

## Electron absorption by complex potentials: One-dimensional case

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(Received 29 March 1996)

We investigate the electronic absorption induced by a complex potential in one-dimensional quantum mechanics. By solving the Schrödinger equation for spatially localized and extended complex potentials, we derive the electronic absorption coefficient  $\alpha$ . Neither a single nor a double Dirac- $\delta$  potential can represent a totally absorbing potential ( $\alpha=1$ ) for any choice of the complex amplitude of the potential. Maximum absorption coefficients for these potential types are 0.5 and  $2(\sqrt{2}-1)\approx 0.828$ , respectively. On the contrary, wall and well potentials do account for total absorption at a particular limit of the potential parameters. [S1050-2947(96)06508-0]

PACS number(s): 03.65.Nk, 72.10.Fk, 73.20.At

### I. INTRODUCTION

Back in 1926, Schrödinger gave to science his equation for the understanding of the behavior of nonrelativistic electrons [1]. It is well known [2,3] that a nonvanishing imaginary part of the potential ( $V_i$ ) can be introduced in the Schrödinger equation in order to describe a stationary electron sink or well (depending on the sign of  $V_i$ ) for an otherwise closed electron system.

In the past, this formulation has been successfully introduced to describe electron and positron inelastic scattering on atoms [4] and molecules [5]. More recently, complex potentials have been applied to model the electron intermixing between Shockley-type surface states and bulk states at adsorbed atoms lying on the surface [6]. Despite the interest in atomic and surface science for complex potentials in the Schrödinger equation, there is, however, to the best of our knowledge, no study in the literature meant to elucidate whether any sort of complex potential can represent, for a particular choice of its parameters, the variety of physical situations ranging from none to full absorption. In particular, we wish to investigate the nontrivial limit, that is, whether any complex potential is suitable to account for total absorption by properly choosing its set of free parameters. Our aim in this paper is to find out what is the absorption upper limit in the case of three different types of complex potentials in one dimension.

The outline of the paper is as follows. After introducing the procedure for calculating the electronic absorption coefficient in Sec. II, we calculate it for the simplest localized zero-dimensional potential—a Dirac  $\delta$  with complex amplitude—in Sec. III. The absorption at two identical Dirac  $\delta$ 's a distance  $a$  apart is studied in Sec. IV, whereas in Sec. V we deal with an extended potential: a one-dimensional (well

or wall) complex potential. Section VI is devoted to a discussion and conclusions.

### II. SCATTERING AT COMPLEX POTENTIALS

By introducing a complex potential ( $V=V_r+iV_i$ ) with a nonvanishing imaginary part ( $V_i\neq 0$ ) in the Schrödinger equation we can describe two totally different physical situations [3]: a stationary and a nonstationary one. Next we discuss them with the help of the conservation equation for the density of probability  $\rho(x)=|\psi(x)|^2$ :

$$\frac{\partial\rho(x)}{\partial t} + \nabla J(x) = \frac{2V_i\rho(x)}{\hbar}, \quad (1)$$

where  $J(x)=(\hbar/2mi)(\psi^*\nabla\psi - \psi\nabla\psi^*)$  is the probability flux and  $\psi(x)$ , the wave-function solution of the Schrödinger equation. When  $\partial\rho/\partial t\neq 0$ , we describe nonstationary (decaying in time) states, while for  $\partial\rho/\partial t=0$  we have a stationary state that describes electron scattering on a constantly leaking potential (for  $V_i<0$ ) [3]. The absorption or leakage out of the electronic system induced by the complex potential with  $V_i<0$  is characterized by the electronic absorption coefficient  $\alpha$ . In one dimension, it equals minus the probability flux lost when a plane wave, going from  $-\infty$  to  $+\infty$ , scatters on the potential, divided by the incident probability flux, i.e.,

$$\begin{aligned} \alpha &= -\frac{J(x=+\infty) - J(x=-\infty)}{J(x=-\infty)} \\ &= -\frac{J(x=+\infty) - J(x=-\infty)}{\hbar k/m}, \end{aligned} \quad (2)$$

where  $0\leq\alpha\leq 1$ , corresponding to  $\alpha=1$  (0) for total (no) absorption, respectively. Integrating Eq. (1) from  $-\infty$  to  $+\infty$ , we obtain an expression equivalent to Eq. (2) for the absorption coefficient:

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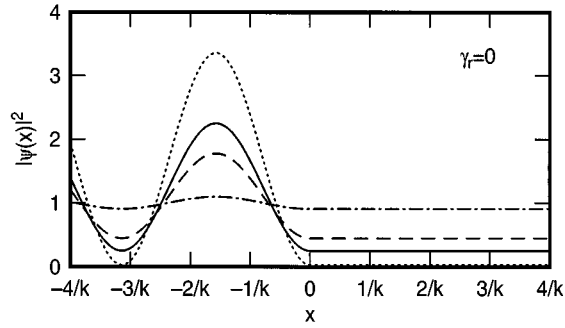


FIG. 1. Density of probability for solutions of the one-dimensional complex potential in Eq. (4) with  $\gamma_r=0$ . Dotted curve,  $k/\gamma_i=0.1$ ; full curve, 0.5, maximum absorption; dashed curve, 1; dot-dashed curve, 10. Note that maximum absorption is not correlated with minimum transmission, but with a global minimum for the sum of the reflected and transmitted densities of probability. For maximum absorption ( $\alpha=0.5$ ), the coefficients of the plane-wave solution of the problem [see Eq. (5)], correspond to  $A=-0.25$  and  $B=0.25$ . Therefore, 25% of the incident probability is reflected, 50% absorbed, and the remaining 25% transmitted.

$$\alpha = \frac{2m \int_{\Gamma} |V_i| \rho(x) dx}{\hbar^2 k}, \quad (3)$$

where  $\Gamma$  is the segment of the real axis with nonvanishing  $V_i$ .

### III. ABSORPTION AT A SINGLE COMPLEX DIRAC $\delta$

The first potential to be investigated is given by

$$V = \frac{\hbar^2}{2m} (\gamma_r - i\gamma_i) \delta(x), \quad (4)$$

with both  $\gamma_r$  and  $\gamma_i > 0$  constant. For this potential, the solution wave function equals

$$\psi(x) = \begin{cases} e^{ikx} + A e^{-ikx} & \text{for } x < 0 \\ B e^{ikx} & \text{for } x > 0, \end{cases} \quad (5)$$

where  $k$  is the wave vector,  $k = \sqrt{2mE/\hbar^2}$ , and  $E$  is the energy of the incident plane wave. Using Eq. (2) or (3) and Eq. (5), we derive the absorption coefficient

$$\alpha = \frac{\frac{4k}{\gamma_i}}{\left(1 + \frac{2k}{\gamma_i}\right)^2 + \left(\frac{\gamma_r}{\gamma_i}\right)^2}. \quad (6)$$

For  $k/\gamma_i$  much smaller or larger than 1, the absorption vanishes; a complex Dirac  $\delta$  is transparent for plane waves with wavelength much shorter or longer than  $\gamma_i$ . Note that the absorption coefficient for this potential type does not depend on the sign of the real part of the potential but only on its absolute value.

From Eq. (6), we find out that the maximum absorption

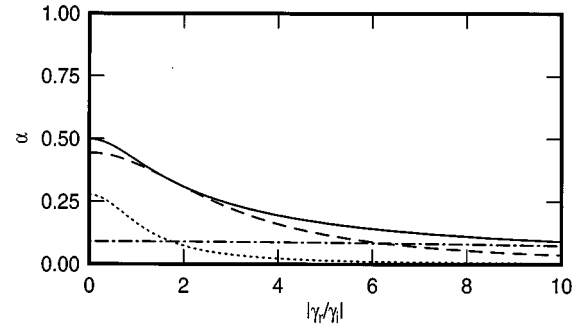


FIG. 2. Electronic absorption coefficient  $\alpha$  as a function of the absolute value of the ratio between real and imaginary parts  $\gamma_r/\gamma_i$  for the single Dirac  $\delta$  scattering potential in Eq. (4) with  $k/\gamma_i=0.5$  (dotted curve), 2 (dashed curve), and 10 (dot-dashed curve). Absolute maximum absorption is achieved when  $k/\gamma_i=0.5$  (full curve) and  $\gamma_r=0$ .

$$\alpha_{\max} = \frac{1}{1 + \sqrt{1 + \left(\frac{\gamma_r}{\gamma_i}\right)^2}} \quad (7)$$

is obtained when

$$\frac{k}{\gamma_i} = \frac{1}{2} \sqrt{1 + \left(\frac{\gamma_r}{\gamma_i}\right)^2}. \quad (8)$$

Therefore, the absolute maximum of the absorption is 0.5, which corresponds to  $\gamma_r=0$ , i.e., an imaginary scattering potential, and  $k/\gamma_i=0.5$ . We compare the density of probability for total absorption with densities of probability for less absorbing situations in Fig. 1. In Fig. 2 the dependence of the absorption coefficient on the ratio between the real and imaginary parts of the potential is shown. We observe that increasing  $|\gamma_r|$ , which means making the potential more attractive or repulsive, leads to less absorption. The dependence of the electronic absorption coefficient on the ratio between  $k$  and  $\gamma_i$  is illustrated in Fig. 3 for different values of the ratio  $\gamma_r/\gamma_i$ .

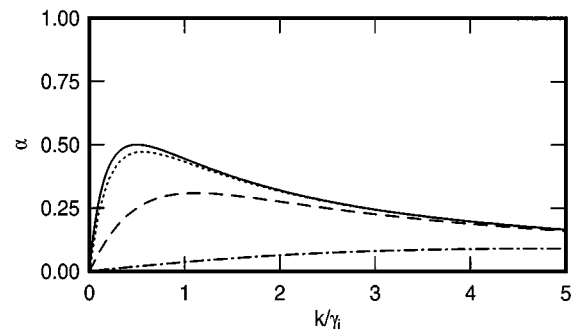


FIG. 3. Electronic absorption coefficient  $\alpha$  as a function of the ratio between the wave vector  $k$  and the imaginary part of the scattering potential in Eq. (4),  $\gamma_i$ , for  $\gamma_r/\gamma_i=0$  (full curve), 0.5 (dotted curve), 2 (dashed curve), and 10 (dot-dashed curve).

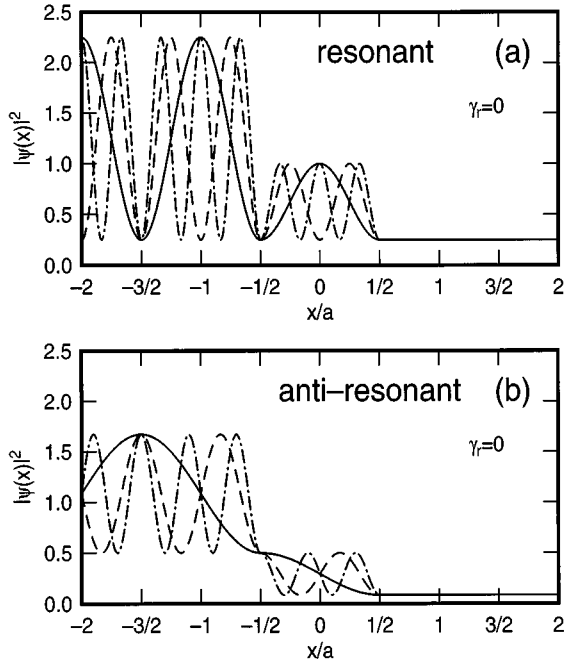


FIG. 4. (a) Density of probability for the  $n_r=1, 2$ , and 3 resonant solutions of the double  $\delta$  potential in Eq. (9) with vanishing  $\gamma_r$ . (b) Same for the  $n_a=0, 1$ , and 2 antiresonant solutions.  $z=2k/\gamma_i$  is taken at maximum absorption, i.e.,  $z=2$  and  $2^{1/2}$ , for resonant ( $\alpha=0.5$ ) and antiresonant [ $\alpha=2(2^{1/2}-1)$ ] conditions, respectively. For the maximum at resonant scattering, we have  $A=-0.5(-1)^{n_r}$  and  $D=0.5$  in Eq. (10), which account for 25% of the incident probability being reflected, 50% absorbed, and 25% transmitted. For the absolute maximum [antiresonant scattering and  $\alpha=2(2^{1/2}-1)$ ], we have  $A=-(-1)^{n_{a,i}}/(2+2^{1/2})$  and  $D=1/(2+2^{1/2})$ , which leads to the following distribution of the impinging probability: 8.58% is reflected, another 8.58% is transmitted, and the remaining 82.84% is absorbed.

#### IV. ABSORPTION AT TWO DIRAC $\delta$ 'S

In order to study how the upper limit for electronic absorption increases when scattering on more than one identical localized complex potential, we introduce the potential

$$V = \frac{\hbar^2}{2m} (\gamma_r - i\gamma_i) \left[ \delta\left(x + \frac{a}{2}\right) + \delta\left(x - \frac{a}{2}\right) \right], \quad (9)$$

where  $\gamma_r$  and  $\gamma_i > 0$  are constants. Later in this section we justify why we first calculate the absorption for  $\gamma_r=0$ . Using Eqs. (2) or (3) and the solution of the Schrödinger equation for the potential in Eq. (9),

$$\psi(x) = \begin{cases} e^{ikx} + Ae^{-ikx} & \text{for } x < -\frac{a}{2} \\ Be^{ikx} + Ce^{-ikx} & \text{for } -\frac{a}{2} < x < \frac{a}{2} \\ De^{ikx} & \text{for } x > \frac{a}{2}, \end{cases} \quad (10)$$

we obtain:

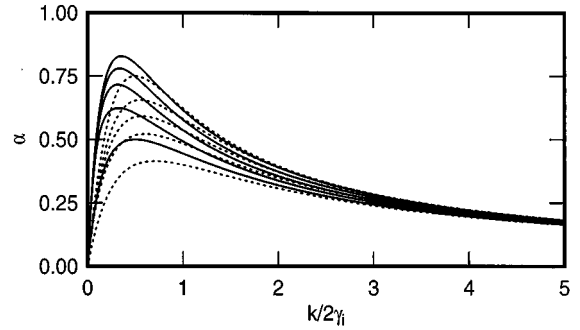


FIG. 5. Electronic absorption coefficient for the potential in Eq. (9) with  $\gamma_r=0$ . (full curve) and  $\gamma_r=1$  (dotted curve). Each curve corresponds to a different value of  $y = \sin^2(ka)$ . From bottom to top: 0.0 (resonant condition), 0.25, 0.5, 0.75, and 1.0 (antiresonant condition).

$$\alpha_{(\gamma_r=0)} = \frac{4z[(1+z)^2 - 2(1+z)(1-y) + 1]}{z^2(2+z)^2 + 4(1+z)^2y}, \quad (11)$$

where  $z = 2k/\gamma_i = (2/\gamma_i)\sqrt{2mE/\hbar^2}$  and  $y = \sin^2(ka)$  ( $0 \leq y \leq 1$ ). The two extreme values of  $y$  correspond to two different physical situations described below by means of the associated wavelengths,  $\lambda = 2\pi/k$ , namely:

$$y=0 \rightarrow \lambda = \frac{2a}{n_r}, \quad n_r = 1, 2, 3, \dots$$

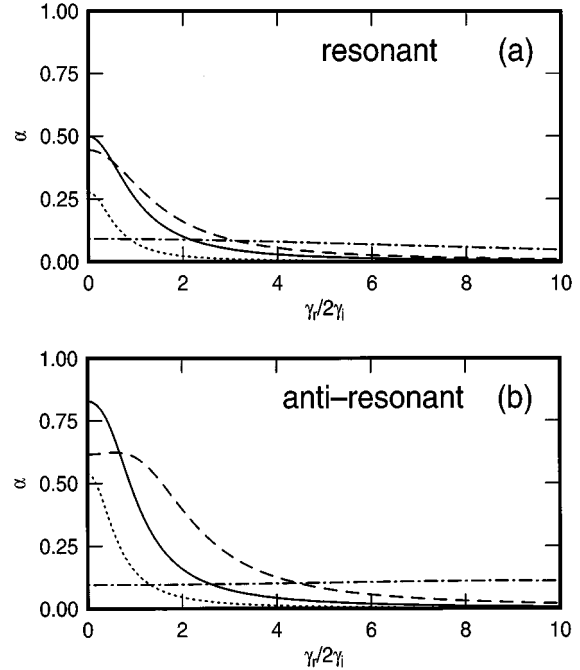


FIG. 6. Electronic absorption coefficient as a function of half of the ratio between the real and the imaginary part of the scattering potential in Eq. (9) with  $k/2\gamma_i=0.1$  (dotted curve), 1 (dashed curve), and 10 (dot-dashed curve). (a)  $ka=2\pi$ , which corresponds to resonant scattering, whereas (b)  $ka=\pi/2$  for antiresonant scattering.  $z$  values are the same as in Fig. 4. Absolute maximum absorption (full curves) is achieved for  $\gamma_r=0$  when  $k/2\gamma_i=0.5$  and  $1/2^{1/2}$  for resonant and antiresonant scattering, respectively.

$$y=1 \rightarrow \lambda = \frac{4a}{2n_a+1}, \quad n_a=0,1,2,\dots \quad (12)$$

For  $y=0$  the wavelength of the density of probability  $|\psi(x)|^2$ , is  $a/n_r$ , either the distance between the two scattering potentials  $a$  or a submultiple of it. We refer to this situation as resonant scattering, whereas antiresonant scattering corresponds to  $y=1$ . Intuitively, we foresee that in the case of resonant scattering the absorption is minimum due to the fact that the density of probability can show minima at the positions where the two potentials are located, therefore reducing absorption. The antiresonant condition represents the opposite situation: if the density of probability happens to be zero at one of the scatterers, it is maximum at the other and, therefore, the absorption increases in comparison with the situation of resonant scattering. In Figs. 4(a) and 4(b) we show the densities of probability for three resonant and antiresonant solutions of the problem. Note that the density of probability is continuous everywhere and has a continuous derivative everywhere but at the location of the  $\delta$ 's.

$$\alpha = \frac{4\{z^4 + 2z^3 + 2z^2(1+w^2) + 2z^2 \cos(ka) + [zw \sin(ka) - \cos(ka)(1+z+w^2)]\}}{z\{z^4 + 4z^3 + 2z^2(3+w^2) + 4z(1+w^2) - 2z[z(1-w^2) + 2(1+w^2)]\cos(2ka) + 4zw(1+z+w^2)\sin(2ka) + 4\sin^2(ka)(1+w^2)^2\}} \quad (13)$$

where  $z=2k/\gamma_i$  and  $w=\gamma_r/\gamma_i$ . A numerical analysis of this expression shows that the maximum absorption found when discussing the case  $\gamma_r=0$  can never be exceeded when  $\gamma_r \neq 0$  (see Fig. 5 for a particular case). Once the absolute maximum of the absorption coefficient for this problem is known, we study how  $\alpha$  changes when varying the real part of the scattering potential. As with the potential in Eq. (4), for the present case more repulsive or attractive potentials do not lead to a larger upper limit for the absorption than the one found for  $\gamma_r=0$ . In Fig. 6 we show how the absorption decreases for nonimaginary repulsive ( $\gamma_r>0$ ) potentials.

## V. WALL AND WELL COMPLEX POTENTIALS

In order to study the electronic absorption in the case of spatially extended potentials, we choose the simplest type

$$V = \frac{\hbar^2}{ma}(\gamma_r - i\gamma_i) \quad \text{for } |x| < \frac{a}{2}, \quad (14)$$

where  $\gamma_r$  and  $\gamma_i > 0$  are constant. For  $\gamma_r > 0$  ( $< 0$ ), we have a wall (well) type of scattering potential. Contrary to the previously presented problems, the study of the problem for  $\gamma_r=0$  (imaginary potential) does not lead to a shortcut in the mathematical formulation needed for the general case (both  $\gamma_r$  and  $\gamma_i$  being different from zero). Thus we deal with the general problem directly. Due to the fact that the solution of the Schrödinger equation is different depending on the value of the incident plane-wave energy with respect to the potential-energy level, we discuss these two cases separately. Note that for a well potential, we only need to study the first case discussed, whereas for a wall potential both are required to account for the entire incident energy range.

We illustrate in Fig. 5 the behavior of the absorption coefficient for  $\gamma_r=0$  when going from resonant to antiresonant scattering. The absorption increases monotonical when approaching the antiresonant condition. From Eq. (11), we find that the maximum absorption coefficient at resonant scattering is 0.5 when  $z=2k/\gamma_i=2$ , while for antiresonant scattering the maximum rises to  $2(\sqrt{2}-1) \approx 0.828$  when  $z=\sqrt{2}$ . For the sake of a comparison with Fig. 3, the absorption coefficient is plotted vs  $k/2\gamma_i$  in Fig. 5, instead of vs  $z=2k/\gamma_i$ . The maximum absorption in the single- $\delta$  problem corresponds, in the present problem, to the resonant condition, assuming that the  $\gamma_i$  in Eq. (4) equals half of the value of  $\gamma_i$  in Eq. (9).

So far we have described the scattering at two Dirac  $\delta$ 's for a vanishing real part of the potential, i.e.,  $\gamma_r=0$ . In the most general case ( $\gamma_r=0$ ), the absorption coefficient is given by

### A. Incident scattering energies larger than the potential barrier ( $E > \hbar^2\gamma_r/ma$ )

Assuming the incident plane wave impinging from  $-\infty$ , the solution of the Schrödinger equation for the potential in Eq. (14) is given by

$$\psi(x) = \begin{cases} e^{ikx} + Ae^{-ikx} & \text{for } x < -\frac{a}{2}, \\ Be^{iqx} + Ce^{-iqx} & \text{for } -\frac{a}{2} < x < \frac{a}{2} \\ De^{ikx} & \text{for } x > \frac{a}{2}, \end{cases} \quad (15)$$

where  $q=q_r+iq_i$ , with

$$q_r = \sqrt{\frac{k^2 - 2\gamma_r/a + \sqrt{(k^2 - 2\gamma_r/a)^2 + 4\gamma_i^2/a^2}}{2}}, \quad (16)$$

$$q_i = \frac{\sqrt{2}\gamma_i/a}{\sqrt{k^2 - 2\gamma_r/a + \sqrt{(k^2 - 2\gamma_r/a)^2 + 4\gamma_i^2/a^2}}}, \quad (17)$$

$$k = \sqrt{\frac{2mE}{\hbar^2}}. \quad (18)$$

As with the previous problems, imposing continuity and derivativity conditions at the boundaries between regions, we determine the coefficients  $A$ ,  $B$ ,  $C$ , and  $D$ . Once the solution of the Schrödinger equation is known, the absorption coefficient can be derived using, for example, Eq. (2):

$$\begin{aligned}\alpha &= 1 - |A|^2 - |D|^2 \\ &= 1 - \frac{|(1 - K^2/Q^2)(e^{iQ} - e^{-iQ})|^2 + |4K/Q|^2}{|(1 - K/Q)^2 e^{iQ} - (1 + K/Q)^2 e^{-iQ}|^2},\end{aligned}\quad (19)$$

where  $K = ka$ ,  $Q = Q_r + iQ_i = qa$ , and therefore

$$Q_r = \sqrt{\frac{K^2 - G_r + \sqrt{(K^2 - G_r)^2 + G_i^2}}{2}},\quad (20)$$

$$Q_i = \frac{G_i}{\sqrt{2} \sqrt{K^2 - G_r + \sqrt{(K^2 - G_r)^2 + G_i^2}}},\quad (21)$$

with  $G_r = 2\gamma_r a$  and  $G_i = 2\gamma_i a$ . Note that the condition  $E > \hbar^2 \gamma_r / ma$  leads to  $K^2 > G_r$ , where  $G_r$  is larger (smaller) than zero for the wall (well) problems.

### B. Incident scattering energies smaller than the potential barrier ( $E < \hbar^2 \gamma_r / ma$ )

The solution of the Schrödinger equation is formally the same as that in Eq. (15) except for the region of the potential ( $-a/2 < x < a/2$ ), where

$$\psi(x) = B e^{qx} + C e^{-qx}.\quad (22)$$

In the present case, we have

$$q_r = \sqrt{\frac{2\gamma_r/a - k^2 + \sqrt{(2\gamma_r/a - k^2)^2 + 4\gamma_i^2/a^2}}{2}},\quad (23)$$

$$q_i = \frac{-\sqrt{2}\gamma_i/a}{\sqrt{2\gamma_r/a - k^2 + \sqrt{(2\gamma_r/a - k^2)^2 + 4\gamma_i^2/a^2}}}.\quad (24)$$

Using Eq. (2), we obtain

$$\begin{aligned}\alpha &= 1 - |A|^2 - |D|^2 \\ &= 1 - \frac{|(1 + K^2/Q^2)(e^Q - e^{-Q})|^2 + |4K/Q|^2}{|(1 + K/iQ)^2 e^Q - (1 - K/iQ)^2 e^{-Q}|^2},\end{aligned}\quad (25)$$

where now

$$Q_r = \sqrt{\frac{G_r - K^2 + \sqrt{(G_r - K^2)^2 + G_i^2}}{2}},\quad (26)$$

$$Q_i = \frac{-G_i}{\sqrt{2} \sqrt{G_r - K^2 + \sqrt{(G_r - K^2)^2 + G_i^2}}},\quad (27)$$

with  $G_r > K^2$  and  $E < \hbar^2 \gamma_r / ma$ . These expressions correspond only to the wall problem and for incident energies smaller than the potential barrier.

### C. Imaginary potential: Long- and short-wavelength limits

As in previous sections, we start our analysis of the absorption coefficient with the case of vanishing  $G_r$ , i.e., when the potential is imaginary and, therefore, the absorption is a

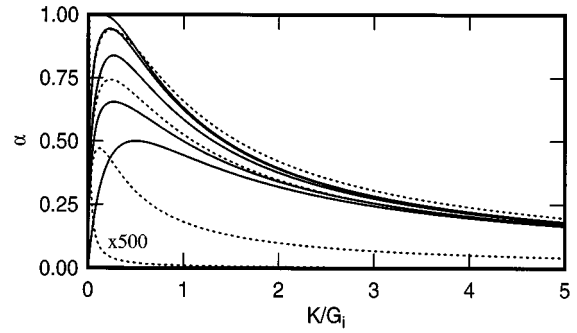


FIG. 7. Electronic absorption coefficient for the potential in Eq. (14) with vanishing  $G_r$  (full curve) and  $G_r = 5$  (dotted curve). Each curve corresponds to a different value of  $K = ka$ . From bottom to top: 0.01, 1, 2, 5, and 1000. The upper limit of the absorption coefficient increases from 0.5 for the long-wavelength limit ( $1 \gg K = 0.5G_i$ ) to 1 for the short wavelength limit ( $1 \ll K \ll G_i$ ).

function of only two variables:  $K$  and  $G_i$ . Using Eq. (19), we calculate the absorption coefficient for fixed  $K$  and varying  $G_i$  as shown in Fig. 7. The upper limit for the electronic absorption coefficient is bound between 0 (no absorption) for  $1 \gg K \gg G_i$  and 1 (total absorption) when  $1 \ll K \ll G_i$ . We focus our interest on the upper part of the absorption regime, i.e., between 0.5, which occurs when  $1 \gg K = 0.5G_i$ , and total absorption ( $1 \ll K \ll G_i$ ). We refer to these two extreme situations as the long- and short-wavelength limits because they are equivalent to  $2\pi/\gamma_i = \lambda \gg a$  and  $1/\gamma_i \ll \lambda \ll a$ , respectively. The corresponding wave functions for the two limits are plotted in Fig. 8. By introducing a nonvanishing part of the complex potential, we do not find any other situation that produces full absorption. As in the previously discussed cases, a complex potential with a nonvanishing real part constrains  $\alpha$  to values smaller than the upper limit for the absorption: in the present case to values smaller than 1. At the short-wavelength limit, however, an increase of the real part

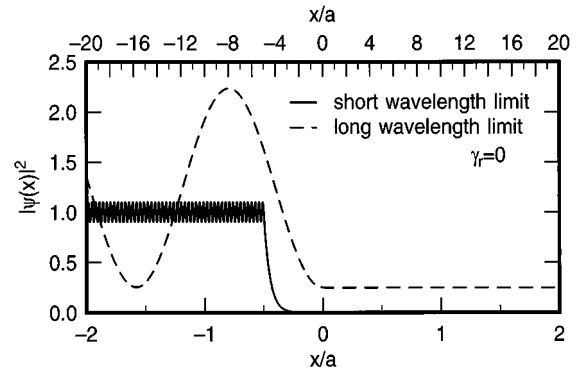


FIG. 8. Density of probability for the complex potential in Eq. (14) and vanishing  $\gamma_r$ . The dashed and solid curves correspond to solutions close to the short and long wavelength limits, respectively. The dashed line was obtained with  $K = 100$  and  $G_i = 2000$ , being  $\alpha = 0.9975$ . The second curve represents the case  $K = 0.5 \times G_i = 0.2$ , which gives  $\alpha = 0.5060$ . Whereas the long-wavelength limit corresponds to an absorption of 50%, total absorption is obtained at the short-wavelength limit. Note that the scale for  $x/a$  at the bottom (top) corresponds to density of probability for the short- (long-) wavelength limits.

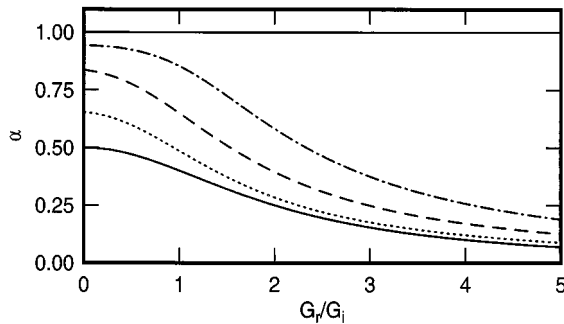


FIG. 9. Electronic absorption coefficient as a function of the ratio between the real and the imaginary part of the scattering potential in Eq. (14). Each curve corresponds to a fixed value of  $K$  and  $G_i$ . From bottom to top:  $K=ka=0.01, 1, 2, 5$ , and  $1000$ . The value for  $G_i$  is fixed for each curve and is such that  $\alpha$  is maximum at  $G_r=0$  for the given  $K$  values. Full curves start at the long- (bottom-) and short- (top) wavelength limits.

of the complex potential reduces the absorption very little compared to situations far away from the limit, as shown in Fig. 9.

## VI. DISCUSSION AND CONCLUSIONS

In this paper we have discussed the maximum possible electronic absorption in one dimension for the cases of three complex potential types: a single Dirac  $\delta$ , two Dirac  $\delta$ 's separated by a distance  $a$ , and a flat positive or negative potential of length  $a$ . For a single Dirac  $\delta$ , the absorption induced by the imaginary part of the potential cannot exceed 0.5 of the incoming probability flux. The absorption coefficient ( $\alpha$ ), defined as the ratio between the absorbed vs the incident flux of probability, is 0.5 at maximum absorption, which occurs when the real part of the potential  $\gamma_r$  vanishes and  $k/\gamma_i=0.5$ . Under these conditions, the incoming probability flux is distributed in the following way: 25% is back-scattered, 50% absorbed, and the remaining 25% transmitted. For a nonvanishing real part of the scattering potential,  $\alpha$  does not depend on the sign of the real part of the potential but only on its magnitude. Therefore two  $\delta$ -like potentials with the same  $|\gamma_r|$  absorb the same electronic density of

probability independent of its repulsive or attractive character.

For the case of two  $\delta$ 's separated a distance  $a$ , the upper limit for  $\alpha$  appears to be higher,  $2(\sqrt{2}-1)\approx 0.828$ , but again the incoming flux of probability cannot be totally absorbed for any particular complex amplitude of the potential. The highest absorption that can be expected for such a potential is reached when the real part of the potential vanishes—as for a single  $\delta$  potential—and when the wavelength of the incident plane wave is given by  $\lambda=4a/(2n_a+1)$  with  $n_a=0,1,2,\dots$ . This corresponds to the aforementioned anti-resonance condition, for which  $|\psi|^2$  is maximum at one  $\delta$  and zero at the other. A nonvanishing part of the potential does not increase the upper limit for absorption for any choice of the parameters in the potential.

The third sort of potential discussed in this work, a constant complex potential that extends along a finite segment of length  $a$ , does describe total absorption for the short-wavelength limit described in Sec. V C. For this limit, the wavelength of the incident electron  $\lambda$  is much smaller than the extent of the potential  $a$ , but much larger than the inverse of the complex amplitude  $1/\gamma_i$ , i.e.,  $1/\gamma_i\ll\lambda\ll a$ . We have found numerically for any set of free parameters in the potential no other situation where  $\alpha=1$  but at the mentioned short-wavelength limit.

To sum up, we have shown that  $\delta$ -like localized potentials do not account for total absorption for any particular choice of the complex amplitude, whereas extended wall or well potentials can describe a physical situation where total absorption occurs.

## ACKNOWLEDGMENTS

We are thankful to M. Fischetti for fruitful discussions and L. Mijares for a careful reading of a draft of this paper. Patrici M.-M. acknowledges the financial support of the Ministerio de Educación y Ciencia of the Spanish Government. His work was done under a joint-study agreement between the Institut de Ciència de Materials de Barcelona (Consejo Superior de Investigaciones Científicas, Spain) and IBM. Pau M.-M. thanks the Centre Europeu de Paral·lelisme de Barcelona for computer support at the Universitat Politècnica de Catalunya.

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