Atomic coherence and bistable lasers without inversion

Guao Qin Ge,[1] Xueli Luo,[2] Ying Wu,[1] and Zaiguang Li[3]

[1]Department of Physics, Huazhong University of Science and Technology, Wuhan 430074, People's Republic of China

[2]Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics, The Chinese Academy of Sciences, Wuhan 430071, People's Republic of China

[3]National Laboratory of Laser Technique, Huazhong University of Science and Technology, Wuhan 430074, People's Republic of China

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We examine a model of a driven two-level system with injected coherent atoms. We find a set of lasing phase diagrams for the resonant excitations. We show that the laser field in the system is related to the initial preparation of atoms, and that when the degree of this coherence is changed, the system exhibits bistability in the inversionless region. We also discuss the crucial role of atomic coherence in achieving lasing action. [S1050-2947(96)07107-7]

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There has been considerable interest recently in the study of lasing without inversion (LWI). Many schemes have been proposed [1–15]. Most of these publications have focused mainly on the conditions for the onset of lasing action, and a few others have shown that inversionless lasers may have some interesting statistical properties, such as narrower linewidths due to reduced spontaneous emission noises [10(a)], and amplitude squeezing in three-level [11] and four-level systems [7(c)]. Most recently, we have shown that [8] it is possible to obtain optical bistability in the inversionless region for a driven two-level system with incoherent atomic injection.

Investigating various schemes for LWI, it is not difficult to find that atomic coherence effects and external pump field(s) are fundamentally important for achieving LWI. Now the following questions are raised. What is the connection between the atomic coherence and the external pump field? What is most important for achieving laser light?

The laser model we analyze here is similar to that in Ref. [8]. N two-level atoms are injected into a cavity and directly driven by a coherent external pump field. The difference is that the atoms are prepared coherently before they are injected into the cavity. Since the external pump field and injected atomic coherence are included in one model, this enables us to discuss the above-raised questions. Some interesting results have been obtained. We obtain a set of ''phase diagrams'' for the model. There are three phase regions: a monostable lasing region and two bistable regions. The bistability can be obtained even when the upper-level population is almost zero. In all regions (i.e., for all choices of parameters), the system lases without the need for population inversion. A detailed analysis shows that atomic coherence (or polarization between transition levels of active atoms) plays a crucial role in achieving laser action.

A quantum-mechanical description of our model is given in the dipole and rotating-wave approximations by the Hamiltonian

$$\begin{aligned}H=&\hbar\omega_c a^+a+\sum_j \frac{\hbar}{2}\omega_a\sigma_{zj}\\&+\hbar\sum_j \theta(t-t_j)(ga\sigma_j^+ + g^*a^+\sigma_j)\\&+\hbar\sum_j \theta(t-t_j)(g_pe^{-i\omega_pt}\sigma_j^+ + g_p^*e^{i\omega_pt}\sigma_j)+H_{\text{loss}},\end{aligned}\tag{1}$$

where a^+ and a are creation and annihilation operators for the cavity field, respectively; $\sigma_j^+=|2\rangle_{jj}\langle 1|$ and $\sigma_j=|1\rangle_{jj}\langle 2|$ are the usual raising and lowering operators of the jth atom; $\sigma_{zj}=|2\rangle_{jj}\langle 2|-|1\rangle_{jj}\langle 1|$ is the inversion operator; g and g_p are coupling constants of atoms with the cavity and with the pump field, respectively; ω_c, ω_a, and ω_p are frequencies corresponding to the cavity, atoms, and pump-field; and the step function $\theta(t-t_j)$ is 1 for $t\geqslant t_j$ (injection time of the jth atom) and zero otherwise.

By the same procedure as that in Ref. [8], we obtain a field equation of motion

$$\begin{aligned}\langle\dot{b}\rangle=&-\left(\frac{\gamma_c}{2}+i\Delta_c\right)\langle b\rangle+\left(\frac{\gamma_c}{2}+i\Delta_c\right)g_r\\&-ig^*\sum_j \theta(t-t_j)\langle\tilde{\sigma}_j\rangle,\end{aligned}\tag{2}$$

where the total field of the system has been introduced:

$$\langle b\rangle=\langle\tilde{a}+g_r\rangle=r_0e^{i\Phi_0}.\tag{3}$$

$\tilde{a}(t)=ae^{i\omega_pt}$, $\tilde{\sigma}_j(t)=\sigma_je^{i\omega_pt}$, $\Delta_c=\omega_c-\omega_p$, $\Delta_a=\omega_a-\omega_p$, $g_r=g_p/g, \gamma_c$ and Γ are the cavity decay rate and the atomic decay rate, respectively, and $\langle\tilde{\sigma}_j(t)\rangle$ can be solved to be

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$$\langle\tilde{\sigma}_j(t)\rangle=\exp[-\Gamma(t-t_j)]\Bigg\{\langle\tilde{\sigma}_j(t_j)\rangle\cos^2|g|r_0(t-t_j)+\frac{g^2}{|g|^2}\langle\tilde{\sigma}^+(t_j)\rangle e^{i2\Phi_0}\sin^2|g|r_0(t-t_j)$$
$$+i\frac{g}{2|g|}\langle\sigma_{zj}(t_j)\rangle e^{i\Phi_0}\sin 2|g|r_0(t-t_j)-i\frac{\Delta_a}{2|g|r_0}\langle\tilde{\sigma}_j(t_j)\rangle\sin 2|g|r_0(t-t_j)$$
$$+\frac{g\Delta_a}{2|g|^2r_0}\langle\sigma_{zj}(t_j)\rangle e^{i\Phi_0}\sin^2|g|r_0(t-t_j)\Bigg\}. \tag{4}$$

Because the atoms are initially prepared coherently, their initial conditions can be assumed to be

$$\langle\tilde{\sigma}_j(t_j)\rangle=\tilde{\rho}_{21},\quad \langle\tilde{\sigma}_j^+(t_j)\rangle=\tilde{\rho}_{12},\quad \langle\sigma_{zj}(t_j)\rangle=\rho_{22}-\rho_{11}, \tag{5}$$

where ρ_{11}, ρ_{22}, and $\tilde{\rho}_{21}=\tilde{\rho}_{12}^*$ are the same for all atoms. By substituting Eqs. (4) and (5) into (2), and replacing the summation in (2) with an integration over the injection time t_j, $\Sigma_j\theta(t-t_j)\langle\tilde{\sigma}_j(t)\rangle\rightarrow R\int_{-\infty}^{t}dt_j\langle\tilde{\sigma}_j(t)\rangle$ (where R is the time-independent injection rate), it is possible to obtain

$$\langle\dot{b}\rangle=\frac{1}{1+\frac{4g^2r_0^2}{\Gamma^2}}e^{i\Phi_0}\Bigg\{\frac{Rg^2r_0}{\Gamma^2}(\rho_{22}-\rho_{11})-i\frac{Rg}{\Gamma}\Bigg[\tilde{\rho}_{21}e^{-i\Phi_0}\Bigg(1+\frac{2g^2r_0^2}{\Gamma^2}\Bigg)+\frac{2g^2r_0^2}{\Gamma^2}\tilde{\rho}_{12}e^{i\Phi_0}+\frac{g^2r_0}{\Gamma^2}\frac{\Delta_a}{\Gamma}(\rho_{22}-\rho_{11})\Bigg]-\frac{Rg\Delta a}{\Gamma^2}\tilde{\rho}_{21}\Bigg\}$$
$$-\left(\frac{\gamma_c}{2}+i\Delta_c\right)r_0e^{i\Phi_0}+\left(\frac{\gamma_c}{2}+i\Delta_c\right)g_r, \tag{6}$$

where g and g_p, for simplicity, are chosen to be real. Separating Eq. (6) into a real part and an imaginary one, and by means of Eq. (3), one obtains an equation for the effective total field amplitude $E=2gr_0/\Gamma$,

$$\dot{E}=\frac{1}{1+E^2}\Bigg\{\frac{g_c}{2}(\rho_{22}-\rho_{11})E+g_c|\tilde{\rho}_{21}|\Bigg[\sin(\theta-\Phi_0)-\frac{\Delta_a}{\Gamma}\cos\theta\Bigg]\Bigg\}-\frac{\gamma_c}{2}E+\frac{2g_pA}{\Gamma}\cos(\Phi_0-\sigma), \tag{7}$$

and an equation for its phase,

$$\dot{\Phi}_0=-g_c\Bigg[\frac{|\tilde{\rho}_{21}|\cos(\theta-\Phi_0)}{E}+\frac{1}{2(1+E^2)}\frac{\Delta_a}{\Gamma}(\rho_{22}-\rho_{11})+\frac{1}{E(1+E^2)}\frac{\Delta_a}{\Gamma}|\tilde{\rho}_{21}|\sin\theta\Bigg]-\Delta_c+\frac{2g_pA}{\Gamma E}\sin(\sigma-\Phi_0), \tag{8}$$

where $g_c=2g^2R/\Gamma^2$, $A=\sqrt{(\gamma_c/2)^2+\Delta_c^2}$, $tg\sigma=\Delta_c/\gamma_c/2$, and $\tilde{\rho}_{21}=|\tilde{\rho}_{21}|e^{i\theta}$. $\tilde{\rho}_{21}$ is determined by the initial atomic preparation. Equations (7) and (8) are general forms of the amplitude and phase of the total lasing field. To deal with the connections of atomic coherence with lasing action and with bistable lasers without inversion, we focus on the resonant excitations, i.e., $\Delta_a=\Delta_c=0$, and seek the steady-state solution of the lasing field. In this case, $\sigma=0$ and $A=\gamma_c/2$. From Eq. (8), one can obtain the steady phase

$$tg\Phi_0=-\frac{G_c|\tilde{\rho}_{21}|\cos\theta}{G_c|\tilde{\rho}_{21}|\sin\theta+G_p}, \tag{9}$$

where we have redefined dimensionless parameters $G_c=g_c/\gamma_c=2g^2R/\Gamma^2\gamma_c$ and $G_p=g_p/\Gamma$. Equation (9) shows that the steady phase of the lasing field is related to the initial preparation of atoms. For a certain preparation (i.e., given a $\tilde{\rho}_{21}$), there are two possible solutions with Φ_0 in the second and fourth quadrants, respectively. The corresponding solutions of the lasing field amplitude, respectively, are

$$\frac{E(\rho_{22}-\rho_{11})-2|\tilde{\rho}_{21}|\frac{1}{M}(G_c|\tilde{\rho}_{21}|+G_p\sin\theta)}{1+E^2}=\frac{E}{G_c}+2\frac{G_p}{G_c}\frac{1}{M}(G_c|\tilde{\rho}_{21}|\sin\theta+G_p), \tag{10}$$

$$\frac{E(\rho_{22}-\rho_{11})+2|\tilde{\rho}_{21}|\frac{1}{M}(G_c|\tilde{\rho}_{21}|+G_p\sin\theta)}{1+E^2}=\frac{E}{G_c}-2\frac{G_p}{G_c}\frac{1}{M}(G_c|\tilde{\rho}_{21}|\sin\theta+G_p), \tag{11}$$

where

$$M=[G_c^2|\tilde{\rho}_{21}|^2+2G_cG_p|\tilde{\rho}_{21}|\sin\theta+G_p^2]^{1/2}. \tag{12}$$

Equations (10) and (11) show that the steady field amplitudes are relative to various possible atomic injection determined by $\tilde{\rho}_{21}=|\tilde{\rho}_{21}|e^{i\theta}$. Now we discuss an interesting case for the atomic preparation examined in [Ref. [6(a)]],

$$|\tilde{\rho}_{21}|=(\rho_{11}\rho_{22})^{1/2},\quad \theta=\frac{\pi}{2}. \tag{13}$$

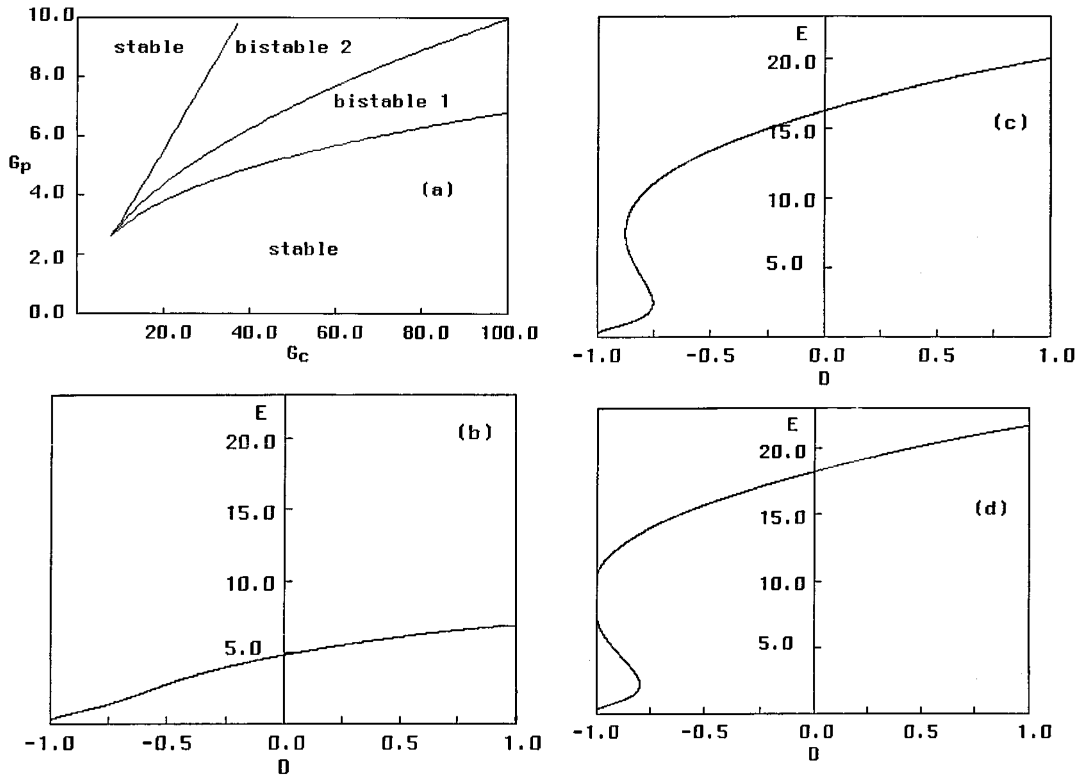

FIG. 1. Lasing phase diagrams determined by Eq. (15). (a) Phase regions corresponding to G_p and G_c, the dimensionless pumping-field-atom and cavity-atom-coupling constants, determined by Eq. (15). There are one (mono-)stable and two bistable regions. Values of the cusp points of the three lines are $G_c=8$ and $G_p=2.598$. (b) Variation of E with $D=\rho_{22}-\rho_{11}$; parameters $G_c=20$ and $G_p=2$ are chosen in the ''stable'' region. (c) Variation of E with D; $G_c=80$ and $G_p=8$ are chosen in the ''bistable 1'' region. (d) Variation of E with D; $G_c=80$ and $G_p=9$ are chosen in the ''bistable 2'' region.

In this case, Eqs. (10) and (11) correspondingly become

$$\frac{ED-\sqrt{1-D^2}}{1+E^2}=\frac{E}{G_c}+2\frac{G_p}{G_c} \tag{14}$$

for $\Phi_0=\pi$, and

$$\frac{ED+\sqrt{1-D^2}}{1+E^2}=\frac{E}{G_C}-2\frac{G_P}{G_C} \tag{15}$$

for $\Phi_0=0$, where $D=\rho_{22}-\rho_{11}$ is the inversion parameter, and the condition $\rho_{11}+\rho_{22}=1$ has been used. Now let us discuss the properties of Eqs. (14) and (15). Equation (14) has the solution only for $D>0$. However, in this case, $\partial\dot{\Phi}/\partial\Phi(\Phi_0=\pi)>0$. This means that the phase Eq. (8) is unstable, and so is the solution of field equation (14). Therefore, we do not discuss it any longer.

Now let us pay attention to Eq. (15), where the solution properties are significantly important. First of all, we will emphasize that the solution of Eq. (15) exists for all possible values of $D(-1<D<1)$, i.e., the solution of the total effective lasing field has no requirement of population inversion. Second, we obtain a complete phase diagram for the lasing field. A detailed analysis shows that the properties of the lasing field are different in different domains of parameters G_c and G_p. In other words, according to the properties of the lasing field, the values of parameters G_c and G_p are divided into three regions: a monostable region and two bistable regions [see Fig. 1(a)]. Three boundaries dividing the regions are analytically obtained and given, from lower to upper respectively, by

$$G_p^{(1)}=\frac{\sqrt{3}}{2}\sqrt{G_c^{2/3}-1}, \tag{16a}$$

$$G_p^2=\frac{1}{2}\left[\left[\frac{G_c}{2}-1+\sqrt{\frac{G_c^2}{4}-2G_c}\right]^{1/2}+G_c\frac{\left[\frac{G_c}{2}-1+\sqrt{\frac{G_c^2}{4}-2G_c}\right]^{1/2}}{\frac{G_c}{2}+\left(\frac{G_c^2}{4}-2G_c\right)^{1/2}}\right], \tag{16b}$$

$$G_p^3=\frac{1}{2}\left[\left[\frac{G_c}{2}-1-\sqrt{\frac{G_c^2}{4}-2G_c}\right]^{1/2}+G_c\frac{\left[\frac{G_c}{2}-1-\sqrt{\frac{G_c^2}{4}-2G_c}\right]^{1/2}}{\frac{G_c}{2}-\left(\frac{G_c^2}{4}-2G_c\right)^{1/2}}\right]. \tag{16c}$$

The three lines converge at cusp points $G_c=8$ and $G_p=2.598$. Now we analyze the properties of the solutions. In Fig. 1(b), it is shown that the lasing field in the (mono)stable region has only one steady-state solution. In the region ''bistable 1,'' however, Fig. 1(c) shows that there are bistable steady states in the inversionless region $(D<0)$. The basic difference of the bistability from the conventional one is that the bistable states here are located in the inversionless region, and that the hysteresis loop varies with respect to the inversion parameter D rather than to the external input field. The solution in the region ''bistable 2'' shows that the bistability exists near where the upper-level population is zero. The bistable curve is cut off at the zero population in the upper levels. This property indicates that as long as there are particles on the upper level, the system lases until the particles on the upper level are decreased to zero. A reasonable deduction for this may be quantum noise quenching or squeezing due to initial atomic coherence [18]. The physical origin of the bistability discussed here is similar to the conventional absorptive optical bistability [16,17]. However, because of the initial atomic coherence, the bistability is changed here with degree of coherence, and the quantum fluctuation properties of the system are also modified [18].

From the point of view of obtaining bistability and LWI, the system of injecting coherent atoms and injecting incoherent atoms have similar properties; however, they have different ''phase diagrams.'' In particular, if the pump field is removed from both cases, another important difference comes to pass. In the case of injecting coherent atoms, the system still lases without the need of population inversion. There is no threshold for such a system [see Fig. 2, where the curve is determined by Eq. (15). However, when injecting incoherent atoms, the laser system must have population inversion [[6(a)], [8]]. On the other hand, if one investigates the coherence condition in this case, it will be found that the inversion condition is nothing but the coherence condition. This can be seen from Eq. (4). [The condition makes Im $\langle\tilde{\sigma}_j(t)\rangle>0$.] It should be borne in mind that the so-called atomic coherence here means nonzero off-diagonal elements of the density matrix of the atoms; in other words, the atoms are polarized. The above facts suggest that atomic coherence plays a crucial role in achieving lasing action. In order to achieve lasing action, atoms should be coherent. The threshold conditions for lasing action in some systems turn out to be required to reach atomic coherence, while threshold conditions are not required in some other systems simply because the atomic coherence has already been achieved. In these systems, atoms are either prepared or driven coherently. Consequently, we deduce that atomic coherence is the most important thing for achieving laser action. So we can say that coherent light originates from the atomic coherence, i.e., from atomic coherence to optical coherence. The most important property of the laser is its coherence, no matter how it is produced, with or without population inversion. The above discussion is also suitable for the multilevel system [7(b)].

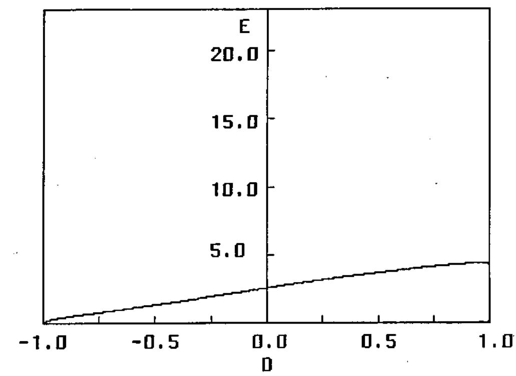

FIG. 2. Variation of E with D. Here the pump field is removed, and $G_p=0$. The inversionless lasing determined by Eq. (15) is also available, where $G_c=20$.

In conclusion, we have discussed a driven two-level system with injecting coherent atoms. We obtained a set of lasing phase diagrams which show that bistability can be observed in the inversionless region. Both stability and fluctuation properties are modified due to the initial atoms coherence. Because the optical bistability has very important applications in optical switches and optical information processing, a study of bistable inversionless lasers may shed further light on the applications of inversionless lasers. We have also discussed the role of atomic coherence in lasing systems. We have shown that it is atomic coherence that plays the crucial role in achieving lasing action, no matter whether it requires population inversion or not.

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