{xc})^2/2\chi^2]}}{\chi\sqrt{2\pi}}\frac{\sin^2\mu_n t}{\mu_n^2}, \tag{18}$$

where

$$\mu_n^2(p_{x0})=|g|^2\frac{(n+l)!}{n!}f^2(n+l)+\left[\frac{\Delta(p_{x0},n)}{2}\right]^2$$

and

$$\Delta(p_{x0},n)=\Delta_0(n)-l\frac{k(p_{x0}-p_{xc})}{m},$$

where we define further

$$l\left\{\Omega+\frac{1}{l}\sum_{j=1}d_j[(n+l)^j-n^j]-\frac{kp_{xc}}{m}\right\}-[(\omega_2-\omega_1)+\varepsilon l^2]=\Delta_0(n). \tag{19}$$

From the above calculation, we show that the operator transformation in Eq. (3) is very useful to study all fundamental questions in the context of a *motion-quantized atom* with a *single* or *multimode traveling-wave fields,* such as the radiation force, the atomic momentum, etc. From Eqs. (13)–(18), it is shown that all the above quantities including the *atomic momentum* and the *radiation force* experience the quantum collapses and revivals due to the quantum properties of a coherent-state field and the monochromaticity of the initial matter wave (an atom initially prepared in the momentum eigenstate), which will be destroyed by the diffusion of the atomic initial momentum. These phenomena would be explained as the results of the atom motion reflecting the variation of the atomic population inversion and the initial momentum diffusion dephasing the Rabi frequency components. The above quantities have been calculated in detail [10] when the atomic momentum is initially prepared in a pure eigenstate. In this paper, the expectation values of atomic population in the excited state, atomic momentum and radiation force as the functions of time t and intensity-dependent medium (or intensity-dependent coupling) are plotted in Fig. 1–4 when the atom is initially in a momentum

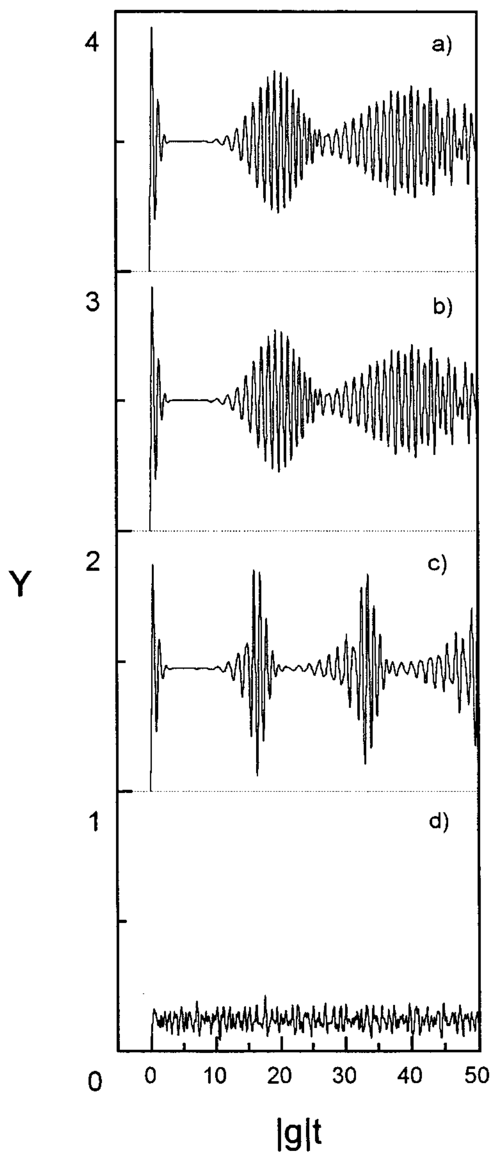

FIG. 1. Y_0 being the expectation value of the atomic population in the excited-state $\langle\hat{\sigma}_{11}\rangle$ and the momentum increment $\langle\Delta\hat{\vec{p}}\rangle$ (unit: $\hbar\vec{k}$) as the function of $|g|t$ and an intensity-dependent medium. (a) $Y=Y_0+3$ for $d_j=0.000|g|$, (b) $Y=Y_0+2$ for $d_2=0.005|g|$ and $d_j=0$, $j\neq 2$; (c) $Y=Y_0+1$ for $d_3=0.005|g|$ and $d_j=0$, $j\neq 3$; (d) $Y=Y_0$ for $d_4=0.005|g|$ and $d_j=0$, $j\neq 4$. Note that $l=1$ (one-photon process), $f(x)=1$, $\Omega-(kp_{xc}/m)-[(\omega_2-\omega_1)+\varepsilon]=0$, $\overline{n}=10$ and the atomic center-of-mass state initially in a pure momentum eigenstates ($\chi=0$).

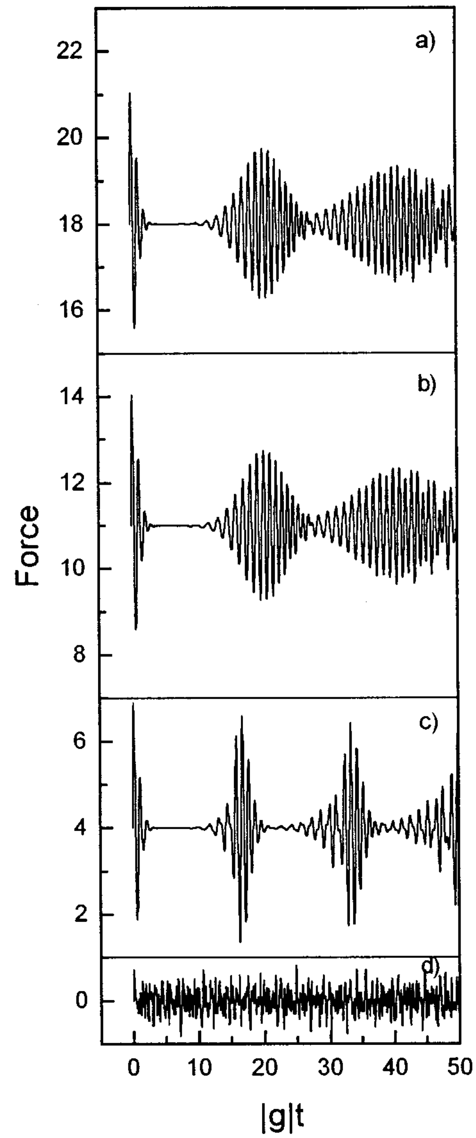

FIG. 2. The radiation force F_0 (unit: $\hbar\vec{k}|g|$) induced by the atom-light interaction as the function of $|g|t$ and an intensity-dependent medium. (a) $F=F_0+18$ for $d_j=0.000|g|$; (b) $F=F_0+11$ for $d_2=0.005|g|$ and $d_j=0$, $j\neq 2$; (c) $F=F_0+4$ for $d_3=0.005|g|$ and $d_j=0$, $j\neq 3$; (d) $F=F_0$ for $d_4=0.005|g|$ and $d_j=0$, $j\neq 4$. The other parameters are same as those in Fig. 1.

eigenstate and the light field in a coherent state. The calculation results have definitely shown that the atomic momentum and the radiation force have the quantum collapse-revival behavior, like the population inversion in the ordinary JCMs. The intensity-dependent medium modifies the effective detuning to effectuate the modification of the collapse-revival phenomena in the atomic motion. But in the intensity-dependent medium, the collapse-revival phenomena are distorted due to the huge change of both effective detuning and effective photon statistics [for example: $f(n+1)$ is related to the statistics].

In the following section, we discuss the generalization of a moving two-level atom interacting with the multimode traveling-wave fields.

IV. A MOTION-QUANTIZED ATOM INTERACTING WITH THE MULTIMODE TRAVELING-WAVE FIELDS

The total Hamiltonian of the system in RWA, considered in this section, is written as

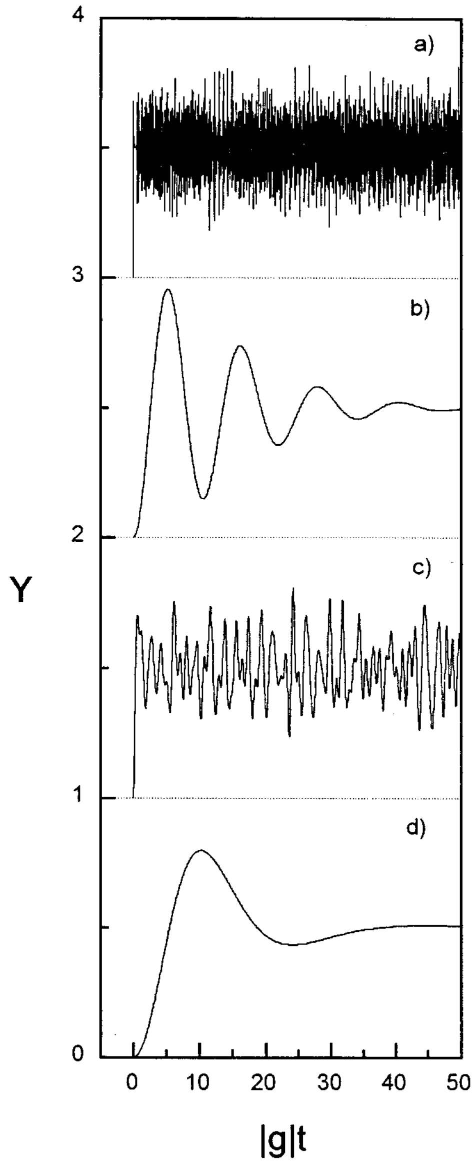

FIG. 3. Y_0 being the expectation value of the atomic population in the excited-state $\langle\hat{\sigma}_{11}\rangle$ and the momentum increment $\langle\Delta\hat{\vec{p}}\rangle$ (unit: $\hbar\vec{k}$) as the function of $|g|t$ and an intensity-dependent coupling. (a) $Y=Y_0+3$ for $f(x)=x$; (b) $Y=Y_0+2$ for $f(x)=1/(x+1)$; (c) $Y=Y_0+1$ for $f(x)=\sin(x)$; (d) $Y=Y_0$ for $f(x)=\exp(-\sqrt{x})$. Note that $l=1$ (one-photon process), $d_j=0(j=1,2,\ldots)$, $\Omega-(kp_{xc}/m)-[(\omega_2-\omega_1)+\epsilon]=0$, $\overline{n}=10$ and the atomic center-of-mass state initially in a pure momentum eigenstates ($\chi=0$).

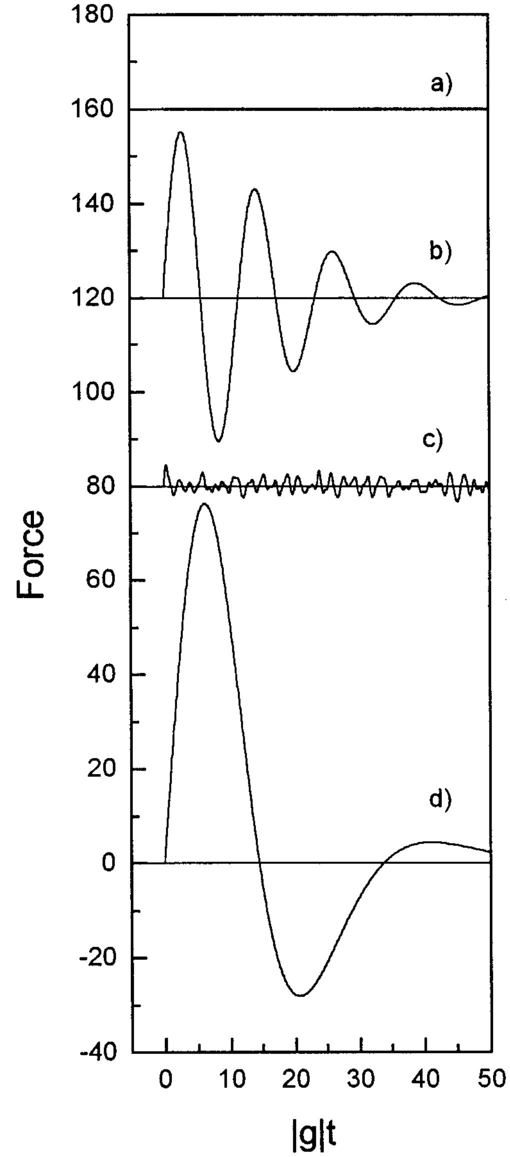

FIG. 4. The radiation force F_0 (unit: $\hbar\vec{k}|g|$) induced by the atom-light interaction as the function of $|g|t$ and an intensity-dependent coupling. (a) $F=F_0+160$ for $f(x)=x$; (b) $F=F_0+120$ for $f(x)=1/(x+1)$; (c) $F=F_0+80$ for $f(x)=\sin(x)$; (d) $F=F_0$ for $f(x)=\exp(-\sqrt{x})$. The other parameters are the same as those in Fig. 3.

$$\hat{H}=\frac{\hat{\vec{p}}^2}{2M}+\hbar\left[\omega_0\hat{\sigma}_0+\omega_1\hat{\sigma}_1+\sum_{l=1}\Omega_l\hat{a}_l^\dagger\hat{a}_l+\sum_{j=1}\left(\sum_{l=1}d_{jl}\hat{a}_l^\dagger\hat{a}_l\right)^j\right]+\hbar\left[g\hat{\sigma}_{10}f(\hat{a}_1^\dagger\hat{a}_1,\hat{a}_2^\dagger\hat{a}_2,\ldots)\times\prod_{l=1}(\hat{a}_l e^{i\vec{k}_l\cdot\hat{\vec{R}}})^{m_l}+\text{H.c.}\right] \tag{20}$$

In this MQJCM Hamiltonian, the conservative quantities (atomic probability, excited numbers, and atomic-field momentum) are as follows:

$$\hat{\sigma}_0+\hat{\sigma}_1=\hat{N},\quad \hat{a}_l^\dagger\hat{a}_l+m_l\hat{\sigma}_1=\hat{N}_l\quad(l=1,2,\ldots),$$

$$\hat{\vec{p}}+\hbar\sum_{l=1}\vec{k}_l\hat{a}_l^\dagger\hat{a}_l=\hat{\vec{N}}_p, \tag{21}$$

where l and m_l represents the lth mode and its corresponding multiphoton transition, respectively. One introduces the *operator transformation* $\hat{T}=\exp[i(\Sigma_{l=1}\hat{a}_l^\dagger\hat{a}_l\vec{k}_l)\hat{\vec{R}}]$. Then, following the same procedure and the same definitions, we have the Hamiltonians, the detuning operator

$$\hat{H}'=\frac{(\hat{\vec{p}}-\Sigma_{l=1}\hbar\vec{k}_l\hat{a}_l^\dagger\hat{a}_l)^2}{2M}+\hbar\Bigg[\omega_0\hat{\sigma}_0+\omega_1\hat{\sigma}_1+\sum_{l=1}\Omega_l\hat{a}_l^\dagger\hat{a}_l+\sum_{j=1}\left(\sum_{l=1}d_{jl}\hat{a}_l^\dagger\hat{a}_l\right)^j\Bigg]+\hbar\left[g\hat{\sigma}_{10}f(\hat{a}_1^\dagger\hat{a}_1,\hat{a}_2^\dagger\hat{a}_2,\ldots)\prod_{l=1}(\hat{a}_l)^{m_l}+\text{H.c.}\right], \tag{22}$$

$$\hat{H}_0=\frac{(\hat{\vec{p}}-\Sigma_{l=1}\hbar\vec{k}_l\hat{N}_l)^2}{2\hbar M}+\sum_{l=1}\Omega_l\hat{N}_l+\sum_{j=1}\left(\sum_{l=1}d_{jl}\hat{N}_l\right)^j+\left(\omega_0-\frac{\hat{\Delta}}{2}\right)\hat{N}, \tag{23}$$

$$\hat{V}=-\frac{\hat{\Delta}}{2}(\hat{\sigma}_1-\hat{\sigma}_0)+\left[g\hat{\sigma}_{10}f(\hat{a}_1^\dagger\hat{a}_1,\hat{a}_2^\dagger\hat{a}_2,\ldots)\times\prod_{l=1}(\hat{a}_l)^{m_l}+\text{H.c.}\right], \tag{24}$$

$$\hat{\Delta}=\sum_{l=1}m_l\Omega_l+\sum_{j=1}\left\{\left(\sum_{l=1}d_{jl}\hat{N}_l\right)^j-\left[\sum_{l=1}d_{jl}(\hat{N}_l-m_l)\right]^j\right\}-\frac{(\Sigma_{l=1}m_l\vec{k}_l)(\hat{\vec{p}}-\Sigma_{l=1}\hbar\vec{k}_l\hat{N}_l)}{M}-\left[(\omega_1-\omega_0)+\frac{\hbar}{2M}\left(\sum_{l=1}m_l\vec{k}_l\right)^2\right] \tag{25}$$

and the time evolution operator

$$\hat{U}_d(t,0)=\begin{pmatrix}\cos\hat{\mu}_1't-i\dfrac{\hat{\Delta}_1'\sin\hat{\mu}_1't}{2\hat{\mu}_1'} & -ig*\dfrac{\sin\hat{\mu}_1't}{\hat{\mu}_1'}f(\hat{a}_1^\dagger\hat{a}_1,\ldots)\prod_{l=1}(\hat{a}_l^\dagger e^{-i\vec{k}_l\cdot\hat{\vec{R}}})^{m_l}\\ -ig\prod_{l=1}(\hat{a}_l e^{i\vec{k}_l\cdot\hat{\vec{R}}})^{m_l}f(\hat{a}_l^\dagger\hat{a}_1,\ldots)\dfrac{\sin\hat{\mu}_1't}{\hat{\mu}_1'} & \cos\hat{\mu}_2't+i\dfrac{\hat{\Delta}_2\sin\hat{\mu}_2't}{2\hat{\mu}_2'}\end{pmatrix}, \tag{26}$$

where

$$\hat{\Delta}_1'=\sum_{l=1}m_l\Omega_l+\sum_{j=1}\left\{\left(\sum_{l=1}d_{jl}\hat{a}_l^\dagger\hat{a}_l\right)^j-\left[\sum_{l=1}d_{jl}(\hat{a}_l^\dagger\hat{a}_l-m_l)\right]^j\right\}-\frac{(\Sigma_{l=1}m_l\vec{k}_l)\hat{\vec{p}}}{M}-\left[(\omega_1-\omega_0)+\frac{\hbar}{2M}\left(\sum_{l=1}m_l\vec{k}_l\right)^2\right],$$

$$\hat{\Delta}_2'=\sum_{l=1}m_l\Omega_l+\sum_{j=1}\left\{\left[\sum_{l=1}d_{jl}(\hat{a}_l^\dagger\hat{a}_l+m_l)\right]^j-\left(\sum_{l=1}d_{jl}\hat{a}_l^\dagger\hat{a}_l\right)^j\right\}-\frac{\left(\sum_{l=1}m_l\vec{k}_l\right)\left(\hat{\vec{p}}-\sum_{l=1}m_l\hbar\vec{k}_l\right)}{M}-\left[(\omega_1-\omega_0)+\frac{\hbar}{2M}\left(\sum_{l=1}m_l\vec{k}_l\right)^2\right],$$

$$\hat{\mu}_1'^2=\left(\frac{\hat{\Delta}_1'}{2}\right)^2+|g|^2f^2(\hat{a}_1^\dagger\hat{a}_1,\ldots)\prod_{l=1}(\hat{a}_l^\dagger)^{m_l}(\hat{a}_l)^{m_l},$$

$$\hat{\mu}_2'^2=\left(\frac{\hat{\Delta}_2'}{2}\right)^2+|g|^2f^2(\hat{a}_1^\dagger\hat{a}_1+m_1,\ldots,\hat{a}_l^\dagger\hat{a}_l+m_l,\ldots)\prod_{l=1}(\hat{a}_l)^{m_l}(\hat{a}_l^\dagger)^{m_l}.$$

Like Sec. III, we assume that the initial statistics of the atomic momentum eigenstates is Gaussian, the initial atomic internal state is the ground state and each initial field is prepared in a corresponding coherent state, and then the initial density operator is

$$\hat{\rho}(0)=\sum_{\substack{n_1,n_2,\cdots=0\\ m_1,m_2,\cdots=0}}^{\infty}e^{-\Sigma_{j=1}\bar{n}_j}\frac{\Pi_{j=1}(\alpha_j)^{n_j}(\alpha_j^*)^{m_j}}{\sqrt{n_1!m_1!n_2!m_2!\cdots}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}d^3\vec{p}_0\cdot d^3\vec{p}_0'|\vec{p}_0,0,n_1,\ldots\rangle\times\langle\vec{p}_0',0,m_1,\cdots|F(\vec{p}_0)F^*(\vec{p}_0'). \tag{27}$$

Let us consider the wave vector of a light field in the x direction, it is easy to get the expectation values of some physical quantities

$$\langle\hat{\sigma}_{11}\rangle=\sum_{n_1,n_2,\cdots=0}^{\infty} e^{-\Sigma_{j=1}\overline{n_j}}\left[\prod_{j=1}\frac{\overline{n_j}^{m_j+m_j}}{(n_j+m_j)!}\frac{(n_j+m_j)!}{n_j!}\right]\times|g|^2f^2(n_1+m_1,n_2+m_2,\ldots)\int_{-\infty}^{\infty}dp_{x0}\times\frac{e^{-[(p_{x0}-p_{xc})^2/2\chi^2]}}{\chi\sqrt{2\pi}}\frac{\sin^2\mu_{\{n_j\}}(p_{x0})t}{[\mu_{\{n_j\}}(p_{x0})]^2},\tag{28}$$

$$\langle\Delta\hat{\vec{p}}\rangle=\langle\hat{\vec{p}}\rangle-(p_{xc}\vec{e}_x+p_{yc}\vec{e}_y+p_{zc}\vec{e}_z)=\hbar\sum_{j=1}m_j\vec{k}_j\langle\hat{\sigma}_{11}\rangle,\tag{29}$$

$$\langle\Delta\hat{n}_j\rangle=\langle\hat{a}_j^\dagger\hat{a}_j\rangle-\overline{n_j}=-m_j\langle\hat{\sigma}_{11}\rangle,\tag{30}$$

$$\langle\hat{\vec{F}}\rangle=\left(\hbar\sum_{j=1}m_j\vec{k}_j\right)\sum_{n_1,n_2,\cdots=0}^{\infty}e^{-\Sigma_{j=1}\overline{n_j}}\times\left[\prod_{j=1}\frac{\overline{n_j}^{m_j+m_j}}{n_j!}\right]|g|^2f^2(n_1+m_1,n_2+m_2,\ldots)\times\int_{-\infty}^{\infty}dp_{x0}\frac{e^{-[(p_{x0}-p_{xc})^2/2\chi^2]}}{\chi\sqrt{2\pi}}\frac{\sin2\mu_{\{n,j\}}(p_{x0})t}{\mu_{\{n_j\}}(p_{x0})},\tag{31}$$

$$\langle(\Delta\hat{\vec{p}})^2\rangle=\chi^2+\left(\sum_{j=1}\hbar m_j\vec{k}_j\right)^2\langle\hat{\sigma}_{11}\rangle(1-\langle\hat{\sigma}_{11}\rangle)+2\left(\sum_{j=1}\hbar m_jk_j\right)\sum_{n_1,n_2,\cdots=0}^{\infty}e^{-\Sigma_{j=1}\overline{n_j}}\times\left[\prod_{j=1}\frac{\overline{n_j}^{m_j+m_j}}{(n_j+m_j)!}\frac{(n_j+m_j)!}{n_j!}\right]|g|^2\times f^2(n_1+m_1,n_2+m_2,\cdots)\int_{-\infty}^{\infty}dp_{x0}(p_{x0}-p_{xc})\times\frac{e^{-[(p_{x0}-p_{xc})^2/2\chi^2]}}{\chi\sqrt{2\pi}}\frac{\sin^2\ \mu_{\{n_j\}}(p_{x0})t}{[\mu_{\{n_j\}}(p_{x0})]^2},\tag{32}$$

$$\langle(\Delta\hat{n}_j)^2\rangle=\overline{n_j}+m_j^2\langle\hat{\sigma}_{11}\rangle(1-\langle\hat{\sigma}_{11}\rangle)+2m_j\sum_{n_1,n_2,\cdots=0}^{\infty}e^{-\Sigma_{j=1}\overline{n_j}}\times\left[\prod_{j=1}\frac{\overline{n_j}^{m_j+m_j}}{(n_j+m_j)!}\frac{(n_j+m_j)!}{n_j!}\right]\times|g|^2f^2(n_1+m_1,n_2+m_2,\ldots)(n_j+m_j-\overline{n_j})\times\int_{-\infty}^{\infty}dp_{x0}\frac{e^{-[(p_{x0}-p_{xc})^2/2\chi^2]}}{\chi\sqrt{2\pi}}\frac{\sin^2\mu_{\{n_j\}}(p_{x0})t}{[\mu_{\{n_j\}}(p_{x0})]^2},\tag{33}$$

where

$$\mu_{\{n_j\}}^2(p_{x0})=|g|^2f^2(n_1+m_1,n_2+m_2,\ldots)\prod_{j=1}\frac{(n_j+m_j)!}{n_j!}+\left[\frac{\Delta'(p_{x0},n_1,n_2,\cdots)}{2}\right]^2$$

and

$$\Delta'(p_{x0},n_1,n_2,\ldots)=\Delta_0'(n_1,n_2,\ldots)-\left(\sum_{j=1}m_jk_j\right)\frac{(p_{x0}-p_{xc})}{M},$$

where we define further

$$\hat{\Delta}_1'(n_1,n_2,\ldots)=\sum_{l=1}m_l\Omega_l+\sum_{j=1}\left\{\left[\sum_{l=1}d_{jl}(n_l+m_l)\right]^j-\sum_{l=1}d_{jl}(n_l)^j\right\}-\left(\sum_{l=1}m_l\vec{k}_l\right)\frac{\hat{\vec{p}}_{xc}}{M}-\left[(\omega_1-\omega_0)+\frac{\hbar}{2M}\left(\sum_{l=1}m_l\vec{k}_l\right)^2\right].$$

From Eq. (28) and Eq. (33), it is shown that: (1) The operator transformation developed in this paper is very effective for solving exactly the single-channel MQJCMs although it is not unique. (2) The quantum collapse-revival phenomena of atomic population, atomic momentum, radiation force, momentum diffusion, and the statistical properties of light fields are predicted due to the quantization of atomic motion and the quantum properties of light fields. (3) The destruction of the above quantum revivals due to the initial superposition of atomic momentum eigenstates can be calculated. (4) The effective effects of the intensity-dependent media is explicitly included in the specially defined detuning and this result can lead to recovering all the consequences of the JCMs in the Kerr-like media [9].

The differences between the ordinary JCMs and the present MQJCMs are: (1) The latter involves three types of physical quantities—field, atomic internal states, and atomic external states, other than two types of physical quantities—field and atomic states in the former. (2) The ordinary JCMs only give the variation of atomic internal states due to the quantum properties of the light fields such as the quantum collapse-revival phenomena of atomic population, and the modulation of the light fields through the interaction such as squeezed states. But in MQJCMs, the variation of atomic external states (momentum, force, momentum statistics, etc.) as a function of atomic internal states and light fields is predicted, which does not appear in the ordinary JCMs. On the other hand, the effects of atomic external states and their statistics on atomic internal states and light fields is elucidated explicitly. In one word, MQJCMs can link the light fields, atomic internal and external states, and give their exact variations and statistics.

V. SUMMARY

In this paper, we introduce the operator transformation to solve the motion-quantized Jaynes-Cumming Model which consists of a two-level moving atom in the arbitrary intensity-dependent media interacting with a single-mode or multimode traveling-wave fields under the quantization of atomic position and momentum. The exact time evolution of the systems is given through which all the dynamics can be calculated. The quantum collapse-revival phenomena of atomic population, atomic momentum, radiation force, momentum diffusion, and the statistical properties of light fields due to the quantization of atomic motion and the quantum properties of light fields are predicted when the light fields and a moving atom is initially prepared in the coherent states and a momentum eigenstate, respectively. If the atom initially in a superposition of atomic momentum eigenstates, the destruction of the quantum revivals due the dephasing properties of the initial momentum statistics appears like the ordinary JCMs with the cavity loss or the spontaneous emission [4]. The effective effects of the intensity-dependent media is explicitly included in the specially defined detuning and this result can lead to recovering all the consequences of the JCMs in the Kerr-like media [9].

The method developed in this paper can be generalized to solve the motion-quantized problems in the traveling-wave fields. The resulting conclusion will lead to well understanding the effects of quantizing the atomic position, momentum and light fields and the relations between fields, atomic internal, and external degrees of freedom, which is very important for atom optics and a single-atom maser.

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